

Is Piketty’s “Second Law of Capitalism” Fundamental?

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Abstract

In *Capital in the Twenty-First Century*, Thomas Piketty uses what he calls “the second fundamental law of capitalism” to predict that wealth-to-income ratios are poised to increase dramatically as economies’ growth rates fall during the twenty-first century. This law states that in the long run the wealth-to-income ratio equals s/g , where s is the economy’s saving rate and g its growth rate. We argue that this law rests on a theory of saving that is hard to justify. First, it holds the net saving rate constant as growth falls, driving the gross savings rate to one as growth goes to zero. Second, it is inconsistent with both the textbook growth model and the theory of optimal saving: in both of these theories the net saving rate goes to zero as growth goes to zero. Third, both of these theories provide a reasonable fit to observed data on gross and net saving rates in the United States, whereas Piketty’s does not. Finally, contrary to Piketty’s second law, both of these theories predict that wealth-to-income ratios increase only modestly as growth falls.

1 Introduction

Thomas Piketty’s recent book *Capital in the Twenty-First Century* is a timely and important contribution that turns our attention to striking long-run trends in economic inequality. A large part of the book is thus a documentation of historical data, going further back in time, and focusing more on the very richest in society, than have most existing economic studies. This work is bound to remain influential. A central theme in the book, however, goes beyond mere documentation: as the title of the book suggests, it makes predictions about the future. Here, Piketty argues forcefully that future declines in economic growth—stemming from slowdowns in technology or drops in population growth—will likely lead to dramatic concentrations of economic and political power through the accumulation of capital (or wealth) by the very richest. These predictions are the subject of the present note.

We emphasize, first, that Piketty’s predictions are not mere extrapolations from past data but, instead, rest importantly on the use of economic theory. This is important, since for the predictions to be reliable, one would want to feel some comfort in the particular theory that is used. Our main point is to make clear that there are strong reasons to doubt the specific theory that Piketty advances.

We argue that one of the key building blocks in this theory—what he calls the second fundamental law of capitalism—is rather implausible, for two reasons. First, we demonstrate that it implies saving behavior that, as the growth rate approaches zero, requires the aggregate economy to save 100% of GDP each year. Such behavior is clearly hard to square with any standard theories of how individuals save, and it is inconsistent with the findings in the empirical literature on how individuals actually save.

Second, we look at aggregate U.S. data to try to compare Piketty’s assumption to standard, alternative theories, and we find that the data speaks rather clearly against Piketty’s theory. Equipped with theories that we find more plausible, we show that even if the rate of economic growth were to decline all the way to zero, inequality would increase only very modestly, in fact so modestly that we would hesitate to use this argument to make a prediction of rising inequality. We think, in contrast, that future developments of other determinants of wealth inequality—such as educational institutions, skill-biased technical change, globalization, and changes in the structure of capital markets—are likely to be much more fundamental.

Piketty’s first law of capitalism is a definition: capital’s share of national income, y , is $r \times k/y$, where r is a measure of the return to capital and k is a measure of the aggregate stock of capital. The second law, on which we focus here, is more than a definition. Though Piketty calls it an “accounting equation”, it really is a theory, because it builds in a certain form of saving behavior. It says that if the economy keeps the saving rate, s , constant over time, then the capital-to-income ratio k/y must, in the long run, become equal to s/g , where g is the economy’s growth rate. Although the second law is entirely about the aggregate k/y , it may indirectly inform us about inequality since the inequality in capital holdings in all modern societies is higher than is the inequality of all of income. So if these relative dispersions of k and y do not change as k/y rises, overall inequality in riches is poised to rise.¹ In particular, were the economy’s growth rate to decline towards zero, as Piketty argues it will, the capital-output ratio would explode, with dramatic consequences for inequality. Another facet of this development would be observed from capital’s share of income: combining the two laws, one obtains one of the central relationships of Piketty’s book: capital’s share of income is $r \times s/g$. Thus, with g dropping toward zero, and without

¹The relative dispersions might of course change. We discuss this point in Section 8.2 below.

a counteracting fall in r —and Piketty argues we should not expect r to change much—we will also see a drastic increase in the capital share.

The argument working through k/y , in its disarming simplicity, may look attractive, but it is worrisome to those of us who have studied basic growth theory based either on the assumption of a constant saving rate—such as in the undergraduate textbook version of Solow’s classical model—or on optimizing growth, along the lines of Cass (1965) and Koopmans (1965) or its counterpart in modern macroeconomic theory. Why? Because we do not quite recognize the second law, $k/y = s/g$. Did we miss something important, even fundamental, that has been right in front of us all along?

There are no errors in the formula Piketty uses, and it is actually consistent with the very earliest formulations of the neoclassical growth model, but it is not consistent with the textbook model as it is generally understood by macroeconomists.² An important purpose of this note is precisely to relate Piketty’s theory to the textbook theory. Those of you with standard modern training have probably already noticed the difference between Piketty’s equation and the textbook version that we are used to. In the textbook model, the capital-to-income ratio is not s/g but rather $s/(g+\delta)$, where δ is the rate at which capital depreciates. With the textbook formula, growth approaching zero would increase the capital-output ratio but only very marginally; when growth falls all the way to zero, the denominator would not go to zero but instead would go from, say 0.08—with g around 0.03 and $\delta = 0.05$ as reasonable estimates—to 0.05.³ As it turns out, however, the two formulas are not inconsistent because Piketty defines his variables, such as income, y , not as the gross income (i.e., GDP) that appears in the textbook model but rather as *net* income, i.e., income net of depreciation. Similarly, the saving rate that appears in the second law is not the gross saving rate as in the textbook model but instead what Piketty calls the “net saving rate”, i.e., the ratio of net saving to net income.

Contrary to what Piketty suggests in his book and papers, this distinction between net and gross variables is quite crucial for his interpretation of the second law when the growth rate falls towards zero. This turns out to be a subtle point, because on an economy’s balanced growth path, for any positive growth rate g , one can map any net saving rate into a gross saving rate, and vice versa, without changing the behavior of capital accumulation. The range of net saving rates constructed from gross saving rates, however, shrinks to zero as g goes to zero: at $g = 0$, the net saving rate has to be zero no matter what the gross rate is, as long as it is less than 100%. Conversely, if a positive net saving rate is maintained as g goes to zero, the gross rate has to be 100%. Thus, at $g = 0$, either the net rate is 0 or the gross

²In the concluding section of this note, we make some remarks on the historical use of different assumptions on saving.

³See, for example, the calibration that Cooley and Prescott (1995) perform.

rate is 100%. As a theory of saving, we maintain that the former is fully plausible whereas the latter is all but plausible.⁴

We look more closely at Piketty’s assumption of a constant (positive) net saving rate from two perspectives: the textbook model with an exogenous gross saving rate and a model in which the saving rate is chosen optimally. In both settings the net saving rate can be derived as an explicit function of g : as g changes, so ought the net saving rate. Moreover, in all cases (even when the production for gross output is the one that Piketty implicitly assumes) the net saving rate has to approach zero when g becomes zero.⁵ The simplest theory of saving in the case without growth is the permanent-income theory (due to Friedman (1957)): with a constant wage rate and a constant return to saving, a consumer maintains his asset holdings at a constant level and consumes his wage plus the interest income on the assets every year. Maintaining a constant asset level precisely means having a net saving rate of zero. Thus, a zero net saving rate when there is no growth is very natural: it is what one would expect. Under optimal saving behavior, this result is, moreover, very robust.

In the event, then, that the net saving rate becomes zero as g goes to zero, the second law of capitalism takes the unusable form $k/y = 0/0$. But it is straightforward to show that in a neoclassical growth context, this ratio is, in fact, finite both in the textbook model of exogenous saving and in the optimal-saving model. Moreover, whether one uses the textbook assumption of a historically plausible saving rate or an optimizing rate, when growth falls drastically—say, from 3% to 1.5% or even all the way to zero—then the capital-to-income ratio, the centerpiece of Piketty’s analysis of capitalism, does not explode but rather increases only modestly. In conclusion, at least from the perspective of the theory that we are more used to and find more a priori plausible, the second law of capitalism turns out to be neither alarming nor worrisome, and Piketty’s argument that the capital-to-income ratio is poised to skyrocket does not seem well-founded.

These theory comparisons are made mostly based on available insights on how individuals consume. We also look at some aggregate historical data in order to compare Piketty’s theory with the obvious alternatives. A full investigation is beyond the scope of the present note but in postwar U.S. data we found that the standard formulations of the theory, especially those based on optimizing saving, line up much better with the evidence. With declines in

⁴To be clear, Piketty does take his second fundamental theory seriously: he does use it—as we do here—for comparative statics and for examining the limiting case where g approaches zero. References for comparative statics exercises varying g but holding the net savings rate constant can be found on pp. 167 and 193 in his book; Piketty and Zucman also conduct similar exercises in their joint papers. References for the limiting case of zero growth can be found following p. 227 in his book, in the section entitled “Back to Marx and the Falling Rate of Profit”, and on p. 840 of Piketty and Saez (2014), in the section entitled “Wealth-to-Income Ratios”.

⁵Homburg (2014) makes a related point, using a two-period OLG setting to argue, as we do, that the “coefficient s is not exogenous but an increasing function of the growth rate . . . running through the origin”.

growth, the net saving rate has fallen historically, and it is currently actually already close to zero. The optimizing model predicts that both the gross and the net saving rates will vary positively with growth, and this prediction is borne out clearly in the U.S. data.

The paper is organized as follows. In Sections 2–5 we describe the textbook model and Piketty’s model in parallel—both their common components and their differences. In Section 6 we look in particular at the case where $g = 0$. In Section 7.1 we show how to map the textbook saving rate into the Piketty saving rate and we show that nothing dramatic occurs when the growth rate drops from 3% on down. Section 7.2 studies the case with an endogenous saving rate derived from standard intertemporal utility maximization. Section 7.3 then compares the different theories of saving from the perspective of U.S. data. Piketty also advances a second theory in his book—the “ $r - g$ theory”—and although this theory is not the focus of this paper it does have some relevant overlap with the theories we do discuss here; we discuss these in Section 8. Section 9 makes some concluding remarks.

2 Ingredients common to the two models

The accounting framework:

$$\begin{aligned}c_t + i_t &= y_t \\k_{t+1} &= (1 - \delta)k_t + i_t \\y_t &= F(k_t, z_t l),\end{aligned}$$

where z displays labor-augmenting technical progress and F has constant returns to scale.

3 The textbook model: a constant gross saving rate

The assumptions:

- The *production function*: output, $F(k, \cdot)$, is positive and increasing in k and satisfies Inada conditions; in particular, $F_1(k, \cdot) \rightarrow 0$ as $k \rightarrow \infty$.⁶
- *Behavior*: investment is a constant fraction $s > 0$ of output. That is, $i_t = sy_t$.

These assumptions deliver

$$k_{t+1} = (1 - \delta)k_t + sF(k_t, z_t l)$$

and if $z_t = (1 + g)^t$ this becomes ($\hat{x}_t \equiv x_t/z_t$)

$$(1 + g)\hat{k}_{t+1} = (1 - \delta)\hat{k}_t + sF(\hat{k}_t, l).$$

⁶This condition on the marginal product of capital could be relaxed to $F_1(k, \cdot) \rightarrow \underline{r} < \delta$ as $k \rightarrow \infty$ without affecting any of our results.

This means that there will be a steady state such that

$$\frac{\hat{k}}{\hat{y}} = \frac{s}{g + \delta}.$$

So along a balanced growth path, $k_t/y_t = s/(g + \delta)$.

4 Piketty's model

Piketty works instead with “net” variables:

- The *production function*: $F(k, \cdot) - \delta k$ is positive and increasing in k and satisfies an Inada condition; in particular, $F_1(k, \cdot) - \delta \rightarrow 0$ as $k \rightarrow \infty$.⁷
- *Behavior*: net investment is a constant fraction $\tilde{s} > 0$ of net output. That is, $i_t - \delta k_t = \tilde{s}(y_t - \delta k_t)$.

Note that F in Piketty's model does not satisfy the Inada condition that the textbook model imposes on F : because the production function for net output, i.e., $\tilde{F}(k, \cdot) \equiv F(k, \cdot) - \delta k$, is strictly increasing, $F_1(k, \cdot)$ is bounded below by δ .

Defining net saving $\tilde{i}_t \equiv i_t - \delta k_t$, these assumptions deliver

$$k_{t+1} = (1 - \delta)k_t + i_t = k_t + \tilde{i}_t = k_t + \tilde{s}\tilde{F}(k_t, z_t l).$$

Thus for all values of g , provided $\tilde{s} > 0$, capital is always increasing, because net output, \tilde{F} , is positive. This is true even if g is 0, or less than 0! Piketty's assumptions, in effect, amount to more “aggressive” saving behavior than in the textbook model.

With the same kind of transformation as in the standard model, we obtain

$$(1 + g)\hat{k}_{t+1} = \hat{k}_t + \tilde{s}\tilde{F}(\hat{k}_t, l)$$

and provided $g > 0$ we now obtain a steady-state level of \hat{k} such that

$$\frac{\hat{k}}{\hat{y}} = \frac{\tilde{s}}{g}.$$

(Note that net output is in the denominator, not gross output.) Along a balanced path, $k_t/\tilde{y}_t = \tilde{s}/g$.

⁷On his blog at econbrowser.com/archives/2014/05/criticisms-of-piketty, James Hamilton discusses Piketty's second law using the standard textbook production function, i.e., that satisfying the usual Inada condition. In matters of substance, his discussion leads to the same conclusions as does our analysis, though the case $g = 0$ has different properties in the two cases: in his setup capital converges to a constant but in Piketty's it diverges, as we discuss below. We adopted Piketty's view on production functions to adhere to the treatment in the book. Along these lines, in some of his discussions Piketty even entertains the possibility that the *net* production function does not satisfy the Inada condition. We comment on this case below.

5 Some simple steady-state comparisons

Do both models satisfy the basic growth facts? Along an exactly balanced path, for positive g they do—and positive g s are what we have observed recently over the last hundred or so years in developed economies. In particular, all ratios in Piketty’s model behave as they do in the textbook model: y_t grows at a constant rate and the ratio k_t/y_t is constant over time, as are $\frac{F_{1t}k_t}{y_t}$ (capital’s share of income) and the return to capital (measured as $F_{1t} - \delta$). In short, even though the production function and the assumption about savings behavior in Piketty’s model differ from their counterparts in the textbook model, both models can be made consistent with the growth facts.

The consumption-output ratio (with output measured in gross terms), though not usually mentioned explicitly among the Kaldor facts, is $1 - s$ in the textbook model, but in Piketty’s model it is

$$\frac{c_t}{y_t} = \frac{F(k_t, z_t l) - i_t}{F(k_t, z_t l)} = \frac{\tilde{F}(k_t, z_t l) - \tilde{i}_t}{F(k_t, z_t l)} = \frac{(1 - \tilde{s})\tilde{F}(k_t, z_t l)}{F(k_t, z_t l)}.$$

So what is the ratio of net to gross output in Piketty’s model? His steady state gives

$$g\hat{k} = \tilde{s}\tilde{F}(\hat{k}, l) = \tilde{s}(F(\hat{k}, l) - \delta\hat{k})$$

so

$$F(\hat{k}, l) = \frac{g + \tilde{s}\delta}{\tilde{s}}\hat{k}$$

With $\frac{\tilde{F}}{F}$ thus equal to $\frac{g}{g + \tilde{s}\delta}$, Piketty’s model implies a steady-state ratio of consumption to gross output of

$$(1 - \tilde{s})\frac{g}{g + \tilde{s}\delta}.$$

That is, the lower is the rate of growth, the lower is the consumption-output ratio. Or, put in terms of the gross saving rate, we obtain

$$s(g) = \frac{\tilde{s}(g + \delta)}{g + \tilde{s}\delta}, \tag{1}$$

an expression which is decreasing in g . Thus, Piketty’s assumption of a constant net saving rate embodies an assumption that the saving behavior is increasingly aggressive as growth falls.

The capital-output ratios also appear very different in the two settings. However, Piketty uses net output in the denominator and the textbook model uses gross output. So as not to compare apples with oranges, let us find the ratio of capital to gross output in Piketty’s model. His steady state has $g\hat{k} = \tilde{s}(F(\hat{k}, l) - \delta\hat{k})$, so the ratio of capital to gross output, $\hat{k}/F(\hat{k}, l)$, becomes $\tilde{s}/(g + \tilde{s}\delta)$, to be compared with the textbook’s $s/(g + \delta)$. Similarly,

one can derive the ratio of capital to net output implied by the textbook model; it can be calculated as:

$$\frac{k_t}{\tilde{y}_t} = \frac{k_t}{y_t - \delta k_t} = \frac{\frac{s}{g+\delta}}{1 - \delta \frac{s}{g+\delta}} = \frac{s}{g + \delta(1 - s)}, \quad (2)$$

where the second equality involves evaluation at steady state.

6 Low growth from the two perspectives

As growth goes to zero, (balanced-growth) capital to net output goes to infinity in Piketty’s model, but his ratio of capital to gross output, like that in the textbook model, stays bounded ($\frac{1}{\delta}$ and $\frac{s}{\delta}$, respectively). However, there are nevertheless sharp differences in the consumption and capital accumulation behavior.

First, the consumption-output ratio is equal to $1 - s$ in the textbook model, but in Piketty’s model it is $(1 - \tilde{s})g/(g + \tilde{s}\delta)$ so it goes to zero as g goes to zero! A different way of saying this is that the conventionally measured saving rate goes to one as g goes to zero in Piketty’s model.⁸

Second, Piketty’s model implies that the ratio of capital to net output goes to infinity as g goes to zero. The textbook model implies, for the same ratio, a limit $s/(\delta(1 - s))$; for reasonable values like $s = 0.26$ and $\delta = 0.05$, we thus obtain a ratio near 7. For the ratio between capital and gross output, the differences between the models are large too; the textbook model delivers s/δ , i.e., a little over 5 for standard parameters, whereas Piketty’s model gives $1/\delta$, i.e., as high a value as 20.

Third, in the textbook model capital goes to a steady state, but in Piketty’s model capital grows without bound. Growth without a bound would not be feasible in the textbook model (without making consumption negative), because of the Inada condition on F_1 . However, it is feasible in Piketty’s model, since F_1 is bounded below by δ : no matter how much capital there is, its return is always high enough so as to replace depreciation. Thus, with weak enough consumption demands, capital will keep accumulating.⁹

⁸When g is exactly zero there is no balanced-growth path in Piketty’s model, but in that case one can show that as $t \rightarrow \infty$ the gross saving rate goes to one (and so the consumption-output ratio goes to zero). The gross saving rate in Piketty’s model equals $(\tilde{s} + \delta(k_t/\tilde{y}_t))/(1 + \delta(k_t/\tilde{y}_t))$. We will show that the ratio $k_t/\tilde{y}_t \rightarrow \infty$ as $t \rightarrow \infty$. First, note that the difference $k_{t+1} - k_t = \tilde{s}\tilde{F}(k_t, l)$ is positive for all t (provided $\tilde{s} > 0$) and increases over time because $\tilde{F}(k, \cdot)$ is increasing in k , implying that $k_t \rightarrow \infty$ as $t \rightarrow \infty$. Second, by l’Hôpital’s rule, $\lim_{k \rightarrow \infty} (\tilde{F}(k, l)/k) = \lim_{k \rightarrow \infty} \tilde{F}_1(k, l)$. By the Inada condition on \tilde{F} , this limit is 0, so $k_t/\tilde{y}_t \rightarrow \infty$.

⁹As pointed out above, we included an Inada condition for Piketty’s model. Whereas he uses this assumption in some of the research papers on which the book is based, he also discusses a different assumption: one where the elasticity of the marginal product of capital with respect to capital is rather low, implying that the Inada condition is not met. With such an assumption, the analysis of balanced growth paths would change, as it would (at least for some parameter values) lead to “endogenous” growth, i.e., asymptotic growth at

7 What ought one assume about the net saving rate?

Having established that for economies at $g = 0$, Piketty's assumptions amount to a gross saving rate of 100%; more generally, as g goes to zero, Piketty's model implies a conventionally measured saving rate approaching one. We contend that this is an extreme and rather implausible implication of his model. But to be fair, a more systematic comparison of models, and their implication for data, is needed, so in this section we review the standard models and look at some data. We look first at the textbook Solow model to see what its implications for the net saving rate are as growth rates fall toward zero. We then look at the framework used in the empirical literature studying individual consumption behavior: that of optimizing saving. In these settings, the saving rates, gross and net, are endogenous and in general not constant. In this context, we also look at optimal saving in a neoclassical growth framework (along the lines of Cass and Koopmans). Given our three settings—Piketty's model, the textbook Solow model, and the optimal-saving model—we summarize the differences, which in principle are testable, and use data from the postwar U.S. for a preliminary assessment.

7.1 Deriving the net saving rate from the textbook model

Assuming that the textbook model describes saving behavior correctly, we can compute the net saving rate as defined by Piketty. It becomes

$$\frac{\tilde{i}_t}{\tilde{y}_t} = \frac{sy_t - \delta k_t}{y_t - \delta k_t} = \frac{s - \delta \frac{k_t}{y_t}}{1 - \delta \frac{k_t}{y_t}}.$$

On a balanced growth path, and using the value of the ratio of capital to gross output from the textbook model, we thus obtain

$$\tilde{s}(g) = \frac{s - \delta \frac{s}{g+\delta}}{1 - \delta \frac{s}{g+\delta}} = \frac{gs}{g + \delta(1 - s)}. \quad (3)$$

Equation (3) shows how the function $\tilde{s}(g)$, i.e., net saving solved out endogenously, behaves as g changes (for a given rate of gross saving, s). As indicated above, we see that the net saving rate goes to zero as g goes to zero, no matter what s is—unless $s = 1$, which is a case that probably no one in the applied literature has taken seriously. Note too that comparing equations (2) and (3), it is straightforward to see that in the textbook model the capital-to-net-output ratio is equal $\tilde{s}(g)/g$, i.e., the net saving rate over the growth rate.

a positive percentage rate, even though there is no technological change. We did not consider this case since the consensus in the empirical growth literature has rejected this case, in favor of human capital and technology accumulation as the sources of long-run growth.

This expression looks similar to Piketty’s second law of capitalism, but the key point is, again, that under the assumption of a constant gross saving rate, the net saving rate is not fixed but rather declines to 0 as g declines to 0.

We summarize our findings in this section in the following two figures. Figure 1 first uses equation (3) to display the net saving rates implied by the textbook model, as g goes between zero and 4%, for the whole range of gross saving rates $s \in [0, 1)$, with δ set to 0.05. Figure 2 does the same for the gross rates implied by Piketty’s model in equation (1) for the whole range of net rates $\tilde{s} \in (0, 1]$, with δ again set to 0.05.

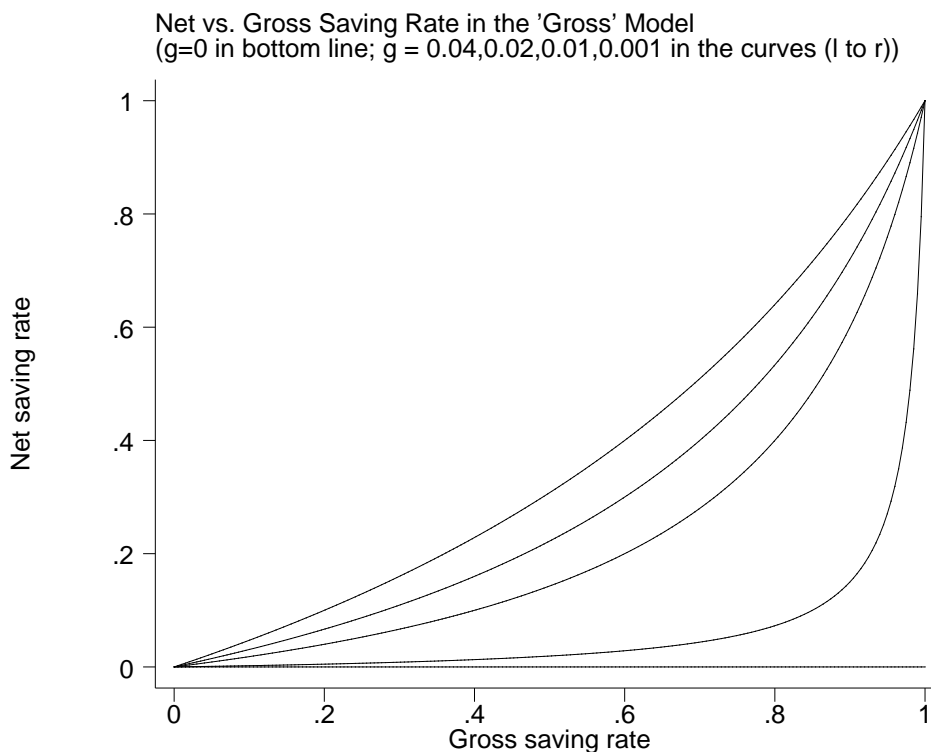


Figure 1

Suppose, finally, that we are interested in Piketty’s measure of capital to net output: could it still be that it rises significantly as g goes to zero? Equation (2) indeed shows that as g falls, while maintaining a constant gross saving rate s , the ratio in Piketty’s second fundamental law indeed rises. But how much does it rise? For example, assume that one period is a year and set $s = 0.26$, $g = 0.03$, and $\delta = 0.05$ so that $\hat{k}/\hat{y} = 3.25$ and $\hat{k}/\hat{\tilde{y}} = 3.88$. If g falls by half to 0.015, then these ratios increase modestly to 4 and 5, respectively. But using the second law holding \tilde{s} constant (as Piketty does in his book and Piketty and Zucman do in their joint papers) would lead one to believe that a halving of the growth rate would lead to a doubling of $\hat{k}/\hat{\tilde{y}}$ rather than less than a 30% increase. This is no trivial matter because a key pillar of Piketty’s argument in his book is that rates of economic growth are slowing

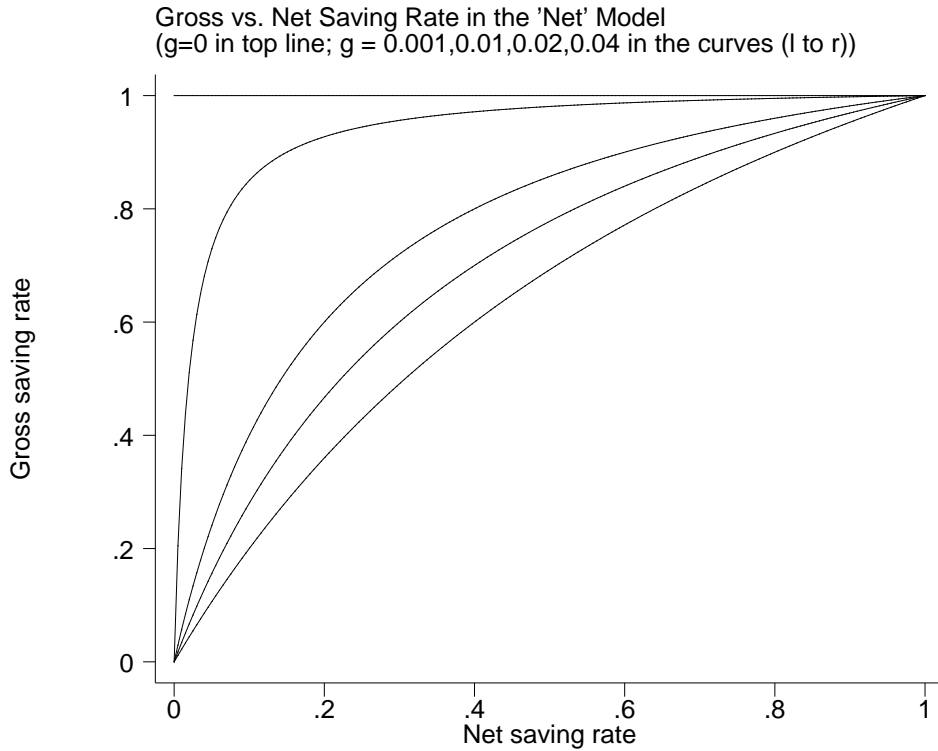


Figure 2

to zero, leading therefore eventually to dramatic increases in the capital-to-net-income ratio, undermining the stability of capitalism.

7.2 Deriving the net saving rate from optimization

We will now briefly discuss whether Piketty's assumed saving behavior can be rationalized based on intertemporal optimization. There are many possible structures one could adopt here but the most commonly used setting is one with infinitely-lived dynasties, and that is the one we will use as well. Since our focus is on the limiting case where $g = 0$, we formally describe optimization for that case only. We look first at the simplest possible optimal-saving problem: one in partial equilibrium where the interest rate as well as the wage rate are constant. We then look at a general-equilibrium economy where the production technology is that assumed by Piketty. In both cases we find that optimal behavior entails setting the net saving rate \tilde{s} , as defined by Piketty, equal to zero when $g = 0$. Later, in Section 7.3, we introduce growth into a general equilibrium model of optimal saving with a standard production technology and report quantitatively how the gross and net savings rates vary with g on a balanced growth path. There we also briefly compare the various models of saving that we consider in this paper with observed U.S. data on gross and net savings rates.

7.2.1 A single consumer

We will assume that the consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where u is an increasing and strictly concave function; concavity here implies consumption smoothing, for which there appears to be strong support in countless empirical studies of individual consumption behavior. The consumer's budget constraint reads

$$c_t + k_{t+1} = (1 + r - \delta)k_t + w,$$

where w is a constant wage, $1 + r$ is the gross return from capital, and δ , as above, is the depreciation rate.¹⁰ We take as implicit a condition preventing the consumer from pyramid-scheme borrowing but otherwise assume no constraints on either saving or borrowing. The consumer thus starts out with some capital k_0 and, given a wage and a net return that are equal to w and $r - \delta$ at all times, we ask ourselves: how will he save?

Substituting c_t into the objective function and taking derivatives with respect to k_{t+1} , we obtain

$$u'(c_t) = \beta u'(c_{t+1})(1 + r - \delta).$$

Consumption behavior here depends critically on whether $\beta(1 + r - \delta)$ is above, below, or equal to 1. Assuming first that it is equal to 1, because this is the only case that allows an exact steady state, we obtain a solution with constant consumption, $c_t = c_{t+1}$, since u' is monotone. This implies, from the budget constraint, that for all t

$$c_t = (r - \delta)k_t + w$$

and

$$k_t = k_0.$$

This is the classical “permanent-income behavior”: the consumer keeps the asset holding constant and consumes the return on the asset only plus the wage income. Here we can identify the consumer's net, or “disposable”, income \tilde{y}_t as $k_t(1 + r - \delta) + w$. Hence, writing

$$c_t = (1 - \tilde{s})\tilde{y}_t$$

and

$$k_{t+1} - k_t = \tilde{s}\tilde{y}_t,$$

¹⁰We can equivalently think of this as an open economy: the interest rate is the world interest rate and $rk_t + w$ is GNP.

we can identify net saving as a fraction of disposable income, \tilde{s} , as 0, just as we just showed the textbook model would imply. One can, of course, depart from $\beta(1 + r - \delta) = 1$. Any such departure would imply that \tilde{s} would depend on time and would either begin positive and eventually turn negative, or the other way around; loosely speaking, the rate would be around zero. Moreover, small departures from $\beta(1 + r - \delta) = 1$ would produce only small departures from \tilde{s} in finite time. Thus, we conclude that this model robustly predicts $\tilde{s} = 0$, along with a bounded value of k/\tilde{y} .

The permanent-income model, thus, suggests that it is not immaterial whether one expresses saving behavior the “textbook way” or the “Piketty way”. The former is consistent with this model but the latter is not.¹¹ Or, rather, it is consistent only if the relevant saving rate is zero, but this is precisely the rate that makes Piketty’s main argument—that the ratio of capital to net income explodes at $g = 0$ —break down.

7.2.2 General equilibrium with Piketty’s production function

We will now show that, using the production function assumed by Piketty, one cannot rationalize his assumed saving behavior. Piketty’s assumption is that \tilde{F} is positive so let us use the production function $F(k, l) = Ak^\alpha + \delta k$, which satisfies his assumption (Cobb-Douglas here is not essential). This makes the economy’s resource constraint read

$$c_t + k_{t+1} = Ak_t^\alpha + \delta k_t + (1 - \delta)k_t = Ak_t^\alpha + k_t.$$

So we essentially have a model with no depreciation, and clearly (as demonstrated above) this model allows unbounded growth. What is, however, reasonable saving behavior for such a model? Let us again use the dynastic setup, and let us for simplicity focus on the planner’s problem, as it deliver quantities that coincide with those of the competitive equilibrium.

$$\max_{\{k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(Ak_t^\alpha + k_t - k_{t+1})$$

a problem that is concave and has a unique solution characterized by the usual, modern-macro Euler equation

$$\frac{1}{Ak_t^\alpha + k_t - k_{t+1}} = \beta \frac{\alpha Ak_{t+1}^{\alpha-1} + 1}{Ak_{t+1}^\alpha + k_{t+1} - k_{t+2}}$$

along with a transversality condition. The Euler equation admits a steady state k defined uniquely by the condition

$$1 = \beta(\alpha Ak^{\alpha-1} + 1)$$

¹¹The textbook saving rate would be defined by $c_t = (1 - s)(rk_t + w)$ and $k_{t+1} - k_t(1 - \delta) = s(rk_t + w)$, implying $s = \delta \frac{k}{rk + w} = \delta \frac{k}{y}$. Depending on the initial capital stock, national income and capital will have different values but the capital-output ratio will be $\frac{s}{\delta}$.

and one can show, using standard methods, that there is convergence to this steady state (with an accompanying convergence of consumption to a constant number).¹²

Put differently, even with the (unusual) production function used by Piketty—one that admits unbounded growth without technical change—standard assumptions on behavior (the optimization of a reasonable-looking utility function) delivers a steady state, quite in contrast with Piketty’s assumption on saving. His assumption on saving is that $\tilde{s} > 0$, which the above analysis shows must not hold, but rather $\tilde{s} = 0$ is optimal in the long run. This, again, must obviously hold since net saving is $k_{t+1} - k_t$ and this is zero whenever the economy reaches a steady state.

7.3 Postwar U.S. data

The data we use is postwar U.S. annual time series for output and net and gross saving rates since 1950.¹³ Although this time period is too short to make truly long-run evaluations, we do find it instructive to look at some of its properties from the perspective of the different models. Our comparisons here use the steady states, or balanced growth paths, of the different models.¹⁴ Recall that in Piketty’s model, the net saving rate is constant over time and independent of the growth rate g , implying that as growth increases, the implied gross saving rate declines (equation (1) above). In contrast, the textbook version of the Solow model has the gross saving rate constant, with the implied net saving rate responding positively to the growth rate (equation (3)). Finally, in the usual optimizing growth model with a standard production function, straightforward calculations show that both saving rates (net and gross) are increasing in g . Figure 3 illustrates this using a standard calibration.

First, plotting the observed saving rates over time as in Figure 4, we observe that at least in percentage terms, the fluctuations in \tilde{s} are larger than those in s . In this sense, if one were to choose between making one of them constant over time, it would make more sense to assume s constant: the textbook version of the Solow model. We also see that \tilde{s} has fallen gradually toward zero; it was below zero during the recent recession and over the last 5 or so years is well approximated by zero. Thus, that \tilde{s} will remain constant and positive in the twenty-first century does not appear like a good assumption at all.

Looking at how saving rates appear to vary with growth rates in the observed data, Figure 5 shows decadal averages (for six decades starting in 1950) of saving rates and growth rates,

¹²Clearly, in a steady state the transversality condition is met, too.

¹³Specifically, from the FRED database we use series A023RX1A020NBEA on real gross national income, series W206RC1A156NBEA on gross saving as a percentage of gross national income, and series W207RC1A156NBEA on net saving as a percentage of gross national income. These series are all come from the Bureau of Economic Analysis; using them it is straightforward to construct a series for net saving as a percentage of net national income.

¹⁴Evaluating the dynamics is far more involved and is best left for future study.

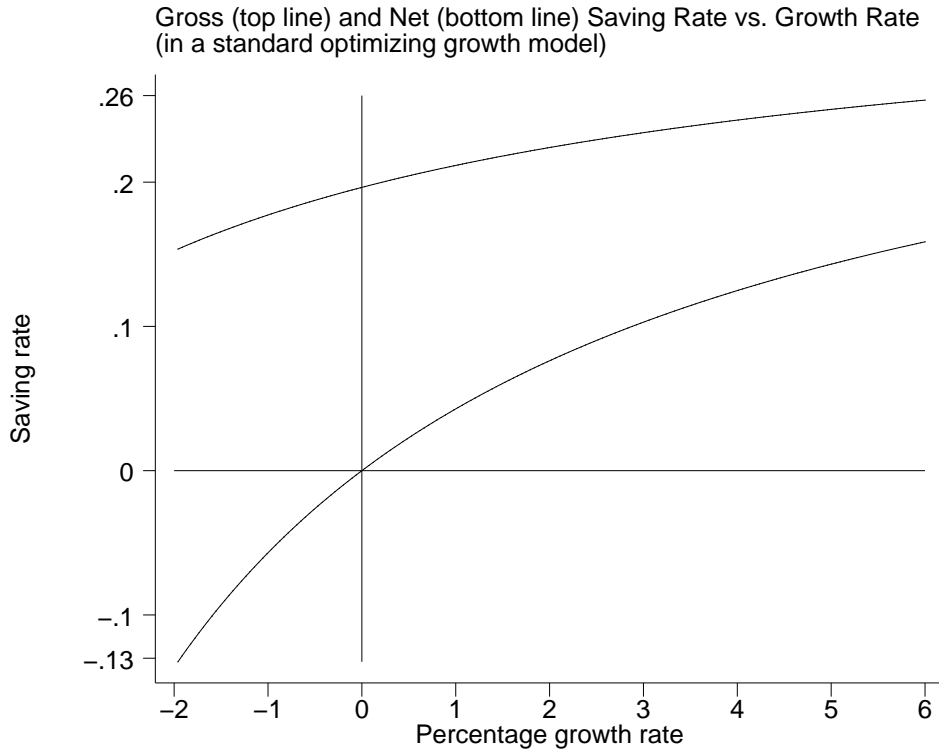


Figure 3

revealing a strong positive relationship. The data is therefore consistent with the optimizing theory in this sense. We see also that the intercept for \tilde{s} , at $g = 0$, is negative, if anything, not positive. Thus, projecting into the future based on Piketty's second fundamental law, with g going to zero, appears unwise.

Let us finally briefly comment on Piketty's point of view; clearly, since his assumptions on saving are non-standard, relative to the applied economics literature, a comparison with the standard model ought to be a main concern in his works, where he does appear to claim that his model allows an accurate account of the historical data. At the very least, one would like to see a comparison between his theory and more standard theory. Piketty and Zucman (2014) studies capital accumulation in a cross-section of countries from the perspective of his formulation of aggregate saving but does not address, to the best of our knowledge, the central question of how net saving rates vary with growth rates.¹⁵ Tables 3 and 4 in this paper report net saving rates and growth rates in a small cross-section of developed countries for the period 1970–2010; here there seems to be no systematic relationship between net saving

¹⁵Instead, this paper uses the growth model to perform an accounting exercise: changes in Piketty and Zucman's broad measure of wealth that cannot be accounted for by the accumulation of savings (given the observed saving rates) are attributed instead to capital gains, i.e., to changes in the market value of capital. We find this accounting exercise interesting but it is not, as far as we can see, a test that can discriminate between different ways of formulating the growth model.

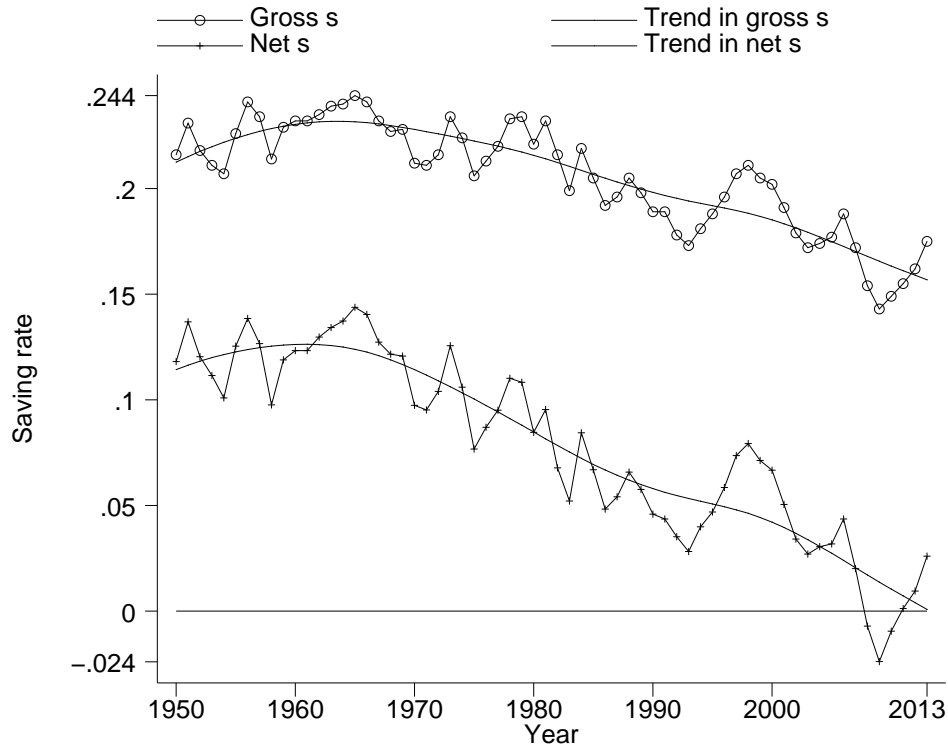


Figure 4

rates and growth rates. But this is not surprising because, as Piketty himself notes in his work, countries differ in all sorts of ways (attitudes towards saving, institutional structures for savings, etc.) that one would want to control for. What would be required instead, expanding on our analysis of U.S. data, is a panel study of a cross-section countries with country-specific fixed effects using, if possible, data going much further back in time. Such an analysis is beyond the scope of this paper, but we guess that such data will imply that low-growth episodes are associated with low, not high, gross saving rates.

8 Connections to the $r - g$ theory

In his book, Piketty in fact presents two theories: the fundamental laws, discussed in the above sections, and a theory in which the difference $r - g$, where, again, r is the return to saving and g is the growth rate, drives inequality. Unlike the fundamental laws, the $r - g$ theory is not about macroeconomics aggregates per se but instead focuses on the right tail of the wealth distribution. Although Piketty does not develop this theory in detail in his book, he does do so in other work; see, example, Piketty and Zucman (2014) or Piketty

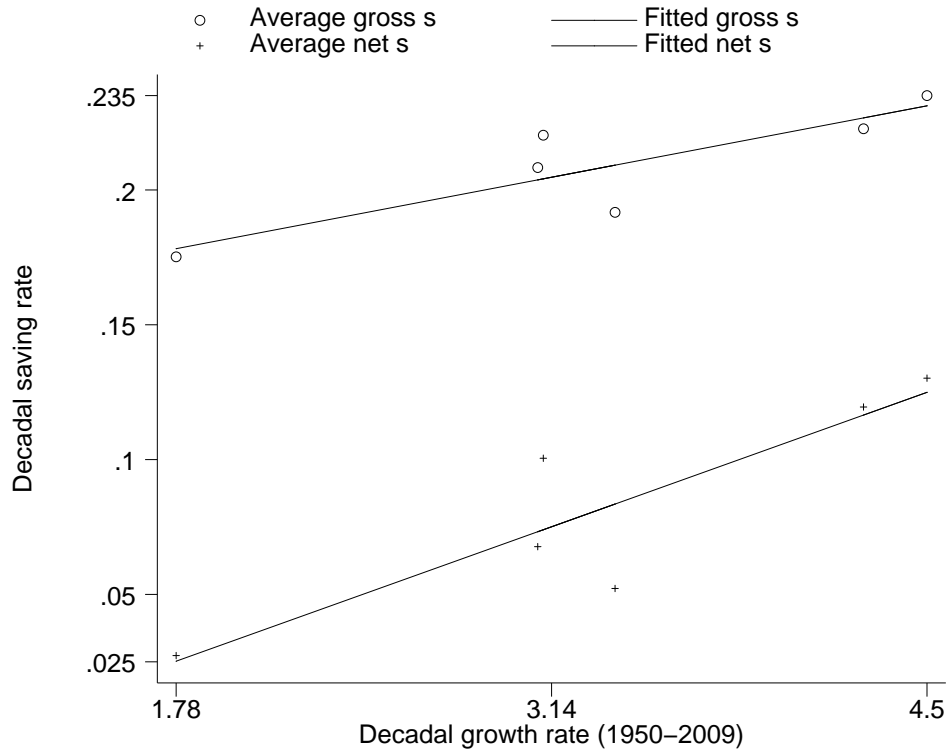


Figure 5

and Saez (2014).¹⁶ Broadly speaking, the $r - g$ theory argues that in models featuring multiplicative shocks to wealth accumulation, the right tail of the wealth distribution looks like a Pareto distribution with Pareto coefficient determined (in part) by $r - g$: other things equal, higher values of $r - g$ lead to thicker tails. These multiplicative shocks could, for example, take the form of random saving rates, as in Piketty and Zucman’s stylized model, or of random returns. The prediction of increasing inequality again has its origins in falling rates of population and technology growth: were g to fall—and if r , in response, were to fall less than g —then the consequent increase in $r - g$ would thicken the right tail of the wealth distribution.

Although the two theories that Piketty puts forth are distinct, in this section we argue that they overlap, especially in how the theories are used to predict the future. First, we argue that the prediction that $r - g$ would go up were g to fall depends not only on whether the marginal return to investment is responsive to capital accumulation but also on the theory of saving. Second, because the optimal saving theory in Section 7.2.1—and its extension to incorporate uncertainty of various forms—is well-established and tested in

¹⁶The expression for capital’s income share coming from the use of Piketty’s second law, rs/g , also suggests that $r - g$ plays a key role, but in this case for determining an aggregate rather than the shape of the distribution of wealth; it is the connection of $r - g$ to this shape that we discuss here and that Piketty emphasizes in his book.

the empirical consumption literature, we use it to obtain specific quantitative predictions jointly for $r - g$ and inequality when g falls. The framework we use shares some elements with Piketty and Zucman’s—though, unlike theirs, it is not a stylized framework but is instead restricted by data on macroeconomic aggregates and inequality—but at its core lies a workhorse equilibrium model of inequality that is by now quite standard in the literature on macroeconomics with heterogeneous consumers.

Our findings, in brief, are, first, that in the textbook growth model $r - g$ falls when g falls, unless either the saving rate is unrealistically large or r responds rather inelastically to variations in capital. Moreover, under the optimal saving theory in Section 7.2.1, $r - g$ again falls when g falls if the elasticity of intertemporal substitution is less than one. Second, in our quantitative model built on optimal saving theory—the theory that we think fits the data best amongst the alternatives we considered here—a fall in g *lowers* inequality somewhat among the poor while hardly changing the right tail of the wealth distribution at all. At the same time, the capital-output ratio rises, thereby refuting the assumption in Piketty’s argument that this ratio and the degree of inequality covary positively.

8.1 How g changes $r - g$: the role of the saving theory

First, it is clear from the perspective of standard growth theory that how g influences r , through changes in the amount of accumulated capital, depends on the nature of the production technology. If output responds rather inelastically to the capital input, as in fact Piketty argues it does, r will not change much as capital increases. We do not focus on this elasticity here, though Rognlie (2014) argues persuasively that some of Piketty’s arguments rely on an unrealistically small elasticity.

Instead, we simply note that even with significant decreasing returns $r - g$ may increase when g falls if the saving rate is high enough. To see this, consider the textbook growth model with a Cobb-Douglas production function ($y = k^\alpha$). Then $r = \alpha \frac{y}{k}$, implying that

$$r - g - \delta = \left(\frac{\alpha}{s} - 1 \right) (g + \delta).$$

So (net) $r - g$ falls as g falls if $s < \alpha$. But this condition is exactly the condition ensuring dynamic efficiency.¹⁷ If instead the rate of saving were to exceed the Golden-Rule rate, $r - g$ would indeed rise as g falls.¹⁸

Second, using the theory of optimal saving one can obtain additional insights. There, the usual Euler equation applies, which is to say that, at least along a balanced growth path, $u'(c) = \beta u'(c(1 + g))(1 + r)$, using the notation above. With a utility function that is such

¹⁷The Golden-Rule saving rate solves $\max_k y - (\delta + g)k$, implying $r = g + \delta$, and hence $s = \alpha$.

¹⁸A similar result holds for Piketty’s model.

that the consumer chooses to grow consumption at net rate g —power utility—we thus obtain $(1 + g)^\sigma = \beta(1 + r)$, where $\sigma > 0$ is the curvature of the utility function (i.e., the inverse of the elasticity of intertemporal substitution). A first-order approximation to this relationship yields $\sigma g = r - \rho$, where $\rho \equiv (1 - \beta)/\beta$ is the time discount rate. Thus, $r - g = \rho + (\sigma - 1)g$. With logarithmic utility ($\sigma = 1$), $r - g$ is invariant to g ; with higher (lower) curvature, $r - g$ rises (falls) as g rises. Much of the consumption literature is concerned precisely with estimating the need for consumption smoothing—the parameter σ —and although there is no consensus most applied researchers use a value of one or somewhat above one. The conclusion from this literature, then, is that $r - g$ does not rise as g falls, and in fact falls if the elasticity of intertemporal substitution is less than one.

8.2 $r - g$ in a quantitative version of Piketty and Zucman (2014)

Following Huggett (1993) and Aiyagari (1994), who build in turn on foundational work by Bewley, there is an extensive literature on macroeconomics with heterogeneous consumers studying the equilibrium determination of wealth and consumption inequality. The central idea of this class of models is that consumers are precautionary savers in the face of uninsurable earnings risk, leading in equilibrium to wealth distributions that display the kind of skewness displayed in the data. The model in Piketty and Zucman (2014) can be viewed as a version of the Bewley-Huggett-Aiyagari setup where consumers have random saving rates stemming from randomness in preferences, such as in discount rates or bequest motives. Piketty and Zucman show, in particular, that such randomness causes the right tail of the wealth distribution to take a Pareto shape.

In Krusell and Smith (1998), we in fact studied, as a second leading example, a model precisely with randomness in discount factors in addition to uninsurable earnings risk. We considered this extension because we wanted to illustrate that small amounts of such randomness could generate wealth distributions that match observed data: they display very high Gini coefficients and have a large mass of households at zero or negative wealth. We did not, however, consider changes in g in that paper, instead simply setting g to zero.¹⁹ In order to obtain a quantitative assessment of how a fall in g might influence both $r - g$ and wealth inequality, we briefly describe some computational experiments that we ran using our 1998 model.²⁰ For details on the calibration of our model we refer the reader back to our

¹⁹We also did not realize that our model might generate wealth distributions with a Pareto-shaped right tail.

²⁰To conduct these experiment we actually ran our original code, available online at aida.wss.yale.edu/smith/code.htm, augmented only to incorporate technology growth at net rate g . We did not shut down the aggregate shocks that we study in the 1998 paper and therefore the numbers reported here are actually averages over simulations generated by a model in which the wealth distribution moves over time, though in a stationary manner.

g (%)	Gini	pct. < 0	% of wealth held by				k/y	r (%)	$r - g$ (%)
			top 0.5%	top 1%	top 5%	top 10%			
0.007	0.795	11.3	14.1	21.0	49.1	66.8	3.504	2.854	0.9537
0.006	0.790	10.3	14.1	21.0	48.9	66.4	3.632	2.754	0.9535
0.005	0.786	9.3	14.1	21.0	48.6	66.0	3.769	2.653	0.9534
0.004	0.781	8.3	14.1	21.0	48.4	65.6	3.917	2.553	0.9533
0.003	0.777	7.4	14.1	21.0	48.2	65.3	4.077	2.453	0.9532
0.002	0.774	6.6	14.2	21.0	48.0	65.0	4.250	2.353	0.9531
0.001	0.770	5.8	14.3	21.0	47.8	64.7	4.439	2.253	0.9530
0.000	0.767	5.1	14.3	21.1	47.7	64.3	4.645	2.153	0.9530

Table 1: The effects of g on wealth inequality

earlier paper; we deviate here only in our settings for the growth rate (0.7% quarterly in the benchmark calibration), the rate of depreciation of capital (1.2% quarterly as in Cooley and Prescott (1995)), and the trio of random quarterly discount factors (here we use 0.9874, 0.9901, and 0.9928).

Table 1 below shows that as growth falls inequality as measured by the Gini coefficient in fact *falls*, though only to a small extent. The share of wealth held by the very richest barely changes; instead the wealth of the poorest households increases substantially (the fraction of households with negative wealth falls from 11.3% to 5.1%), largely accounting for the fall in inequality. These results do not stem from changes in $r - g$, which barely moves, in accord with our discussion in Section 8.1 (we use logarithmic utility in these experiments). To understand the fall in inequality, note instead that k/y , and hence y , rises. In this economy the worst income realization for a consumer is unemployment, in which case the consumer does receive welfare assistance/UI benefits. But in our calibration these benefits are not indexed to overall income. As overall income rises, the relative value of being unemployed falls, and hence poorly-insured households save more for precautionary reasons so that they can maintain higher consumption precisely when they are unemployed.²¹ Finally, the table also points to another weakness of the presumptions made in Piketty’s analysis of the second law. The fall in g here does increase but wealth inequality still falls, even though k is more unevenly distributed than y . What is happening, of course, is that as g falls the dispersion of k falls too. Put differently, it is hard to construct an experiment in which k/y changes but the dispersions of k and y do not, precisely because all of these objects are endogenous. Simply lowering g in this environment thus leads to a conclusion opposite to Piketty’s.

²¹If UI benefits were indexed to overall income, we expect that the effect on inequality would vanish, but more generally the effects on inequality will depend on the design of welfare payments and other institutional features.

We also consider how variations in the duration of households’ earnings shocks influences inequality and $r - g$. Table 2 shows the effects of increasing the length of an average unemployment spell from 1/3 of a year to 1 year. We see that longer unemployment spells cause inequality fall, because again poorer consumers self-insure against these longer spells by accumulating additional wealth. At the same time, wealth concentration drops substantially among the rich too, consistent with the findings in Krusell and Smith (1997) that as shocks become more persistent—hence inhibiting risk sharing through borrowing and lending between consumers—the dispersion of the wealth distribution tends to shrink (virtually to a singleton when shocks are permanent). Although $r - g$ does fall the changes are again quite small. Here g is constant so all of the change in $r - g$ stems from higher saving in aggregate; this higher saving, in turn, reflects the greater severity of the shocks rather than changes in $r - g$, which are anyway small. In sum, in these experiments there is a correlation between $r - g$ and inequality like the one Piketty emphasizes in his book but the mechanism is quite different, with causality running from changes in the environment to changes in $r - g$ rather than the other way around.

duration (quarters)	Gini	pct. < 0	% of wealth held by				k/y	r (%)	$r - g$ (%)
			top 0.5%	top 1%	top 5%	top 10%			
4/3	0.859	25.1	17.1	24.4	53.9	72.1	3.501	2.856	0.9563
2	0.795	11.3	14.1	21.0	49.1	66.8	3.504	2.854	0.9537
8/3	0.747	6.5	12.6	19.1	45.8	62.7	3.508	2.851	0.9509
4	0.678	3.6	11.1	16.8	41.1	57.0	3.515	2.845	0.9454

Table 2: The effects of income shock duration on wealth inequality and $r - g$

In sum, (i) quantitative versions of the $r - g$ theory do not deliver the kinds of results emphasized in Piketty and Zucman (2014) and, to the extent that changes in g have an effect on inequality, the sign is different and the mechanism operates through effects among the poorest; (ii) there are other, nontrivial reasons why wealth inequality and $r - g$ might still correlate positively, and we provide such an example in the context of our quantitative model.

9 Concluding remarks

In conclusion, Piketty’s “second fundamental law of capitalism” and the central theme of his book—that when growth falls toward zero, the capital-income share increases dramatically—appear very difficult to justify, at least in light of our view of how savings decisions are made. These views are based, first, on the fact that we find a 100% gross saving rate—the

implication of Piketty’s model when growth approaches zero—implausible; and, second, on the large empirical literature studying individual consumption behavior. We also take a first look at U.S. postwar data and find, roughly speaking, that the optimal-saving model—that is, the model used in the applied microeconomics literature and by Cass and Koopmans in a growth context—seems to fit the data the best, somewhat better than the textbook Solow model. Piketty’s model, on the other hand, does not appear consistent with this data. Equipped with the models we thus deem better capable of describing actual saving behavior, we then revisit Piketty’s main concern: the evolution of inequality in the 21st century. Using these models as a basis for prediction, we robustly find very modest effects of a declining growth rate on the capital-output ratio and on inequality. Thus, we find Piketty’s second law quite misleading, and certainly not fundamental; we in fact think that the fundamental causes of wealth inequality are to be found elsewhere.

As a matter of history of economic thought, in making his assumptions on saving Piketty has respectable forerunners, to say the least. Solow’s celebrated 1956 paper on economic growth, in fact, assumes from the outset that the net saving rate is constant (and positive), as do Swan’s analysis from the same year and Domar’s pioneering study in 1946. Later, in 1953, Domar posits two formulations of a growth model, one in terms of net saving rates and one in terms of gross rates, as does Johansen in 1959.²² Phelps (1961) develops his famous “Golden Rule” in a model with a constant net saving rate, following Solow. Finally, to round out our quick (and surely incomplete) review of the early literature on growth models with fixed saving rates, Uzawa (1961) posits a capitalist-laborer model in which capitalists save all and laborers save none of their income; in this model he accounts explicitly for depreciation of the capital stock, unlike Solow (1956).

Turning to models of growth based on optimizing model, the pioneering studies of Cass (1965) and Koopmans (1965) both explicitly account for depreciation, as does their forerunner Uzawa (1964) in a model with linear utility.²³ Drawing clear lessons from this whirlwind tour of the historical development of growth models would no doubt require much more detailed and careful study. But we conjecture that it was not until researchers started to take “microfoundations” seriously—in the sense of specifying explicitly the decision problem faced by savers and identifying the state variables, such as capital, upon which their decisions depend—that some of the confusions (apparent in our brief review of the literature) about how to specify saving rules began to clear up. Subsequently, the modern textbook version of the Solow growth model has universally assumed a constant gross saving rate, and we have

²²Domar (1953) calls his two models the α and β models, where α and β are the gross and net saving rates, respectively. He suggests that the β model “is more applicable to a centrally directed economy . . . rather than to a society like the United States”. For such countries he prefers the α model. Domar also references Duesenberry (1949), who suggests that the saving rate be a function of the rate of growth of income.

²³An even earlier forerunner, Ramsey (1928), however, does not.

argued that in an environment without growth in either population or technology this is the only assumption that makes sense.

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