Scarcity of Safe Assets, Inflation, and the Policy Trap[*]

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Abstract

A goal of this paper is to make sense of the seemingly puzzling behavior of interest rates and inflation - and the role of central banks in that behavior - during and after the Great Recession, particularly in the United States. To this end, we construct a model in which government debt plays a key role in exchange, and can bear a liquidity premium. If asset market constraints bind, then there need not be deflation under an indefinite zero interest rate policy (ZIRP). Further, ZIRP may not be optimal under these circumstances. A Taylor-rule central banker could be subject to a ZIRP trap and persistently undershoot target inflation. As well, a liquidity premium on government debt creates additional Taylor rule perils, because of a persistently low real interest rate. We make a case that this is the key policy predicament currently faced by many central banks in the world.

1   Introduction

In this paper, we start with a basic idea – that modeling the role of all government and central bank liabilities as liquidity can give us important insights into the behavior of inflation, interest rates, and the effects of monetary policy. We then show how this can matter for our understanding of the Great Recession and its aftermath, and for the performance of conventional monetary policy rules.

In the United States, short-term nominal interest rates have been close to zero since late 2008. Thus, the zero lower bound has been a reality for the Fed for more than six years. In standard monetary models, a central bank policy rule that keeps the nominal interest rate at zero forever is a Friedman rule (Friedman 1969). Typically, however, the Friedman rule is associated with deflation. For example, in versions of the neoclassical growth model with no aggregate uncertainty and a role for money, the Friedman rule will imply deflation at the rate of time preference. But, since at least early 2010, the inflation rate in the U.S. has varied roughly between 1% and 3% on a year-over-year basis. The flip side of those two observations – near-zero short-term nominal interest rates and positive inflation – is that real interest rates have been persistently low since the Great Recession.

What are we to make of these observations, and what are the implications for monetary policy? A typical approach to explaining the persistence of low real interest rates in New Keynesian (NK) models (e.g. Werning 2011) is to introduce a preference shock – an increase in the representative agent's discount factor – which lowers the "natural real rate of interest." Confronted with such a shock, it can be optimal for a central banker correcting sticky price frictions to set the nominal interest rate at the zero lower bound. The zero lower bound then represents a constraint on policy, and NK models are thus used to argue that the real interest rate is too high relative to what is optimal. The NK approach is then to find policy remedies in central bank forward guidance (Werning 2011, Woodford 2012) or increases in government spending (Eggertsson and Krugman 2012). But baseline NK models have difficulty in explaining recent inflation experience in the United States. A cornerstone of NK models is the Phillips curve, which posits a negative relationship between the inflation rate and the output gap – the difference between output if prices were flexible, and actual output. Given the size of perceived output gaps during the Great Recession, inflation appears to have been too high to be explained by New Keynesian models.[1] As well, since 2012 in the United States, inflation has been falling while the unemployment rate is falling, and inflation expectations have been "anchored" in the Fed's view. These facts seem hard to reconcile with the New Keynesian Phillips curve.

Does standard Monetarism help us understand post-Great Recession experience in the United States? The quantity theory view is that we can decide, on empirical grounds, on an appropriate monetary aggregate – a collection of assets with a high velocity of circulation that is judged to have a stable demand function. That demand function, according to this view, should have few arguments, for example a short-term nominal interest rate, the price level, and real income. But, for the United States, whatever monetary aggregate was deemed by a quantity theorist to be appropriate prior to the financial crisis would surely be judged inappropriate now, with the benefit of hindsight. For example, suppose we consider the period from the beginning of 2009 to the present – a period

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[1] Though Christiano et al. (2014) argue that including the effects of higher costs can explain the Great Recession inflation data for the U.S.

over which the nominal interest rate has been essentially zero. If money demand were stable over this period, the velocity of money – defined in terms of the relevant monetary aggregate – should have been roughly constant (cutting some slack for the possibility that money demand may not be proportional to nominal income). However, from first quarter 2009 to fourth quarter 2014, in the United States the velocity of the monetary base dropped by 49%, the velocity of M1 by 32.5%, and the velocity of M2 by 11.6%. Thus, in general, standard monetary aggregates have become disconnected from the behavior of nominal income and the price level.

Any reconciliation between the quantity theory of money and the behavior of monetary quantities during the Great Recession and its aftermath requires revising our views about what money is, or delving more deeply into the roles played by assets in the economy. Either way, we would be taking an approach inconsistent with standard Monetarism, which has had difficulty in the past with financial disruption, changes in financial regulations, and technological change. There are theories that can potentially make sense of the behavior of asset quantities during the Great Recession and after. For example, Williamson (2012, 2014a, 2014b) studies economies in which an expansion in interest-bearing reserves – even away from the zero lower bound on the nominal interest rate – need not be inflationary. As well Ennis (2014), in a different type of model, shows how a large increase in excess reserves need not be connected to an increase in the price level. But those theories are not what old-school monetarism is about.

To address these issues, we build on ideas from Williamson (2012, 2014a, 2014b), in a somewhat different modeling framework from what was used in those papers, allowing us to extend the ideas in new directions. The model we construct is highly tractable, and has the property that exchange is intermediated by an array of assets. In the model, economic agents are arranged in large households – a device in the spirit of Lucas (1990) or Shi (1997), for example, which permits us to capture the key functions of sophisticated financial market arrangements, without the modeling difficulties associated with intermediated structures and complicated contracts. Households trade in asset markets and goods markets, and can make transactions using money, government bonds, and credit, though money can be used in a wider array of transactions than can other assets. The model is constructed to capture how assets are intermediated and used by the banking system for transactions purposes, though the large household construct allows us to abstract from the details of banking arrangements, which are considered explicitly in Williamson (2012, 2014a, 2014b).

If the asset market constraints of households in the model do not bind, the model behaves in a conventional way, i.e. much like Lucas and Stokey (1987), in terms of how assets are priced, and the relationship between real and nominal interest rates.[2] As well, if asset constraints do not bind, even at the zero lower bound on the nominal interest rate, then a Friedman rule for monetary policy is optimal. However, if asset market constraints bind, the behavior of

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[2] See also Bansal and Coleman (1996), which has a transactions role for government bonds.

the model is quite different. The binding asset market constraint imparts a liquidity premium to government bonds, and bonds bear a low real return to reflect that. In general equilibrium, the asset market constraint binds because private credit is limited, government bonds can be used to substitute for limited private credit, and the supply of government bonds is determined by exogenous fiscal policy. We assume that the fiscal authority acts to set the real value of the consolidated government debt, and then the job of the central bank is to determine the composition of that consolidated government debt, through open market operations.

When the asset market constraint binds, lower nominal interest rates will reduce output and consumption, and will reduce welfare when the nominal interest rate is close to zero. Thus, a binding asset market constraint implies that the zero lower bound is not optimal. A financial shock (which we can interpret as a financial crisis shock) can make the asset market constraint bind, or will tighten the asset market constraint if it binds in the absence of the shock. A financial crisis shock will then lower the real interest rate, but the optimal monetary policy response (given fiscal policy) is *not* to go to the zero lower bound, in contrast to what occurs in NK models. The reason we get these results is that, when asset market constraints bind, an open market sale of government bonds at the zero lower bound involves an exchange of bonds, which are useful in exchange, for money, which is also useful in exchange, but in a different way. On net, this is beneficial.

If the asset market constraint binds, at the zero lower bound the inflation rate is higher the tighter is the asset market constraint. Basically, the liquidity premium on government debt implies the real interest rate is low, so the inflation rate is higher at the zero lower bound the higher is the liquidity premium. Thus, for a sufficiently tight asset market constraint, the inflation rate need not be negative at the zero lower bound, and the inflation rate will fluctuate if there are fluctuations in factors that make the asset market constraint more or less tight. Thus, the ideas represented in this model can potentially explain inflation behavior during and after the financial crisis – behavior that might otherwise appear anomalous.

Central bankers – particularly in the United States – tend to formulate policy in terms of Taylor rules. That is, central banks set a short term nominal interest rate target in response to the inflation rate and some measure of the inefficiency of real outcomes (an output gap, for example). The Taylor rule has also become a cornerstone of NK theory (see Woodford 2003). But the Taylor rule has also been shown to have poor properties in standard monetary models. For example, Benhabib et al. (2001) demonstrate that an aggressive Taylor rule, under which the central bank adjusts the nominal interest rate more than one-for-one with changes in the inflation rate (the Taylor principle), can lead to multiple dynamic equilibria converging to a liquidity trap steady state. In the liquidity trap steady state, the nominal interest rate is zero, the central bank undershoots its target inflation rate, and the central bank would like to lower the nominal interest rate but is constrained by the zero lower bound.

In this paper, we explore further the "perils" of Taylor rules, in cases where

the Taylor rule (as in Benhabib et al. 2001) is a function only of the inflation rate. An important issue here is that the economy behaves much differently if asset market constraints do not bind than if they do. In the former case, typical Taylor rules will yield performance in the spirit of Benhabib et al. (2001). The Taylor principle implies that there are two equilibria, one in which the central bank achieves its inflation target, and another in which the central bank persistently undershoots its target – a liquidity-trap zero-lower-bound equilibrium. Under some versions of the Taylor rule, there also exists a continuum of equilibria, each of which converges in finite time to the liquidity trap steady state.

When asset market constraints bind, however, there are additional perils associated with Taylor rules. First, given binding asset market constraints, the real interest rate is endogenous even in the long run. That is, the long run real interest rate in this case depends in particular on the real stock of government debt outstanding and households' credit limits. This is problematic for the standard Taylor rule, as the rule needs to account for endogeneity in the long run real interest rate to support the central banker's inflation target as an equilibrium. Therefore, a Taylor rule with good operating characteristics must be considerably more complicated.

As well, with binding asset market constraints, a given Taylor rule can yield a more formidable multiple equilibrium problem than when asset market constraints are non-binding. Even if there is a steady state equilibrium in which the central bank achieves its inflation target, there can be other equilibria with positive nominal interest rates in which the inflation target is not achieved, and a liquidity trap equilibrium may also exist. Further, Taylor rules can lead to multiple dynamic equilibria converging to the liquidity trap equilibrium, just as in the case with nonbinding asset market constraints. But in contrast to the unconstrained case, in the constrained case this need not arise only when the Taylor principle holds. Indeed, there are Taylor rules under which there are multiple dynamic equilibria converging to the liquidity trap equilibrium even when the central bank does not aggressively respond to inflation, if asset market constraints do not bind.

All of this illustrates a general tendency for Taylor rules to lead to policy traps. The central bank can have an inflation target in mind, and attempt to hit the target by way of a Taylor rule. But under various rules, there is a tendency (though not in all cases) for there to exist a stable liquidity trap steady state, with inflation below the central bank's target. Thus, the economy can be drawn to the liquidity trap steady state and become stuck there, unless the central bank changes its rule. It seems fair to argue that this describes what is currently occurring in the Euro area, the United States, Sweden, Switzerland, the U.K., and Japan, among other countries. In these cases, central banks appear stuck at the zero lower bound, inflation is lower than the central bank's target, and the central bank appears powerless to raise the inflation rate.

The key novel results in the paper follow from the binding asset market constraint, which gives rise to a liquidity premium on government debt. That is, the price of government debt is greater than its fundamental value as dictated by

the present value of the payoffs on government debt, appropriately discounted. There is an inefficiency, and the low real interest rate reflects it. A similar inefficiency occurs, for example, in Williamson (2014a, 2014b), but there the liquidity premium is associated with a binding collateral constraint. Though the role for government debt that arises in our model is a transactions role, the model permits an interpretation of this as a role for collateral, and of the binding asset market constraint as a collateral scarcity.[3]

The collateral scarcity is critical for our key results, and we want to relate these results to U.S. experience in the Great Recession and its aftermath. Therefore, we would like some assurance that collateral scarcity is an empirically relevant phenomenon. What is the empirical evidence, beyond the fact that real interest rates on government debt have been historically low since the Great Recession? Figure 1 presents some evidence on the quantities of safe assets, potentially useful as collateral in financial markets, in the world, from 2001-2013.[4] Here, before the financial crisis, we include U.S. government debt, Euro-area government debt, U.S. agency debt, and asset-backed securities, as safe collateral. After the financial crisis, we drop asset-backed securities from the stock of safe collateral, as the financial crisis effectively decimated these asset categories. Then, in 2011, we drop Euro-area debt other than French and German debt, as European sovereign debt crises implied that the debt of some European countries was no longer perceived as safe. While these calculations are crude, they give us an idea of the magnitude of the effects of these world financial events on the stock of safe collateral.

[Figure 1 here.]

Furthermore, Gorton (2010) documents how high-grade collateral assets facilitate transactions among institutional investors. The practice of tranching asset pools allows the private sector to create safe assets, and the rehypothecation of collateral permits it to be used in multiple credit contracts.[5] Much of this latter type of activity occurred in the "shadow banking" sector. Thus, the destructive effect of the financial crisis on the stock of collateral could be much larger than what we see in Figure 1, due to a multiplier effect – houses are used as collateral to back mortgages, the mortgages are sold and repackaged as asset-backed securities which are then used as collateral in financial markets, and the asset-backed securities are rehypothecated, etc. Therefore, we think there is good direct evidence that collateral scarcity during and following the Great Recession was an important phenomenon.

The remainder of the paper proceeds as follows. In the second section the baseline model is set up, and equilibria are constructed and their properties

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[3] The model also admits an interpretation financial arrangements as banking with deposit liabilities backed 100% by government bonds (at least in the case where credit limits are zero).

[4] This figure is an updated version of Exhibit 137 in Credit Suisse (2011). Their figures exclude U.S. treasury debt held by the Fed, but we include it in our calculations since we count Fed reserve liabilities as safe assets.

[5] See Andolfatto, Martin and Zhang (2014).

studied in Section 3. Section 4 involves a study of the model's behavior under Taylor rules for monetary policy. Section 5 is a conclusion.

2 Model

There is a continuum of households with unit mass, each of which consists of a continuum of consumers with unit mass, and a worker/seller. Each household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 u(c_t(i))di - \gamma n_t \right], \tag{1}$$

where $c_t(i)$ denotes the consumption of the ith consumer in the household, and $i \in [0,1]$, with consumer names uniformly distributed over the unit interval. In (1), $0 < \beta < 1$, $\gamma > 0$, and n_t is the labor supply of the worker in the household. The household possesses a technology that permits one unit of the perishable consumption good to be produced with each unit of labor supplied by the worker in the household. The consumers in the household cannot consume the household's own output.

The household enters each period with a portfolio of assets, and then trades on a competitive asset market. The worker/seller then supplies labor and produces output. Then, the worker/seller takes the produced output of the household, n_t, and chooses one of two distinct competitive markets on which to sell it. In market 1, only money is accepted in exchange for goods, as there is no technology available for verifying the existence of other assets that the buyer of goods may hold in his or her portfolio, and no technology for collecting on debts. In market 2, government bonds and money are accepted in exchange, and buyers of goods can also use a limited amount of within-period credit.

An individual consumer cannot decide which market he or she will visit (1 or 2), nor does the household decide this. At the beginning of each period, a consumer in the household knows that he or she has a probability θ of going to market 1 and probability $1 - \theta$ of going to market 2. Then, after the asset market has closed, and before goods markets open, the household learns which consumers will visit which markets, and the law of large numbers dictates that a fraction θ of consumers in the household will visit market 1, and a fraction $1 - \theta$ will visit market 2. After the household learns the markets in which consumers will trade, it can allocate assets to consumers in a manner that maximizes household utility. Consumers then take the assets they receive from households, they trade on goods markets, and consume on the spot. It is not possible for consumers in the household to share consumption among themselves. We can then write the preferences of the household as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\theta u(c_t^1) + (1 - \theta)u(c_t^2) - \gamma n_t \right], \tag{2}$$

where c_t^j denotes the consumption of consumers in the household who trade in market j.

The household begins each period with m_t units of money carried over from the previous period, along with b_t^a maturing one-period government bonds acquired in the asset market of the previous period, and b_t^g maturing one-period government bonds acquired in the goods market of the previous period, by the worker/seller. Here, m_t, b_t^a, and b_t^g are measured in units of period $t-1$ consumption good 1. The household also receives a money transfer τ_t from the government in the asset market, defined in units of current consumption good 1. The household then takes beginning-of-period wealth, and trades on the asset market to obtain the money and bonds that it will distribute to consumers in the household to make purchases. The asset market constraint for the household is

$$\theta c_t^1 + q_t b_t^2 + q_t b_{t+1}^a + m_t^2 \leq \frac{p_{t-1}}{p_t}\left(m_t + b_t^a + b_t^g\right) + \tau_t, \tag{3}$$

where q_t denotes the price of government bonds in terms of money, b_t^2 and m_t^2 are government bonds and money, respectively, that are given to consumers in the household who purchase goods in competitive market 2, and b_{t+1}^a denotes bonds that will be held over by the household until period $t+1$. The price p_t denotes the price of good 1 in terms of money. Note that θc_t^1 appears on the left-hand side of (3), as this is the quantity of money, in real terms, that is distributed to consumers who will purchase goods in market 1.

In the analysis that follows, the nonnegativity constraint on bonds held over from the current asset market to the next period is critical (all other nonnegativity constraints remain implicit):

$$b_{t+1}^a \geq 0. \tag{4}$$

Constraint (4) is implied by limited commitment, in that the household cannot commit to pay off debt in future periods that is acquired in the current period.

The household can borrow on behalf of consumers who purchase goods in market 2, and these consumers can also make purchases with money and bonds. The household's within-period debt is constrained, in that it can pay back at most κ_t at the end of the period, where κ_t is exogenous. As well, credit transactions are not feasible in market 1. Total purchases by consumers who purchase in market 2 are then constrained by

$$(1-\theta)c_t^2 = b_t^2 + m_t^2 + \kappa_t. \tag{5}$$

Note that bonds are not discounted when accepted in exchange, since either one bond or one unit of money is a claim to one unit of money at the beginning of period $t+1$, from the point of view of the seller in the goods market. However, bonds can trade at a discount on the asset market, i.e. we can have $q_t < 1$. We also assume in (5) that the household always borrows up to its credit limit, and we will later derive a condition that assures this in equilibrium. In (5), a claim to one unit of market 1 consumption goods at the end of the period exchanges for one unit of goods in market 2.

The household's budget constraint is

$$\theta c_t^1 + q_t(1-\theta)c_t^2 + m_{t+1} + b_{t+1}^g + q_t b_{t+1}^a = \frac{p_{t-1}}{p_t}(m_t + b_t^a + b_t^g) + \tau_t + n_t + q_t \kappa_t - \kappa_t$$

(6)

In equation (6), m_{t+1} denotes money held over until period $t+1$, and the quantity b_{t+1}^g denotes bonds received in payment for goods sold by the household, or in settlement of within-period credit. Note that the price of good 2 in terms of good 1 is q_t, which is implicit from (3) and (5). An additional unit of credit implies that the household can purchase one more unit of consumption goods in market 2. Thus, since the price of market 2 consumption goods in terms of market 1 consumption goods is q_t, the term $q_t \kappa_t$ appears on the right-hand side of equation (6). But one unit of credit is a promise to pay one unit of market 1 consumption goods at the end of the period, which accounts for the $-\kappa_t$ term on the right-hand side of (6). Further, in equation (6), as we have already pointed out for equation (5), we have assumed that the household borrows up to its within-period credit limit.

The government's budget constraints are

$$\bar{m}_0 + q_0 \bar{b}_0 = \tau_0$$

(7)

$$\bar{m}_t - \frac{p_{t-1}}{p_t}\bar{m}_{t-1} + q_t \bar{b}_t - \frac{p_{t-1}}{p_t}\bar{b}_{t-1} = \tau_t, \ t = 1, 2, 3, ...,$$

(8)

where $\bar{m}_t$ and $\bar{b}_t$ are, respectively, the quantities of money and and bonds outstanding (net of government bonds held by the central bank) after asset market transactions in period t. Note that we have assumed that there are no government liabilities (money or bonds) outstanding at the beginning of period 0.

In the model, the heterogeneous-agent representative household is a convenient device that allows us to avoid some complications that might ensue if we were to model financial intermediation arrangements at a more fundamental level. For example, in Williamson (2012, 2014a, 2014b), which works from a Lagos-Wright (2005) base, there are transactions in which different types of assets can be used, and the role of financial intermediaries is essentially to provide insurance against the need for cash – instances in which sellers of goods will not accept intermediary liabilities or other assets. Williamson (2014a, 2014b) also includes a role for assets as collateral. In our model, government bonds are used directly in transactions, rather than as collateral, but economically that is not fundamentally different from an arrangement in which financial intermediary deposits are used in transactions, and those deposits are backed by government bonds, perhaps with the government bonds functioning as collateral for the financial intermediary, as in Williamson (2014a, 2014b). Indeed, the model we work with here has operating characteristics that are similar to the models in Williamson (2012, 2014a, 2014b).

3 Equilibrium

Let λ_t^1, λ_t^2, and μ_t denote, respectively, the multipliers associated with constraints (3), (5), and (6). The consumer chooses c_t^1, c_t^2, n_t, b_t^2, m_t^2, b_{t+1}^a, m_{t+1}, and b_{t+1}^g, in the current period. Then, from the household's optimization problem, we get

$$u'(c_t^1) - \lambda_t^1 - \mu_t = 0, \tag{9}$$

$$u'(c_t^2) - \lambda_t^2 - q_t\mu_t = 0, \tag{10}$$

$$-\gamma + \mu_t = 0, \tag{11}$$

$$-q_t\lambda_t^1 + \lambda_t^2 = 0, \tag{12}$$

$$-\lambda_t^1 + \lambda_t^2 \le 0, \tag{13}$$

$$-q_t(\lambda_t^1 + \mu_t) + \beta E_t\left[\frac{p_t}{p_{t+1}}\left(\lambda_{t+1}^1 + \mu_{t+1}\right)\right] \le 0, \tag{14}$$

$$-\mu_t + \beta E_t\left[\frac{p_t}{p_{t+1}}\left(\lambda_{t+1}^1 + \mu_{t+1}\right)\right] = 0. \tag{15}$$

First, from (12) and (13), note that if $q_t < 1$, then consumers will not purchase good 2 with money, as it is cheaper to pay with bonds if bonds trade at a discount on the asset market. Reducing (9)-(15) to something we can work with, we get

$$-\gamma + \beta E_t\left[\frac{p_t}{p_{t+1}}u'(c_{t+1}^1)\right] = 0, \tag{16}$$

$$u'(c_t^2) - q_t u'(c_t^1) = 0, \tag{17}$$

$$u'(c_t^2) - \gamma \ge 0, \tag{18}$$

and from (16) and (17) we can derive

$$q_t = \underbrace{\frac{u'(c_t^2)}{\gamma}}_{\text{liquidity premium}} \times \underbrace{\beta E_t\left[\frac{p_t}{p_{t+1}}\frac{u'(c_{t+1}^1)}{u'(c_t^1)}\right]}_{\text{fundamental}} \tag{19}$$

$$1 = \underbrace{\frac{u'(c_t^1)}{\gamma}}_{\text{liquidity premium}} \times \underbrace{\beta E_t\left[\frac{p_t}{p_{t+1}}\frac{u'(c_{t+1}^1)}{u'(c_t^1)}\right]}_{\text{fundamental}} \tag{20}$$

Equations (19) and (20) price bonds and money, respectively. In each equation, the left-hand side is the price of the asset, in units of money, and the right-hand side is a liquidity premium multiplied by the "fundamental," which would be the value of the asset if it were not useful in exchange. Note that the liquidity premium for bonds is the inefficiency wedge for good 2, $\frac{u'(c_t^2)}{\gamma}$ (the ratio of marginal utility of the good to the disutility of producing it), while the liquidity premium for money is the inefficiency wedge for good 1, $\frac{u'(c_t^1)}{\gamma}$. Under any circumstances, at the zero lower bound on the nominal interest rate ($q_t = 1$),

the liquidity premia on bonds and money must be equal, as from (19) and (20), $c_t^1 = c_t^2$. As we will show later, though, liquidity premia on bonds and money need not be unity at the zero lower bound.

We can also determine the real interest rate, as follows. Suppose a real bond that sells at price s_t^a, in units of consumption good 1, in the asset market of period t, and pays off one unit of consumption good 1 in the asset market of period $t+1$. Also suppose that this asset is accepted in exchange, just as nominal bonds are. Its price at the end of the period – the price a firm is willing to take for the real bond in exchange for consumption good 2 – is given by s_t^g. Then, optimization by the household implies

$$-s_t^a \lambda_t^1 + \lambda_t^2 s_t^g = 0 \tag{21}$$

$$-s_t^g \mu_t + \beta E_t \left(\lambda_{t+1}^1 + \mu_{t+1} \right) = 0. \tag{22}$$

Therefore, from (9), (11), (12), (17), (21), and (22), we can determine the price of the real bond as

$$s_t^a = \underbrace{\frac{u'(c_t^2)}{\gamma}}_{\text{liquidity premium}} \times \underbrace{\beta E_t \left[\frac{u'(c_{t+1}^1)}{u'(c_t^1)} \right]}_{\text{fundamental}}. \tag{23}$$

Note, in equation (23), as in (19), that we can write the price of the real bond as a liquidity premium, $\frac{u'(c_t^2)}{\gamma}$, multiplied by the fundamental.

We will specify fiscal policy as setting the real value of the consolidated government debt each period, V_t, i.e.

$$V_t = \bar{m}_t + q_t \bar{b}_t, \tag{24}$$

where V_t is exogenous. Then, from (8),

$$\tau_0 = V_0,$$

$$\tau_t = V_t - \frac{p_{t-1}}{p_t} V_{t-1} - \bar{b}_{t-1} \frac{p_{t-1}}{p_t} \left(1 - q_{t-1} \right), \tag{25}$$

so the period 0 real transfer to the private sector is exogenous, but the transfer in each succeeding period is endogenous, and in general will depend on monetary policy, which affects prices. Thus, fiscal policy responds passively to monetary policy so as to achieve a particular time path for the total real value of the consolidated government debt. Monetary policy consists of setting a target q_t for the price of government bonds, and this target is then supported by open market operations. The relationship between fiscal and monetary policy here is the same as in Williamson (2014a, 2014b). As it turns out, the real value of the consolidated government debt will play a critical role in our model, and for the key results, so it proves convenient (and realistic, we think) to specify the fiscal policy rule as setting the real value of the consolidated government debt exogenously. Then, monetary policy is about determining the composition of the consolidated government debt so as to achieve a particular price for government

debt in financial markets. But, a key element in how monetary policy affects inflation, for example, will be determined by the nature of the fiscal policy rule.

Let $\pi_t = \frac{p_t}{p_{t-1}}$ denote the gross inflation rate. Then, from (3), (5), (6), (16)-(18), (24) and market clearing, an equilibrium is a stochastic process $\{c_t^1, c_t^2, \pi_{t+1}\}_{t=0}^{\infty}$ solving

$$-\gamma + \beta E_t \left[\frac{u'(c_{t+1}^1)}{\pi_{t+1}} \right] = 0, \tag{26}$$

$$u'(c_t^2) - q_t u'(c_t^1) = 0, \tag{27}$$

$$u'(c_t^2) - \gamma = 0 \text{ and } V_t + q_t \kappa_t \geq \theta c_t^1 + (1-\theta) q_t c_t^2, \tag{28}$$

or

$$u'(c_t^2) - \gamma \geq 0 \text{ and } V_t + q_t \kappa_t = \theta c_t^1 + (1-\theta) q_t c_t^2, \tag{29}$$

given a stochastic process $\{V_t, q_t, \kappa_t\}_{t=0}^{\infty}$, with $q_t \leq 1$. In (28) and (29) note that, if $u'(c_t^2) - \gamma = 0$, then the real value of government debt plus the credit limit is more than sufficient to finance purchases in market 2, so this is the case in which constraint (4) does not bind. But, if $u'(c_t^2) - \gamma > 0$, i.e. if exchange in market 2 is inefficient, then the value of consumption of both goods consumed is constrained by the real quantity of consolidated government debt plus the credit limit, and the nonnegativity constraint (4) binds.

Filling in the remaining equilibrium details, the price level at the initial date is determined by

$$p_0 = \frac{M_0 + q_0 B_0}{V_0}, \tag{30}$$

where M_0 and B_0 denote, respectively, the nominal quantity of money and the number of bonds outstanding in period 0, where each bond is a claim to one unit of money in period 1. Thus, in period 0, the central bank announces q_0, and the fiscal authority issues bonds with a total current nominal market value of $M_0 + q_0 B_0$. The central bank then issues M_0 units of money, in nominal terms, and exchanges this money for government bonds in the initial open market operation, at the market price q_0. Then, given p_0 from (30), the sequence of inflation rates $\{\pi_{t+1}\}_{t=0}^{\infty}$ determines the price level path. Further, the central bank must choose a sequence of open market purchases and sales consistent with its nominal interest rate targets. First, if $q_t < 1$, so that households wish to use all money balances during the period in transactions in market 1, then

$$\bar{m}_t = \theta c_t^1. \tag{31}$$

In this case, the size of the central bank's open market purchase in period t must be sufficiently large that consumers in market 1 can purchase their desired quantity of consumption goods at market prices. As well, the open market operation cannot be too large, as households would not want to hold money balances over until the next period, given market prices. Next, if $q_t = 1$, then there is a liquidity trap and

$$\bar{m}_t \geq \theta c_t^1. \tag{32}$$

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In this case, the household is indifferent between allocating money or bonds to consumers who make purchases in market 2. Thus, the open market purchase in period t needs to be sufficiently large, so that consumers in market 1 have enough money, but as money and bonds are perfect substitutes in market 2 transactions, the central bank could purchase the entire stock of government debt and this would not matter for equilibrium quantities and prices.

The way we have specified policy here shares ideas with the literature on the fiscal theory of the price level (FTPL), if only because we recognize the importance of the interaction between monetary and fiscal policy (see for example Leeper 1991 and Woodford 1995). The FTPL emphasizes the consolidated government budget constraint, and the role of the fiscal authority in determining the price level. In this model, the fiscal policy rule we specify bears some resemblance to the one that Leeper (1991) studies, but in our context it would not be correct to say that fiscal policy determines the price level. Given the fiscal policy rule, monetary policy will matter for the prices of goods in terms of money. For example, if the fiscal authority fixes the total quantity of nominal bonds issued in period 0, then from (30), the choice of q_0 by the central bank matters for p_0. Then, in each periods 1, 2, 3, ..., monetary policy will matter for the inflation rate, in general, so fiscal and monetary policy jointly determine prices – and quantities.

For now we model $\{q_t, V_t, \kappa_t\}$ as an exogenous stochastic process, but later we will permit endogeneity in the central bank's choice of q_t. To solve for an equilibrium, note first, given our specification for monetary policy, that from (27)-(29) consumption quantities c_t^1 and c_t^2 depend only on q_t, V_t, and κ_t, i.e. we can solve period-by-period for consumption, and then for output $y_t = \theta c_t^1 + (1-\theta)c_t^2$ and labor supply $n_t = y_t$. We then need to solve for a stochastic process for the gross inflation rate $\{\pi_t\}$, which must solve (26) given our solution for the stochastic process for c_t^1. It is straightforward to verify that one solution to (26) is always

$$\pi_t = \frac{\beta u'(c_t^1)}{\gamma}. \tag{33}$$

But is this a unique solution? Suppose, for example, that $\{q_t, V_t, \kappa_t\}$ is a two-state Markov process. Then, it is straightforward to show that, if $\{q_t, V_t, \kappa_t\}$ is i.i.d., the solution for the inflation rate is indeterminate, with (33) describing only one solution in a continuum. But, if $\{q_t, V_t, \kappa_t\}$ is serially correlated, then we can also show that (33) describes the unique solution. If we are concerned with uniqueness of equilibrium, then the fact that there exist stochastic processes for the exogenous variables which imply a unique solution is reassuring. In any case though, we can only be assured of uniqueness within the class of equilibria for which the endogenous variables depend only on the exogenous state. It is well-known that in standard monetary models there can exist multiple periodic or sunspot equilibria, for example.

Let *unconstrained equilibrium* and *constrained equilibrium*, denote the cases where (28) and (29) apply, respectively, i.e. in which the nonnegativity constraint (4) binds, and does not bind, respectively. For a given stochastic process

$\{q_t, V_t, \kappa_t\}$ it is possible that (28) will apply in some periods and (29) in other periods, but it will prove useful to consider two special cases: (i) (28) applies in all periods – the unconstrained equilibrium – and (ii) (29) applies in all periods – the constrained equilibrium.

3.1 Unconstrained Equilibrium

An unconstrained equilibrium has standard properties that we would find in typical cash-in-advance cash good/credit good models, e.g. Lucas and Stokey (1987). In an unconstrained equilibrium, from (28), exchange in market 2 is efficient as $u'(c_t^2) - \gamma = 0$. For convenience, let c^* denote the efficient consumption quantity, which solves

$$u'(c^*) = \gamma. \tag{34}$$

Thus, from (19) and (20), there is no liquidity premium associated with bonds, but there is a standard liquidity premium associated with money. From (33) and (26), c_t^1 solves

$$u'(c_t^1) = \frac{\gamma}{q_t}, \tag{35}$$

i.e. the inefficiency in market 1, and the liquidity premium on money (from equation (20)) are associated with a positive nominal interest rate ($q_t < 1$). From (33) and (35), we can solve for the gross inflation rate:

$$\pi_t = \frac{\beta}{q_t}. \tag{36}$$

Equation (36) is the Fisher relation – there is an approximately one-for-one relationship between the inflation rate and the nominal interest rate. Further, from (23), we get

$$s_t^a = q_t \beta E_t \left[\frac{1}{q_{t+1}} \right], \tag{37}$$

where the real interest rate is $\frac{1}{s_t^a} - 1$.

Note that, from (28),

$$V_t + q_t \kappa_t \geq \theta c_t^1 + (1 - \theta) q_t c^* \tag{38}$$

must be satisfied for the unconstrained equilibrium to exist. This equilibrium is unconstrained, as the consolidated government debt V_t and the credit limit κ_t are irrelevant for the solution, at the margin. So, Ricardian equivalence holds, and the solution has the property that growth in the money supply determines the inflation rate. For example, if $\kappa_t = \kappa$, $q_t = q < 1$ for all t, and (38) holds, then c_t^1 is constant for all t, and so from (31) and (36), the price level and the money stock both grow at the constant rate $\frac{\beta}{q} - 1$. But V_t could fluctuate in arbitrary ways, as long as (38) holds for all t, and so the growth rate in the nominal quantity of government debt could fluctuate in ways that are disconnected from the inflation rate. Further, from (37), the real interest rate is determined solely by monetary policy.

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We can illustrate the unconstrained equilibrium in Figure 2. With a positive nominal interest rate, i.e. $q_t = q_1 < 1$, the equilibrium is at point A, where the locus defined by (27), dropping time subscripts, intersects $c_t^2 = c^*$. An efficient allocation is B, which will be the equilibrium allocation when $q_t = 1$. At point B, $c_t^1 = c_t^2 = c^*$, which is the conventional Friedman rule allocation.

[Figure 2 here.]

3.2 Constrained Equilibrium

In the constrained equilibrium, (4) binds for all t, so the quantity of government bonds, plus the credit limit, constrains the quantity of consumption in market 2. The constrained equilibrium is the unconventional case, in which, from (3), (4) with equality, (5), (24), and market clearing, c_t^1 and c_t^2 solve

$$V_t + q_t \kappa_t = \theta c_t^1 + (1 - \theta) q_t c_t^2 \tag{39}$$

and (27). Again, note that the solution for c_t^1 and c_t^2 depends only on the current exogenous variables V_t, q_t, and κ_t, and then we can solve for π_t from (33). The model solves period-by-period, so comparative statics is straightforward. In a constrained equilibrium, dropping t subscripts for convenience, from (39) and (27) we get

$$\frac{dc_1}{dq} = \frac{\kappa y u''(c_2) - u'(c_2)(1 - \theta)\left[\frac{c_2 u''(c_2)}{u'(c_2)} + 1\right]}{\theta u''(c_2) + q^2(1 - \theta)u''(c_1)} \tag{40}$$

$$\frac{dc_2}{dq} = \frac{\theta u'(c_1) + q u''(c_1)\left[\kappa - (1 - \theta)c_2\right]}{\theta u''(c_2) + q^2(1 - \theta)u''(c_1)} \tag{41}$$

Note that (5) implies that, if (4) binds, then

$$\kappa < (1 - \theta)c_2, \tag{42}$$

so that the credit limit does not permit all the goods supplied in market 2 to be purchased with credit in equilibrium. Then, (23) and (41) imply that $\frac{dc_2}{dq} < 0$. The sign of $\frac{dc_1}{dq}$ is in general ambiguous, but, if $-\frac{cu''(c)}{u'(c)} < 1$, then $\frac{dc_1}{dq} > 0$. Thus, provided there is not too much curvature in the utility function, a lower nominal interest rate (higher q) reduces consumption in market 2 and increases consumption in market 1. Essentially, an increase in q is an increase in the relative price of consumption in market 2, from the household's point of view. Then, $-\frac{cu''(c)}{u'(c)} < 1$ implies that the substitution effect of the relative price change dominates the income effect in terms of the the net effect on consumption in market 1. Further, if $-\frac{cu''(c)}{u'(c)} < 1$, then (31) implies that, if the central bank wants the nominal interest rate to fall, i.e. wants q to rise, then consumption in market 1 must rise, which requires that the central bank conduct an open market purchase of government bonds. This will increase the quantity of cash in market 1, in real terms, and reduce the quantity of assets exchanged in market 2,

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as the quantity of bonds outstanding falls. As a result, consumption in market 2 falls.

We can illustrate the constrained equilibrium in Figure 2. Here, V_0 is the locus defined by (39) for the case in which $q = 1$, while V_1 is the same locus when $q = q_1 < 1$. Then, a reduction in q from 1 to q_1 shifts the equilibrium from D to E. Consumption in market 2, c_2, must increase, while c_1 may rise or fall.

As well, it is useful to look at the effect of an increase in the nominal interest rate on real GDP, which we can express as

$$y_t = \theta c_t^1 + (1 - \theta)c_t^2. \tag{43}$$

Then, from (40) and (41),

$$\frac{dy}{dq} = \frac{[\kappa - (1 - \theta)c_2]\left[\theta u''(c_2) + (1 - \theta)qu''(c_1)\right] + u'(c_1)\theta(1 - \theta)(1 - q)}{\theta u''(c_2) + q^2(1 - \theta)u''(c_1)}.$$

Therefore, given (42), $\frac{dy}{dq} < 0$, so output goes down when the nominal interest rate goes down. We could also do a welfare calculation to find the optimal monetary policy in this economy under a constrained equilibrium, taking fiscal policy as given. We can do a period-by-period maximization of period utility for the representative household, so drop t subscripts and write the welfare measure as

$$W = \theta u(c_1) + (1 - \theta)u(c_2) - \gamma\left[\theta c_1 + (1 - \theta)c_2\right]$$

Then, differentiating, we get

$$\frac{\partial W}{\partial q} = \theta\left[u'(c_1) - \gamma\right]\frac{dc_1}{dq} + (1 - \theta)\left[u'(c_2) - \gamma\right]\frac{dc_2}{dq} \tag{44}$$

So, at the zero lower bound ($q = 1$), where $c_1 = c_2 = V + \kappa$,

$$\frac{\partial W}{\partial q} = [u'(V + \kappa) - \gamma]\frac{dy}{dq} < 0.$$

Therefore, given (42), $q < 1$ is optimal, so optimal monetary policy is not a zero-lower-bound policy if the equilibrium is constrained. In general, from (44), (40), and (41), we can write

$$\frac{\partial W}{\partial q} = \frac{[\theta u''(c_2) + q(1 - \theta)u''(c_1)]\left[u'(c_1) - \gamma\right]\left[\kappa - (1 - \theta)c_2\right] - \gamma\theta(1 - \theta)u'(c_1)(1 - q)}{\theta u''(c_2) + q^2(1 - \theta)u''(c_1)}.$$

Therefore, we can say, in general, that welfare increases as the nominal interest rate increases, so long as the nominal interest rate is close to zero, i.e. q is close to 1. Thus, close to the zero lower bound, an open market sale of government bonds by the central bank will increase consumption in market 2 – the goods purchased with bonds – and reduce consumption in market 1 – goods purchased with money. On net, this will increase output and, provided the nominal interest rate is close enough to zero, we can be assured that welfare will increase. One's intuition might lead one to believe that an open market purchase should imply

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less liquidity and lower welfare, but that is not the case here, as bonds and money are both liquid assets.

Thus, since c_2 is strictly decreasing in q, from (41), which implies that the inefficiency wedge in market 2, $\frac{u'(c_2)}{\gamma}$, is strictly increasing in q, a constrained equilibrium exists if and only if the equilibrium is constrained at the zero lower bound. In a constrained equilibrium at the zero lower bound, $c_1 = c_2 = V + \kappa$. Therefore, from (29), a constrained equilibrium exists for some q if and only if

$$\frac{u'(V + \kappa)}{\gamma} > 1, \tag{45}$$

i.e. if and only if $V + \kappa$ is sufficiently small. Furthermore, if (45) holds, then if q is sufficiently small, the equilibrium will be unconstrained. To be more precise, if (45) holds, then the equilibrium is constrained for $q \in (\hat{q}, 1]$, and unconstrained for $q \leq \hat{q}$, where $(\hat{q}, \hat{c}_1)$ solve

$$u'(\hat{c}_1) = \frac{\gamma}{\hat{q}}, \tag{46}$$

$$V + \hat{q}\kappa = \theta \hat{c}_1 + (1 - \theta)\hat{q}c^*. \tag{47}$$

How does monetary policy affect the real interest rate in a constrained equilibrium? One way to think about this is to consider an equilibrium that is constrained, with $q_t = q$, $V_t = V$ and $\kappa_t = \kappa$ for all t, so that $c_t^1 = c_1$ and $c_t^2 = c_2$ for all t. Then, from (23),

$$s_t^a = \frac{u'(c_2)\beta}{\gamma}. \tag{48}$$

Therefore, from (48), the real interest rate depends on the inefficiency wedge (the ratio $u'(c_2)/\gamma$) in market 2, i.e. on the liquidity premium on real bonds. Thus, from (41), a decrease in the nominal interest rate by the central bank also reduces the real interest rate, as this reduces the supply of bonds, tightens the finance constraint, and increases the liquidity premium on government debt.

We can interpret a constrained equilibrium as one in which there is a scarcity of government debt, since the quantity of government debt plus private credit is insufficient to finance an efficient level of consumption in market 2. This scarcity is reflected in a low real interest rate, and it manifests itself only when the nominal interest rate is low. Further, reducing the nominal interest rate when the asset scarcity exists only exacerbates the inefficiency, reducing aggregate output and welfare. This contrasts with results from the New Keynesian literature, for example Eggertsson and Krugman (2012) and Werning (2011). In New Keynesian models a low real interest rate, whether caused by a preference shock or a tighter borrowing constraint, can imply that there is a welfare improvement from lowering the nominal interest rate to zero, with the zero lower bound constraining optimal policy.

Fiscal policy matters for the equilibrium allocation and prices in the constrained equilibrium, in contrast to the non-constrained equilibrium. In particular, from (27) and (39), the real quantity of consolidated government debt V_t

matters for the equilibrium consumption allocation, so the equilibrium is non-Ricardian. As well, the constrained equilibrium has the property that nominal growth in the quantity of government debt will matter for inflation. For example, suppose a constant quantity of real consolidated government debt V, a constant credit limit κ, and a constant price for government debt, q, as determined by monetary policy. Then, c_t^1 is constant for all t, and from (33) the inflation rate is a constant, $\pi - 1$. If $q < 1$, then, from (31), the stock of money grows at rate $\pi - 1$. But, given (39), it will be true in general that, with $q_t < 1$, and given (31), that

$$(1 - \theta)c_t^2 = \bar{b}_t + \kappa_t, \tag{49}$$

i.e. consumption in market 2 is equal to the end-of-period value of government debt plus the credit limit. So, since we are considering an experiment in which all exogenous variables are constant for all t, so that c_t^2 is constant in equilibrium, therefore, from (49) $\bar{b}_t = \bar{b}$, a constant for all t, and the real value of government debt is constant over time. Therefore, the nominal stock of government debt also grows at the rate of inflation, as does the nominal consolidated government debt.

Note that it would not be correct to say that fiscal policy determines prices or inflation in the constrained equilibrium with $q_t < 1$. Indeed, fiscal policy and monetary policy jointly determine prices and quantities.

However, suppose, as above, that all exogenous variables are constant for all t, and $q_t = 1$ for all t, in a constrained equilibrium. Then, (32) must hold, but this could be consistent with money growth that does not match the inflation rate over long periods of time. But, since (39) holds in this equilibrium, the rate of growth in nominal consolidated government debt is equal to the inflation rate for all t in equilibrium. Thus, in a liquidity trap, monetary policy becomes irrelevant and fiscal policy determines the inflation rate.

3.3 Credit Constraints and Government Debt

What happens if there is a change in the credit constraint κ_t? For example, we might think of a decrease in κ_t as capturing some of what occurred during the financial crisis. In an unconstrained equilibrium, a change in κ_t has no effects at the margin, as the credit limit and government debt are large enough to support efficient exchange in market 2. Therefore, given monetary policy, there is no change in consumption or output.

However, from (46) and (47), a decrease in κ acts to reduce $\hat{q}$, so there is an increase in the critical value for the nominal interest rate, below which the equilibrium will be constrained. Therefore, a discrete decrease in κ could result in a constrained equilibrium in a case in which the equilibrium was unconstrained before the change in κ. Further, given q, the constrained equilibrium will have lower consumption in both markets and lower real output if κ decreases, from (27) and (39). In Figure 3, the equilibrium is initially at A, at the intersection between the upward-sloping locus defined by (27), and the downward-sloping locus defined by (39). Then, holding monetary policy constant, so q is constant,

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the locus defined by (39) shifts from V_0 to V_1 with a decrease in κ. As a result, consumption in both markets, c_1 and c_2, must fall, and from (43), total real GDP falls as well.

[Figure 3 here.]

A decrease in κ acts directly in a constrained equilibrium to reduce the demand for consumption in market 2, given the quantity of government bonds (in real terms) outstanding. This would tend to increase q, which is the price of consumption in market 2 relative to consumption in market 1. But the central bank, which targets q, offsets the incipient increase in q with an open market sale of government bonds, which tends to reduce consumption in market 1 and increase consumption in market 2. On net, the effect of the increase in government bonds outstanding does not completely offset the reduction in the credit limit, so consumption in market 2 falls. Consumption in market 1 falls because there is less money outstanding, in real terms.

More formally, from (27) and (39),

$$\frac{dc_1}{d\kappa} = \frac{qu''(c_2)}{\theta u''(c_2) + q^2(1-\theta)u''(c_1)} > 0, \tag{50}$$

$$\frac{dc_2}{d\kappa} = \frac{q^2 u''(c_2)}{\theta u''(c_2) + q^2(1-\theta)u''(c_1)} > 0. \tag{51}$$

Therefore, a reduction in κ reduces consumption in both markets, and lowers real output. As well, from (27) and (39), we get

$$\frac{dc_1}{dV} = \frac{1}{q}\frac{dc_1}{d\kappa}, \tag{52}$$

$$\frac{dc_2}{dV} = \frac{1}{q}\frac{dc_2}{d\kappa}. \tag{53}$$

Therefore, a decrease in the real quantity of consolidated government debt has the same qualitative effect as a reduction in the credit limit. Put another way, a reduction in the credit limit can be mitigated or eliminated if the fiscal authority acts to increase the quantity of government debt. However, one of our goals in this paper is to examine the effects of monetary policy in the face of suboptimal behavior of the fiscal authority which, in this case, will not act to relax asset market constraints even if that is appropriate.

We can also examine how changes in κ and V affect the real interest rate. If, as above, we look at cases where $\kappa_t = \kappa$ and $V_t = V$ for all t, then from (48), (51), and (53), a decrease in κ or V will lower the real interest rate, because this increases the inefficiency wedge in market 2, and therefore increases the liquidity premium on government bonds.

As well, note from (33) that a decrease in κ or V will increase the inflation rate, given q. Because the asset market constraint tightens, increasing the liquidity premium on government bonds and lowering the real interest rate, the

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inflation rate must rise since the nominal interest rate is being held constant in these experiments.

These results – that a reduction in credit limits will reduce consumption and output, reduce the real interest rate, and lead to an increase in the inflation rate (given the nominal interest rate) – are consistent with observations on the U.S. economy following the financial crisis. Post-2008, the real interest rate on government debt was low, and it may seem surprising, given the zero-lower-bound policy of the Fed, that the inflation rate was still positive. However, this is consistent with what our model predicts.

To see more clearly where these results are coming from, we consider in the next subsection what happens at the zero lower bound.

3.4   Liquidity Trap

It is useful to examine the properties of the model when the nominal interest rate is set to zero by the central bank, or $q_t = 1$, so that we have a liquidity trap equilibrium. From (27), this implies that $c_t^1 = c_t^2$, so consumption is equalized across markets. If (45) does not hold, so that the real value of the consolidated government debt plus the credit limit is sufficiently large, then the liquidity trap equilibrium is unconstrained, so from (28), we have $u'(c_t^1) = u'(c_t^2) = \gamma$ and, from (33), $\pi_t = \beta$. Therefore, if assets used in exchange and credit are sufficiently plentiful, then a liquidity trap equilibrium has conventional properties. Exchange in markets 1 and 2 is efficient, and there is deflation at the rate of time preference. This is a standard Friedman-rule equilibrium.

However, a constrained liquidity trap equilibrium has very different properties. If (45) holds, then from (39),

$$c_t^1 = c_t^2 = y_t = V_t + \kappa_t \tag{54}$$

so consumption and output are determined by the value of the consolidated government debt plus the credit limit. This shows, in the most obvious way, the non-Ricardian nature of the constrained equilibrium. In a liquidity trap, increases in the quantity of government debt (in real terms) are not neutral, and will increase output and consumption one-for-one. As well, from (33), the inflation rate in a liquidity trap, if the equilibrium is constrained, is given by

$$\pi_t = \frac{\beta u'(V_t + \kappa_t)}{\gamma}, \tag{55}$$

so the inflation rate increases with the inefficiency wedge in goods markets, which determines the liquidity premium on all assets – money and bonds. Basically, a lower quantity of government debt plus credit limit implies a larger inefficiency wedge in goods markets, a larger liquidity premium on assets used in exchange, and a larger inflation rate. There need not be deflation in a liquidity trap, in contrast to the standard Friedman-rule unconstrained equilibrium.

This is a key property of the model, and in principle it can help us explain, for example, why a long-period of near-zero short-term nominal interest rates, as

in the United States since late 2008, need not produce deflation. But could the mechanism at work in this model – a large liquidity premium on government debt – account quantitatively for inflation rates as high as 3% (as in the United States after the Great Recession) when the short-term nominal interest rate is near zero? Exploring this issue properly would be the subject of another paper, but the evidence of Krishnamurthy and Vissing-Jorgensen (2013) provides support for our case. These authors find empirical evidence of a significant liquidity effect on the prices of short-maturity and long-maturity Treasury debt, measured from Treasury debt yields and the ratio of Treasury debt to GDP.

To better understand the role of fiscal policy in determining the inflation rate in a constrained liquidity trap equilibrium, consider the special case where $\kappa_t = 0$ for all t, $V_t = V$, and $q_t = 1$, where V is a nonnegative constant. Then, from the government budget constraint, (25),

$$\tau = V\left(1 - \frac{1}{\pi}\right), \tag{56}$$

and (55) gives

$$\pi = \frac{\beta u'(V)}{\gamma}. \tag{57}$$

Given our specification of policy, the real value of the consolidated government debt V is exogenous, and (56) and (57) determine the gross inflation rate π and the real transfer (i.e. the real government deficit) in periods $t = 1, 2, ...$, denoted τ. Thus, given V, π is uniquely determined by (57). Suppose that $u(c) = \frac{c^{1-\delta}}{1-\delta}$, where $\delta > 0$ is the coefficient of relative risk aversion. Then from (56) and (57), we can obtain a closed form solution for the government deficit

$$\tau = V - \frac{\gamma}{\beta}V^{\delta+1}, \tag{58}$$

for $V \in [0, V^*)$, where

$$V^* = \gamma^{-\frac{1}{\delta}}$$

is the value for the consolidated government debt above which the constrained equilibrium does not exist. In Figure 4, we illustrate the determination of τ, as in equation (58). Note that the function on the right-hand side of (58), as plotted in Figure 4, is nonmonotonic in V. To understand this, note that the right-hand side of (58) is the seigniorage revenue from the inflation tax on the consolidated government debt at the zero lower bound. The tax base is V, and the inflation tax rate is $1 - \frac{1}{\pi}$. An increase in V increases the tax base, but from (57) this reduces the inflation rate and reduces the inflation tax rate. Thus, essentially Figure 4 is a Laffer curve. An increase in V could result in a decrease or an increase in the government deficit, depending on what side of the Laffer curve we start on but, in the neighborhood of zero inflation, an increase in V is associated with a reduction in the government deficit.

[Figure 4 here.]

We could explore alternative fiscal policy rules. For example, in (56) and (57), suppose that the real government deficit τ is exogenous, with (56) and (57) determining π and V. Again assuming a utility function with constant relative risk aversion, and letting $\rho = \frac{1}{\pi}$ for convenience, use (56) and (57) to obtain

$$\tau = \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\delta}} \left(\rho^{\frac{1}{\delta}} - \rho^{\frac{1}{\delta}-1}\right). \tag{59}$$

In equation (59), the right-hand side is again the revenue from the inflation tax, but with this policy rule we can have two solutions. Figure 5 illustrates equation (59). Clearly, for any exogenous $\tau \geq 0$ that is feasible, there are two solutions for ρ, one with a high value for ρ and a low inflation rate, and one with a low value for ρ and a high inflation rate. If $\left(\frac{1}{\gamma}\right)^{\frac{1}{\delta}} (1 - \frac{1}{\beta}) < \tau < 0$, then there is deflation, and only one value for ρ that solves (59).

So, when we specify fiscal policy in this way, we get another type of conventional Laffer curve. If the fiscal authority wants to generate a given quantity of revenue from the inflation tax, then in general there are two ways to do that – with a high tax rate (a high inflation rate) or a low tax rate (a low inflation rate). Of course there is nothing new about this, as such Laffer curves exist in other monetary models, for example in pure-currency overlapping generations models (see Sargent 1987 for example). In a liquidity trap, we would be surprised if this model did not behave like a pure currency model, and exhibit a Laffer curve relationship under this type of fiscal policy rule.

[Figure 5 here.]

A question we might ask is what policy can do to raise the inflation rate at the zero lower bound. We have been treating q_t as our policy variable, so the question is whether, given $q_t = 1$, there is any policy that can raise the inflation rate. To be more concrete, in the circumstances we outlined above, in which exogenous variables are constant for all time and $q_t = 1$ for all t, what options are open to policymakers if an increase in the inflation rate is desired? From (32), we know that the central bank must assure that a given quantity of money – in real terms – is outstanding in order to support a zero nominal interest rate. But the central bank could go so far as to purchase the entire stock of government debt outstanding, and this would have no effect on the consumption, output, labor supply, or the inflation rate – that is the nature of a liquidity trap. However, fiscal policy is not powerless. From (57), the fiscal authority can set V to achieve any inflation rate $\pi - 1 \geq \beta - 1$ that it wants. Of course, we also know that a reduction in V reduces welfare at the zero lower bound, so higher inflation implies lower welfare.

4 Taylor Rules

Thus far, we have established the operating characteristics of this model economy, and have characterized optimal monetary policy, given a fiscal policy rule

which is in general suboptimal. In this section, we want to understand what will happen in this economy if a central banker adopts a standard type of policy rule – a Taylor rule. We know at the outset that the Taylor rule will be suboptimal here, in general, given fiscal policy, but we wish to understand what types of pitfalls would meet a Taylor rule central banker in this context. Real-world central bankers certainly take an interest in Taylor rules, and such rules seem to fit historical central banking behavior well, particularly in the United States. If this is what central banks do, we want to know exactly what is wrong with it. Is the Taylor rule a good approximation to optimal central bank behavior, or does it go radically wrong?

For simplicity we assume that the central banker cares only about inflation, and the Taylor rule takes the form

$$\frac{1}{q_t} = \max[\pi_t^\alpha \left(\pi^*\right)^{1-\alpha} x_t, 1] \tag{60}$$

Here, $\frac{1}{q_t}$ is the gross nominal interest rate, π^* is the central bank's target gross inflation rate, and x_t is the adjustment the central bank makes for the real rate of return on government debt. In general, $\alpha > 0$, and if $\alpha > 1$ the rule follows the "Taylor principle," whereby deviations of the inflation rate from its target are met with an aggressive nominal interest rate response by the central bank.

Some of the details in the Taylor rule – whether $\alpha < 1$ or $\alpha > 1$, the form that x_t takes, or whether it is π_t, or gross inflation at some other date that appears on the right-hand side of (60) – will matter for our results. In addition, we want to explore how the nature of the equilibrium – unconstrained or constrained – will make a difference. We already know from the work of Benhabib et al. (2001) that there can be "perils" that result from adherence to Taylor rules by central bankers, including multiple equilibria, in some of which the central bank does not achieve its inflation target in the limit. Here, we want to explore some of the pitfalls of Taylor rules in more depth.

4.1 Unconstrained Equilibrium

First, for the unconstrained equilibrium, we will examine the behavior of the model under four different versions of the Taylor rule. These four cases are: (i) constant adjustment for the long-run real interest rate; (ii) endogenous real interest rate; (iii) forward-looking rule; and (iv) backward-looking rule.

4.1.1 Constant Long-Run Real Interest Rate

Assume that (45) holds, so the equilibrium is unconstrained for all $q_t \leq 1$. From (23), if all exogenous variables are constant forever, then the gross real interest rate will be $\frac{1}{\beta}$. Therefore, in a standard fashion, if the real interest rate adjustment in the Taylor rule is set to match the long-run behavior of the model,

then $x_t = \frac{1}{\beta}$. Then from (34)-(36) and (60), we can solve for equilibrium q_t from

$$1 = \max\left[\left(\frac{q_t\pi^*}{\beta}\right)^{1-\alpha}, q_t\right]. \tag{61}$$

Note in particular that there are no dynamics associated with this Taylor rule, which is in part due to our assumption of quasilinear preferences. Assume $\pi^* \geq \beta$, so that the target inflation rate is larger than minus the rate of time preference. Then, if the Taylor rule follows the Taylor principle, so $\alpha > 1$, there are two equilibrium solutions to (61), as depicted in Figure 6. The two solutions are $q_t = \frac{\beta}{\pi^*}$, which implies that $\pi = \pi^*$ and the central bank achieves its target rate of inflation, and $q_t = 1$, which is the liquidity trap solution for which $\pi = \beta \leq \pi^*$. In the liquidity trap equilibrium the central banker sees a low inflation rate, and responds aggressively by setting the nominal interest rate to zero, which has the effect of producing a low inflation rate. This is a well-known property of monetary models (see Benhabib et al. 2001) – under the Taylor principle there are multiple steady states, including the liquidity-trap steady state. In this particular model, in an unconstrained equilibrium, the Taylor rule does not impart any dynamics to the economy (in contrast to Benhabib et al. 2001), but we will show in what follows how dynamic equilibria arise with other forms of the Taylor rule.

[Figure 6 here.]

If, however, $\alpha < 1$, then, as in Figure 7, there is a unique equilibrium with $q = \frac{\beta}{\pi^*}$, and the central banker always achieves his or her inflation target. Thus, given this form for the Taylor rule, in an unconstrained equilibrium the Taylor principle is not a good idea, as this implies an equilibrium in which the central banker will not achieve his or her inflation target. Note that, if $\pi^* = \beta$, then this maximizes welfare in the unconstrained equilibrium, and the equilibrium is unique for any $\alpha > 0$. In this case the Taylor rule gives us the Friedman rule solution as a unique equilibrium.

[Figure 7 here.]

4.1.2   Endogenous Real Interest Rate

In the Taylor rule the term x_t makes an adjustment for the real interest rate, typically in line with some view about the long-run Fisher relation. With an appropriately chosen x_t term, there is at least the possibility that the Taylor rule will lead to convergence to the central bank's inflation target in the long run. But, what if the central bank accounted explicitly for endogeneity in the real interest rate? In particular, suppose that the central bank chooses

$$x_t = \frac{1}{s_t^a}, \tag{62}$$

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where s_t^a is the price of a real bond, as determined in (23). In this case, the central bank recognizes that the real interest rate is endogenous, and sets the nominal interest rate in line with fluctuations in that rate. Suppose for convenience that we consider only deterministic dynamic equilibria. Substituting in (60) using (23), (28), (33), (36) and (62), we get

$$\pi_{t+1} = \max \left[\pi_t^\alpha \left(\pi^* \right)^{1-\alpha}, \beta \right],$$ (63)

which is a nonlinear first-order difference equation in the gross inflation rate π_t, which we can use to solve for an equilibrium. An unconstrained dynamic equilibrium satisfying this version of the Taylor rule is a sequence $\{c_t^1, q_t, c_t^2, \pi_t\}_{t=0}^\infty$ satisfying (33), (36), (63), and $c_t^2 = c^*$ for all t.

First, suppose that $\alpha > 1$. Then there are two steady states, just as for the Taylor rule with $x_t = \beta^{-1}$, and these are the same steady states as for the simpler Taylor rule – a high-inflation steady state in which the central bank achieves its inflation target, and the liquidity trap equilibrium in which the central bank falls short of its inflation target.

In contrast to the case with $x_t = \beta^{-1}$ though, there are nonstationary equilibria. With $\alpha > 1$, there exists a continuum of nonstationary equilibria, indexed by the initial inflation rate, that converge to the liquidity trap steady state in finite time. In each of these equilibria, $\beta < \pi_0 < \pi^*$, and

$$\pi_{t+1} = \max \left\{ \pi_t^\alpha \left(\pi^* \right)^{1-\alpha}, \beta \right\},$$ (64)

for $t = 1, 2, ...$, with

$$q_t = \frac{\beta}{\pi_t}.$$ (65)

In Figure 8, B is the high-inflation steady state, A is the liquidity trap steady state, and we have depicted one of the nonstationary equilibria, for which the initial gross inflation rate is π_0, and there is convergence to the liquidity trap steady state in period 4.

[Figure 8 here.]

Second, if $\alpha < 1$, then there is a unique steady state with $\pi_t = \pi^*$ and $q_t = \frac{\beta}{\pi^*}$. As well, there exists a continuum of nonstationary equilibria that converge in the limit to the steady state equilibrium. For each of these equilibria, $\beta \leq \pi_0 < \infty$,

$$\pi_{t+1} = \pi_t^\alpha \left(\pi^* \right)^{1-\alpha}$$

for $t = 1, 2, ...$, c_t^1 solves (64), and q_t is given by (65). In Figure 9, we show the case $\alpha < 1$, where A is the steady state, and we show one of the nonstationary equilibria, for which the initial gross inflation rate is π_0, and there is convergence in the limit to the steady state.

[Figure 9 here.]

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4.1.3 Forward Looking Taylor Rule

Alternatively, suppose that we specify the Taylor rule in a forward-looking manner, as

$$\frac{1}{q_t} = \max[\pi_{t+1}^{\alpha} \left(\pi^*\right)^{1-\alpha} x_t, 1], \tag{66}$$

so now the central bank targets the current nominal interest rate as a function of the inflation rate in the next period, again restricting attention to deterministic cases. Suppose also that $x_t = \frac{1}{\beta}$. Then, from (66) and (36), we can construct a difference equation that solves for the equilibrium sequence, $\{\pi_t\}_{t=0}^{\infty}$, i.e.

$$\pi_{t+1} = \max[\pi_t^{\frac{1}{\alpha}} \left(\pi^*\right)^{1-\frac{1}{\alpha}}, \beta], \tag{67}$$

with $\pi_0 \geq \beta$. Then, from (67), the liquidity trap steady state, with $\pi_t = \beta$ for all t, exists if and only if

$$\left(\pi^*\right)^{1-\frac{1}{\alpha}} \leq \beta^{1-\frac{1}{\alpha}}. \tag{68}$$

Therefore, since $\pi^* \geq \beta$, (68) holds if $\alpha < 1$, and does not hold if $\alpha > 1$. Thus, in contrast to the Taylor rule given by (60), the forward-looking Taylor rule (66) implies that, under the Taylor principle, there is a unique steady state with $\pi = \pi^*$, but if $\alpha < 1$ there are two steady states: the liquidity trap with $q_t = 1$ and $\pi_t = \beta$ for all t, and the high-inflation steady state with $\pi = \pi^*$ and $q_t = \frac{\beta}{\pi^*}$.

Finally, exploring the dynamics of the forward-looking Taylor rule (66), if $\alpha < 1$ then there exists a continuum of dynamic equilibria with $\pi_0 \in [\beta, \pi^*)$, each of which converges in finite time to the liquidity trap steady state. However, if $\alpha > 1$, then there exists a continuum of dynamic equilibria with $\pi_0 \geq \beta$, each of which converges to the high-inflation equilibrium for which the central bank achieves its inflation target. Therefore, the forward-looking Taylor rule (66) has good properties under the Taylor principle.

4.1.4 Backward-Looking Taylor Rule

The last Taylor rule we consider is a backward-looking rule of the form

$$\frac{1}{q_t} = \max \left[\pi_{t-1}^{\alpha} \left(\pi^*\right)^{1-\alpha} \frac{1}{\beta}, 1\right], \tag{69}$$

which implies, from (36), the difference equation

$$\pi_{t+1} = \max \left[\pi_t^{\alpha} \left(\pi^*\right)^{1-\alpha}, \beta\right].$$

This is then identical to (64), so this rule implies equilibria identical to the Taylor rule with an endogenous real interest rate.

Given the four alternative Taylor rules we considered here, the "Taylor principle" (the case $\alpha > 1$) sometimes has bad properties, and sometimes not, in this standard unconstrained case. The Taylor principle, which implies an aggressive

reaction of the central bank to deviations of the inflation rate from its target, can yield a liquidity trap steady state in which the central banker falls short of his or her inflation target. If this steady state exists, it is stable, in the sense that there exists a continuum of nonstationary equilibria that converge to the liquidity trap steady state in finite time. When the Taylor rule is badly behaved under the Taylor principle, the results have the flavor of those in Benhabib et al. (2001).

4.2 Constrained Equilibrium

The results in the previous confirm the spirit of previously known results, but our main interest is in what happens in the constrained equilibrium case. Here, suppose that (45) holds, so that the equilibrium will be constrained for sufficiently large q_t. Just as for the unconstrained case, we will examine the four alternative Taylor rules and their implications for the equilibrium allocation.

4.2.1 Constant Long Run Real Interest Rate

First, let $x_t = x$, a constant, so that there will be a period-by-period equilibrium solution (c_1, c_2, q, π) solving, from (27), (29), (33), and (60),

$$\frac{1}{q} = \max \left[\pi^\alpha \left(\pi^*\right)^{1-\alpha} x, 1 \right],$$ (70)

$$u'(c_2) - q u'(c_1) = 0,$$ (71)

$$V + q\kappa = \theta c_1 + (1 - \theta) q c_2,$$ (72)

$$\pi = \frac{\beta u'(c_1)}{\gamma}$$ (73)

Then, from (54), a liquidity trap equilibrium, with $q = 1$, has $c_1 = c_2 = V + \kappa$. Therefore, from (70) and (73) this is an equilibrium if and only if

$$\left[\frac{\beta u'(V + \kappa)}{\gamma} \right]^\alpha (\pi^*)^{1-\alpha} x \leq 1$$ (74)

Note that (74) does not hold if $V + \kappa$ is sufficiently small, i.e. if government debt is sufficiently scarce and the credit limit is sufficiently low. From (73), smaller $V + \kappa$ lowers consumption at the zero lower bound, and increases the inflation rate. Thus, if $V + \kappa$ is small, so that π is large at the zero lower bound, then at the zero lower bound the Taylor-rule central banker wants to raise the nominal interest rate, so the zero lower bound is not an equilibrium. This is one way in which our results differ from the unconstrained case, in which there is an equilibrium at the zero lower bound with an aggressive Taylor rule ($\alpha > 1$). When government debt is scarce and the credit limit is low, there is a liquidity premium on government debt, and the inflation rate can be high enough at the zero lower bound that, even if the central banker reacts very aggressively to inflation, the zero lower bound is not an equilibrium.

To characterize equilibria, it will be useful again to restrict attention to the utility function $u(c) = \frac{c^{1-\delta}}{1-\delta}$, where $\delta > 0$ is the coefficient of relative risk aversion. Then, from (70)-(73), we get

$$\frac{1}{q} = \max\left[\left(\frac{\beta}{\gamma}\right)^\alpha \left[\frac{\theta + (1-\theta)q^{1-\frac{1}{\delta}}}{V + q\kappa}\right]^{\delta\alpha} (\pi^*)^{1-\alpha} x, 1\right], \tag{75}$$

which solves for equilibrium q. Previously, we determined, from (40), that $\delta < 1$ is sufficient to guarantee that a reduction in q (an increase in the nominal interest rate) implies a decrease in c_1, and an increase in the inflation rate, supported by an open market sale of government bonds by the central bank. But, if $\delta < 1$, then the first argument under the max operator on the right-hand side of (75) is strictly decreasing in q. Since the left-hand side of (75) is also strictly decreasing in q, this opens up the possibility of multiplicity of equilibria satisfying $q > 1$. A convenient example that illustrates this is $u(c) = \log c$. Note that, from (40), the log case implies that, if $\kappa > 0$, then c_1 will decline when q decreases (just as for any $\delta < 1$), which is a key property that makes the log example "standard." From equations (70)-(73) it is convenient to characterize the equilibrium solution in terms of an equation that solves for the gross inflation rate π,

$$\frac{\pi\gamma\kappa}{\beta - V\pi\gamma} = \max\left[\pi^\alpha (\pi^*)^{1-\alpha} x, 1\right] \tag{76}$$

First, for this example, a liquidity trap equilibrium exists if and only if, from (74),

$$\left[\frac{\beta}{\gamma(V + \kappa)}\right]^\alpha (\pi^*)^{1-\alpha} x \leq 1 \tag{77}$$

Next, if there is an equilibrium in which the central banker achieves his or her inflation target away from the zero lower bound, then from (76), existence of such an equilibrium requires

$$x = \frac{\gamma\kappa}{\beta - \pi^*\gamma V}, \tag{78}$$

if we restrict attention to Taylor rules for which x is a function only of exogenous variables. As well, it must be the case, from (70) that $\pi^* x \geq 1$, or from (78),

$$\pi^* \geq \frac{\beta}{\gamma(V + \kappa)}. \tag{79}$$

Condition (79) states that, if there is an equilibrium in which the central bank achieves its inflation target π^*, then the inflation target must be greater than the inflation rate when the nominal interest rate is zero. Then, (78) and (79) are necessary and sufficient for such an equilibrium to exist.

Therefore, for the central bank to achieve its inflation target requires that x be set correctly in the Taylor rule (70). Further, x is a function of the exogenous variables κ and V, i.e. the credit limit and the real quantity of consolidated

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government debt. Also, x depends on the inflation target itself. Thus, in the constrained equilibrium, in which the real interest rate is endogenous, one cannot assure that the Taylor rule will achieve the inflation target (in the short run or the long run) as in the unconstrained case in this model.

However, even if the central banker sets x appropriately, so that there exists an equilibrium in which the inflation target is achieved, there may exist other equilibria. For example, if (78) holds, then from (76), an equilibrium away from the zero lower bound satisfies

$$\frac{\beta - \pi^* \gamma V}{\beta - \pi \gamma V} = \left(\frac{\pi}{\pi^*} \right)^{\alpha - 1} \tag{80}$$

Suppose that $\alpha = 2$ and $\beta - \pi^* \gamma V > 0$. Then, there can exist two equilibria away from the zero lower bound, and the equilibrium with the higher rate of inflation has the property that $\pi = \pi^*$.

For arbitrary x, we can construct examples for which there are three equilibria. For example, if $\alpha = 2$, then we can write (76) as

$$\frac{\gamma \kappa}{\beta - V \pi \gamma} = \max \left[\pi \left(\pi^* \right)^{-1} x, \frac{1}{\pi} \right], \tag{81}$$

and then it is clear from Figure 10 that there can be three equilibria, denoted by A, B, and C in the figure: one at the zero lower bound, and two other equilibria for which the nominal interest rate is strictly positive.

[Figure 10 here.]

However, note in this example that, if $\alpha < 1$, then the equilibrium must be unique, and we could either have a liquidity trap equilibrium at the zero lower bound, or one with a positive nominal interest rate. Further, if (78) and (79) hold, then the unique equilibrium has the property that the central banker achieves his or her inflation target.

The log utility example illustrates two problems with standard Taylor rules. The first is particular to the constrained equilibrium. A scarcity of government debt creates the problem that the real interest rate is endogenous, so simple Taylor rules will not achieve the central banker's inflation target unless he or she accounts appropriately for this real interest rate endogeneity. As well, multiplicity of equilibria can potentially be a worse problem in the constrained equilibrium case. In circumstances in which there could be two equilibria in the unconstrained case, we constructed an example in which there could be three. Finally, as in the unconstrained case, the Taylor principle may not help matters. In the example, there is a unique equilibrium when the Taylor principle does not hold ($\alpha < 1$), but if $\alpha > 1$, there could be multiple equilibria.

4.2.2 Endogenous Real Interest Rate

Next, suppose that (45) holds, and the central bank follows a Taylor rule that allows for the endogeneity in the real interest rate. Then, from (23), (27), and

(33),

$$x_t = \frac{1}{q_t \pi_{t+1}}. \tag{82}$$

Therefore, from (61), (82), (27), and (39), assuming that $V_t = V$ and $\kappa_t = \kappa$ for all t, we can express the Taylor rule as

$$\pi_{t+1} = \max \left[\pi_t^\alpha \left(\pi^* \right)^{1-\alpha}, \frac{\beta u'(V + \kappa)}{\gamma} \right]. \tag{83}$$

Which is identical to (63) for the unconstrained case, except that the second argument in the max operator on the right-hand side of (63) is a different constant than in (83), because the inflation rate at the zero lower bound is different in the unconstrained and constrained cases. The results here are identical to the constrained case, except that, if

$$\pi^* < \frac{\beta u'(V + \kappa)}{\gamma},$$

then the only equilibrium is the liquidity trap equilibrium, and the central bank will perpetually exceed its inflation target, as

$$\pi_t = \frac{\beta u'(V + \kappa)}{\gamma} > \pi^*,$$

in the liquidity trap equilibrium. However, if

$$\pi^* \geq \frac{\beta u'(V + \kappa)}{\gamma}, \tag{84}$$

then the results will be qualitatively the same as in the unconstrained case in which the Taylor rule adjusts for endogeneity in the real interest rate. There exist two steady state constrained equilibria, a high-inflation equilibrium in which the central bank achieves its inflation target, and a liquidity trap steady state in which the central bank perpetually undershoots its inflation target.

Also, as in the unconstrained case, there exists a continuum of nonstationary equilibria, which either converge to the liquidity trap steady state (if $\alpha > 1$) or the high-inflation steady state (if $\alpha < 1$).

This Taylor rule solves the problem which occurs due to endogeneity of the real interest rate in a constrained equilibrium, and it does no worse than the same Taylor rule in the unconstrained case. But the same problems that occurred in the unconstrained equilibrium are present here. In particular, the Taylor principle leads to multiple steady states, and the liquidity trap steady state is stable. As well, there are multiple dynamic equilibria, even in the absence of the Taylor principle.

4.2.3 Forward Looking Taylor Rule

Next, consider the forward-looking Taylor rule, as specified by (66). Then, similar to (76), in the case where $u(c) = \log c$, $V_t = V$ for all t, and $\kappa_t = \kappa$ for

all t, we obtain

$$\frac{\pi_t \gamma \kappa}{\beta - V\pi_t \gamma} = \max\left[\pi_{t+1}^\alpha \left(\pi^*\right)^{1-\alpha} x, 1\right]. \tag{85}$$

Then, $\{\pi_t\}_{t=0}^\infty$ is a solution to the difference equation

$$\pi_{t+1} = \max\left[\left(\frac{\pi_t \gamma \kappa}{\beta - V\pi_t \gamma}\right)^{\frac{1}{\alpha}} \left(\pi^*\right)^{1-\frac{1}{\alpha}} x^{-\frac{1}{\alpha}}, \frac{\beta}{\gamma\left(\kappa + V\right)}\right] \tag{86}$$

with $\pi_0 \geq \frac{\beta}{\gamma(\kappa+V)}$.

Suppose that we consider the case $\alpha = 1$. If

$$\beta - \gamma\left(\kappa + V\right)x^{-1} \geq 0, \tag{87}$$

then there exist two steady state equilibria: the liquidity trap equilibrium, and a high-inflation equilibrium. In the high-inflation equilibrium, the central bank achieves its inflation target if and only if

$$x = \frac{\gamma\kappa}{\beta - \pi^* V\gamma},$$

and

$$\pi^* \geq \frac{\beta}{\gamma\left(\kappa + V\right)}.$$

Thus, due to real interest rate endogeneity in the constrained case, the adjustment x for the real rate must depend on exogenous variables, including the inflation target, in the example. If (87) does not hold, then there is only one steady state equilibrium, which is the liquidity trap equilibrium. As well, if (87) holds, then there exists a continuum of dynamic equilibria with $\pi_0 \in \left[\frac{\beta}{\gamma(V+\kappa)}, \frac{\beta - \gamma\kappa x^{-1}}{V\gamma}\right)$, and each these equilibria converges in finite time to the liquidity trap equilibrium.

An interesting feature of the example is that, in contrast to some of the other cases we have examined involving constrained and unconstrained equilibria, the dynamic properties of the model do not differ according to whether $\alpha < 1$ or $\alpha > 1$. In the example, by continuity the qualitative characteristics of the dynamics are the same in the neighborhood of $\alpha = 1$.

4.2.4 Backward looking Taylor Rule

Now, consider the backward looking rule

$$\frac{1}{q_t} = \max\left[\pi_{t-1}^\alpha \left(\pi^*\right)^{1-\alpha} x, 1\right], \tag{88}$$

As in the previous case, consider the example $u(c) = \log c$, $V_t = V$ for all t, and $\kappa_t = \kappa$ for all t. Then, similar to (85), we obtain

$$\frac{\pi_t \gamma \kappa}{\beta - V\pi_t \gamma} = \max\left[\pi_{t-1}^\alpha \left(\pi^*\right)^{1-\alpha} x, 1\right]. \tag{89}$$

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Then, as in the previous case, restrict attention to $\alpha = 1$, so (89) gives a difference equation

$$\pi_{t+1} = \max\left[\frac{\pi_t x\beta}{\gamma\kappa + \pi_t xV\gamma}, \frac{\beta}{\gamma(V+\kappa)}\right]. \tag{90}$$

There exists a steady state in which the central bank achieves its inflation target if and only if

$$x = \frac{\gamma\kappa}{\beta - \pi^*V\gamma} \tag{91}$$

and

$$\frac{\beta}{\gamma V} > \pi^* \geq \frac{\beta}{\gamma(V+\kappa)}. \tag{92}$$

Then, (90)-(92) imply that the liquidity trap steady state does not exist. From (90), if (91) and (92) hold, then there exists a continuum of dynamic equilibria with $\pi_0 \in \left[\frac{\beta}{\gamma(V+\kappa)}, \pi^*\right)$, each of which converges to the steady state with $\pi_t = \pi^*$ for all t.

Therefore, this example turns the previous one on its head. Again, the properties of the model will be qualitatively identical for α in the neighborhood of unity. In contrast to the case with the previous Taylor rule, the backward looking Taylor rule in this example has good properties.

4.2.5 Discussion: Taylor Rules in the Constrained Case

A key difficulty that adherence to a Taylor rule presents in the constrained case is that the real interest rate is endogenous, even in the long run. Part of the logic behind the Taylor rule – as written down by Taylor himself – is that monetary policy cannot affect the real interest rate in the long run, and the steady state real interest rate is a constant, independent of exogenous variables. But that is not the case in a constrained equilibrium, in which monetary policy matters for the long run real rate, and the long run real rate depends on exogenous variables, including the stock of government debt outstanding.

There are solutions to this constrained-equilibrium Taylor rule problem, which involve modifying the rule to account for real interest rate endogeneity, as we showed. But any of these solutions involve making the rule more complicated. Either additional variables need to go into the rule, or the central banker needs to include in the rule a measure of the real interest rate. Ultimately, though, even if such steps are taken, adherence to the Taylor rule can lead to a policy trap. The Taylor-rule central banker can be drawn to – and become stuck indefinitely in – a liquidity trap equilibrium. Alternatively, there may exist steady state equilibria for which the nominal interest rate is positive, but the central bank does not achieve its inflation target.

5   Conclusion

During and after the Great Recession, the behavior of asset prices, inflation, and the role of central banks in that behavior, appears puzzling, both in the United States and elsewhere. Short-term real interest rates have been persistently low, and deflation is not yet in evidence in the United States, in spite of the fact that the Fed has kept short-term interest rates at or close to zero for more than six years. In the 1970s and 1980s, a common view was that central banks have a tendency to create excessive inflation, and that commitment to policy rules will solve that problem. But, in the United States, Europe, the U.K., Switzerland, and Sweden, for example, the inflation rate has fallen below central bank inflation targets. These central banks seem well aware of policy rules and what they are about, but they seem powerless to create more inflation. The post-Great Recession central banking problem seems to be a lack of inflation rather than too much of it.

In this paper, we have attempted to come to grips with these puzzles, in part by recognizing the special role that government debt plays in financial markets. In the model, government debt literally serves a transactions role, but this can be taken to stand in for a broader role for government debt as collateral in financial markets. Government debt is then a substitute for private credit and private assets that serve as collateral. A financial shock that impairs private credit and effectively destroys private collateral can then create a shortage of government debt, in the sense that the liquidity premium on government debt increases, and the short-term real rate of interest falls. The existence of a large liquidity premium can imply that, even if the short-term nominal interest rate is at the zero lower bound, the inflation rate can be positive, just as we have been observing in the United States.

Further, if asset market constraints bind in the model, which makes the real rate of interest low, then a zero interest rate policy (ZIRP) is not optimal. An open market purchase of government bonds by the central bank implies a higher quantity of outside money outstanding relative to government debt. But if government debt is scarce, an open market purchase that lowers the nominal interest rate will lower the amount of government debt available to support aggregate economic activity, and this can be bad for economic performance, on net.

A lesson that central bankers appear to have absorbed from the 1970s is that commitment to policy rules is important. John Taylor's work (see Taylor 1993), and interest in the Taylor rule by New Keynesians (Woodford 2003), focused central bankers on this particular rule as a way to structure how they think about monetary policy. It has been known for some time (see Benhabib et al. (2001)) that the Taylor rule has poor properties in standard monetary models. In particular, under Taylor rules that prescribe an aggressive response of the target nominal interest rate to a change in the inflation rate, there are multiple equilibria that converge to an undesired ZIRP under which the central bank persistently undershoots its inflation target.

Our model is consistent with the results of Benhabib et al. (2001) when

asset market constraints do not bind, but if asset market constraints bind this further adds to the problem. With scarce government debt, the central bank must modify the Taylor rule in potentially complicated ways, adjusting for endogeneity in the short-term real interest rate so as to achieve its inflation target. And the liquidity premium on government debt can create added multiplicities. There may not only be multiple equilibria that yield convergence to ZIRP, but steady state equilibria with a positive nominal interest rate for which the central bank does not achieve its inflation target.

A central banker following a Taylor rule, when confronted with a financial crisis, would be inclined – as, for example, the Fed was – to resort to ZIRP. Our model tells us that, if the financial shock is severe enough, and if government debt is in short enough supply, then the central bank could continue to see an inflation rate at or above its target, even given a prolonged period of ZIRP. Thus, the central bank could become complacent about its inflation target. But, as the private economy repairs itself and asset market constraints are relaxed, the inflation rate will tend to fall if ZIRP continues. Then, if the central banker is following a Taylor rule that dictates an aggressive response to inflation, that will serve to maintain ZIRP indefinitely, unless the policy rule changes. The central banker continues to see low inflation, and the logic behind his or her Taylor rule says that ZIRP will eventually produce more inflation. But that never happens.

Thus ZIRP is a policy trap – one which central banks in the world may now be falling into. One could be hopeful that the nature of the trap would become obvious. But Japan's experience – short-term nominal interest rates close to zero since 1995, with an inflation rate of roughly zero over that period – should give one pause.

6   References

Andolfatto, D., Martin, F. and Zhang, S. 2014. "Rehypothecation," working paper, St. Louis Federal Reserve Bank.

Benhabib, J., Schmitt-Grohe, S., and Uribe, M. 2001. "The perils of Taylor rules," *Journal of Economic Theory* 96, 40-69.

Bansal, R. and Coleman, W.J. 1996. "A monetary explanation of the equity premium, term premium, and risk-free rate puzzles," *Journal of Political Economy* 104, 1135-1171.

Christiano, L., Eichenbaum, M., and Trabandt, M. 2014. "Understanding the great recession," working paper, Northwestern University.

Eggertsson, G. and Krugman, P. 2012. "Debt, deleveraging, and the liquidity trap: a Fisher-Minsky-Koo approach," *The Quarterly Journal of Economics* 127, 1469-1513.

Ennis, H. 2014. "A simple general equilibrium model of large excess reserves," working paper, Richmond Federal Reserve Bank.

Friedman, M. 1969. *The optimum quantity of money and other essays*, Aldine Publishing Company, New York.

Gorton, G. 2010. *Slapped by the invisible hand: The panic of 2007*, Oxford University Press, Oxford, U.K.

Kehoe, T. and Levine, D. 1993. "Debt-constrained asset markets," *Review of Economics Studies* 60, 865-888.

Krishnamurthy, A. and Vissing-Jorgensen, A. 2013. "Short-term debt and financial crises: What we can learn from U.S. Treasury supply?" working paper, Northwestern University.

Lagos, R. and Wright, R. 2005. "A unified framework for monetary theory and policy analysis." *Journal of Political Economy* 113, 463–484.

Leeper, E. 1991. "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics* 27, 129-147.

Lucas, R. and Stokey, N. 1987. "Money and interest in a cash-in-advance economy," *Econometrica* 55, 491-513.

Sanches, D., and Williamson, S. 2010. "Money and credit with limited commitment and theft," *Journal of Economic Theory* 145, 1525-1549.

Sargent, T. 1987. *Dynamic macroeconomic theory,* Harvard University Press, Cambridge, MA.

Shi, S. 1997. "A divisible search model of fiat money," *Econometrica* 65, 75-102.

Taylor, J. 1993. "Discretion versus policy rules in practice," *Carnegie-Rochester Conference Series on Public Policy* 39, 195-214, December.

Werning, I. 2011. "Managing a liquidity trap: monetary and fiscal policy," NBER Working Papers 17344, National Bureau of Economic Research, Inc.

Williamson, S. 2012. "Liquidity, monetary policy, and the financial crisis: A new monetarist approach," *American Economic Review* 102, 2570-2605.

Williamson, S. 2014a. "Scarce collateral, the term premium, and quantitative easing," working paper, St. Louis Federal Reserve Bank.

Williamson, S. 2014b. "Central bank purchases of private assets," working paper, St. Louis Federal Reserve Bank.

Woodford, M. 1995. "Price-level determinacy without control of a monetary aggregate," *Carnegie-Rochester Conference Series on Public Policy* 43, 1-46.

Woodford, M. 2003. *Interest and prices*, Princeton University Press, Princeton, NJ.

Woodford, M. 2012. "Methods of policy accommodation at the interest-tate lower bound," *Proceedings - Economic Policy Symposium - Jackson Hole*, Federal Reserve Bank of Kansas City, 185-288.

Figure 1: Safe Assets: World

$ Billions
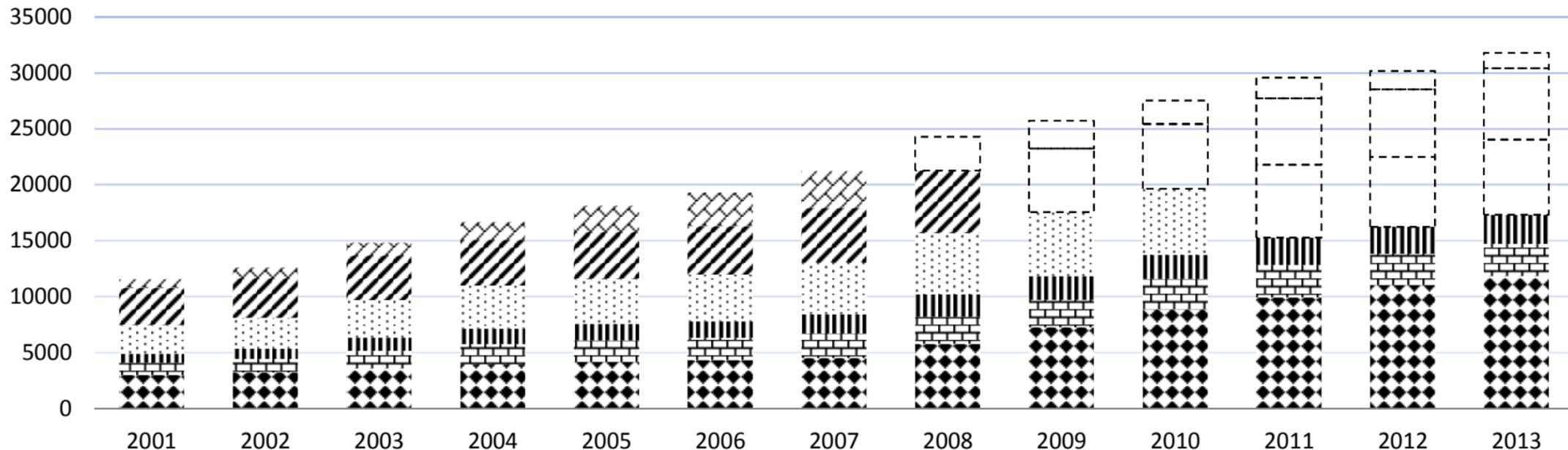
Source: FRB/Eurostat/ECB/Haver

Legend:
- US Treasuries
- German Bunds
- French Bonds
- Euro Sovereigns (ex Germany and France)
- US Agency Debt
- US Private ABS
- No Longer Safe Euro
- No Longer Safe Agency
- No Longer Safe ABS

Figure 2: Equilibrium Allocations
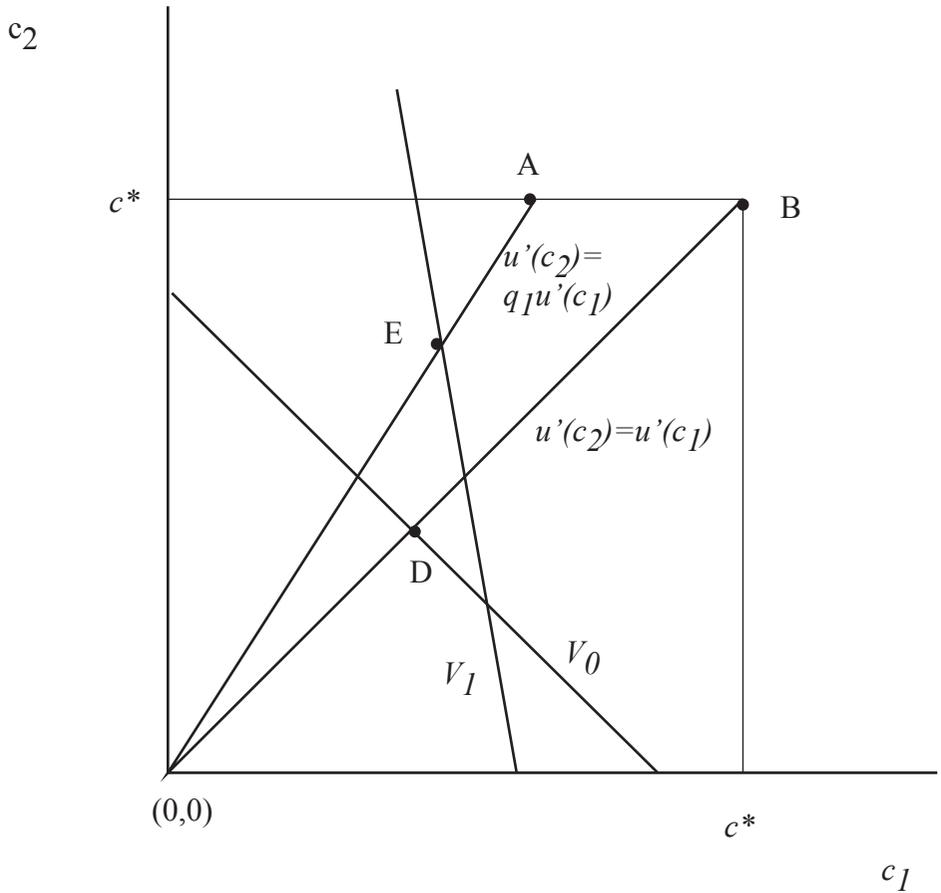
Figure 3: A Decrease in V or κ, Constrained Equilibrium
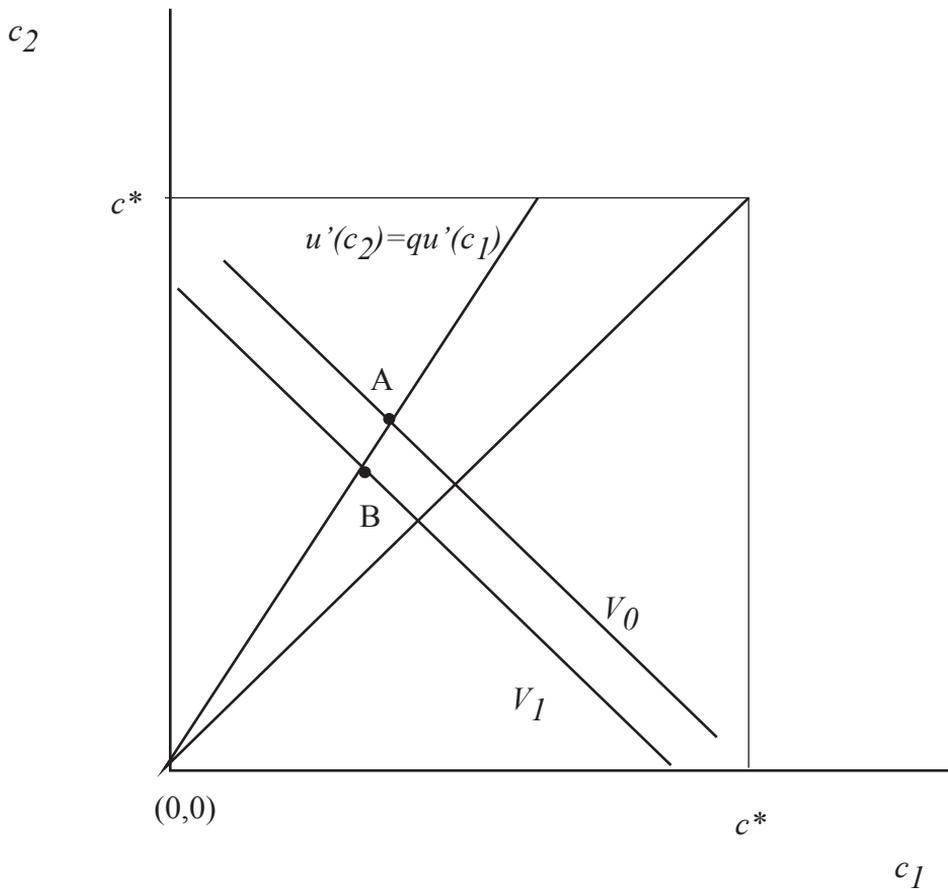
Figure 4: Example: Government Deficit as a Function of Outstanding Consolidated Government Debt at the Zero Lower Bound
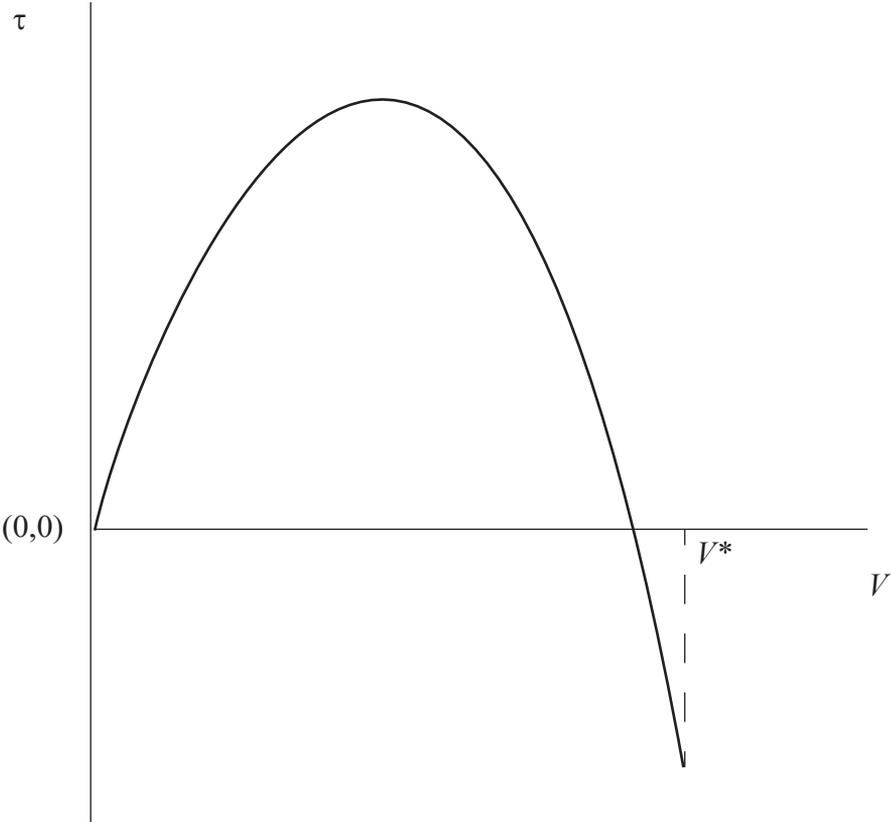
Figure 5: Example: Government Deficit as a Function of the Inverse of the Gross Inflation Rate at the Zero Lower Bound
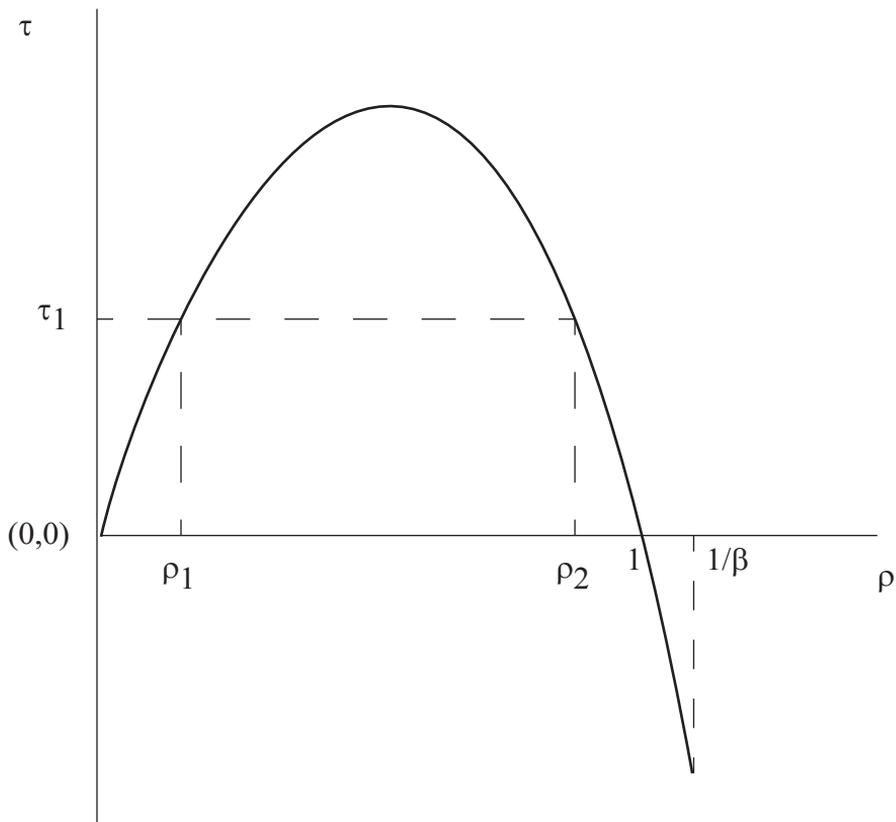
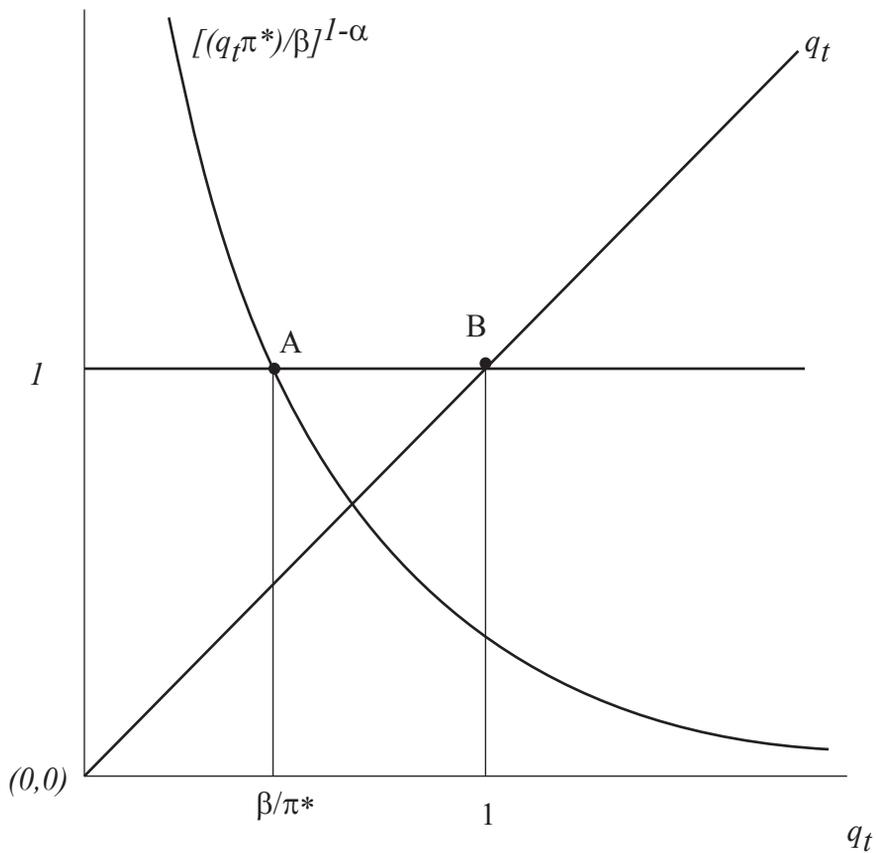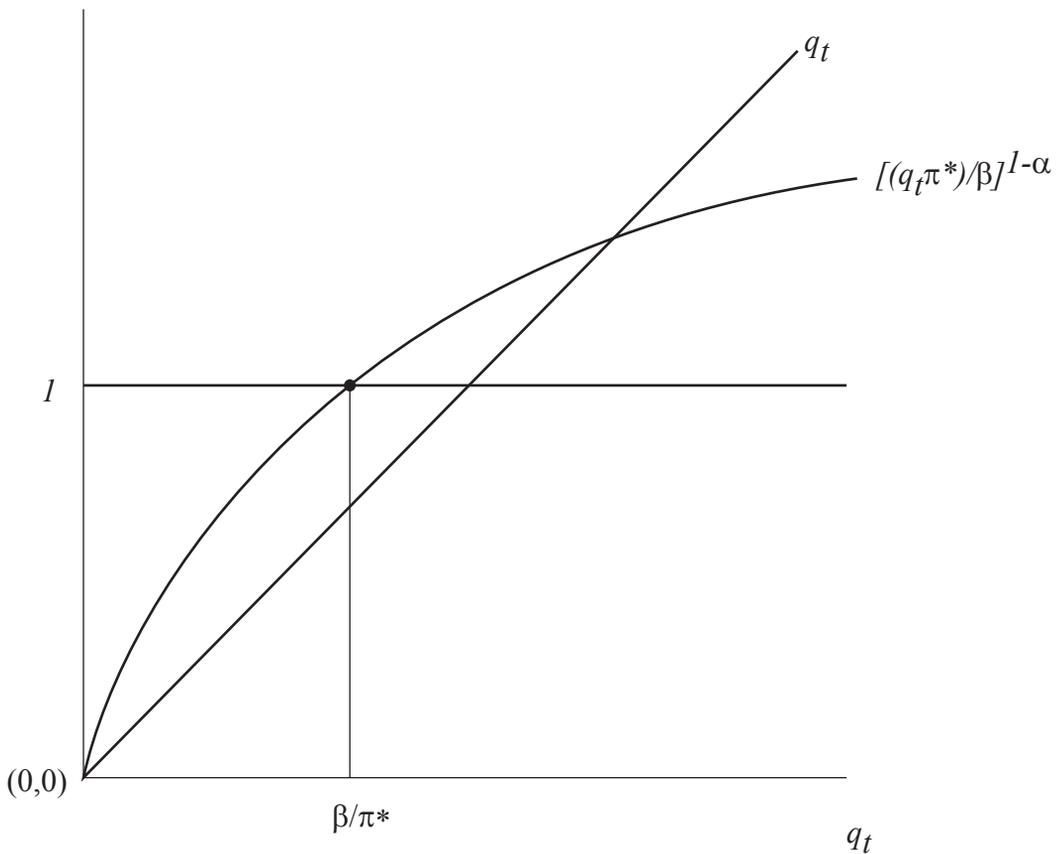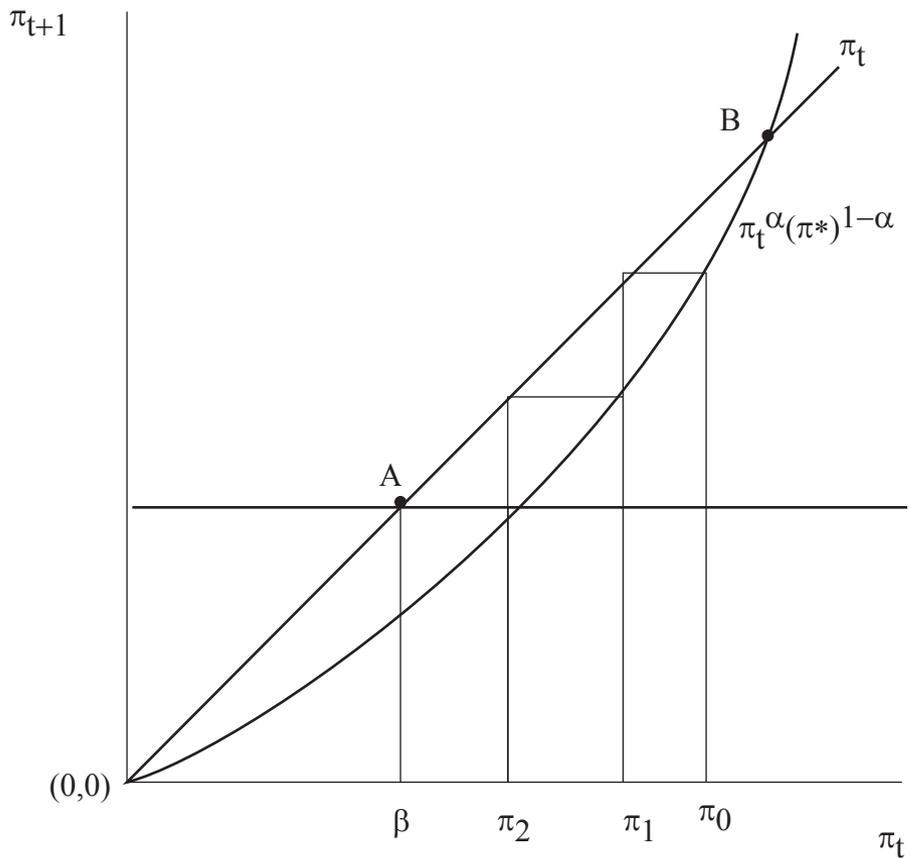**Figure 6: Taylor Rule Equilibrium, Unconstrained,** $\alpha > 1$

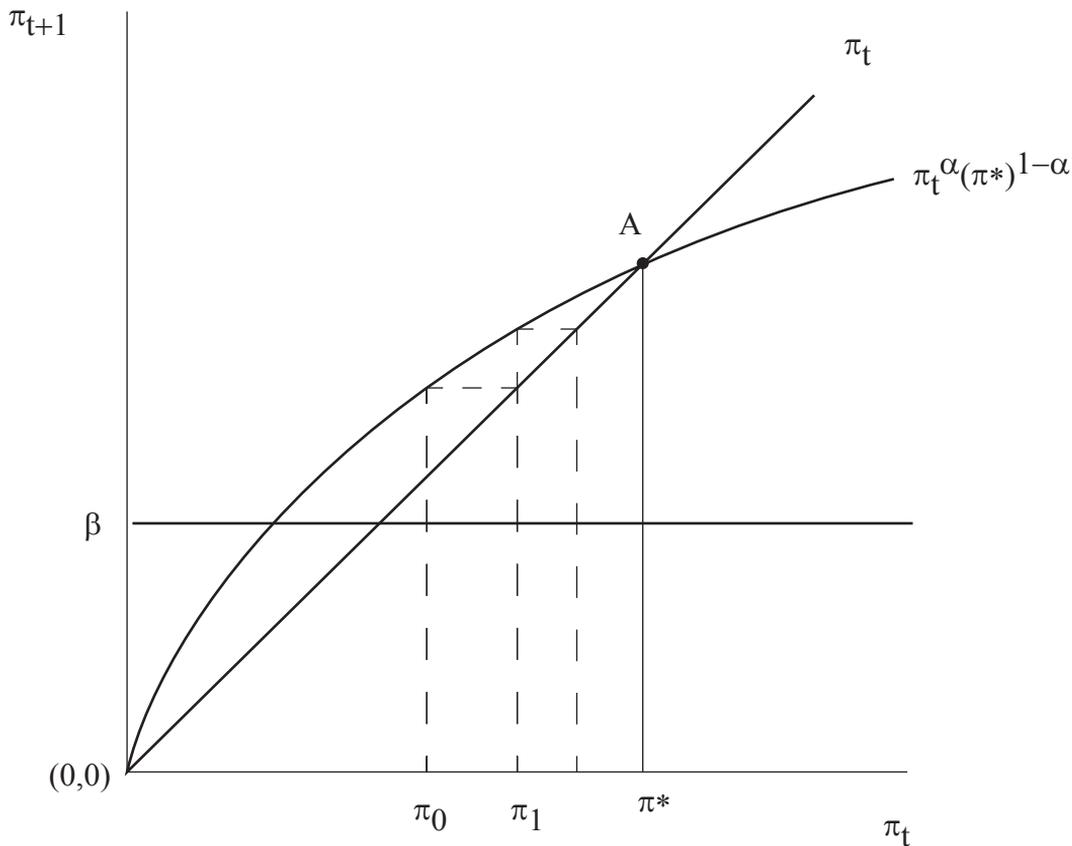**Figure 7: Taylor Rule Equilibrium, Unconstrained,** $\alpha < 1$

Figure 8: Taylor Rule Equilibrium, Unconstrained, $\alpha > 1$, Endogenous Real Interest Rate

Figure 9: Taylor Rule Equilibrium, Unconstrained, $\alpha < 1$, Endogenous Real Interest Rate

Figure 10: Example: Taylor Rule Equilibrium, Constrained Case, Constant Real Interest Rate