Inflation Through the Lens of the Fiscal Theory*

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Abstract

We develop the fiscal theory of the price level in a range of models using both *ad hoc* policy rules and jointly optimal monetary and fiscal policies. The article is prepared for the *Handbook of Macroeconomics*, volume 2 (John B. Taylor and Harald Uhlig, editors, Elsevier Press).

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1 Introduction

There is a long tradition in macroeconomics of modeling inflation in stable economies by focusing on monetary policy and abstracting from fiscal policy.[1] As the global financial crisis and its aftermath rocked the world economy, the tenability of that modeling approach has been strained. Let's start with a few observations of economic developments since 2008.

1. Many countries reacted to the financial crisis and recession that began in 2008 by sharply reducing monetary policy interest rates and implementing large fiscal stimulus packages.

2. Central banks reacted to the financial crisis by purchasing large quantities of private assets and government bonds in actions that more closely resemble fiscal than monetary policy [Brunnermeier and Sannikov (2013), Leeper and Nason (2014)].

3. Sovereign debt crises in the euro zone culminated in the European Central Bank's "outright monetary transactions," a promise to purchase sovereign debt in secondary markets in unlimited quantities for countries that satisfied conditionality restrictions.

4. Rapid adoption of fiscal austerity measures beginning in 2010 and 2011 created challenges for central banks that were already operating at or near the lower limits for nominal interest rates.

5. Exploding central bank balance sheets also grew riskier, increasing concerns about whether the requisite fiscal backing or support for monetary policy is guaranteed [Del Negro and Sims (2015)].

6. In 2013, Japan's newly elected prime minister Shinzō Abe adopted "Abenomics," a mix of fiscal stimulus, monetary easing and structural reforms designed to re-inflate a Japanese economy that has languished since the early 1990s.

7. Table 1 reports that government debt expansions during the recession were significant: net debt as a share of GDP rose between 37 and 79 percent among four advanced-economy country groups. As central banks begin to raise interest rates toward more normal levels, these debt expansions will carry with them dramatically higher debt service to create fresh fiscal pressures. The Congressional Budget Office (2014) projects that U.S. federal government net interest payments will rise from 1.3 to 3.0 percent of GDP from 2014 to 2024.[2] Evidently, there are substantial fiscal consequences from central bank exits from very low policy interest rates.

[1] Focusing on stable economies rules out hyperinflations, which are widely believed to have fiscal origins.

[2] The CBO expects a relatively modest interest in treasury interest rates over that period, with the 10-year

	2008	2015
Euro area	54.0	74.0
Japan	95.3	140.0
United Kingdom	47.5	85.0
United States	50.4	80.9

Table 1: Net general government debt as percent of GDP. Projections for 2015. Source: International Monetary Fund (2014)

8. With an increasing number of central banks now paying interest on reserves, one important distinction between high-powered money and short-term nominal government bonds has disappeared, removing a principle distinction between monetary and fiscal policy [Cochrane (2014)].

9. Sovereign debt troubles in the euro area and political intransigence in many countries remind us that every country faces a fiscal limit, which is the point at which the adjustments in primary surpluses needed to stabilize debt are not assured. Uncertainty about future fiscal adjustments can untether fiscal expectations, making it difficult or impossible for monetary policy to achieve its objectives [Davig, Leeper, and Walker (2010, 2011)].

10. Exacerbating the fiscal fallout from the crisis, aging populations worldwide create long-run fiscal stress whose resolution in most countries is uncertain. This kind of uncertainty operates at low frequencies and may conflict with the long-run objectives of monetary policy.

It is hard to think about these developments without bringing monetary and fiscal policy *jointly* into the analysis. Several of these examples also run counter to critical maintained assumptions in monetarist/new Keynesian perspectives, including:

- fiscal policies will usually adjust government revenues and expenditures as needed to finance and stabilize government debt; this ensures that fiscal actions are "self-correcting" and need not concern monetary policymakers;

- sufficiently creative monetary policies—which include interest rate settings, quantitative easing, credit easing, government debt management, forward guidance—can always achieve desired inflation and macroeconomic objectives;

rate rising from 2.8 to 4.7 percent and the average rate on debt held by the public rising from 1.8 to 3.9 percent.

- impacts of monetary policy on fiscal choices are small enough to be of negligible importance to monetary policy decisions, freeing central banks to focus on a narrow set of goals.

As even this handful of examples makes clear, it is unlikely to be fruitful to interpret recent macroeconomic policy issues by studying monetary or fiscal policy in isolation. This article takes that premise as given to explore how macro policies interact to determine equilibrium.

1.1 What is the Fiscal Theory?

In this chapter we consider models with monetary policy, a maturity structure for nominal government debt, taxes—distorting or lump-sum—government expenditures—purchases or transfers—and a government budget constraint. In models of this kind, four key features of equilibrium may emerge:

1. There is a prominent role for nominal government debt revaluations that stabilize debt through surprise changes in inflation and bond prices.

2. A breakdown in Ricardian equivalence arises through a monetary-fiscal policy mix that permits nominal government debt expansions or increases in the monetary policy interest rate instrument to increase nominal private wealth and nominal aggregate demand.

3. Expectations of fiscal policy are equally important to those of monetary policy in determining macroeconomic equilibria, as in Tobin (1980), Brunner and Meltzer (1972) and Wallace (1981).[3]

4. Debt management policies matter for equilibrium dynamics, contributing an additional instrument to the standard macroeconomic policy toolkit, as Tobin (1963) argued.

Analyses of the implications of these features in this class of models constitute what we call the "fiscal theory of the price level."[4]

To the extent that this theory differs from conventional theories of inflation or the price level, those differences arise because the fiscal theory explicitly fills out the details of the fiscal side of the models. By doing so, the fiscal theory extracts what assumptions about

[3] Brunner and Meltzer anticipate the fiscal theory by showing that a government debt expansion unaccompanied by higher base money is inflationary when the fiscal deficit is held constant. But they dismiss this result on the grounds that "Price-level changes of this kind have not been important [foonote 13]."

[4] Early contributors to the theory include Begg and Haque (1984), Auernheimer and Contreras (1990) Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (1998).

fiscal behavior are required to deliver conventional results. More importantly, filling in monetary and fiscal behavior reveals that a far richer set of equilibria may emerge from the previously suppressed, but undeniable, fact that monetary and fiscal policies are intrinsically intertwined.

Cochrane (2011, 2014) and Sims (1999, 2013) two leading proponents of the fiscal theory, explore a wide range of issues through the lens of the fiscal theory to reach conclusions that contrast sharply with conventional perspective. This chapter also re-examines some practical issues in the light of the fiscal theory.

Most of the article focuses on the nature of equilibrium, including price-level determination, in models with nontrivial specifications of monetary and fiscal policy behavior. In this sense, the article, like the fiscal theory itself, echoes Wallace's (1981) insight that the effects of central bank open-market operations hinge on the precise sense in which fiscal policy is held constant. Under some assumptions on fiscal behavior, open-market operations are neutral, but different fiscal behavior permits monetary policy actions to have different impacts. Wallace did not explore the nature of price-level determination in the presence of nominal government bonds, which the fiscal theory emphasizes, but his results nonetheless foreshadow the newer literature. We also examine interactions in the opposite direction: how monetary policy behavior can influence the impacts of fiscal actions.

Several themes run through this paper. First, it is always the *joint* behavior of monetary and fiscal policies that determine inflation and stabilize debt. While this point might seem obvious—echoing, as it does, a viewpoint that dates back at least to Friedman (1948)—it is easily missed in the classes of models and policy rules typically employed in modern policy analyses. In those models, inflation appears to be determined entirely by monetary policy behavior—specifically, by the responsiveness of monetary policy to inflation—while debt dynamics seem to be driven only by fiscal behavior—the strength of primary surplus responses to debt. Of course, *in equilibrium* the two policies must interact in particular ways to deliver a determinate equilibrium with bounded debt, but this point is often swept under the carpet in order to focus the analysis solely on monetary policy.[5]

In those frameworks, macroeconomic policies have two fundamental tasks to achieve: determine the price level and stabilize debt. Two distinct monetary-fiscal policy mixes can accomplish those tasks. A second theme is that it is useful for some purposes to categorize those policy mixes in terms of "active" or "passive" policy behavior.[6] An active authority pursues its objectives unconstrained by the state of government debt and is free to set its control variables as it sees fit. But then the other authority must behave passively to

[5]See, for example, Woodford (2003) and Galí (2008).

[6]Leeper (1991) develops this categorization to study bounded equilibria.

stabilize debt, constrained by the active authority's actions and private sector behavior. A determinate bounded equilibrium requires the mix of one active and one passive policy; that mix achieves the two macroeconomic objectives of delivering unique inflation and stable debt processes.[7] The combination of active monetary and passive fiscal policies delivers the usual monetarist/new Keynesian setup in which monetary policy can target inflation and fiscal policy exhibits Ricardian equivalence. An alternative combination of passive monetary and active fiscal policies gives fiscal policy important effects on inflation, while monetary policy ensures that debt is stable. The latter policy regime has been given the unfortunate label "the fiscal theory of the price level." It is convenient to call the usual policy mix Regime M and the fiscal theory mix Regime F.

Third, Regime F policies produce equilibria in which the maturity structure of government debt affects equilibrium dynamics, as Cochrane (2001) and Sims (2011) emphasize. In contrast, without frictions that make short and long debt imperfect substitutes, maturity structure is irrelevant in Regime M. Under the fiscal theory, long debt permits both current and future inflation (bond prices) to adjust to shocks that perturb the market value of debt, which serves to make inflation and, if prices are sticky, real activity less volatile than they would be if all debt were one-period.

Fourth, it is only under the very special cases of flexible prices and lump-sum fiscal shocks/surplus adjustments that standard simple active monetary policy rules can hit their inflation target in regime M. More generally, with sticky prices and distortionary taxation we observe revaluation effects and pervasive interactions between monetary and fiscal policy across both the M and F regimes.

Fifth, the "active/passive" rubrics also lose their usefulness once one considers optimal policies. Jointly optimal monetary and fiscal policies generally combine elements of both regimes M and F: when long-maturity government debt is outstanding, it is always optimal to stabilize debt partly through distorting taxes and partly through surprise changes in inflation and bond prices [Cochrane (2001), Sims (2013), Leeper and Zhou (2013)]. How important inflation is as a debt stabilizer—or in Sims's (2013) terminology, a "fiscal cushion"—depends on model specifics: the maturity structure of debt, the costliness of inflation variability, the level of outstanding government debt, whether optimal policy is with commitment or discretion, proximity of the economy to its fiscal limit, and so forth.

The fact that key features of the fiscal theory emerge as jointly optimal monetary and fiscal policy elevates the theory from a theoretical oddity to an integral part of macroeconomic

[7]There are unbounded equilibria also. Sims (2013) and Cochrane (2011) emphasize the possibility of solutions with unbounded inflation; McCallum (1984) and Canzoneri, Cumby, and Diba (2001) display solutions with unbounded debt that hinge on the presence of non-distorting taxes.

policies that deliver desirable outcomes.

1.2 Brief Overview of the Paper

As we progress through the chapter we gradually widen the extent of monetary and fiscal policy interactions. We start with a simple flexible-price endowment economy subject to shocks to lump-sum transfers. This environment limits the extent of monetary and fiscal interactions to the revaluation effects emphasized by the fiscal theory and supports the strong dichotomy between the M and F regimes. Even in this simple environment, though, there are important spillovers between monetary and fiscal policy under either regime when we allow for either government spending or monetary policy shocks.

We then turn to consider the same rules in a production economy subject to nominal rigidities, but where we retain the assumption that taxes are lump sum. This adds a new channel for monetary and fiscal interactions because monetary policy can affect real interest rates when prices are sticky which, in turn, influence debt dynamics through debt service costs. We then generalize this further by adding distortionary taxation to a new Keynesian economy. Then tax policy affects inflation through its impact on marginal costs, government spending feeds into aggregate demand, and monetary policy affects real interest rates to impact the size of the tax base. In this richer specification equilibrium outcomes are always the result of interactions between monetary and fiscal policy and a key issue is the balance between monetary and fiscal policy in the control of inflation and stabilization of debt. We shall see that the conventional assignment of delegating monetary policy to achieve an inflation target and fiscal policy to stabilize debt is not always optimal.

Most expositions of the fiscal theory posit simple *ad hoc* rules for monetary and fiscal behavior and characterize the nature of equilibria under alternative settings of those rules. This article follows that path in the next two sections in order to derive clean analytical results that explain how the fiscal theory operates and how it differs from alternative policy mixes. Then the paper turns to study jointly optimal monetary and fiscal policies as an alternative vehicle for describing the economic mechanisms that underlie the fiscal theory. Optimal policies make clear that the distinguishing features of the fiscal theory are generally part of a policy mix that produces desirable economic performance.

After those purely theoretical explorations, the paper turns to practical applications of the theory. Those applications include: assessing when revaluation of debt through current and future inflation is likely to be important, fiscal prerequisites for successful inflation targeting, consequences of alternative fiscal reactions to a return to more normal levels of interest rates, monetary-fiscal interactions during episodes of fiscal consolidation, and a discussion of why the central bank needs understand the prevailing monetary-fiscal regime in order to conduct monetary policy.

2 Endowment Economies with Ad Hoc Policy Rules

This section aims to present the distinguishing features of the fiscal theory listed in section 1.1 in the simplest possible model. A representative consumer lives forever and receives a constant endowment of goods, Y, each period. The economy is cashless and financial markets are complete.

2.1 A Simple Model

The consumer optimally chooses consumption, C_t, may buy or sell nominal assets, D_t, at price $Q_{t,t+1}$, receives lump-sum transfers from the government, z_t, and pays lump-sum taxes, τ_t.[8] The representative household maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t) \right\} \tag{1}$$

with $0 < \beta < 1$, subject to the sequence of flow budget constraints

$$P_t C_t + P_t \tau_t + E_t[Q_{t,t+1} D_t] = P_t Y + P_t z_t + D_{t-1} \tag{2}$$

given D_{-1}. $Q_{t,t+1}$ is the nominal price at t of an asset that pays \$1 in period $t+1$ and P_t is the general price level in units of mature government bonds required to purchase one unit of goods. Government bonds sold at t, which are included in D_t, pay gross nominal interest R_t in period $t+1$. Letting $m_{t,t+1}$ denote the real contingent claims price, a no-arbitrage condition implies that

$$Q_{t,t+1} = m_{t,t+1} \frac{P_t}{P_{t+1}} \tag{3}$$

The short-term nominal interest rate, R_t, which is also the central bank's policy instrument, is linked to the nominal bond price: $1/R_t = E_t[Q_{t,t+1}]$.

Setting government purchases of goods to zero, the primary surplus is simply $s_t \equiv \tau_t - z_t$. The household's intertemporal budget constraint comes from iterating on (2) and imposing the no-arbitrage condition, (3), and the transversality condition

$$\lim_{T \to \infty} E_t \left[m_{t,T} \frac{D_{T-1}}{P_T} \right] = 0 \tag{4}$$

[8] D_t consists of privately-issued, B_t^p, and government issued, B_t, assets. Government bonds cost $\$1/R_t$ per unit and are perfectly safe pure discount bonds.

to yield

$$E_t \sum_{j=0}^{\infty} m_{t,t+j} C_{t+j} = \frac{D_{t-1}}{P_t} + E_t \sum_{j=0}^{\infty} m_{t,t+j} (Y_{t+j} - s_{t+j}) \tag{5}$$

where $m_{t,t+j} \equiv \prod_{k=0}^{j} m_{t+k,t+k+1}$ is real discount factor, with $m_{t,t} = 1$.

After imposing equilibrium in the goods market, $C_t = Y$, the real discount factor is constant, $m_{t,t+1} = \beta$, and the nominal interest rate obeys a Fisher relation

$$\frac{1}{R_t} = \beta E_t \frac{P_t}{P_{t+1}} = \beta E_t \frac{1}{\pi_{t+1}} \tag{6}$$

where $\pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate. In equilibrium there will be no borrowing or lending among private agents, so the household's bond portfolio consists entirely of government bonds. Imposing both bond and goods market clearing and the constant real discount factor the household's intertemporal constraint produces the ubiquitous equilibrium condition

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{7}$$

Cochrane (2001) refers to (7) as an "equilibrium valuation equation" because it links the market value of debt outstanding at the beginning of period t, B_{t-1}/P_t, to the expected present value of the cash flows that back debt, primary surpluses. Notice that we derived this valuation equation entirely from private optimizing behavior and market clearing, without reference to government behavior or to the government's budget constraint. The valuation equation imposes no restrictions on the government's choices of future surpluses, in the same way that the Fisher relation does not limit the central bank's choices of the nominal interest rate.

For each date t, equations (6) and (7) constitute two equilibrium conditions in four unknowns: $R_t, P_t, E_t(1/P_{t+1}), E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$. Private sector behavior alone cannot uniquely determine the equilibrium. We turn now to a class of monetary and fiscal policy rules that may deliver determinate equilibria.

2.1.1 Policy Rules

The central bank obeys a simple interest rate rule, come to be called a Taylor (1993) rule, that makes deviations of the nominal interest rate from steady state proportional to deviations of inflation from steady state

$$\frac{1}{R_t} = \frac{1}{R^*} + \alpha_\pi \left(\frac{1}{\pi_t} - \frac{1}{\pi^*} \right) + \varepsilon_t^M \tag{8}$$

where ε_t^M is an exogenous shock to monetary policy. The government sets deviations of the primary surplus from steady state proportional to steady-state deviations of debt

$$s_t = s^* + \gamma \left(\frac{1}{R_{t-1}} \frac{B_{t-1}}{P_{t-1}} - \frac{b^*}{R^*} \right) + \varepsilon_t^F \tag{9}$$

where ε_t^F is an exogenous fiscal shock to the primary surplus. The inverse of the nominal interest rate is the price of nominal debt so $\frac{1}{R_{t-1}} \frac{B_{t-1}}{P_{t-1}}$ is the real market value of debt issued at $t-1$. Policy choices must be consistent with the government's flow budget constraint

$$\frac{1}{R_t} \frac{B_t}{P_t} + s_t = \frac{B_{t-1}}{P_t} \tag{10}$$

where the steady state of the model is

$$\frac{B}{P} = b^*, \quad s^* = (\beta^{-1} - 1) \frac{b^*}{R^*}, \quad R^* = \frac{\pi^*}{\beta}, \quad m^* = \beta$$

It is convenient to express things in terms of the inverse of inflation and real debt, so let $\nu_t \equiv \pi_t^{-1}$ and $b_t \equiv B_t / P_t$. Combining the monetary policy rule with the Fisher equation yields the difference equation in inflation

$$E_t(\nu_{t+1} - \nu^*) = \frac{\alpha_\pi}{\beta} (\nu_t - \nu^*) + \frac{1}{\beta} \varepsilon_t^M \tag{11}$$

Combining the fiscal rule and the government's flow budget constraint, taking expectations and employing the Fisher relation yields real debt dynamics

$$E_t \left(\frac{b_{t+1}}{R_{t+1}} - \frac{b^*}{R^*} \right) = (\beta^{-1} - \gamma) \left(\frac{b_t}{R_t} - \frac{b^*}{R^*} \right) - E_t \varepsilon_{t+1}^F \tag{12}$$

Equations (11) and (12) constitute a system of expectational difference equations in inflation and real debt, which is driven by the exogenous policy disturbances ε^M and ε^F. Given the consumer's discount factor, β, this system appears as though inflation dynamics depend only on the monetary policy choice of α_π, while debt dynamics hinge only on the fiscal policy choice of γ: it is not obvious that monetary and fiscal behavior *jointly* determine inflation and real debt. This apparent separation of the system is deceptive. Because the government issues *nominal* bonds, B_t, the price level appears in both equations and $1/P_t$ is the value of bonds maturing at t.

2.1.2 Solving the Model

We focus on bounded solutions.[9] Stability of inflation depends on α_π/β and stability of debt depends on $\beta^{-1} - \gamma$.

2.1.2.1 Regime M

If $\alpha_\pi/\beta > 1$, then the bounded solution for inflation is

$$\nu_t = \nu^* - \frac{1}{\alpha_\pi} \sum_{j=0}^{\infty} \left(\frac{\beta}{\alpha_\pi}\right)^j E_t \varepsilon_{t+j}^M \tag{13}$$

which delivers a solution for $\{P_{t-1}/P_t\}$ for $t \geq 0$ and the equilibrium nominal interest rate is

$$\frac{1}{R_t} = \frac{1}{R^*} - \sum_{j=1}^{\infty} \left(\frac{\beta}{\alpha_\pi}\right)^j E_t \varepsilon_{t+j}^M \tag{14}$$

In this simple model, both actual and expected inflation depend on the monetary policy parameter and shock, but they appear not to depend in any way of fiscal behavior.

This appearance is deceiving because (13) does not constitute a complete solution to the model; we also need to ensure that there is a bounded solution for real debt. If fiscal policy chooses $\gamma > \beta^{-1} - 1$, then when real debt rises, future surpluses rise by more than the net real interest rate with the change in debt in order to cover both debt service and a little of the principal. In this case, the debt dynamics in (12) imply that for arbitrary deviations of real debt from steady state, $\lim_{T\to\infty} E_t b_{T+1} = b^*$, so debt eventually returns to steady state.

Digging into exactly what fiscal policy does to stabilize debt reveals the underlying policy interactions. Suppose that at time t news arrives of a higher path for $\{\varepsilon_{t+j}^M\}$. This news reduces ν_t, raising the price level P_t. With fiscal rule (9), in the first instance the monetary news leaves s_t unaffected, but household holdings of outstanding bonds, B_{t-1}/P_t, decline. From the government budget constraint, this implies that the market value of debt issued

[9] Unbounded solutions for inflation also exist, as Benhabib, Schmitt-Grohé, and Uribe (2001) show. Sims (1999), Cochrane (2011) and Del Negro and Sims (2015) thoroughly explore those equilibria to argue that a determinate price level requires appropriate fiscal backing. As Del Negro and Sims (2015, p. 3) define it: "Fiscal backing requires that explosive inflationary or deflationary behavior of the price level is seen as impossible because the fiscal authority will respond to very high inflation with higher primary surpluses and to near-zero interest rates with lower, or negative, primary surpluses." Solutions with unbounded debt inevitably rely on non-distorting taxes, which permit revenues to grow forever at the same rate as interest receipts on government bond holdings. Although such paths for revenues are equilibria in the present model, because they are infeasible in economies where taxes distort, we find them to be uninteresting.

at t also falls, even if there is no change in the price of bonds, $1/R_t$

$$\frac{B_t}{P_t R_t} = -s_t + \frac{B_{t-1}}{P_t}$$

In the absence of future fiscal adjustments—such as those in which $\gamma > \beta^{-1} - 1$—household wealth would decline, reducing aggregate demand and counteracting the inflationary effect of the monetary expansion. But when fiscal policy reduces surpluses with debt by more than the real interest rate, surpluses are expected to fall by an amount equal in present value to the initial drop in the value of household bond holdings. This eliminates the negative wealth effect to render monetary policy expansionary.

When the news of higher $\{\varepsilon^M_{t+j}\}$ extends beyond the current period, the nominal interest rate rises, reducing the price of new bonds at t. Lower bond prices implicitly raise interest yields on these bonds that mature in period $t+1$ to create a second channel by which monetary policy affects household wealth. As with the first channel, though, these wealth effects evaporate with the expected adjustments in surpluses.

These fiscal adjustments connect to Wallace's (1981) point that the impacts of open-market operations hinge on the sense in which fiscal policy is "held constant." In Regime M, the "constancy" of fiscal policy is quite specific: it eliminates any monetary effects on balance sheets. By neutralizing the fiscal consequences of monetary policy actions, this regime leaves the impression that, in Friedman's (1970) famous aphorism, "inflation is always and everywhere a monetary phenomenon." Of course, it is the *joint* behavior of monetary and fiscal policies that delivers this impression.

Regime M also delivers the fiscal counterpart to Friedman's aphorism: Ricardian equivalence.[10] A fiscal shock at t that reduces the surplus by one unit is financed initially by an expansion in nominal debt of P_t units. With inflation pinned down by expression (13), real debt also increases by P_t units. Higher real debt, through the fiscal rule, triggers higher future surpluses whose present value equals the original debt expansion. Even in this completely standard Ricardian experiment, it is the joint policy behavior—monetary policy's aggressive response to inflation and fiscal policy's passive adjustment of surpluses—that produces the irrelevance result.

[10]Tobin (1980, p. 53) made this point: "Thus the Ricardian equivalence theorem is fundamental, perhaps indispensable, to monetarism."

2.1.2.2 Regime F

Consider the case in which fiscal policy is active, with exogenous surpluses, so $\gamma = 0$ to make the fiscal rule is $s_t = s^* + \varepsilon_t^F$. The solution for real debt is[11]

$$\frac{b_t}{R_t} = \frac{b^*}{R^*} + \sum_{j=1}^{\infty} \beta^j E_t \varepsilon_{t+j}^F \tag{15}$$

which implies that the value of debt at t depends on the expected present value of surpluses from $t+1$ onward.

We can solve for inflation by combining this solution for b_t with the government's flow budget constraint, noting that $B_{t-1}/P_t = \nu_t b_{t-1}$

$$\nu_t = \frac{(1-\beta)^{-1} s^* + \sum_{j=0}^{\infty} \beta^j E_t \varepsilon_{t+j}^F}{b_{t-1}} \tag{16}$$

where at t, b_{t-1} is predetermined, which produces the solution for the price level

$$P_t = \frac{B_{t-1}}{(1-\beta)^{-1} s^* + \sum_{j=0}^{\infty} \beta^j E_t \varepsilon_{t+j}^F} \tag{17}$$

News of lower surpluses raises the price level and reduces the value of outstanding debt. In contrast to regime M equilibria, in regime F *nominal* government debt is an important state variable.[12] Higher nominal debt or higher debt service raise the price level next period. These results reflect the impacts of higher nominal household wealth. Lower future surpluses—stemming from either lower taxes or higher transfers—or higher initial nominal assets, raise households' demand for goods when there is no prospect that future taxes will rise to offset the higher wealth. Unlike regime M, now equilibrium inflation, as given by (16), depends explicitly on current and expected fiscal choices—through the steady state surplus, s^*, and fiscal disturbances, $\sum_{j=0}^{\infty} \beta^j E_t \varepsilon_{t+j}^F$.

Expression (15) gives the real market value of debt. But in the absence of any stabilizing response of surpluses to real debt ($\gamma = 0$), debt's deviations from steady state are expected to grow over time at the real rate of interest, $1/\beta$, according to (12). Such growth in debt would violate the household's transversality condition, which is inconsistent with equilib-

[11] To derive (15), define $\tilde{b}_t \equiv B_t/P_t R_t$ to write the flow government budget constraint as $\tilde{b}_t + s_t = R_{t-1}\nu_t \tilde{b}_{t-1}$. Take expectations at $t-1$, apply the Euler equation $\beta^{-1} = E_{t-1} R_{t-1} \nu_t$, iterate forward, and impose transversality to obtain (15).

[12] Debt is also a state variable in regime M because it contains information about future surpluses. But in M, changes in the *real* value of debt induce changes in expectations of future *real* government claims on private resources.

rium. To reconcile these seemingly contradictory implications of the equilibrium, we need to understand the role that monetary policy plays in regime F.

Monetary policy ensures that actual debt, as opposed to expected debt, is stable by preventing interest payments on the debt from exploding and permitting surprise inflation to revalue government debt. In regime F, higher interest payments raise nominal wealth, increasing nominal aggregate demand and future inflation, as both (16) and (17) indicate. To understand monetary policy behavior, substitute the solution for ν_t from (16) into the monetary policy rule, (8). To simplify the expression, assume that the policy shocks are *i.i.d.* so that

$$\frac{1}{R_t} - \frac{1}{R^*} = \frac{\alpha_\pi}{\beta}\left[\frac{\beta(1-\beta)^{-1}s^* + \beta\varepsilon_t^F}{b_{t-1}} - \frac{1}{R^*}\right] + \varepsilon_t^M \tag{18}$$

In response to a fiscal expansion—$\varepsilon_t^F < 0$—the central bank reduces $1/R_t$ by $\alpha_\pi \varepsilon_t^F$ to lean against the fiscally-induced inflation. A serially uncorrelated fiscal disturbance leaves the market value of debt at its steady state, $b_{t+j}/R_{t+j} = b^*/R^*$ for $j \geq 0$. This greatly simplifies the time $t+1$ version of (18) to yield

$$\frac{1}{R_{t+1}} - \frac{1}{R^*} = \frac{\alpha_\pi}{\beta}\left(\frac{1}{R_t} - \frac{1}{R^*}\right) \tag{19}$$

If monetary policy were to respond aggressively to inflation by setting $\alpha_\pi/\beta > 1$, $1/R$ would diverge to positive or negative infinity, both situations that violate lower bound conditions on the net, $R-1$, nominal interest rate. Economically, these exploding paths stem from strong wealth effects that arise from ever-growing interest receipts to holders of government bonds. When $\alpha_\pi/\beta > 1$ the central bank raises the nominal interest rate by a factor that exceeds the real interest rate, which increases private agents' nominal wealth and inflation in the next period; this process repeats in subsequent periods. Active monetary policy essentially converts stable fiscally-induced inflation into explosive paths.

Existence of equilibrium requires that the monetary reaction to inflation not be too strong—specifically, that $\alpha_\pi/\beta < 1$, what is called "passive monetary policy." A pegged nominal interest rate, $\alpha_\pi = 0$, is the easiest case to understand. By holding the nominal rate fixed at R^*, monetary policy prevents the fiscal expansion from affecting future inflation by fixing interest payments on the debt. A one-time reduction in s_t that is financed by new nominal bond sales, raises P_t enough to keep B_t/P_t unchanged. But the higher price level also reduces the real value of existing nominal debt, B_{t-1}/P_t, and in doing so reduces the implicit interest payments. In terms of the flow budget constraint

$$\frac{b^*}{R^*} + s_t = \frac{B_{t-1}}{P_t}$$

where real debt remains at steady state because $\gamma = 0$ implies that expected surpluses are unchanged. The larger is the stock of outstanding debt, the less the price level must rise to keep the budget in balance.

More interesting results emerge when there is some monetary policy response to inflation—$0 < \alpha_\pi < \beta$. When monetary policy tries to combat fiscal inflation by raising the nominal interest rate, inflation is both amplified and propagated. Pegging R_t forces all inflation from a fiscal shock to occur at the time of the shock. Raising R_t permits the inflation to persist and the more strongly monetary policy reacts to inflation, the longer the inflation lasts.

Difference equations (18) and (19) make the monetary policy impacts clear. When $\alpha_\pi = 0$, a shock to ε_t^F has no effect on the nominal interest rate. But the larger is α_π, though still less than β, the stronger are the effects of ε_t^F on future nominal interest rates and, through the Fisher relation, future inflation.

Even though the transitory fiscal expansion has no effect on real debt, higher nominal rates bring forth new nominal bond issuances that are proportional to the increases in the price level. Higher nominal debt coupled with higher interest on the debt increase interest payments that raise household nominal wealth in the future. Because future taxes do not rise to offset that wealth increase, aggregate demand and the price level rise in the future.

Expression (18) reveals that an exogenous monetary contraction—lower ε_t^M that raises R_t—triggers exactly the same macroeconomic effects as an exogenous fiscal expansion. Higher interest rates raise debt service and nominal wealth, which increases inflation in the future. In this simple model with a fixed real interest rate, only this perverse implication for monetary policy obtains.[13]

2.2 The Role of Maturity Structure

Tobin (1963) discusses debt management in the context of the "monetary effect of the debt," contrasting this to the "direct fiscal effect" that is determined by the initial increase in the bond-financed deficit. The monetary effect stems from the maturity structure of the debt, which Tobin reasons outlasts the direct effects because it endures over the maturity horizon of the debt. Changes in the maturity composition of debt operate through impacts on the size and composition of private wealth and such changes can affect the macro economy, even if they do not entail changing the overall size of the debt. This section obtains closely related impacts from maturity structure in regime F.

[13]The result that a monetary contraction raises future inflation is reminiscent of Sargent and Wallace's (1981) unpleasant monetarist arithmetic, but the mechanism is completely different. In Sargent and Wallace, tighter money today implies looser money in the future and the higher future inflation can feed back to reduce money demand today. Their result does not stem from wealth effects of monetary policy.

The section introduces a full maturity structure of government debt in general form to derive the bond valuation equation and develop some intuition about the role that maturity plays in the endowment economy in regime F. It then uses a simple special case to make transparent the mechanisms at work in regime F.[14]

2.2.1 A General Maturity Structure

Let $B_t(t+j)$ denote the nominal quantity of zero-coupon bonds outstanding in period t that matures in period $t+j$ and let the dollar-price of those bonds be $Q_t(t+j)$. The government's flow budget constraint at t is

$$B_{t-1}(t) - \sum_{j=1}^{\infty} Q_t(t+j)[B_t(t+j) - B_{t-1}(t+j)] = P_t s_t \tag{20}$$

In a constant-endowment economy, the bond pricing equations are

$$Q_t(t+k) = \beta^k E_t \frac{P_t}{P_{t+k}} \tag{21}$$

for $k = 1, 2, \ldots$. These pricing equations imply the no-arbitrage condition that links the price of a k-period bond to the expected sequence of k 1-period bonds

$$Q_t(t+k) = E_t[Q_t(t+1)Q_{t+1}(t+2) \cdot \ldots \cdot Q_{t+k-1}(t+k)] \tag{22}$$

To derive the bond valuation equation with a general maturity structure, define

$$B_{t-1} \equiv B_{t-1}(t) + \sum_{j=1}^{\infty} Q_t(t+j) B_{t-1}(t+j)$$

as the portfolio of bonds outstanding at the end of period $t-1$, and rewrite the government budget constraint as

$$\frac{B_{t-1}}{P_t} = Q_t(t+1)\frac{B_t}{P_t} + s_t$$

Iterating on this bond portfolio version of the constraint, taking expectations and imposing the bond-pricing relations and the consumer's transversality condition yields the valuation equation

$$\frac{B_{t-1}}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t s_{t+j}$$

[14] These derivations draw on Cochrane (2001, 2014).

or, in terms of the underlying bonds

$$\frac{B_{t-1}(t)}{P_t} + \sum_{j=1}^{\infty} \beta^j E_t \frac{B_{t-1}(t+j)}{P_{t+j}} = \sum_{j=0}^{\infty} \beta^j E_t s_{t+j} \tag{23}$$

Use (23) to repeatedly substitute out future price levels to make explicit how maturity structure enters the valuation equation

$$\frac{B_{t-1}(t)}{P_t} = E_t \left\{ s_t + \beta \underbrace{\left[1 - \frac{B_{t-1}(t+1)}{B_t(t+1)}\right]}_{\text{weight on } t+1} s_{t+1} \right.$$
$$\left. + \beta^2 \underbrace{\left\{1 - \left[\frac{B_{t-1}(t+2)}{B_{t+1}(t+2)} \frac{B_{t-1}(t+1)}{B_t(t+1)} \left(1 - \frac{B_t(t+2)}{B_{t+1}(t+2)}\right)\right]\right\}}_{\text{weight on } t+2} s_{t+2} + \ldots \right\} \tag{24}$$

We write this valuation equation more compactly by defining

$$\Lambda_t(t+k) \equiv \frac{B_t(t+k) - B_{t-1}(t+k)}{B_{t+k-1}(t+k)}$$

as newly issued debt that matures in period $t+k$ as a share of total outstanding debt in period $t+k-1$ that matures at $t+k$. We can now define the maturity weight on the surplus at $t+k$, $L_{t,t+k}$, as depending recursively on these ratios

$$\begin{aligned}
L_{t,t} &= 1 \\
L_{t,t+1} &= \Lambda_t(t+1) \\
L_{t,t+2} &= \Lambda_{t+1}(t+2)L_{t,t+1} + \Lambda_t(t+2) \\
L_{t,t+3} &= \Lambda_{t+2}(t+3)L_{t,t+2} + \Lambda_{t+1}(t+3)L_{t,t+1} + \Lambda_t(t+3) \\
&\vdots \\
L_{t,t+k} &= \sum_{j=0}^{k-1} \Lambda_{t+j}(t+k)L_{t,t+j}
\end{aligned}$$

The compact form of valuation equation (24) is now

$$\frac{B_{t-1}(t)}{P_t} = \sum_{j=0}^{\infty} \beta^j E_t[L_{t,t+j} s_{t+j}] \tag{25}$$

Given a sequence of surpluses, $\{s_t\}$, discount factors and maturity determine the expected present value of surpluses. Shortening maturity (e.g., reducing $\frac{B_{t-1}(t+1)}{B_t(t+1)}$) raises the weights on $s_{t+1}, s_{t+2}, s_{t+3}$, raising that present value—the backing of debt—and the value of debt. Shortening maturity of bonds due at $t+k$ raises weights on all $s_{t+j}, j \geq k$. In this sense, shortening maturity can offset a decline in surpluses.

Surprise changes in future maturity structure appears as innovations in the weights, $L_{t,t+j}$, in valuation equation (38). If primary surpluses are given, an unanticipated shortening of maturity of bonds held by the public would, by raising the value of outstanding debt, reduce the current price level. Viewed through the lens of the fiscal theory, the Federal Reserve's "operation twist" in 2011 would have a contractionary effect on the economy initially.[15] As the example to which we now turn illustrates, the lower price level at t would ultimately be offset by a higher future price level.

2.2.1.1 An Illustrative Example

To cleanly illustrate the role that changes in maturity structure play in determining the timing of inflation, we examine an example from Cochrane (2014). We use the same constant endowment economy, but it operates only in periods $t = 0, 1, 2$ and then ends; we set the real interest rate to zero, so the discount factor is $\beta = 1$. The government issues one- and two-period nominal bonds at the beginning of time, $t = 0$, denoted by $B_0(1)$ and $B_0(2)$, and uses surpluses in periods 1 and 2, s_1 and s_2, to retire the debt. At date $t = 1$ the government may choose to issue new one-period debt, $B_1(2)$, so the change in debt at $t = 1$ is $B_1(2) - B_0(2)$. The three potentially different quantities of bonds sell at nominal prices $Q_0(1), Q_0(2), Q_1(2)$ that obey (21) with $\beta = 1$.[16]

Given initial choices of debt, $B_0(1)$ and $B_0(2)$, the government's budget constraints in periods 1 and 2 are

$$B_0(1) = P_1 s_1 + Q_1(2)[B_1(2) - B_0(2)] \tag{26}$$

$$B_1(2) = P_2 s_2 \tag{27}$$

When primary surpluses are given at $\{s_1, s_2\}$, expression (27) immediately yields the price level in period 2 as

$$\frac{B_1(2)}{P_2} = s_2 \tag{28}$$

[15] The premise of the Fed's actions was that if short and long bonds are imperfect substitutes, then increasing demand for long bonds would reduce long-term interest rates. Lower long rates, it was hoped, would stimulate business investment and the housing market.

[16] We normalize the initial price level to be $P_0 = 1$.

because $B_1(2)$ is predetermined in period 2.

Now impose the asset-pricing relations on the bond prices in the period 1 government budget constraint, (26), to obtain the bond valuation equation

$$\frac{B_0(1)}{P_1} = s_1 + \left[\frac{B_1(2) - B_0(2)}{B_1(2)}\right] E_1 s_2 \tag{29}$$

P_1 depends on the choice of newly issued bonds in period 1.

Solving for expected inflation and bond prices yields

$$E_0\left(\frac{1}{P_2}\right) = Q_0(2) = E_0\left(\frac{s_2}{B_1(2)}\right) = E_0\left[\frac{1}{B_0(2) + (B_1(2) - B_0(2))}\right] s_2 \tag{30}$$

$$E_0\left(\frac{1}{P_1}\right) = Q_0(1) = \frac{E_0[s_1]}{B_0(1)} + \frac{1}{B_0(1)} E_0\left[\frac{B_1(2) - B_0(2)}{B_1(2)}\right] s_2 \tag{31}$$

So the term structure of interest rates also depends on choices about maturity structure.

We can derive more explicitly solutions for the actual or realized price level at $t = 1$ in terms of innovations

$$B_0(1)(E_1 - E_0)\left(\frac{1}{P_1}\right) = (E_1 - E_0)s_1 + (E_1 - E_0)\left(\frac{B_1(2) - B_0(2)}{B_1(2)}\right) s_2 \tag{32}$$

Surprise increases in the price level in period 1 depend negatively on innovations in time 1 and time 2 surpluses and on unexpected lengthening of the maturity of bonds due in period 2.

These derivations show that the government can achieve any path of the nominal term structure—and in this example, expected inflation—that it wishes by adjusting maturity structure. By unexpectedly selling less time-2 debt, the government reduces the claims to time-2 surpluses, which reduces the revenues that can be used to payoff period-1 bonds. This raises inflation in period 1. That increase in inflation comes from reducing $B_1(2)$, which lowers the price level in period 2, as seen from

$$(E_1 - E_0)\left(\frac{B_1(2)}{P_2}\right) = (E_1 - E_0)s_2 \tag{33}$$

If s_2 is given, selling less $B_1(2)$ requires P_2 to fall.

2.2.2 A Useful Special Case

Suppose that the maturity structure declines at a constant rate $0 \leq \rho \leq 1$ each period so that the pattern of bonds issued at $t-1$ obeys

$$B_{t-1}(t+j) = \rho^j B^m_{t-1}$$

where B^m_{t-1} is the portfolio of these specialized bonds in $t-1$. When $\rho = 0$ all bonds are one-period, whereas when $\rho = 1$ all bonds are consols. The average maturity of the portfolio is $1/(1-\beta\rho)$.

With this specialization, the government's flow constraint is

$$B^m_{t-1}\left[1 - \sum_{j=1}^{\infty} Q_t(t+j)\rho^j\right] = P_t s_t + B^m_t \sum_{j=1}^{\infty} Q_t(t+j)\rho^{j-1}$$

If we define the price of the bond portfolio as

$$P^m_t \equiv \sum_{j=1}^{\infty} Q_t(t+j)\rho^{j-1}$$

then the government's budget constraint becomes

$$B^m_{t-1}(1+\rho P^m_t) = P_t s_t + P^m_t B^m_t \tag{34}$$

Bond portfolio prices obey the recursion

$$P^m_t = Q_t(t+1)[1+\rho E_t P^m_{t+1}] = R_t^{-1}[1+\rho E_t P^m_{t+1}] \tag{35}$$

This shows that a constant geometric decay rate in the maturity structure of zero-coupon bonds is equivalent to the interpretation of bonds that pay geometrically decaying coupon payments, as in Woodford (2001) and Eusepi and Preston (2013).

Let R^m_{t+1} denote the gross nominal return on the bond portfolio between t and $t+1$. Then $R^m_{t+1} = (1+\rho P^m_{t+1})/P^m_t$ and the no-arbitrage condition implies that

$$\frac{1}{R_t} = \beta E_t \nu_{t+1} = E_t\left(\frac{1}{R^m_{t+1}}\right) \tag{36}$$

Combining (35) and (36) and iterating forward connects bond prices to expected paths of the short-term nominal interest rate and inflation

$$P_t^m = \sum_{j=0}^{\infty} \rho^j E_t \left(\prod_{i=0}^{j} R_{t+i}^{-1} \right) = \beta \sum_{j=0}^{\infty} (\beta\rho)^j E_t \left(\prod_{i=0}^{j} \nu_{t+i+1} \right) \tag{37}$$

2.3 Maturity Structure in Regime F

Ricardian equivalence in regime M makes the maturity structure of debt irrelevant for inflation, so in this section we focus solely on regime F. When surpluses are exogenous ($\gamma = 0$), the debt valuation equation becomes[17]

$$\frac{(1+\rho P_t^m)B_{t-1}^m}{P_t} = (1-\beta)^{-1} s^* + \sum_{j=0}^{\infty} \beta^j E_t \varepsilon_{t+j}^F \tag{38}$$

In contrast to the situation with only one-period debt ($\rho = 0$) when fiscal news appeared entirely in jumps in the price level, now there is an additional channel through which debt can be revalued: bond prices that reflect expected inflation over the entire duration of debt. News of lower future surpluses reduces the value of debt through both a higher P_t and a lower P_t^m. By (37), the lower bond price portends higher inflation and higher one-period nominal interest rates. The ultimate mix between current and future inflation is determined by the monetary policy rule. Long-term debt opens a new channel for monetary and fiscal policy to interact.

No-arbitrage condition (37) reveals a key aspect of regime F equilibria with long debt. With the simplified maturity structure, ρ determines the average maturity of the zero-coupon bond portfolio. A given future inflation rate has a larger impact on the price of bonds, the larger is ρ or the longer is the average maturity of debt. The maturity parameter serves as an additional discount factor, along with β, so more distant inflation rates have a smaller impact on bond prices than do rates in the near future. Of course, the date t expected present value of inflation influences only the price of bonds that are outstanding at the beginning of t, namely, B_{t-1}^m.

To understand monetary policy's influence on the timing of inflation, note that when monetary policy is passive, $\alpha_\pi/\beta < 1$, (11) implies that k-step-ahead expected inflation is

$$E_t \nu_{t+k} = \left(\frac{\alpha_\pi}{\beta} \right)^k (\nu_t - \nu^*) + \nu^*$$

[17] To derive (38), convert the nominal budget constraint in (34) into a difference equation in the real value of debt, $P^m B^m / P$, impose pricing equations (35) and (36), using the fact that $\beta^{-1} = E_{t-1}[\nu_t(1+\rho P_t^m)/P_{t-1}^m]$, iterate forward and impose the household's transversality condition for debt.

which may be substituted into pricing equation that links P_t^m to the term structure of inflation rates, (37), to yield[18]

$$\rho P_t^m = \sum_{j=1}^{\infty}(\beta\rho)^j E_t \left\{ \prod_{i=0}^{j-1} \left[\left(\frac{\alpha_\pi}{\beta} \right)^{i+1} (\nu_t - \nu^*) + \nu^* \right] \right\} \tag{39}$$

Monetary policy's reaction to inflation—through α_π—interacts with the average maturity of debt—ρ—to determine how current inflation—ν_t, which is given by (16) in regime F—affects the price of bonds. More aggressive monetary policy and longer maturity debt both serve to amplify the impact of current inflation on bond prices, suggesting that higher α_π and higher ρ permit fiscal disturbances to have a smaller impact on current inflation at the cost of a larger impact on future inflation.

Consider two polar cases of passive monetary policy. When $\alpha_\pi = 0$, so the central bank pegs the nominal interest rate and bond prices at $\rho P_t^m = \beta\rho\nu^*/(1-\beta\rho\nu^*)$, the valuation expression becomes

$$\left(\frac{1}{1-\beta\rho\nu^*} \right) \nu_t b_{t-1}^m = (1-\beta)^{-1} s^* + \sum_{j=0}^{\infty} \beta^j E_t \varepsilon_{t+j}^F \tag{40}$$

where we define $b_{t-1}^m \equiv B_{t-1}^m / P_{t-1}$. In this case, expected inflation returns to target immediately, $E_t \nu_{t+j} = \nu^*$ for $j \geq 1$.

The second case is when monetary policy reacts as strongly as possible to inflation, while still remaining passive: $\alpha_\pi = \beta$. Then $\rho P_t^m = \beta\rho\nu_t/(1-\beta\rho\nu_t)$ and the valuation equation is[19]

$$\left(\frac{\nu_t}{1-\beta\rho\nu_t} \right) b_{t-1}^m = (1-\beta)^{-1} s^* + \sum_{j=0}^{\infty} \beta^j E_t \varepsilon_{t+j}^F \tag{41}$$

Now inflation follows a martingale with $E_t \nu_{t+j} = \nu_t$ for $j \geq 1$.

The two polar cases are starkly different. By pegging the nominal interest rate, monetary policy anchors expected inflation on the steady state (target) inflation rate and bond prices are constant. The full impact of a lower present value of surpluses must be absorbed by higher current inflation—lower ν_t—alone. But when monetary policy raises the nominal rate with current inflation by a proportion equal to the discount factor, higher current inflation is expected to persist indefinitely. Bond prices fall by the expected present value of that higher inflation rate, discounted at the rate $\beta\rho$. With the required change in inflation spread evenly

[18] Here we shut down the exogenous monetary policy shock, $\varepsilon_t^M \equiv 0$.

[19] To obtain this result, we require that $\beta\rho\nu_t < 1$ for all realizations of ν_t, or $\pi_t > \beta\rho$, so there cannot be "too much" deflation.

over the term to maturity of outstanding debt, when fiscal news arrives, inflation needs to rise by far less than it does when bond prices are pegged. Of course, the "total"—present value—inflation effect of the fiscal shock is identical in the two cases. Although aggressive monetary policy cannot diminish the total inflationary impact, it can influence the timing of when inflation occurs.

We can consider both these polar cases and the intermediate case where $0 < \alpha_\pi < \beta$, by solving the model numerically in the presence of transfer shocks.[20] These are calibrated following Bi, Leeper, and Leith (2013). We assume that the steady-state ratio of transfers-to-GDP is 0.18, government spending is 21 percent of GDP and taxes amount to 41 percent of GDP implying an (annualized) steady state debt-GDP ratio of 50 percent. Transfers fluctuate according to an autoregressive process with persistence parameter of $\rho_z = 0.9$, and variance of $(0.005z^*)$. In this simple model with an active fiscal policy that does not respond to debt levels, the equilibrium outcome depends on the maturity of the debt stock and the responsiveness of monetary policy to inflation.

Figure 1 plots the response to an increase in transfers. Each column represents a different value of the response of monetary policy to inflation. Monetary policy pegs the nominal rate in the first column so the paths of all variables are the same across maturities: the entire adjustment occurs through surprise inflation in the initial period. In the second column $\alpha_\pi = 0.5$. Now differences emerge across maturities. With one-period debt the magnitude of the initial jump in inflation is the same as under a pegged interest rate because this is the price level jump that is required to reduce the real value of debt to be consistent with lower surpluses. But the monetary policy reaction keeps inflation high for a prolonged period even though it is only the initial jump in inflation that serves to reduce the debt burden. As average maturity increases, the initial jump in inflation becomes smaller. A sustained rise in interest rates depresses bond prices, which allow the bond valuation equation to be satisfied at lower initial inflation rates. It is the surprise change in the *path* of inflation that occurs over the life of the maturing debt stock that reduces the real value of debt. With a positive value of α_π, any jump in inflation is sustained, which unexpectedly reduces the real returns that bondholders receive before that debt is rolled over. As we increase the responsiveness of interest rate to inflation further to $\alpha_\pi = 0.9$, the surprise inflation needed to deflate the real value of debt remains unchanged for single period debt, but is dramatically reduced for longer period debt. When $\alpha_\pi = 0.99$, as demonstrated analytically above, and $\rho > 0$, the rate of inflation follows a near-random walk, jumping to the level needed to satisfy the valuation equation.

[20]The solution procedure follows that of Leith and Liu (2014) which relies on the use of Chebyshev collocation methods and Guass-Hermite quadrature to evaluate the expectations terms.

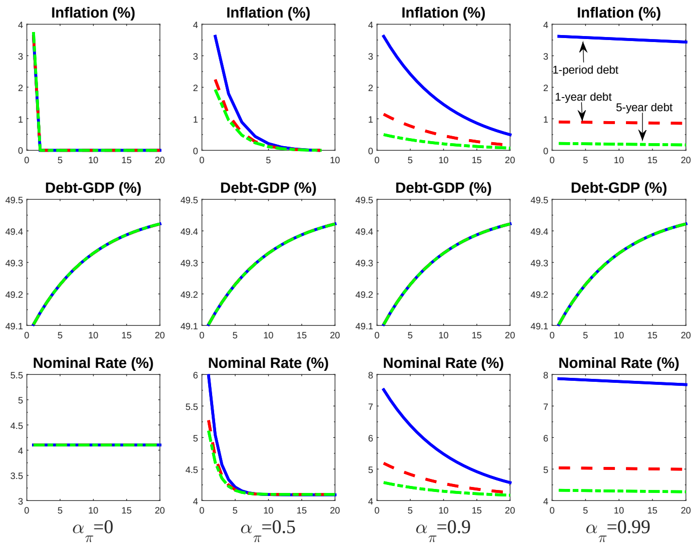

Figure 1: Responses to an increase in transfers under alternative monetary policy rules and alternative maturity structures.

The timing of the transfer shock—whether it is *i.i.d.* or persistent, realized immediately or in the future—doesn't matter beyond the change in the expected discounted value of surpluses that it produces. That present value must be financed with a path of inflation that combines current inflation surprises, and through bond prices, future inflation surprises, to ensure solvency. An anticipated increase in transfers produces surprise inflation today that reduces the current value of the outstanding debt stock, but whose value increase after the increase in transfers is realized.

This result foreshadows an important aspect of optimal policy, which sections 4 and 5 explore: monetary policy can smooth the distortionary effects of fiscally-induced inflation. The above analysis uses an endowment economy subjected to transfer shocks. That environment has the feature that under regime M, monetary policy can perfectly control inflation, while under regime F, prices are determined by the needs of fiscal solvency—the dichotomy across regimes that was emphasized in the original fiscal theory. The more general case breaks the dichotomy to produce interactions between monetary and fiscal policy in both policy regimes. This situation can arise even in the endowment economy when we consider government spending shocks rather than shocks to lump-sum transfers.

2.3.1 Increase in Government Spending

Government spending has implications for both monetary and fiscal policy. The direct impact on the government's finances is obvious. But by affecting the stochastic discount factor, government purchases carry additional effects on inflation and debt dynamics. Again we distinguish between the M and F regimes, although monetary and fiscal policy will interact under both.

2.3.1.1 Policy Under Regime M

When monetary policy is active and fiscal policy is passive, the analysis of the case of transfer shocks largely carries through, although with some additional monetary and fiscal interactions. Substituting the Fisher relation into the monetary policy rule yields the deflation dynamics[21]

$$v_t - v^* = \frac{\beta}{\alpha_\pi} E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} v_{t+1} - v^* \right] \tag{42}$$

which can be solved forward as

$$v_t = \frac{\alpha_\pi - \beta}{\alpha_\pi} E_t \sum_{i=0}^{\infty} \left(\frac{\beta}{\alpha_\pi} \right)^i \frac{u'(c_{t+i})}{u'(c_t)} v^* \tag{43}$$

Inflation deviates from target in proportion to the deviations of the real interest rate path from steady state. Higher government spending raises the real interest rate and inflation.

Debt dynamics emerge from three distinct impacts of government spending: the direct effect on the fiscal surplus, the surprise inflation that arises in conjunction with the monetary policy rule, and movements in real interest rates. Monetary policy can insulate inflation from government spending shocks by reacting to real interest rates, as well as inflation, with the rule

$$\frac{1}{R_t} = \frac{1}{R^*} E_t \frac{u'(c_{t+1})}{u'(c_t)} + \alpha_\pi (\nu_t - \nu^*) \tag{44}$$

By this rule, the policymaker accommodates changes in the natural rate of interest caused by fluctuations in public consumption without deviating from the inflation target. To see this, combine this rule with the Fisher equation to get

$$v_t - v^* = \frac{\beta}{\alpha_\pi} E_t \frac{u'(c_{t+1})}{u'(c_t)} (v_{t+1} - v^*) \tag{45}$$

Policy rule (44) implies that inflation/deflation is always equal to target, $v_t = v^*$. If the monetary policy rule does not respond to fiscal variables, inflation will be influenced by gov-

[21] When the real interest rate can vary, the Fisher relation is

$$\frac{1}{R_t} = \beta E_t \frac{u'(c_{t+1})}{u'(c_t)} \nu_{t+1}$$

ernment spending shocks. Inflation can be insulated from fiscal shocks by allowing monetary policy to directly respond to the effects of fiscal on the natural rate of interest.

2.3.1.2 Policy Under Regime F

In regime F government spending shocks require jumps in inflation to satisfy the bond valuation equation[22]

$$
\begin{aligned}
(1+\rho P_t^m)\frac{B_{t-1}^m}{P_t} &= E_t \sum_{i=0}^{\infty} \beta^i \frac{u'(c_{t+i})}{u'(c_t)} s_{t+i} \\
&= E_t \sum_{i=0}^{\infty} \beta^i \frac{u'(c_{t+i})}{u'(c_t)} s^* - E_t \sum_{i=0}^{\infty} \beta^i \frac{u'(c_{t+i})}{u'(c_t)} \varepsilon_{t+i}^G
\end{aligned} \tag{46}
$$

An increase in government spending increases the marginal utility of consumption, which increases real interest rates and requires a larger initial jump in inflation and drop in bond prices. Bond prices themselves are directly affected by the change in private consumption that arises when the government absorbs a larger share of resources, as the bond-pricing equation shows

$$
P_t^m = \beta E_t (1+\rho P_{t+1}^m) v_{t+1} \frac{u'(c_{t+1})}{u'(c_t)} \tag{47}
$$

Bond prices fall initially and then gradually increase as the period of raised public consumption passes.

Adopting a specific form of utility, $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$, with $\sigma = 2$, we can solve the model in the face of autocorrelated government spending shocks with $\rho_g = 0.9$, and variance of $0.005g^*$. As before, the stochastic model is solved non-linearly using Chebyshev collocation methods [see Leith and Liu (2014)]. Figure 2 reflects the response to government spending shocks which are broadly consistent with the impacts of transfer shocks that appear in figure 1. The main difference is that the growth in consumption as government spending returns to steady state is equivalent to an increase in the real interest rate. However the main message that single period debt requires an initial jump in inflation to stabilize debt and that this jump is unaffected by the description of the monetary policy parameter α_π remains. However, once debt maturity extends beyond a single period prolonging the initial jump in inflation can serve to reduce the magnitude of that initial jump. That is a sustained rise in inflation can also serve to satisfy the government's intertemporal budget constraint through reducing bond prices. Essentially the inflation surprise is spread throughout the life of the outstanding debt stock.

[22] Shutting down shocks to lump-sum taxes and transfers, the surplus is defined as $s_t = \tau^* - z^* - g_t$, where $g_t - g^* \varepsilon_t^G$, so $s_t = s^* - \varepsilon_t^G$.

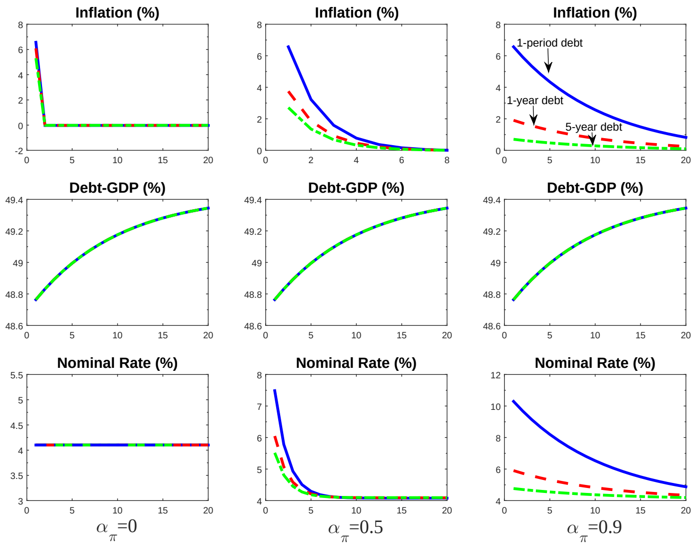

Figure 2: Responses to an increase in government purchases under alternative monetary policy rules and alternative maturity structures.

3 Production Economies with Ad Hoc Policy Rules

The endowment economy is useful for understanding the mechanisms that underlie the fiscal theory. But the exogeneity of the real interest rate and the constancy of output limit a complete understanding of the theory and, in some case, distort that understanding. We now turn to a conventional model in which inflation and output are determined jointly. In extending the analysis to the new Keynesian model we are widening the potential channels through which monetary and fiscal policy interact. To do so incrementally, we assume that taxes remain lump sum so that the effects of monetary policy on output do not affect the tax base to which a distortionary tax is applied. This means that the extra channel we are adding by introducing nominal inertia to a production economy is that monetary policy has influence over ex-ante real interest rates as well as nominal interest rates. This in turn means that the policymaker can ensure the bond valuation equation holds following fiscal shocks through a reduction in ex-ante real interest rates and not just ex-post real interest rate through inflation surprises. When we consider optimal policy in the new Keynesian model we shall allow taxes to distort behavior.

3.1 A Conventional New Keynesian Model

Endogenous output together with sticky prices allow both monetary policy and, in the case of regime F, fiscal policy to have real effects on the economy. We use a textbook version of a new Keynesian model of the kind that Woodford (2003) and Galí (2008) present. Because existing literature, including those two textbooks, thoroughly examines the nature of regime M equilibria, our exposition focuses exclusively on regime F.[23]

The model's key features include: a representative consumer and firm; monopolistic competition in final goods; Calvo (1983) sticky prices in which a fraction $1-\phi$ of goods suppliers sets a new price each period; a cashless economy with one-period nominal bonds, B_t, that sell at price $1/R_t$, where R_t is also the monetary policy instrument; for now, government purchases are zero, so the aggregate resource constraint is $C_t = Y_t$; an exogenous primary government surplus, s_t, with lump-sum taxes; and shocks only to monetary and fiscal policies.[24] We solve a version of the model that is log-linearized around the deterministic steady state with zero inflation.

Let $\hat{x}_t \equiv \ln(x_t) - \ln(x^*)$ denote log-deviations of a variable x_t from its steady state value. Private-sector behavior reduces to a consumption-Euler equation

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma(\hat{R}_t - E_t\hat{\pi}_{t+1}) \tag{48}$$

and a Phillips curve

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{Y}_t \tag{49}$$

where $\sigma \equiv -\frac{u'(Y^*)}{u''(Y^*)Y^*}$ is the intertemporal elasticity of substitution, $\omega \equiv \frac{w'(Y^*)}{w''(Y^*)Y^*}$ is the elasticity of supply of goods, $\kappa \equiv \frac{(1-\phi)(1-\phi\beta)}{\phi}\frac{\omega+\sigma}{\sigma(\omega+\theta)}$ is the slope of the Phillips curve, and θ is the elasticity of substitution among differentiated goods. The parameters obey $0 < \beta < 1, \sigma > 0, \kappa > 0$.

3.1.1 Policy Rules

Monetary policy follows a conventional interest rate rule

$$\hat{R}_t = \alpha_\pi\hat{\pi}_t + \alpha_Y\hat{Y}_t + \varepsilon_t^M \tag{50}$$

and fiscal policy sets the surplus process, $\{\hat{s}_t\}$, exogenously, where $\hat{s}_t \equiv (s_t - s^*)/s^*$. By setting the surplus exogenously, we are implicitly assuming that taxes are lump sum so that any variations in real activity do not impact on the size of the tax base.

Policy choices must satisfy the flow budget constraint

$$\hat{b}_t - \hat{R}_t + \left(\beta^{-1} - 1\right)\hat{s}_t = \beta^{-1}\left(\hat{b}_{t-1} - \hat{\pi}_t\right) \tag{51}$$

[23] We draw from Woodford (1998), but Kim (2003), Cochrane (2011) and Sims (2011) study closely related models.

[24] Because these shocks have no effects on the natural rate of output, there is no distinction between deviations in output from steady state and the output gap.

where b_t is real debt at the end of period t and π_t is the inflation rate between $t-1$ and t. Although this linearized budget constraint does not appear to contain the steady-state debt to GDP ratio, the calibration of the surplus shock does implicitly capture the underlying steady-state level of debt.

3.1.2 Solving the Model in Regime F

The four-equation system—(48)–(51)—together with exogenous $\{\hat{s}_t\}$ yields solutions for $\{\hat{Y}_t, \hat{\pi}_t, \hat{R}_t, \hat{b}_t\}$. Woodford (1998) shows that a unique equilibrium requires that monetary policy react relatively weakly to inflation and output: α_π and α_Y must satisfy

$$-1 - \frac{1+\beta}{\kappa}\alpha_Y - \frac{2(1+\beta)}{\kappa\sigma} < \alpha_\pi < 1 - \frac{1-\beta}{\kappa}\alpha_Y$$

For practical reasons, we restrict α_π's lower bound to 0. In this case, when monetary policy does not respond to output, this reduces to the condition that passive monetary policy requires $0 \leq \alpha_\pi < 1$. In the analytical results that follow, we use this simplified policy rule; numerical results will bring the output response of monetary policy back in.

Substituting a simplified version of the monetary policy rule into the government budget constraint and iterating forward immediately yields several robust features of regime F equilibria

$$E_t \sum_{j=0}^{\infty} \beta^j \hat{\pi}_{t+j} = \left(\frac{1}{1-\alpha_\pi\beta}\right)\left[\hat{b}_{t-1} - (1-\beta)E_t \sum_{j=0}^{\infty}\beta^j \hat{s}_{t+j} + \beta E_t \sum_{j=0}^{\infty}\beta^j \varepsilon^M_{t+j}\right] \tag{52}$$

Although expression (52) is not an equilibrium solution to the model, it highlights several features that the solution displays. First, higher initial debt, a lower expected path of surpluses or a higher expected path of the monetary shock all raise the present value of inflation. Second, a stronger response of monetary policy to inflation, but still consistent with existence of a bounded equilibrium, *amplifies* those inflationary effects. Dependence of inflation on the debt stock and surpluses is ubiquitous in regime F. Perversely, a higher path of the monetary shock or a higher value for α_π constitute a tightening of policy, yet they raise inflation.

In the flexible-price case, $\kappa = \infty$, so $\hat{Y}_t \equiv 0$, and a solution for equilibrium inflation is immediate. This case collapses back to the endowment economy in section 2.1.2.2 with a constant real rate and the simple Fisher relation $\hat{R}_t = E_t\hat{\pi}_{t+1}$. Combine the monetary policy rule with $\alpha_Y = 0$ with the Fisher relation to solve for expected inflation

$$E_t\hat{\pi}_{t+j} = \alpha_\pi^j \hat{\pi}_t + \alpha_\pi^{j-1}\varepsilon^M_t + \alpha_\pi^{j-2}E_t\varepsilon^M_{t+1} + \ldots + \alpha_\pi E_t\varepsilon^M_{t+j-2} + E_t\varepsilon^M_{t+j-1}$$

and use this expression to replace expected inflation rates in (52). Equilibrium inflation is

$$\hat{\pi}_t = \hat{b}_{t-1} + \beta(1-\alpha_\pi\beta)E_t\sum_{j=0}^{\infty}\beta^j\varepsilon^M_{t+j} - (1-\beta)E_t\sum_{j=0}^{\infty}\beta^j\hat{s}_{t+j} \tag{53}$$

Actual inflation rises with initial debt, a higher path of the monetary policy shock or a lower path for surpluses. The effects of surpluses on inflation are independent of the monetary policy choice of α_π, although we saw above that those fiscal effects on expected inflation are amplified by more aggressive monetary policy.

Solving the sticky-price new Keynesian model is more complicated. When $0 < \kappa < \infty$, both output and the real interest rate are endogenous. Defining the real interest rate as $\hat{r}_{t+j} \equiv \hat{R}_{t+j-1} - \hat{\pi}_{t+j}$, write the bond valuation equation as

$$\hat{\pi}_t - E_t\sum_{j=1}^{\infty}\beta^j\hat{r}_{t+j} = \hat{b}_{t-1} - (1-\beta)E_t\sum_{j=0}^{\infty}\beta^j\hat{s}_{t+j} \tag{54}$$

News about lower future surpluses shows up as a mix of higher current inflation and a lower path for the real interest rate. Lower real rates, in turn, transmit into higher output. Fiscal expansions have the old-Keynesian effects—higher real activity and inflation—and monetary policy behavior determines the split between them.

Combining the Euler equation, the Phillips curve and the monetary policy rule produces a second-order difference equation in inflation

$$E_t\hat{\pi}_{t+2} - \frac{1+\beta+\sigma\kappa}{\beta}E_t\hat{\pi}_{t+1} + \frac{1+\alpha_\pi\sigma\kappa}{\beta}\hat{\pi}_t = -\frac{\sigma\kappa}{\beta}\varepsilon^M_t \tag{55}$$

One can show that, given the restrictions on the underlying model parameters, this difference equation has two real roots, one inside $|\lambda_1| < 1$ and one outside $|\lambda_2 > 1|$ the unit circle, which yields the solution for expected inflation[25]

$$E_t\hat{\pi}_{t+1} = \lambda_1\hat{\pi}_t + (\beta\lambda_2)^{-1}\sigma\kappa E_t\sum_{j=0}^{\infty}\lambda_2^j\varepsilon^M_{t+j} \tag{56}$$

We can now solve for the j-step-ahead expectation of inflation by defining the operator

[25] Letting $\gamma_1 \equiv (1+\beta+\sigma\kappa)/\beta$ and $\gamma_0 \equiv (1+\alpha_\pi\sigma\kappa)/\beta$, the roots are $\lambda_1 = (1/2)(\gamma_1 - \sqrt{\gamma_1^2 - 4\gamma_0})$ and $\lambda_2 = (1/2)(\gamma_1 + \sqrt{\gamma_1^2 - 4\gamma_0})$. These derivations owe much to Tan (2015) who employs the techniques that Tan and Walker (2014) develop.

$\mathcal{B}^{-j}x_t \equiv E_t x_{t+j}$ and iterating on (56)

$$\mathcal{B}^{-j}\hat{\pi}_t = \lambda_1^j \hat{\pi}_t + \frac{\sigma\kappa}{\lambda_2\beta}\frac{1}{1-\lambda_2^{-1}\mathcal{B}^{-1}}\left(\lambda_1^{j-1} + \lambda_1^{j-2}\mathcal{B}^{-1} + \ldots + \mathcal{B}^{-j+1}\right)\varepsilon_t^M$$

This yields the solution for the present value of inflation that appears in (52)

$$E_t \sum_{j=0}^{\infty} \beta^j \hat{\pi}_{t+j} = \frac{1}{1-\lambda_1\beta}\hat{\pi}_t + \frac{\sigma\kappa}{\lambda_2(1-\lambda_1\beta)}\frac{1}{(1-\lambda_2^{-1}\mathcal{B}^{-1})(1-\beta\mathcal{B}^{-1})}\varepsilon_t^M$$

Using this expression for the present value of inflation in (52) delivers a solution for equilibrium inflation

$$\begin{aligned}\hat{\pi}_t = &\left(\frac{1-\lambda_1\beta}{1-\alpha_\pi\beta}\right)\left[\hat{b}_{t-1} - \left(\frac{1-\beta}{1-\beta\mathcal{B}^{-1}}\right)\hat{s}_t\right] \\ &+ \left[\frac{1-\lambda_1\beta}{1-\alpha_\pi\beta} - \frac{\sigma\kappa}{\lambda_2}\frac{1}{(1-\lambda_2^{-1}\mathcal{B}^{-1})}\right]\frac{1}{1-\beta\mathcal{B}^{-1}}\varepsilon_t^M \end{aligned} \tag{57}$$

It is straightforward show how the monetary policy parameter affects inflation

$$\frac{\partial\lambda_1}{\partial\alpha_\pi} > 0, \quad \frac{\partial\lambda_2}{\partial\alpha_\pi} < 0, \quad \frac{\partial[\lambda_2(1-\lambda_1\beta)]}{\partial\alpha_\pi} < 0 \quad \frac{\partial\left(\frac{1-\lambda_1\beta}{1-\alpha_\pi\beta}\right)}{\partial\alpha_\pi} > 0 \tag{58}$$

More aggressive monetary policy—larger α_π—affects the equilibrium in the following ways

- amplifies the impacts on inflation from outstanding debt and exogenous disturbances to monetary policy and surpluses
- makes the effects of these shocks on inflation more persistent

Evidently, if fiscal policies set surpluses exogenously, monetary policy is impotent to offset fiscal effects on inflation. And adopting a more hawkish monetary policy stance has the perverse effect of amplifying and propagating the effects of shocks on inflation.

In this basic new Keynesian model, fiscal disturbances are transmitted to output through the path of the ex-ante real interest rate, as the consumption-Euler equation, (48), makes clear. Define the one-period real interest rate as $\hat{r}_t \equiv \hat{R}_t - E_t\hat{\pi}_{t+1}$. To simplify expressions, temporarily shut down the monetary policy shock, $\varepsilon_t^M \equiv 0$. Date the solution for inflation from (57) at $t+1$, take expectations, and substitute the monetary policy rule for the interest rate. After some tedious algebra, the equilibrium real interest rate is

$$\hat{r}_t = \frac{(\alpha_\pi - \lambda_1)(1-\lambda_1\beta)}{1-\alpha_\pi\beta}\left[\hat{b}_{t-1} - (1-\beta)\sum_{j=0}^{\infty}\hat{s}_{t+j}\right] \tag{59}$$

The lead coefficient, $\alpha_\pi - \lambda_1$, depends on monetary policy behavior and on all the model parameters. Because its sign can be positive or negative, lower expected surpluses may lower or raise the short-term real interest rate on impact.

Substituting the monetary policy rule into the definition of the real interest rate and suppressing the monetary policy shock, yields

$$\hat{r}_t \equiv \alpha_\pi\hat{\pi}_t - E_t\hat{\pi}_{t+1}$$

Using the Phillips curve to eliminate inflationary expectations we obtain

$$\hat{r}_t \equiv (\alpha_\pi - \beta^{-1})\hat{\pi}_t - \beta^{-1}\kappa\hat{Y}_t$$

which shows that a given level of positive inflation and output deviations from steady state will be consistent with lower real interest rates the smaller is the monetary policy response to inflation. The intuition is very similar to that in the endowment economy: a passive monetary policy that responds to inflation generates a sustained rise in inflation which does not facilitate the stabilization of single-period debt. In the new Keynesian case such a policy response mitigates the reduction in debt service costs which are an additional channel through which the passive monetary policy stabilizes debt in a sticky-price economy.

3.2 Maturity Structure in Regime F

We introduce the simplified maturity structure that section 2.2.2 describes, in which government debt maturity decays at the constant rate ρ each period, into the new Keynesian model of section 3.1. The no-arbitrage condition links bond prices to the one-period nominal interest rate

$$\hat{P}^m_t = -\hat{R}_t + \beta\rho E_t\hat{P}^m_{t+1} \tag{60}$$

which implies the term structure relation

$$\begin{aligned}\hat{P}^m_t &= -E_t\sum_{j=0}^{\infty}(\beta\rho)^j\hat{R}_{t+j} \\ &= -\frac{1}{1-\beta\rho\mathcal{B}^{-1}}\left[\alpha_\pi\hat{\pi}_t + \varepsilon^M_t\right]\end{aligned} \tag{61}$$

where we have substituted the simpler monetary policy rule in for the nominal interest rate.

The government's flow budget constraint is

$$\beta(1-\rho)\hat{P}_t^m + \beta\hat{b}_t^m + (1-\beta)\hat{s}_t + \hat{\pi}_t = \hat{b}_{t-1}^M \tag{62}$$

where we are defining $b_t^m \equiv B_t^m/P_t$ to be the real face value of outstanding debt.[26] Because bond prices depend on the expected infinite path of inflation and the monetary policy shock, analytical solutions along the lines of section 3.1.2, though feasible, are cumbersome. For example, the analog to the expected present value of inflation expression, (52), is

$$\frac{1}{1-\beta\mathcal{B}^{-1}}\left[1-\frac{\alpha_\pi\beta(1-\rho)}{1-\beta\rho\mathcal{B}^{-1}}\right]\hat{\pi}_t = \hat{b}_{t-1}^m - \left(\frac{1-\beta}{1-\beta\mathcal{B}^{-1}}\right)\hat{s}_t + \frac{\beta(1-\rho)}{(1-\beta\mathcal{B}^{-1})(1-\beta\rho\mathcal{B}^{-1})}\varepsilon_t^M$$

which collapses to (52) when $\rho = 0$ so all debt is one period. The solution for equilibrium inflation, like that when there is only one-period debt in equation (57), depends on all the parameters of the model through the eigenvalues λ_1 and λ_2, but the analytical expression for inflation is too complex to offer useful intuition.

One-period debt makes the value of debt depend only on the current nominal interest rate and, through the monetary policy rule, current inflation. A maturity structure makes that value depend on the entire expected path of nominal interest rates. This gives monetary policy an expanded role in debt stabilization, allowing expected future monetary policy to affect the value of current debt. This additional channel operates through terms in $1/(1-\beta\rho\mathcal{B}^{-1})$ that create double infinite sums in the equilibrium solution.

3.2.1 Impacts of Fiscal Shocks

Figures 3 and 4 illustrate the impacts of a serially correlated increase in the primary fiscal deficit financed by nominal bond sales.[27] Figure 3 maintains that all debt is one period to focus on how different monetary policy rules alter the impacts of a fiscal expansion.

When monetary policy pegs the nominal interest rate—$\alpha_\pi = \alpha_Y = 0$—it fixes the bond price, which front loads fiscal adjustments through current inflation and the real interest rate. Inflation rises, the real rate falls and output increases. Responses inherit the serial correlation properties of the fiscal disturbance. As monetary policy becomes progressively less passive, reacting more strongly to inflation and output, it amplifies and propagates the fiscal shock (dashed lines in figure 3). By reacting more strongly to inflation, monetary

[26] The real market value is $P_t^m B_t^m/P_t$. To derive (62), we use the steady-state relationships $P^{m*} = 1/(\beta^{-1}-\rho)$ and $s^*/b^{m*} = (1-\beta)/(1-\beta\rho)$ in log-linearizing the government budget constraint.

[27] We calibrate the model to an annual frequency, setting $\beta = 0.95, \sigma = 1, \kappa = 0.3$. The surplus is $AR(1)$, $\hat{s}_t = \rho_{FP}\hat{s}_{t-1} + \varepsilon_t^F$, with $\rho_{FP} = 0.6$.

policy ensures that the real interest rate declines by less, tempering the short-run output increases.

The figure makes clear the role that debt plays in propagating shocks in regime F. Stronger and more persistent nominal interest rate increases transmit directly into stronger and more persistent growth in the nominal market value of debt.[28] And persistently higher nominal debt keeps household nominal wealth and, therefore, nominal demand elevated, creating strong serial correlation in inflation and output. This internal propagation mechanism through government debt is absent from regime M, where higher debt carries with it the promise of higher taxes that eliminate wealth effects.

Figure 4 holds the monetary policy rule fixed, setting $\alpha_\pi = \alpha_Y = 0.5$, to reveal how changes in maturity affect fiscal impacts. The figure contrasts one-period debt (dotted-dashed lines) to an average of 5-year maturity (solid lines) and consol debt (dashed lines). Longer maturities force more of the adjustment to higher deficits into lower bond prices, which push more of the impacts into low frequency movements in long-run inflation and real interest rates.[29]

Although short-run inflation is higher with one-period debt, in the long run inflation is lower with shorter maturity bonds. With long debt, bond prices reflect anticipated inflation rates farther into the future, in essence spreading inflationary effects over longer horizons. The cost of doing so is to raise the long-run inflation impacts of fiscal policy.

Another way to summarize the dynamic impacts of fiscal disturbances is to ask how a shock that raises primary deficits by a certain amount gets financed intertemporally, as a function of various model parameters. Underlying the calculations in table 2 are two basic mechanisms that stabilize debt in the face of the surplus shock. First are the revaluation effects that we can summarize by examining the ex-post real return to holding government bonds in any period

$$r_t^m = \frac{(1+\rho P_t^m)}{P_{t-1}^m}\frac{1}{\pi_t}$$

or in linearized form

$$\hat{r}_t^m = \rho\beta \hat{P}_t^m - \hat{\pi}_t - \hat{P}_{t-1}^m$$

By contrasting this with the ex-ante returns the bond holders were expecting when they purchased the bonds in period $t-1$ we can identify the scale of the revaluation effects, which linearized, are

$$\hat{r}_t^m - E_{t-1}\hat{r}_t^m = -(\hat{\pi}_t - E_{t-1}\hat{\pi}_t) + \rho\beta(\hat{P}_t^m - E_{t-1}\hat{P}_t^m) \tag{63}$$

[28] Growth in the nominal market value of debt is $P_t^m B_t^M / P_{t-1}^m B_{t-1}^m$.

[29] The long-term real interest rate, $\hat{r}_t^L$, comes from combining the bond-pricing equation and the Fisher relation to yield the recursion $\hat{r}_t^L = \hat{r}_t + \beta\rho E_t \hat{r}_{t+1}^L$. The long-run inflation rate, $\hat{\pi}_t^L$, which is the present value of inflation discounted by $\beta\rho$, may be computed as $\hat{\pi}_t^L = -\hat{r}_t^L - \hat{P}_t^m$.

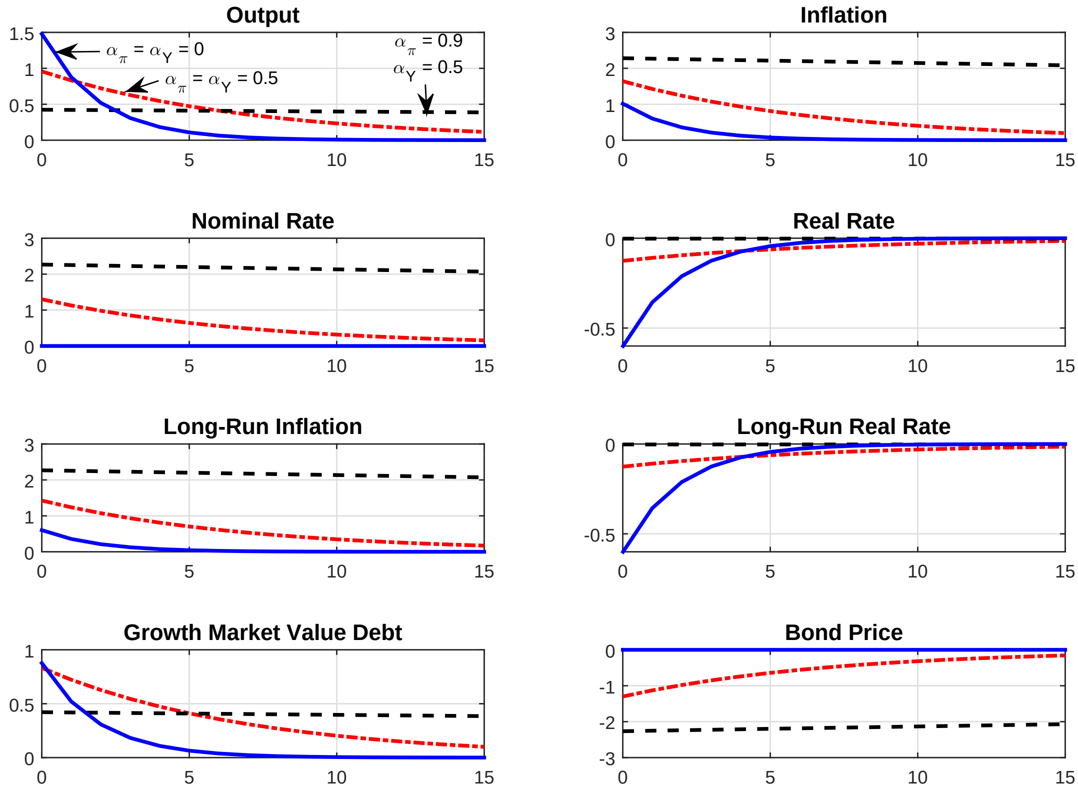

Figure 3: Responses to a 20 percent increase in the initial deficit under alternative monetary policy rules when all debt is one period. Calibration reported in footnote 27.

The first term on the right in (63) gives the losses suffered by bondholders due to surprise inflation in the initial period. The second term gives the losses suffered by holders of mature debt ($\rho > 0$) arising from jumps in bond prices caused by innovations to the expected future path of inflation. These latter revaluation effects are borne by the existing holders of government debt and arise for innovations to the path of inflation over the time to maturity of the debt stock they hold. In the sticky price economy these effects can be complemented by reductions in the ex-ante real rates of return received by future bondholders, which reduce effective debt service costs to create an additional channel through which debt can be stabilized.[30]

In the case of one-period debt it is only the surprise inflation in the initial period that

[30] An equivalent interpretation comes from thinking about the value of debt in the "forward" direction, as being determined by the expected present value of surpluses. Lower real interest rates raise real discount factors to increase the present value of a given strem of surpluses.

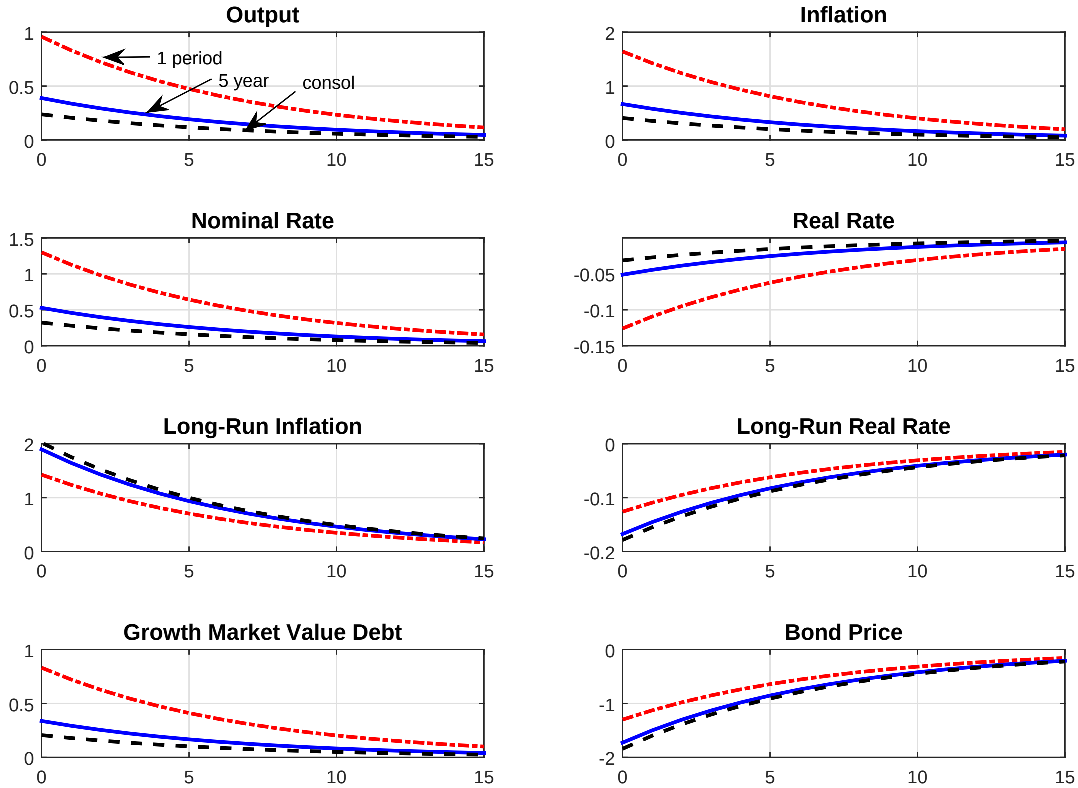

Figure 4: Responses to a 20 percent increase in the initial deficit under alternative maturity structures. Calibration reported in footnote 27.

reduces the real value of government debt. This is then combined with reductions in ex-ante real interest rates to stabilize debt. As α_π increases, there is less reliance on the latter effect and larger jumps in the initial rate of inflation are required to satisfy the bond valuation equation. When we move to longer period debt, there is an additional revaluation effect through the impact of innovations to the path of inflation on bond prices. With bond prices adjusting, we can have smaller, but more sustained increases in inflation that reduce the real market value of debt. These continue to be combined with reductions in ex-ante real interest rates to satisfy the bond valuation equation with these debt service cost effects falling as monetary policy becomes less passive.

To see how this affects the decomposition of the adjustment required to stabilize the debt stock in the face of a surplus shock consider the evolution of the market value of government debt

$$\widetilde{b}_t = r_t^m \widetilde{b}_{t-1} - s_t$$

where $\widetilde{b}_t \equiv \frac{P_t^m Bt}{P_t}$. This can be linearized as

$$\beta\widehat{\widetilde{b}}_t = \hat{r}_t^m + \widehat{\widetilde{b}}_{t-1} - (1-\beta)\widehat{s}_t$$

Using the expected value of surpluses, $\xi_t \equiv (1-\beta)E_t\sum_{j=0}^{\infty}\beta^j\widehat{s}_{t+j}$ which implies $(1-\beta)\widehat{s}_t = \xi_t - \beta E_t\xi_{t+1}$, this becomes

$$\beta(\widehat{\widetilde{b}}_t - E_t\xi_{t+1}) - \hat{r}_t^m = \widehat{\widetilde{b}}_{t-1} - \xi_t$$

Iterating forward we obtain

$$\begin{aligned} \xi_t &= \widehat{\widetilde{b}}_{t-1} + \hat{r}_t^m + E_t\sum_{j=1}^{\infty}\beta^j\hat{r}_{t+j}^m \\ &= \widehat{\widetilde{b}}_{t-1} - \hat{P}_{t-1}^m + \beta\rho\hat{P}_t^m - \hat{\pi}_t + E_t\sum_{j=1}^{\infty}\beta^j\hat{r}_{t+j}^m \end{aligned} \tag{64}$$

The required adjustment is made up of surprise changes in the returns to existing bond holders $\hat{r}_t^m$ as well as expected future returns on bond holdings, $E_t\sum_{j=1}^{\infty}\beta^j\hat{r}_{t+j}^m$. The former is made up of jumps in the initial rate of inflation combined with changes in bond prices to the extent that bonds have a maturity greater than one period, $\rho > 0$. The latter captures the reduction in ex-ante real interest rates which can occur in our sticky price economy.

Table 2 computes the objects in (64) from impulse responses to a deficit innovation. When debt is single period, bond prices do not contribute to financing the deficit. If monetary policy pegs the nominal interest rate, current inflation and future real interest rates play nearly equally important roles. As monetary policy reacts more aggressively to inflation and output, real interest rate responses are tempered, and an increasing fraction of the adjustment occurs through inflation at the time of the fiscal innovation. Longer maturity debt brings bond prices into the adjustment process, and their role grows with both the maturity of debt and the aggressiveness of monetary policy. As a consequence, current inflation moves much less. Consol bonds, together with aggressive monetary policy, push nearly all the adjustment into bond prices, with contemporaneous inflation playing only a minimal role, as the last row of the table reports.

3.2.2 Impacts of Monetary Shocks

Section 2.1.2.2 describes the effects of exogenous monetary policy disturbances in an endowment economy under regime F. Because future surpluses do not adjust to neutralize the

α_π	α_Y	Maturity	% due to $\hat{\pi}_t$	% due to $\hat{P}^m_t$	% due to $\hat{r}^m_{t+j}$
0	0	1 period	44	0	56
0.5	0.5	1 period	71	0	29
0.9	0.5	1 period	98	0	2
0.5	0.5	5 year	29	59	12
0.9	0.5	5 year	20.4	79.2	0.4
0.5	0.5	consol	18	75	7
0.9	0.5	consol	6	94	0

Table 2: The fiscal shock initially raises the deficit by 20 percent. "% due to" are the ratios of the right-hand components of (64) to ξ_t, which is computed from the impulse response of $\hat{s}_{t+j}$, as described in the text. Calibration reported in footnote 27.

wealth effects of monetary policy, contractionary policy—a higher path for the nominal interest rate—raises household interest receipts and wealth, raising nominal aggregate demand. A similar phenomenon can arise in the new Keynesian model, though the dynamics are more interesting.

Figure 5 reports the impacts of an exogenous monetary policy action that raises the nominal interest rate. To highlight the bheavior of monetary policy in regime F, we consider three different monetary policy rules. A rule that pegs the nominal rate (solid lines) raises the short-term real interest rate and depresses output in the short run. Despite the drop in output, inflation rises immediately, even in a model where the Phillips curve implies a strong positive relationship between output and inflation contemporaneously ($\kappa = 0.3$).

This seemingly anomalous outcome underscores the centrality of wealth effects in regime F. Higher nominal interest rates raise households' interest receipts in the future, triggering an expectations of higher future demand and inflation.[31] Through the Phillips curve, the higher expected inflation dominates the deflationary effects of lower output to raise inflation on impact. Expectations are critical to output effects as well. After an initial decline, output always eventually rises because the real interest rate declines at longer horizons.

More aggressive monetary policy behavior (dashed lines) transforms the transitory increase in the policy rate into larger and more persistent increases. Those higher nominal interest rates raise both the growth rate of the nominal market value of debt and real interest receipt. The resulting wealth effects raise and prolong the higher inflation.

That an exogenous monetary policy "contraction," which raises the nominal interest rate, also raises inflation may seem to contradict evidence from the monetary VAR literature. This pattern, dubbed the "price puzzle" by Eichenbaum (1992), is sometimes taken to indicate that monetary policy behavior is poorly identified, perhaps by misspecifying the central

[31]Real interest receipts are defined as $[(1 + \rho P^m_t)/P^m_{t-1}](b^m_{t-1}/\pi_t)$.

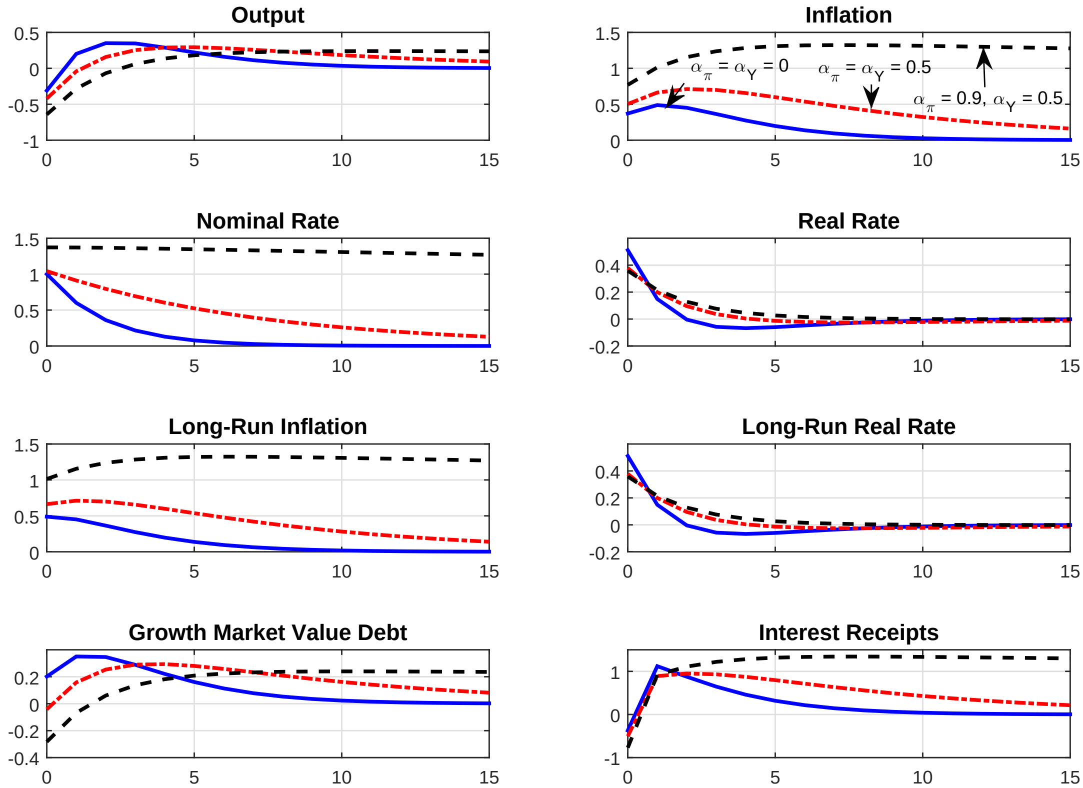

Figure 5: Responses to a 1 percent monetary contraction under alternative monetary policy rules with only one-period government debt. Calibration reported in footnote 27. The monetary policy shock follows the $AR(1)$ process $\varepsilon_t^M = \rho_{MP}\varepsilon_{t-1}^M + \zeta_t^M$ with $\rho_{MP} = 0.6$.

bank's information set, as Sims (1992) argues. Figure 5 makes clear that there is nothing puzzling about the pattern from the perspective of the fiscal theory.

Introducing long debt makes impulse responses accord better with VAR evidence because bond prices absorb much of the monetary shock. Figure 6 contrasts one-period (dotted-dashed lines) with 5-year (solid lines) and consol debt (dashed lines). By reducing growth in the market value of debt, longer maturities attenuate the inflationary effects and make the short-run decline in output longer lasting. Inflation does eventually rise, as it must if bond prices are lower. Sims (2011) calls the pattern of falling, then rising inflation following a monetary contraction "stepping on a rake."

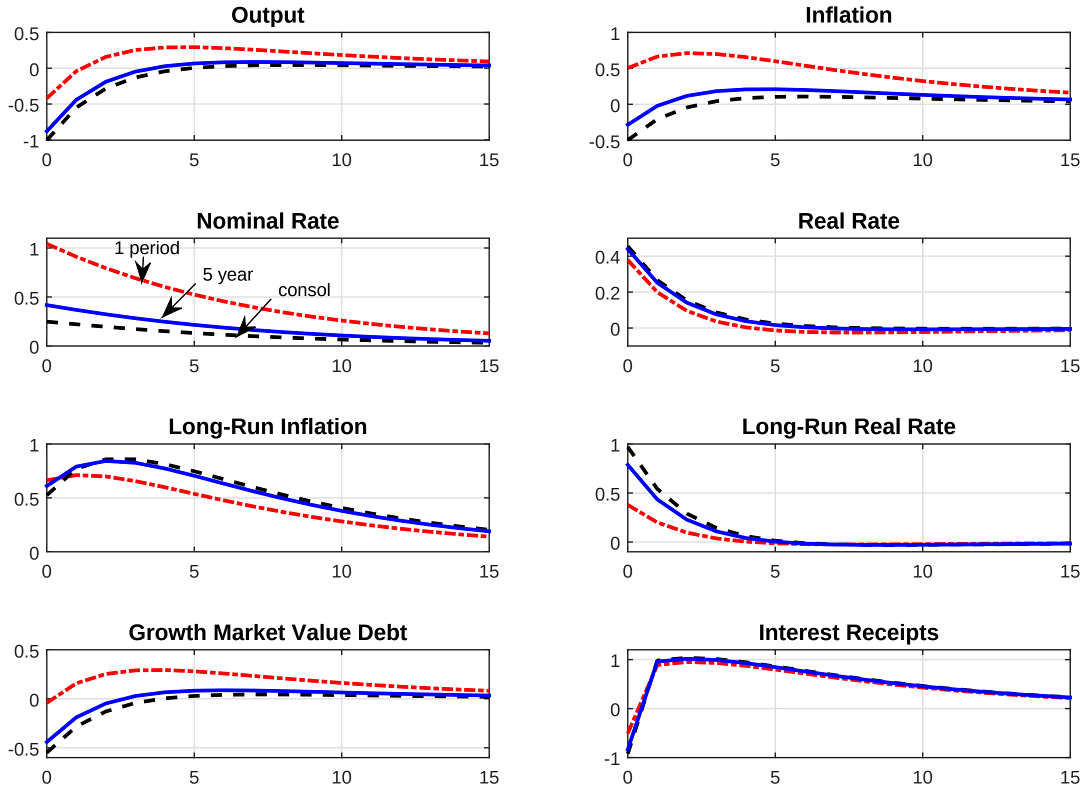

Figure 6: Responses to a 1 percent monetary contraction under alternative maturity structures. Calibration reported in footnote 27. The monetary policy shock follows the $AR(1)$ process $\varepsilon_t^M = \rho_{MP}\varepsilon_{t-1}^M + \zeta_t^M$ with $\rho_{MP} = 0.6$.

4 Endowment Economies with Optimal Monetary and Fiscal Policies

In this section we turn to consider the nature of optimal policy in our simple endowment economy. In doing so we cut across various strands of the literature addressing optimal monetary and fiscal policy issues.

4.1 Connections to the Optimal Policy Literature

We begin by considering Ramsey policies where the policymaker has an ability to make credible promises about how they will behave in the future, before turning to time-consistency issues below. We start by building on Sims's (2013) analysis. He considers a simple linearized model of tax smoothing under commitment in the face of transfer shocks and long-term debt where the policymaker can use costly inflation surprises as an alternative to distortionary taxation in to ensure fiscal solvency. We extend that work in several ways. Specifically, we allow for a geometric maturity structure which nests single-period debt and consols as special cases, employ non-linear model solution techniques and allow for anticipated and unanticipated government spending shocks, in addition to transfer shocks. Non-linear solutions allow us to consider the way in which the size of the debt stock, together with its maturity structure, influences the optimal combination of monetary and fiscal policy in debt stabilization. Innovations to the expected path for inflation can affect bond prices in a way which helps to satisfy the bond valuation equation even without any fiscal adjustment. These bond price movements are effective only if applied to a non-zero stock of outstanding liabilities such that the optimal balance between inflation and tax financing of fiscal shocks is likely to depend on both the level of government debt and its maturity structure.

Without an ability to issue state-contingent debt or use inflation surprises to stabilize debt, Barro (1979) showed that debt and taxes should follow martingale processes to minimize the discounted value of tax distortions. While Barro did consider the impact of surprise inflation on the government's finances, these were treated as exogenous shocks rather than something that can be optimally employed to further reduce tax distortions. Lucas and Stokey (1983) is an equally influential paper that reaches quite different conclusions on the optimal response of tax rates to shocks. Lucas and Stokey consider an economy where the government can issue real state-contingent debt and show that it is optimal for a government to issue a portfolio of debt where the state-contingent returns to that debt isolate the government's finances from shocks so that there is no need for taxes to jump in the manner of Barro's tax smoothing result. Instead, taxes are largely flat and inherit the dynamic properties of the exogenous shocks hitting the economy.

A large part of the post-Lucas and Stokey literature considers the implications of debt that is not state-contingent, as well as ways of converting the payoffs from portfolios of non-state contingent debt into state-contingent payoffs. A key result is that when debt payoffs are not (or cannot be made) state contingent, then the optimal policy looks more like Barro's tax smoothing result. Aiyagari, Marcet, Sargent, and Seppälä (2002) show this by assuming that debt is single period and non-contingent in a model otherwise identical to that of Lucas and Stokey. How might non-contingent debt instruments be made to mimic the payoffs that would be generated by state-contingent debt? Two approaches have been suggested in the literature. First, surprise inflation can render the real payoffs from risk-free nominal bonds state contingent. For example, Chari, Christiano, and Kehoe (1994) use a model where surprise inflation is costless to show that the real contingencies in debt exploited by Lucas and Stokey could be created through monetary policy via the mechanism of surprise inflation when government debt is nominal. This underpins Sims's (2001) results in a model with costless inflation in which tax rates should be held constant to finances any fiscal shocks solely with surprise movements in inflation.

When we start to introduce a cost to surprise inflation, the optimal policy can be strikingly different. For a jointly determined optimal monetary and fiscal policy operating under commitment, Schmitt-Grohé and Uribe (2004) show that in a sticky-price stochastic production economy, even a miniscule degree of price stickiness will result, under the optimal policy, in a steady-state rate of inflation marginally less than zero, with negligible inflation volatility. In other words, although the optimal policy under flexible prices would be to follow the Friedman rule and use surprise inflation to create the desired state-contingencies in the real pay-offs from nominal debt, even a small amount of nominal inertia heavily tilts optimal policy towards zero inflation with little reliance on inflation surprises to insulate the government's finances from shocks. As in Benigno and Woodford (2004), Schmitt-Grohé and Uribe (2004) return to the tax smoothing results of Barro (1979) thanks to the effective loss of state-contingent returns to debt when prices are sticky. Sims (2013) argues that this may be due to fact that Schmitt-Grohe and Uribe only consider single-period debt, and that with longer term debt the efficacy of using innovations to the expected path of inflation to affect bond prices would be enhanced. This is the first issue to which we turn: to what extent will the optimizing policymaker rely on fiscal theory-type revaluations of debt through innovations to the expected path of prices?

While the state-contingencies in real bond payoffs can be generated through the impact of surprise inflation on nominal bonds, an alternative approach when bonds are real is to exploit variations in the yield curve to achieve the same contingencies for the government's whole bond portfolio. With single period risk-free real bonds, Ramsey policy in the Lucas and Stokey model possesses a unit root as in Barro. Angeletos (2002) and Buera and Nicolini (2004) use the maturity structure of non-state contingent real bonds to render the overall portfolio state contingent. With two states for government spending, for example, a portfolio of positive short-term assets funded by issuing long-term debt can insulate the government's finances from government spending shocks. More generally, with a sufficiently rich maturity structure the policymaker can match the range of the stochastic shocks hitting the economy and achieve this hedging. The second broad optimal policy question we consider is: what is the role of debt management in insulating the government's finances from shocks?

Having looked at the ability of the Ramsey policymaker to both hedge against shocks and utilize monetary policy as a debt stabilization tool when complete hedging is not possible, we turn to consider the time-inconsistency problem inherent in such policies. We find that constraining policy to be time consistent radically affects the policymaker's ability to hedge against fiscal shocks and generates serious "debt stabilization bias" problems, as in Leith and Wren-Lewis (2013), that are akin to the inflationary bias problems analyzed in the context of monetary economies.

We begin by considering the role inflation surprises play in optimal policy in our simple endowment economy with a geometrically declining maturity structure. We shall then generalize these results to a more general maturity structure and consider the role of debt management in hedging for fiscal shocks. We then turn to a simple example where complete hedging is feasible.

4.2 The Model

We follow Sims (2013) in defining the inverse of inflation as $\nu_t = \pi_t^{-1}$, and assume the policymaker's objective function is given by

$$-E_0 \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\tau_t^2 + \theta(\nu_t - 1)^2 \right] \tag{65}$$

which the policymaker maximizes subject to the constraints given by the bond valuation equation (after assuming a specific form for per-period utility, $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$)

$$\beta E_t \frac{(1 + \rho P_{t+1}^m)}{P_t^m} \nu_{t+1} \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} = 1 \tag{66}$$

the government's flow budget constraint

$$b_t P_t^m = (1 + \rho P_t^m) b_{t-1} \nu_t + g_t - \tau_t - z_t \tag{67}$$

and the associated transversality condition

$$\lim_{j \to \infty} E_t \left(\prod_{i=0}^{j} \frac{1}{R_{t+i+1}^m \nu_{t+i+1}} \right) \frac{P_{t+j}^m B_{t+j}^m}{P_{t+j}} \geq 0 \tag{68}$$

where $R_{t+1}^m \equiv (1 - \rho P_{t+1}^m)/P_t^m$, and government spending and/or transfers follow exogenous stochastic processes.

4.3 Ramsey Policy

We analyze the time-inconsistent Ramsey policy for our endowment economy given the policymaker's objective function by forming the following Lagrangian

$$\begin{aligned} L_t &= E_0 \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[-\frac{1}{2}(\tau_t^2 + \theta(\nu_t - 1)^2) \right] \\ &\quad + \mu_t \left[\beta E_t \frac{(1+\rho P_{t+1}^m)}{P_t^m} \nu_{t+1} \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} - 1 \right] \\ &\quad + \lambda_t (b_t P_t^m - (1+\rho P_t^m) b_{t-1} \nu_t - g_t - z_t + \tau_t) \end{aligned}$$

which yields the first-order conditions

$$\begin{aligned} \tau_t &: -\tau_t + \lambda_t = 0 \\ \nu_t &: -\theta(\nu_t - 1) + \mu_{t-1} \frac{(1+\rho P_t^m)}{P_{t-1}^m} \left(\frac{c_t}{c_{t-1}} \right)^{-\sigma} - (1+\rho P_t^m) \lambda_t b_{t-1} = 0 \\ P_t^m &: -\frac{\mu_t}{P_t^m} + \mu_{t-1} \rho \frac{\nu_t}{P_{t-1}^m} \left(\frac{c_t}{c_{t-1}} \right)^{-\sigma} + \lambda_t (b_t - \rho \nu_t b_{t-1}) = 0 \\ b_t &: \lambda_t P_t^m - \beta E_t (1+\rho P_{t+1}^m) \nu_{t+1} \lambda_{t+1} = 0 \end{aligned}$$

Defining $\tilde{\mu}_t \equiv \frac{\mu_t}{P_t^m c_t^{-\sigma}}$ the system to be solved for $\{P_t^m, \tilde{\mu}_t, \nu_t, \tau_t, b_t, c_t\}$ is given by

$$\begin{aligned} -\theta(\nu_t - 1) + \tilde{\mu}_{t-1}(1+\rho P_t^m) c_t^{-\sigma} - (1+\rho P_t^m)\tau_t b_{t-1} &= 0 \\ \tau_t b_t - \tilde{\mu}_t c_t^{-\sigma} - \rho \nu_t (\tau_t b_{t-1} - \tilde{\mu}_{t-1} c_t^{-\sigma}) &= 0 \\ \tau_t P_t^m - \beta E_t (1+\rho P_{t+1}^m) \nu_{t+1} \tau_{t+1} &= 0 \\ \beta E_t \frac{(1+\rho P_{t+1}^m)}{P_t^m} \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \nu_{t+1} - 1 &= 0 \\ b_t P_t^m - (1+\rho P_t^m) b_{t-1} \nu_t - g_t + \tau_t - z_t &= 0 \\ g_t - (1-\rho_g) g^* - \rho_g g_{t-1} - \varepsilon_t^g &= 0 \\ z_t - (1-\rho_z) z^* - \rho_z z_{t-1} - \varepsilon_t^z &= 0 \\ y - c_t - g_t &= 0 \end{aligned}$$

with two exogenous shocks describing the evolution of government consumption, g_t, and transfers, z_t and two endogenous state variables, $\tilde{\mu}_{t-1}$ and b_{t-1}, where the former captures the history dependence in policy making under commitment.

To obtain some intuition for how policy operates under commitment, it is helpful to consider three polar cases. First, where inflation is costless, so that $\theta = 0$. Second, where inflation is so costly that the economy can be considered to be real, $\theta \to \infty$. Third, we allow inflation to be costly $\theta > 0$, but assume that taxes have reached the peak of the Laffer curve so that they are no longer available to engage in tax smoothing and instead are held constant, $\tau_t = \tau^*$.

4.3.1 Costless Inflation

In the former case, where inflation is costless, the first two frist-order conditions imply

$$\widetilde{\mu}_{t-1} c_t^{-\sigma} = \tau_t b_{t-1} \tag{69}$$

and

$$\tau_t b_t - \widetilde{\mu}_t c_t^{-\sigma} = \rho \nu_t (\tau_t b_{t-1} - \widetilde{\mu}_{t-1} c_t^{-\sigma}) \tag{70}$$

Substituting the first into the second, lagging one period and comparing the first condition yields

$$\tau_t = \left(\frac{c_t}{c_{t-1}} \right)^{-\sigma} \tau_{t-1} \tag{71}$$

In the absence of government spending shocks (the only source of variation in private consumption in our simple endowment economy) taxes are unchanged. But taxes are rising (falling) whenever government spending is rising (falling). In the case of transfer shocks, inflation jumps to satisfy the bond valuation equation and this is a pure case of the fiscal theory. But when bonds have a maturity beyond a single period, there are an infinite number of patterns of inflation which can satisfy this, due to the impact inflation has on bond prices. While there is a unique required discounted magnitude of surprise inflation needed to satisfy the government debt valuation condition, there are a variety of paths which can achieve that magnitude. When the fiscal shock is a shock to government consumption, this affects real interest rates so that even although inflation can costlessly stabilize debt at its initial steady state level, there is still tilting of tax rates: during periods of high real interest rates, it is desirable to suffer the short-run costs of higher taxation to avoid the longer run costs of supporting the higher steady-state level of debt that would emerge when higher interest rates raise the rate of debt accumulation. In this case it is only because of the commitment to honour the past promises not to deflate away the government's outstanding liabilities that there are positive tax rates at all.

4.3.2 Real Economy

In the second case, inflation is so costly it would never be used under the optimal policy, $\theta \to \infty$ and $\nu_t = 1$. As a result, we rely on jumps in the tax rate to satisfy government solvency and we return to a world of pure tax smoothing, where the tax rate follows the path implied by the first order condition

$$\tau_t P_t^m = \beta E_t(1 + \rho P_{t+1}^m)\tau_{t+1} \tag{72}$$

Under a perfect foresight equilibrium this reduces to

$$\tau_t = \tau_{t+1}\frac{u'(c_{t+1})}{u'(c_t)} \tag{73}$$

This tax rate is constant in the face of transfer shocks, but will be tilted in the presence of government spending shocks—the tax rate at t is increasing (decreasing) when public consumption is anticipated to rise (fall). The fact that it is purely forward looking captures the usual tax smoothing result that the tax rate will jump to the level required to satisfy the government's budget constraint, although we have tilting in the tax rate to capture changes in real interest rates induced by government spending shocks. Eventually the tax rate will achieve a new long-run value consistent with servicing the new steady-state level of debt.

4.3.3 Intermediate Case

In the intermediate case where $0 < \theta < \infty$, the tax smoothing condition remains as above, but will be combined with a pattern of inflation described by

$$\begin{aligned}
-\theta(\nu_t - 1) + \frac{\mu_{t-1}}{P_{t-1}^m}(1 + \rho P_t^m) - (1 + \rho P_t^m)\tau_t b_{t-1} &= 0 \\
\tau_t b_t - \frac{\mu_t}{P_t^m} - \rho\nu_t\left(\tau_t b_{t-1} - \frac{\mu_{t-1}}{P_{t-1}^m}\right) &= 0 \\
b_t P_t^m - (1 + \rho P_t^m)b_{t-1}\nu_t - g_t - z_t + \tau_t &= 0
\end{aligned}$$

which will deliver initial jumps in inflation, bond prices and tax rates to ensure fiscal solvency. These first-order conditions also imply that inflation returns to 1 in steady state, so the optimal commitment policy makes any inflation only temporary. But there is a continuum of steady state debt levels, each with an associated optimal tax rate, that would be consistent with the steady state of the first-order conditions under commitment.

When we consider the third case where taxes are no longer available for tax smoothing, either for political reasons or because the tax rate has reached the peak of the Laffer curve, the relevant optimality conditions become

$$\begin{aligned}
\lambda_t P_t^m - \beta E_t(1+\rho P_{t+1}^m)\nu_{t+1}\lambda_{t+1} &= 0 \\
-\theta(\nu_t - 1) + \widetilde{\mu}_{t-1}(1+\rho P_t^m)c_t^{-\sigma} - (1+\rho P_t^m)\lambda_t b_{t-1} &= 0 \\
\lambda_t b_t - \widetilde{\mu}_t c_t^{-\sigma} - \rho\nu_t(\lambda_t b_{t-1} - \widetilde{\mu}_{t-1}c_t^{-\sigma}) &= 0 \\
b_t P_t^m - (1+\rho P_t^m)b_{t-1}\nu_t - g_t - z_t + \overline{\tau} &= 0
\end{aligned}$$

where the tax rate is fixed at $\bar{\tau}$.

Here the unit root in government debt is no longer present because taxes cannot adjust to support a new steady state debt level, and inflation cannot influence future surpluses. Instead, inflation must be adjusted to ensure fiscal solvency by returning debt to the steady state level consistent with the unchanged tax rate. The pattern of inflation also depends on the maturity structure of the inherited debt stock. To see this more clearly we consider the perfect foresight solution in which the first-order condition for debt implies that $\lambda_t = \lambda_{t+1}$. Combining the second and third conditions yields

$$\nu_t(\nu_t - 1) = \left[1 + (\rho P_t^m)^{-1}\right]\nu_{t+1}(\nu_{t+1} - 1) \tag{74}$$

which describes the dynamics of inflation. Inflation rises following a fiscal shock that would otherwise make debt initially higher and then decline towards its steady state value. The rate of convergence depends on the inverse of the maturity parameter multiplied by the bond price, which initially falls, but then recovers as the period of inflation passes. When $\rho = 0$ the inflation only occurs in the initial period, but becomes more protracted the longer is the maturity of government debt. Similar inflation dynamics are observed when taxes are smoothed, although the magnitude of the initial jump in inflation will be reduced to the extent that tax rates rise to stabilize debt at a higher level in the face of a given shock.

4.4 Numerical Results

The grid-based approach to solving the stochastic version of the model under the simple rules works well when the economy has a well defined steady-state to which it returns. With commitment policies the model enters a new steady state following the realization of a shock, which makes the model difficult to solve using these techniques. For this reason, when considering commitment we restrict attention to perfect foresight equilibrium paths following an initial shock. These paths are computed as follows. We guess the new steady-state value of debt and solve the steady state of the Ramsey problem conditional on that case. This serves as a terminal condition on the model solution 800 periods in the future.

The Ramsey first order conditions are then solved for 800 periods conditional on this guess for the ultimate steady state. If the solution exhibits a discontinuity between the final period of the solution and the imposed terminal condition, the steady state guess is revised. This process continues until the guessed new steady state is indeed the steady state to which the economy now settles.

We begin by considering the same transfers shock considered above for various degrees of maturity and different initial debt to GDP ratios. The autocorrelated shock to transfers reduces the discounted value of future surpluses and requires a monetary and/or fiscal adjustment. These adjustments are plotted in figure 7 for various initial debt-to-GDP ratios and debt maturities. The first column starts from an initial debt to GDP ratio of zero. When debt is initially zero and the initial tax rate of $\tau = 0.39$ can support the initial level of transfers and public consumption, under the optimal policy there is no inflation, regardless of the maturity of debt. This is due to the fact that surprise changes in inflation or bond prices only help satisfy the government's intertemporal budget constraint if there is already an initial debt stock for them to act on. Even though the debt that will be issued as a result of the transfer shock is of different maturities across the experiments reported in the first column of the figure, this will not affect the optimal policy response to the transfers shock when there is initially no debt. The tax rate jumps to a permanently higher level to support a higher steady state debt level, as under Barro's (1979) original tax smoothing result.

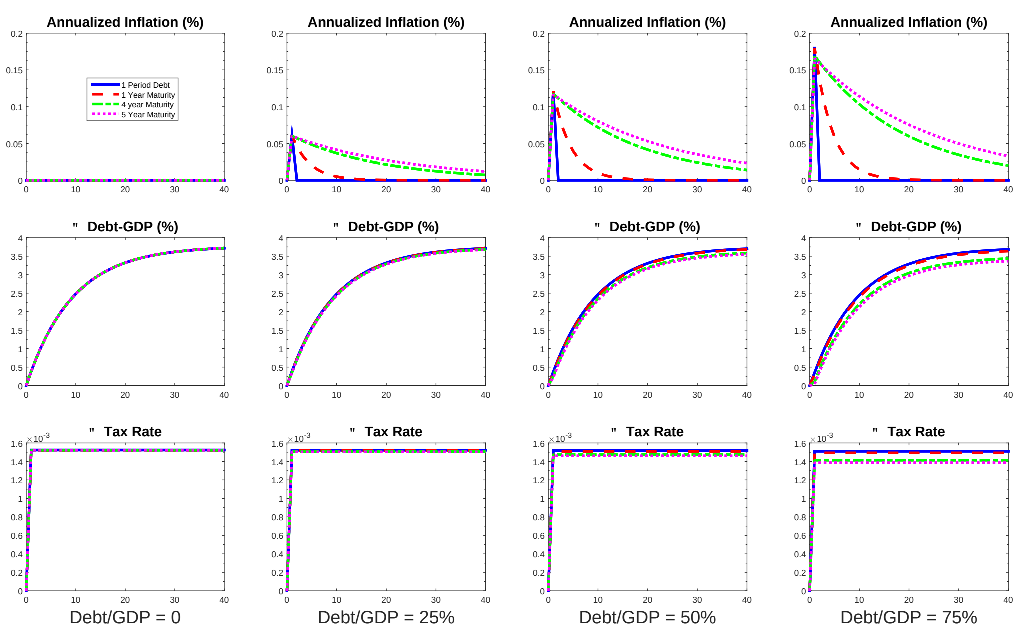

Figure 7: Optimal policy in response to higher transfers with different debt levels and maturities.

The second column begins from an initial steady state with a debt to GDP ratio of 25 percent (and a supporting initial tax rate of $\tau = 0.4$). Now there is mild use of inflation to offset the effects of the transfers shock. Inflation is smaller but more sustained the longer is the average maturity of debt. As maturity lengthens, inflation surprises play an increasingly important role in stabilizing debt, with smaller adjustments in taxes. At higher debt levels, the role of inflation and maturity grow in importance as substitutes for distorting taxes. Ultimately, the increase in inflation is unwound (it serves no purpose as the initial debt stock matures) and there is a permanent increase in both the debt stock and tax rates. These examples underscore that optimal policy is highly state dependent, particularly with respect to the level and maturity of debt at the time the shock hits.

When we turn to government spending shocks in figure 8, the story is similar except that now, through the stochastic discount factor, public consumption tilts the optimal path of taxes and affects the magnitude of the fiscal and inflation adjustments needed to satisfy the debt valuation equation. With no initial stock of debt, the subsequent debt maturity structure is irrelevant and the optimal policy does not generate any inflation. But for a positive initial debt level, the spike in inflation for one-period debt is several orders of magnitude larger than for the portfolio of bonds with an average maturity of 8 years. With only short debt, the inflation is immediately eliminated, while the slight rise in inflation is sustained in the presence of longer term debt. Sustained inflation decreases bond prices that reduce the value of debt to for the more mature bonds, permitting the policymaker to reduce the required jump in the tax rate needed to support the higher level of steady-state debt. Interestingly, the higher tax rates during the period of raised public consumption end up reducing the new steady-state level of debt so that the new steady-state tax rate is actually lower than before the shock. This contrasts to the case of the transfer shock where debt levels were raised following the shock.

Figure 9 reports optimal responses to news of a sustained increase in government spending five year in the future. Initially inflation falls and the tax rate jumps down in support of a debt level that is ultimately lower, despite the increase in government spending. This occurs because the policymaker raises the tax rate for the duration of the rise in public consumption to avoid the rapid accumulation of government debt in a period when real interest rates are relatively high. Bond prices rise as the anticipated increase in government spending approaches and then drop dramatically when the spending is realized.

In this experiment the cost of inflation is quite high, $\theta = 10$. A lower cost would lead to greater reliance on the use of monetary policy and innovations in the anticipated path of prices to stabilize debt. As we show below, even this relatively conservative weight on the costs of inflation still generates a sizeable inflation bias when we consider time-consistent

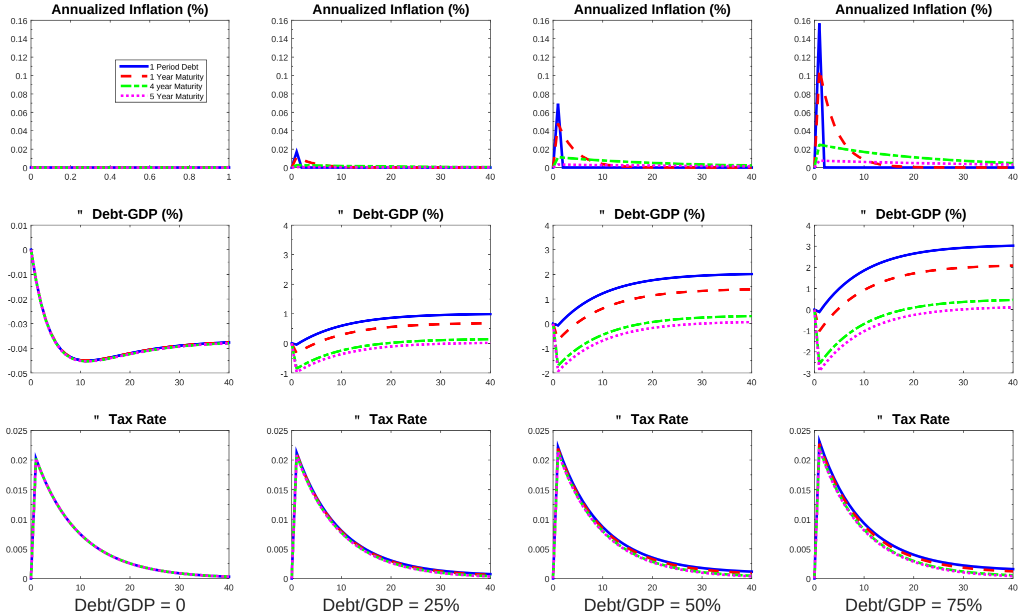

Figure 8: Optimal policy in response to higher government spending with different debt levels and maturities.

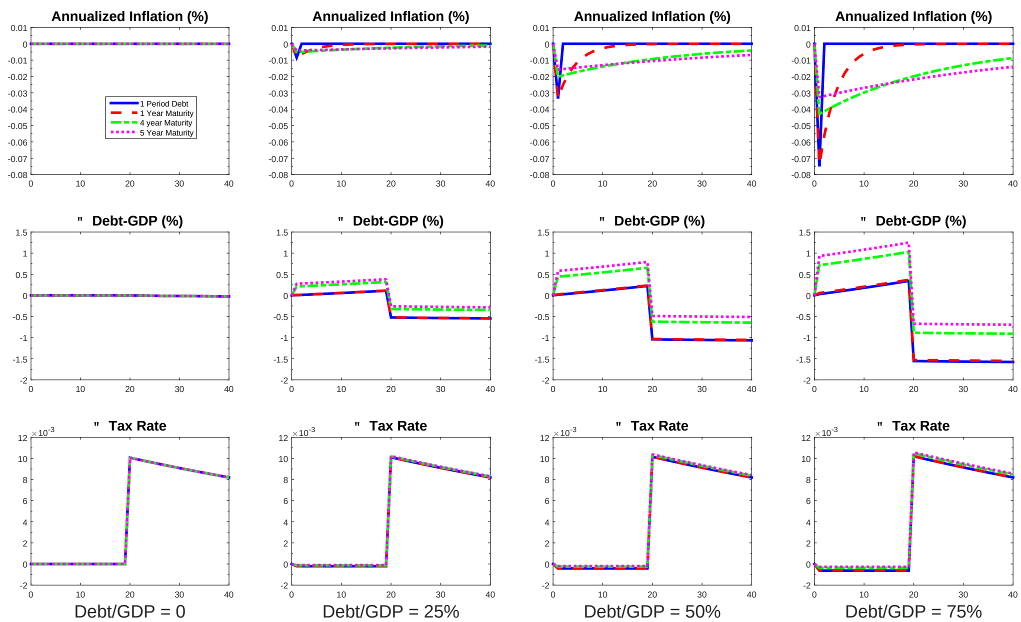

Figure 9: Optimal policy in response to an anticipated increase in government spending with different debt levels and maturities.

policy.

4.5 Ramsey Policy with a General Maturity Structure

Although the geometrically declining maturity structure is a tractable and plausible description of the profile of government debt for many economies, it is useful to broaden the analysis with a more general description of the maturity structure. This generalization refines the description of the role of optimal inflation surprises in stabilizing debt and begins to consider the role of debt management in insulating the government's finances from fiscal shocks. We employ Cochrane's (2001) notation, allowing the bond valuation equation to be written as in equation 24 in section 2.2.1. The government's optimization problem becomes

$$\begin{aligned} L_0 &= E_0 \sum_{t=0}^{\infty} \beta^t [-\frac{1}{2}(\tau_t^2 + \theta(\nu_t - 1)^2) \\ &\quad + \lambda_t (-\sum_{j=0}^{\infty} E_t[\beta^j u'(c_{t+j}) \prod_{s=0}^{j} \nu_{t+s}][\frac{B_t(t+j)}{P_{t-1}} - \frac{B_{t-1}(t+j)}{P_{t-1}}] - u'(c_t)(\tau_t - g_t - z_t)] \end{aligned}$$

The first-order condition for taxation is

$$-\tau_t = u'(c_t)\lambda_t$$

The debt management problem optimally chooses the maturity structure of debt issued in period t which is repayable at future dates, $B_t(t+j)$, to yield the optimality condition

$$\begin{aligned} & -\beta^t \lambda_t \beta^j E_t u'(c_{t+j}) \prod_{s=0}^{j} \nu_{t+s} \frac{1}{P_{t-1}} \\ &= -\beta^{t+1} E_t \lambda_{t+1} \beta^{j-1} u'(c_{t+j}) \prod_{s=0}^{j} \nu_{t+s} \frac{1}{P_{t-1}} \end{aligned} \tag{75}$$

which can be simplified as

$$u'(c_t)\tau_t E_t u'(c_{t+j}) \prod_{s=0}^{j} \nu_{t+s} = E_t u'(c_{t+1})\tau_{t+1} u'(c_{t+j}) \prod_{s=0}^{j} \nu_{t+s} \tag{76}$$

which implies

$$E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} \tau_{t+1} - \tau_t \right] \frac{u'(c_{t+j})}{u'(c_t)} \frac{P_{t-1}}{P_{t+j}} = 0 \tag{77}$$

The covariance between the payoff of debt instrument of maturity j periods and next period's tax rate is zero [see Bohn (1990)]. This is the hedging across states that Angeletos (2002) and Buera and Nicolini (2004) explore. By structuring debt in this way the policymaker minimizes the fiscal and monetary adjustments required in the face of shocks; those policy adjustments then depend on the magnitude and maturity of the outstanding debt stock. To see how debt management can mitigate the need for adjusting tax rates and generating inflation in the face of fiscal shocks, we construct a simple example in the next sub-section where the policymaker can completely insulate the government's finances from government spending shocks.

The final first-order condition is for deflation

$$-\beta^t\theta(\nu_t-1)\nu_t+\sum_{i=0}^{t}\beta^i\lambda_i(-\sum_{j=0}^{\infty}[\beta^j u'(c_{i+j})\prod_{s=0}^{j}\nu_{i+s}][\frac{B_i(i+j)}{P_{i-1}}-\frac{B_{i-1}(i+j)}{P_{i-1}}]) \tag{78}$$

This can be combined with the condition for debt management and quasi-differenced to obtain

$$(\nu_t-1)\nu_t=\beta E_t(\nu_{t+1}-1)\nu_{t+1}+\theta^{-1}\lambda_t E_t u'(c_{t+1})[\frac{B_{t-1}(t+1)}{P_{t+1}}] \tag{79}$$

This expression highlights more clearly the link between inflation and the maturity structure of the pre-determined debt stock, than does the geometrically declining maturity structure. The inflation dynamics under the optimal policy are in a very similar form to the non-linear new Keynesian Phillips curve when price stickiness results from Rotemberg (1982) quadratic adjustment costs. The key difference is that the forcing variable is the element of the predetermined debt stock that matures in period $t+1$. Deflation/inflation anticipates the rate at which the debt stock issued at time $t-1$ when the plan was formulated, matures. This makes current inflation reflect the discounted value of future debt as it matures. As debt matures, the effectiveness of inflation diminishes and inflation falls: the optimal rate of inflation jumps and gradually erodes until all the initial outstanding debt stock has matured. Notice that this Ramsey plan for inflation is only affected by debt dated at time $t-1$, and the maturity structure of debt issued after this initial period is irrelevant in a perfect foresight environment. Future maturities will affect the government's ability to insure against fiscal shocks in a stochastic environment. We can see this latter point more clearly by considering a simple example.

4.6 Commitment and Hedging

Angeletos and Buera and Nicolini argue that debt maturity should be structured to insure the economy against shocks by having the government issue long-term liabilities, but hold an almost offsetting portfolio of short term assets (the net difference being the government's overall level of indebtedness). In the face of fluctuating spending needs and interest rates, bond prices adjust to help finance debt without requiring any change in taxation. In these papers the short and long positions are constant over time, so that they do not require active management, although numerically they are extremely large positions (5 or 6 times the value of GDP in Buera and Nicolini (2004)). This approach amounts to another way to introduce the contingency in overall debt payments even although these individual assets/liabilities are not state contingent.

To construct a simple example of the use of debt management for hedging purposes we consider an environment where taxes and transfers are at their steady state values ($\tau_{t+j} = \tau^* = 0.39y$ and $z_{t+j} = z^* = 0.18y$). Government spending can either take the value of $g^h = 0.22y > g^*$, with probability $1/2$, or $g^l = 0.2y < g^*$ with complementary probability. Government debt takes the form of a single-period bond of quantity b^s issued in period t, repayable in period $t+1$ and a portfolio of longer term bonds of geometrically declining maturity, so that the quantity of debt issued in period t maturing in period $t+j$ is $\rho^j b^m$. With a single *i.i.d.* shock all that is required for complete hedging is that the maturity structure contain both one- and two-period debt to enable us to perfectly hedge, as in Buera and Nicolini. With additional *i.i.d.* shock processes, complete hedging is not possible, as we would require some persistence in the shock process and longer term debt. Because we wish to contrast this case with a scenario where a time-consistent policymaker seeks to use debt management for the purposes of hedging and mitigating time-consistency problems, we allow for a combination of longer term bonds and short-term bonds in which varying proportions of the two types can act as a proxy for changes in average debt maturity. In this example, transfer shocks, which amount to shocks that do not directly affect bond prices and interest rates, cannot be completed hedged, although movements in inflation as part of the optimal policy response could provide some hedging opportunities.

Generalizing the Ramsey policy considered above to include a single-period nominal bond as well as the portfolio of bonds with geometrically declining maturity, the system of first-order conditions to be solved as part of the Ramsey problem is

$$
\begin{aligned}
-\theta(\nu_t-1)+\widetilde{\mu}_{t-1}(1+\rho P_t^m)c_t^{-\sigma}+\widetilde{\gamma}_{t-1}c_t^{-\sigma}-(1+\rho P_t^m)\tau_t b_{t-1}-\tau_t b_{t-1}^s &= 0\\
\tau_t b_t-\widetilde{\mu}_t c_t^{-\sigma}-\rho\nu_t(\tau_t b_{t-1}-\widetilde{\mu}_{t-1}c_t^{-\sigma}) &= 0\\
\tau_t b_t^s-\widetilde{\gamma}_t c_t^{-\sigma} &= 0\\
\tau_t P_t^m-\beta E_t(1+\rho P_{t+1}^m)\nu_{t+1}\tau_{t+1} &= 0\\
\tau_t P_t^s-\beta E_t\nu_{t+1}\tau_{t+1} &= 0\\
\beta E_t\frac{(1+\rho P_{t+1}^m)}{P_t^m}\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\nu_{t+1}-1 &= 0\\
\beta E_t\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\nu_{t+1}-P_t^s &= 0\\
b_tP_t^m+b_t^sP_t^s-(1+\rho P_t^m)b_{t-1}\nu_t-b_{t-1}^s\nu_t-g_t-z^*+\tau_t &= 0
\end{aligned}
$$

$$g_t=g^i,\ i=h,l \text{ with prob } 1/2$$

where $\widetilde{\mu}_{t-1}=\frac{\mu_{t-1}}{P_{t-1}^m c_{t-1}^{-\sigma}}, \widetilde{\gamma}_{t-1}=\frac{\gamma_{t-1}}{P_{t-1}^s c_{t-1}^{-\sigma}}$ and γ_t is the Lagrange multiplier associated with the pricing of single-period bonds, $P_t^s=\beta E_t\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\nu_{t+1}$. There are four state variables—$\widetilde{\mu}_{t-1},\widetilde{\gamma}_{t-1},b_t,b_t^s$—the first two of which capture the history dependence in policymaking under commitment. Despite the complexity of these first-order conditions, the policymaker can fulfill this Ramsey program with a constant tax rate and no inflation by buying an appropriate quantity of single-period assets paid for by issuing longer-term bonds. Shocks to public consumption then induce fluctuations in the prices of these assets/liabilities which perfectly insulate the government's finances.

With *i.i.d.* fluctuations in government spending, the current level of spending is also a state variable: we are either in the high- or low-government spending regime and may exit that regime with a probability of 1/2 each period.

The pricing equation for geometrically declining coupon bonds is

$$P_t^m=\beta E_t(1+\rho P_{t+1}^m)\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\nu_{t+1}$$

With government spending fluctuating between high and low states, bond prices will fluctuate depending on the spending state. Define $u_{ij}=\frac{u'(1-g^i)}{u'(1-g^j)}=\frac{(1-g^i)^{-\sigma}}{(1-g^j)^{-\sigma}}$, $i,j=l,h$, and i$\neq j$ bond prices in spending regime i, $i=h,l$ are given by

$$
\begin{aligned}
P_i^m &= \beta\frac{1}{2}(1+\rho P_i^m)+\beta\frac{1}{2}(1+\rho P_j^m)u_{ji}\\
&= A_i+B_iP_j^m
\end{aligned}
$$

where $A_i = (1 - \frac{1}{2}\beta\rho)^{-1}(\frac{1}{2}\beta + \frac{1}{2}\beta u_{ji})$ and $B_i = (1 - \frac{1}{2}\beta\rho)^{-1}\frac{1}{2}\beta\rho u_{ji}$, $i, j = l, h$, and $i \neq j$, which can be solved as

$$P_i^m = \frac{A_i + B_i A_j}{1 - B_i B_j}$$

For one-period debt this reduces to

$$P_i^s = \frac{1}{2}\beta + \frac{1}{2}\beta u_{ji}$$

Optimal hedging uses these fluctuations in bond prices to construct of portfolio of government debt that negates the need to vary taxes or induce inflation surprises, despite the random movements in government consumption.

The flow budget constraint conditional on the government spending regime, but with constant tax rates and no inflation, is

$$P_i^m b^m + P_i^s b^s = (1 + \rho P_i^m) b^m + b^s - (\tau^* - g^i - z^*)$$

We choose b^m and b^s to ensure this equation holds regardless of the government spending regime, so that the government does not need to issue or retire debt as it moves between low and high spending regimes. This portfolio is given by

$$\begin{bmatrix} b^m \\ b^s \end{bmatrix} = -\begin{bmatrix} P_i^m(1-\rho) - 1 & P_i^s - 1 \\ P_j^m(1-\rho) - 1 & P_j^s - 1 \end{bmatrix}^{-1} \begin{bmatrix} \tau^* - g^i - z^* \\ \tau^* - g^j - z^* \end{bmatrix}$$

We can achieve the same portfolio by considering the debt valuation equation in a given period, which is contingent on the government spending state. If government spending is currently high, that equation is

$$\begin{aligned} & b^s(u'(c^h)) + b^m(u'(c^h)) + \sum_{j=1}^{\infty}(\rho\beta)^j \left[\frac{1}{2}u'(c^l) + \frac{1}{2}u'(c^h)\right] b^m \\ = \;& u'(c^h)(\tau^* - g^h - z^*) + \sum_{j=1}^{\infty}\beta^j \left[\frac{1}{2}u'(c^l)(\tau^* - g^l - z^*) + \frac{1}{2}u'(c^h)(\tau^* - g^h - z^*)\right] \end{aligned}$$

and if government spending is low it is

$$\begin{aligned} & b^s(u'(c^l)) + b^m(u'(c^l)) + \sum_{j=1}^{\infty}(\rho\beta)^j \left[\frac{1}{2}u'(c^l) + \frac{1}{2}u'(c^h)\right] b^m \\ = \;& u'(c^l)(\tau^* - g^h - z^*) + \sum_{j=1}^{\infty}\beta^j \left[\frac{1}{2}u'(c^l)(\tau^* - g^l - z^*) + \frac{1}{2}u'(c^h)(\tau^* - g^h - z^*)\right] \end{aligned}$$

subtracting one from the other implies

$$[b^s + b^m](u'(c^h) - u'(c^l)) = u'(c^h)(\tau^* - g^h - z^*) - u'(c^l)(\tau^* - g^l - z^*) \tag{80}$$

Without any change in taxation or inflation, government solvency is ensured, provided that debt maturing in the current period has the value implied by this equation. Assuming a sufficiently low level of net indebtedness, the primary budget will swing between deficit and surplus as government spending moves from high to low regimes, implying that the right side of (80) is negative. Since $u'(c^h) > u'(c^l)$, this condition requires that the Ramsey policymaker buys short term assets to such an extent that $b^s < -b^m$. The budget constraint is insulated from the effects of government spending shocks, which can be absorbed by bond prices without any need to issue new debt, change taxes or generate inflation surprises.

The size of the longer term liabilities must, equivalently, satisfy the solvency conditions conditional on the current level of government consumption. For example

$$\begin{aligned} & b^s(u'(c^l)) + b^m(u'(c^l)) + \sum_{j=1}^{\infty}(\rho\beta)^j \left[\frac{1}{2}u'(c^l) + \frac{1}{2}u'(c^h)\right] b^m \\ = \quad & u'(c^l)(\tau^* - g^h - z^*) + \sum_{j=1}^{\infty}\beta^j \left[\frac{1}{2}u'(c^l)(\tau^* - g^l - z^*) + \frac{1}{2}u'(c^h)(\tau^* - g^h - z^*)\right] \end{aligned}$$

which can be written as

$$\begin{aligned} & \frac{\rho\beta}{1-\rho\beta}\left[\frac{1}{2}u'(c^l) + \frac{1}{2}u'(c^h)\right] b^s + b^s u'(c^h) + b^m u'(c^h) \\ = \quad & \frac{\beta}{1-\beta}\left[\frac{1}{2}u'(c^l)(\tau^* - g^l - z^*) + \frac{1}{2}u'(c^h)(\tau^* - g^h - z^*)\right] + u'(c^h)(\tau^* - g^h - z^*) \end{aligned}$$

This expression can either define the steady-state level of long term debt given the tax rate or the tax rate given the long term debt stock. Either interpretation is consistent with a steady-state solution to the Ramsey tax smoothing plan where the solution of the remainder of the Ramsey problem is $\tau_t = \tau^*$, $z_t = z^* = 0.18y$, $\nu_t = 1$, $\frac{\gamma_i}{P_i^s} = \tau^* b^s$, $\frac{\mu_i}{P_i^m} = \tau^* b^m$, $i = h, l$ with probability of 1/2. In other words, the steady-state tax rate which can support the average level of government spending, steady-state transfers and the steady-state net debt stock, while fluctuations in bond prices mitigate the need for further tax adjustments to compensate for fluctuations in government spending.

Figure 10 reveals the pattern of bond returns and the underlying asset positions for a series of random draws across the two spending regimes. The figure's bottom right panel describes a particular realization of the government spending shocks. Despite these movements in spending the budget constraint can be satisfied with a constant tax rate and no inflation surprises by buying short-term assets that are funded by issuing longer term debt. The portfolio that achieves this implies that the government holds short-term assets of around 22 percent of GDP, with longer term liabilities of around 70 percent of GDP and a net debt of around 48 percent. Although large, these positions are less than those typically found for richer stochastic processes, where positions often exceed the economy's total endowment by several factors [see Buera and Nicolini (2004)]. Since the ability to hedge relies on variation in the yield curve having longer term liabilities to set against the short-term assets is most effective. Then a portfolio of single-period assets matched with 1-year liabilities requires far more short-term assets, compared to a portfolio made up of the same assets and bonds with an average maturity of five years. Hedging in this way implies that a positive shock to government spending, which raises the primary deficit actually leads to a reduction in the value of government indebtedness, rather than to an increase. This is a general prediction of models that have achieved financial market completeness which Marcet and Scott (2009) use as the basis of an empirical test, but the data strongly reject.

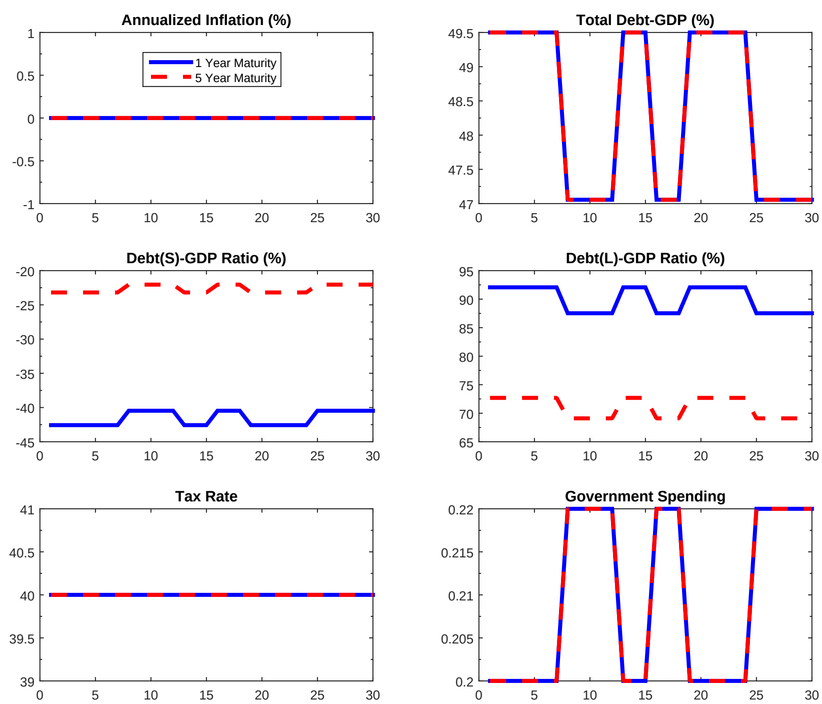

Figure 10: Optimal hedging under commitment.

Faraglia, Marcet, and Scott (2008) extend Buera and Nicolini's analysis to move away from an endowment economy to consider a production economy with capital. This makes the size of the extreme portfolio positions even larger, and now the liability/asset positions are no longer constant, but highly volatile, possibly even reversing the issue-long-buy-short recommendation. Because yield premia are not very volatile, they are therefore not very effective as a source of insurance. They then consider what happens if the government is unsure about the specification of some element of the model. The sensitivity of results to small changes in model specification means that it is often better to run a balanced budget than run the risk of getting the portfolio composition wrong. Similarly, even modest transaction costs would make it undesirable to construct such huge portfolios.

4.7 Discretion

A large part of the literature that extends Lucas and Stokey's (1983) analysis focuses on the importance of having access to state-contingent debt either directly or by using inflation surprises and debt management to render state dependent the real payoffs from government debt. When the policymaker can replicate the Ramsey policy in Lucas and Stokey through such devices, there remains the issue of whether the underlying policy is time consistent. In the original Lucas and Stokey model, the Ramsey policy can be made time-consistent by adhering to a particular debt maturity structure. Lucas and Stokey then conjecture that allowing debt to be nominal would make the policy problem trivial: positive debt would be costlessly deflated by positive surprise inflation and negative debt would be adjusted by surprise deflation to the level sufficient to support the first-best allocation (the interest on the debt paying for government consumption, consistent with any fiscal taxes/subsidies required by offset other market distortions). This reasoning suggests that the only interesting case is when the outstanding debt stock is zero.

Persson, Persson, and Svensson (1987) initiated a debate to explore the Lucas and Stokey conjecture.[32] Alvarez, Kehoe, and Neumeyer (2004) conclude that the Lucas and Stokey structure of state-contingent indexed debt, in combination with a condition that net nominal debt is zero so that government debt liabilities equal the stock of money, can ensure the time-consistency of the original Lucas and Stokey Ramsey policy in a monetary economy that follows the Friedman rule. As Persson, Persson, and Svensson (2006) note, these conditions essentially reduce the monetary version of the Lucas and Stokey economy to its real version.

Bohn (1988) argues that in issuing nominal debt the policy maker trades-off the ability to use inflation surprises as a hedging device when debt is nominal against the inflation bias that a positive stock of debt creates. In models where the problem is not constructed to mimic the Lucas and Stokey Ramsey policy, the time-consistent policy typically implies a mean reverting steady state level of debt. Debt can be positive or negative, depending on

[32]Persson, Persson, and Svensson (2006) chart the course of this debate.

the nature of the time-inconsistency problem. The issue of the time-consistency of policy is also dependent on the cost of inflation surprises. Persson, Persson, and Svensson (2006) use beginning- rather than end-of-period money balances in the provision of liquidity services to make unexpected inflation costly, which allows them to construct a time-consistent portfolio of indexed and nominal debt. Martin (2009) uses the cash-credit good distinction in Lucas and Stokey to generate a cost to inflation which is then balanced against the gains from using inflation to reduce the value of single-period nominal debt. This generates a mean reverting steady-state level of debt under discretion, rather than the random walk in steady-state debt, which is a feature of the Ramsey tax smoothing policy without state contingent debt. Martin (2011) combines the Lagos and Wright (2005) monetary search model with fiscal policy and explores the time-consistency problem to find that the welfare costs of an inability to commit are small. This conclusion likely reflects the nature of the costs of surprise inflation; as noted above, when Schmitt-Grohé and Uribe (2004) introduce even a tiny degree of nominal inertia, the time-inconsistent Ramsey policy tilts very firmly in favour of price stability, away from the Friedman rule and the use of inflation surprises.

We now turn to consider the impact on the balance between monetary and fiscal policy of constraining the policymaker to be time-consistent. We continue to use the endowment economy where inflation is assumed to be costly as a short-cut to introducing nominal inertia.

The policymaker cannot make credible promises about how they will behave in the future in order to improve policy trade-offs today. Even in this simple model there is an endogenous state variable in the form of government debt, so that policy actions today will affect future expectations through the level of debt that the policy bequeaths to the future. We define the auxiliary variable

$$M(b_{t-1}, g_{t-1}) = (1+\rho P_t^m)\nu_t \left(c_t\right)^{-\sigma}$$

to write the Bellman equation of the associated policy problem as

$$\begin{aligned} V(b_{t-1}, g_{t-1}) \;\; = \;\; & -\frac{1}{2}(\tau_t^2 + \theta(\nu_t - 1)^2) + \beta E_t V(b_t, g_t) \\ & + \mu_t(\beta \frac{c_t^\sigma}{P_t^m} E_t M(b_t, g_t) - 1) \\ & + \lambda_t(b_t P_t^m - (1+\rho P_t^m) b_{t-1}\nu_t - g_t - z_t + \tau_t) \end{aligned}$$

We have replaced the expectations in the bond-pricing equation with the auxiliary variable to indicate that the policymaker cannot influence those expectations directly by making policy commitments. But those expectations are a function of the state variables. We take government spending and transfers to be exogenous autoregressive processes.

The implies the first-order conditions

$$
\begin{aligned}
\tau_t &: -\tau_t + \lambda_t = 0 \\
\nu_t &: -\theta(\nu_t - 1) - \lambda_t(1 + \rho P_t^m) b_{t-1} = 0 \\
P_t^m &: -\frac{\mu_t}{P_t^m} + \lambda_t(b_t - \rho b_{t-1}\nu_t) = 0 \\
b_t &: \frac{\mu_t}{P_t^m} c_t^\sigma \beta E_t \frac{\partial M(b_t, g_t)}{\partial b_t} + \lambda_t P_t^m + \beta E_t \frac{\partial V(b_t, g_t)}{\partial b_t} = 0
\end{aligned}
$$

From the envelope theorem

$$
\frac{\partial V(b_{t-1}, g_{t-1})}{\partial b_{t-1}} = -(1 + \rho P_t^m)\nu_t \lambda_t
$$

which can be led one period and substituted into the first-order condition for government debt

$$
\frac{\mu_t}{P_t^m} c_t^\sigma \beta E_t \frac{\partial M(b_t, g_t)}{\partial b_t} + \lambda_t P_t^m - \beta E_t (1 + \rho P_{t+1}^m)\nu_{t+1}\tau_{t+1} = 0
$$

Combining the condition for the bond price P_t^m with the Fisher equation implies

$$
\frac{\mu_t}{P_t^m} = \lambda_t(b_t - \rho \nu_t b_{t-1})
$$

which can be used to eliminate $\frac{\mu_t}{P_t^m}$ from the condition for debt. The system to be solved for $\{P_t^m, \nu_t, \tau_t, b_t, g_t\}$ is

$$
\begin{aligned}
\nu_t &: -\theta(\nu_t - 1) - \tau_t(1 + \rho P_t^m) b_{t-1} = 0 \\
b_t &: \tau_t(b_t - \rho \nu_t b_{t-1})\beta c_t^\sigma E_t \frac{\partial M(b_t, g_t)}{\partial b_t} + \tau_t P_t^m - \beta E_t(1 + \rho P_{t+1}^m)\nu_{t+1}\tau_{t+1} = 0
\end{aligned}
$$

The first-order condition for inflation is now

$$
-\theta(\nu_t - 1) = (1 + \rho P_t^m) b_{t-1} \tau_t
$$

Under commitment, inflation persisted only for as long as the maturity structure of the predetermined debt stock at the time a shock hit. Under time-consistent policy, outside of the policymaker's bliss point (of zero inflation and no taxation), with a non-zero debt stock there will always be a state-dependent mix of taxation and inflation. A positive stock of debt delivers positive inflation, regardless of the maturity structure of that debt. This reflects the inflation bias inherent in the time-consistent policy in the presence of nominal debt.

We can see some more differences between discretion and commitment by contrasting the equivalent expressions describing the evolution of the tax rate. Under commitment we obtain the standard tax-smoothing result adjusted for the tilting implied by variations in the stochastic discount factor

$$\tau_t P_t^m = \beta E_t(1 + \rho P_{t+1}^m)\nu_{t+1}\tau_{t+1}$$

The equivalent condition under discretion is

$$\tau_t P_t^m = \beta E_t(1 + \rho P_{t+1}^m)\nu_{t+1}\tau_{t+1} - \tau_t(b_t - \rho\nu_t b_{t-1})\beta c_t^\sigma E_t \frac{\partial M(b_t, g_t)}{\partial b_t}$$

The additional term captures the effects of the tax rate on expectations of inflation and bond prices through the level of debt carried into the future. Increased debt raises expected inflation and lowers expected bond prices, so $E_t \frac{\partial M(b_t,g_t)}{\partial b_t} < 0$. This additional term raises the tax rate above the level implied by the tax-smoothing condition observed under commitment. Where the tax rate under commitment was carefully constructed to allow debt levels to permanently rise, under discretion the tax rate prevents debt from rising permanently.[33]. Instead a discretionary policymaker returns debt to a steady-state value that is very close to zero, but slightly negative. This cannot be seen analytically, so we need to analyze the numerical solution to the time-consistent policy problem.

The numerical solution under discretion is radically different from that under commitment (see figure 11). Under commitment, policy allows the steady state level of debt to follow a random walk and the use of inflation to offset shocks is relatively modest. Under discretion there is a unique steady state at which the policy supporting the steady-state debt level is time consistent, and this occurs at a slightly negative debt stock with a mild deflation. The negative steady-state debt stock falls far short of the negative debt levels that would be needed to support the first-best allocation—that is, the stock of government-held assets generates interest income sufficient to pay for all transfers and government spending without levying any distortionary taxes. Private-sector expectations ensure that the policymaker does not accumulate such a level of assets. Bondholders know that once the government has accumulated a positive stock of assets, it has an incentive to introduce surprise deflation to increase the real value of those assets. This knowledge reduces agents' inflation expectations until the policymaker no longer wishes to introduce such deflationary surprises. Accumulating more assets would then worsen this incentive to deflate to confront the policymaker with a trade-off between accumulating assets to reduce tax rates and the expected deflation that the accumulation of assets implies. In the steady state a balance is struck with a mild

[33] Calvo and Guidotti (1992) label this the "debt aversion" effect and Leith and Wren-Lewis (2013) call it the "debt-stabilization bias."

deflation and small negative debt stock, although both are extremely close to zero.

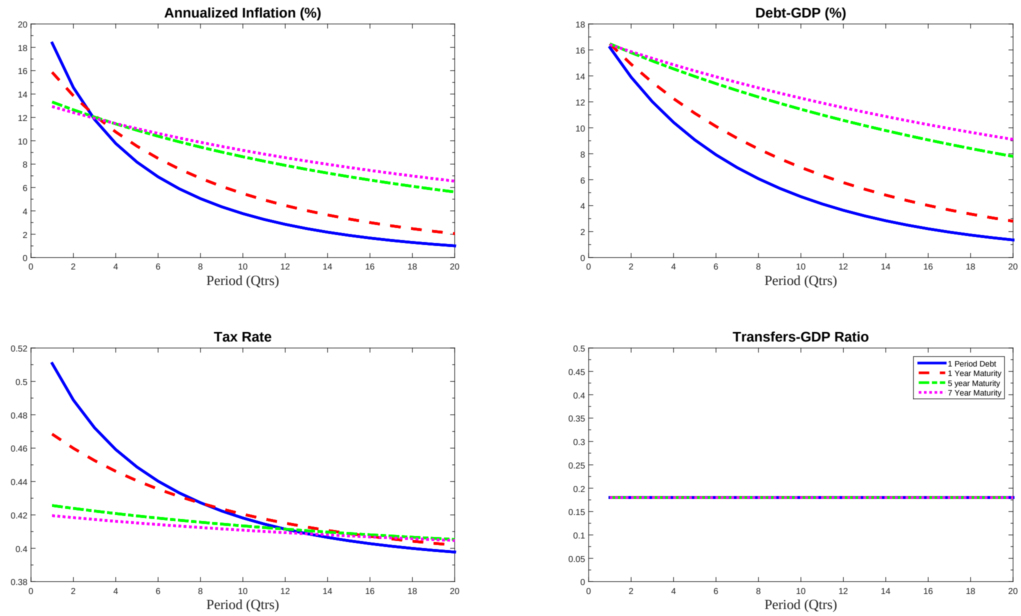

Figure 11: Optimal time-consistent policy when debt is above its steady-state level.

At positive debt levels there is a significant desire to reduce debt through inflation surprises. Economic agents anticipate this and raise their inflationary expectations. Positive debt levels raise inflation in a highly nonlinear way because they introduce a state-dependent inflationary bias which can be very large. Even modest debt-to-GDP ratios can imply double digit inflation. This is a surprising outcome because the same model and parameterization under commitment implies no inflation at all in the absence of shocks and only small inflation with shocks and positive debt levels.

Debt maturity lessens this debt stabilization bias problem so that for a given debt-to-GDP ratio, inflation will be lower the longer is debt maturity. With short maturity debt, surprise inflations have to take place in the current period and cannot be smoothed, so the incentives to inflate today are greater and the resultant inflationary bias problem reflects this. With longer maturity debt, smaller sustained increases in inflation are equally powerful, so the inflationary bias problem in a given period is smaller. This debt stabilization bias is heavily dependent on the magnitude of the government debt stock. When debt is high, the efficacy of surprise inflation—either current inflation or through bond prices—is also much higher and this raises government's incentives to use this device to stabilize debt. As a result the debt stabilization bias rises dramatically with debt levels.

In the absence of innovations to the fiscal surplus, this higher inflation does not actually stabilize debt. As in the original inflation bias problem, there is a pure cost in the form of higher inflation which does not generate any reduction in debt.[34] But unlike the original inflation bias problem, in our case the magnitude of the bias is endogenous and depends on the size and maturity of the government debt. The policymaker can choose to reduce debt through taxation to gradually reduce the bias. Under discretion the reduction in debt can be a quite rapid, particularly when the debt stock is large and of short maturity. The costs of the policymaker being unable to commit in this context are not that debt is unstable, but that the policymaker too aggressively returns government debt to its steady state level following shocks. This message resonates when thinking about actual fiscal austerity policies in many countries after the 2008 global financial crisis.

4.8 Debt Management under Discretion

The above results highlight the time consistency issues created by nominal debt. The existing optimal policy literature also considers time-consistency issues in relation to debt management issues. Specifically, in the Lucas and Stokey model with state-contingent debt, the maturity structure is key in ensuring that the Ramsey policy described in Lucas and Stokey is time consistent. At the same time, the optimal hedging analysis shows that the maturity structure can create a portfolio of government bonds that features the right state-contingent payoffs even when the underlying bonds are not state contingent. In the context of a real model, Debortoli, Nunes, and Yared (2014) also allow the government to hold short-term assets and longer term liabilities (which are individually not state contingent), but require the policy to be time-consistent. They show that the optimal policy results in a relatively flat maturity structure that offsets the costs of not being able to commit even although this removes the tilting in maturity that is beneficial in terms of insurance effects.

To assess the trade-offs between optimal hedging and time-consistency, we use the same model that delivered complete hedging of government expenditure shocks under commitment, and solve that model under discretion. In introducing single-period bonds to time-consistent policy problem we need to defined an additional auxiliary variable

$$N(b_{t-1}, b^s_{t-1}, g_{t-1}) = \nu_t \left(c_t\right)^{-\sigma}$$

All expectations are now a function of three state variables, longer term bonds, b_{t-1}, single period bonds, b^s_{t-1} and government spending, g_{t-1}, which will either equal $0.22y$ in the high spending regime, or $0.2y$ in the low spending case.

[34] Analogously, in Barro and Gordon (1983) this additional inflation does not reduce unemployment.

The policy problem is

$$
\begin{aligned}
V(b_{t-1}, b^s_{t-1}, g_{t-1}) \quad = \quad & -\frac{1}{2}(\tau_t^2 + \theta(\nu_t - 1)^2) + \beta E_t V(b_t, b^s_t, g_t) \\
& +\mu_t(\beta \frac{c_t^\sigma}{P_t^m} E_t M(b_t, b^s_t, g_t) - 1) \\
& +\gamma_t(\beta E_t \frac{c_t^\sigma}{P_t^s} E_t N(b_t, b^s_t, g_t) - 1) \\
& +\lambda_t(b_t P_t^m + b^s_t P^s_t - (1 + \rho P_t^m) b_{t-1}\nu_t - b^s_{t-1}\nu_t - g_t + \tau_t - z_t)
\end{aligned}
$$

which has an additional constraint associated with the pricing of short-term bonds, and the government's flow budget constraint contains both single-period and declining coupon bonds. After applying the envelope theorem this implies the first-order conditions. For inflation

$$
-\theta(\nu_t - 1) = \tau_t[(1 + \rho P_t^m) b_{t-1} + b^s_{t-1}]
$$

The level of inflation depends on the total level of indebtedness across short and long bonds, so that a positive level of net indebtedness implies an inflationary bias. As before, this bias serves no purpose in terms of reducing the real debt burden, but reflects economic agents' expectations than if inflation were any lower, the policy would be tempted to introduce a surprise inflation to facilitate debt reduction.

The tax smoothing conditions are

$$
\begin{aligned}
\tau_t P_t^m \quad = \quad & \beta E_t (1 + \rho P^m_{t+1}) \nu_{t+1} \tau_{t+1} - \tau_t (b_t - \rho \nu_t b_{t-1}) \beta c_t^\sigma E_t \frac{\partial M(b_t, b^s_t, g_t)}{\partial b_t} \\
& -\tau_t b^s_t \beta c_t^\sigma E_t \frac{\partial N(b_t, b^s_t, g_t)}{\partial b_t}
\end{aligned}
$$

and

$$
\begin{aligned}
\tau_t P_t^s \quad = \quad & \beta E_t \nu_{t+1} \tau_{t+1} - \tau_t (b_t - \rho \nu_t b_{t-1}) \beta c_t^\sigma E_t \frac{\partial M(b_t, b^s_t, g_t)}{\partial b^s_t} \\
& -\tau_t b^s_t \beta c_t^\sigma E_t \frac{\partial N(b_t, b^s_t, g_t)}{\partial b^s_t}
\end{aligned}
$$

The first two terms of these expressions reflect the same tax-smoothing conditions found under commitment, where the choice of short-term assets and longer-term bonds could satisfy these conditions while perfectly insulating the government's finances from the fluctuations in government spending. The final two terms in each condition capture the impact that another unit of short or long debt has on long- and short-term bond prices through the impact of debt on inflation expectations. These effects highlight the incentives that the policymaker has to reduce indebtedness to reduce inflation, given the inflationary bias problem created by a positive stock of government debt. The magnitude of the effect of reducing either short or long-term debt by one bond may vary depending on the relative proportions of the two bonds. In other words, by varying the relative proportions of single period and longer term debt, the policymaker can vary the average debt maturity and thereby influence the inflationary bias problem implied by a given level of indebtedness.

Solving the model without switching in government spending generates a steady state with near zero debt and inflation (see figure 12). Introducing government spending switches induces fluctuations in all variables. The movements in spending are largely matched with movements in tax rates (even although these could have been eliminated by issuing an appropriately constructed portfolio of short-term assets and longer-term liabilities), although with some increase in the debt/deficit when we are in the high spending regime. The stochastic steady state asset and liability positions are only slightly positive for assets, and slightly negative for liabilities, but quite distant from the magnitude of the positions required for perfect hedging. Inflation follows the level of indebtedness, giving rise to a positive (negative) inflation bias when the level of indebtedness is positive (negative).

Starting from a positive level of indebtedness, figure 13 plots the mix of short- and long-term debt as the economy transitions toward the stochastic steady-state. Calvo and Guidotti's (1992) debt aversion appears as the policymaker fairly rapidly reduces indebtedness in an attempt to eliminate the inflationary bias that debt induces. The fluctuations in debt induced by the changing spending regime are small relative to the general debt dynamics implied by the transition to steady state. The fact that the single-period debt does not rise dramatically when overall indebtedness increases implies that there is an effective lengthening of maturity as overall debt levels increase. This echoes the results of Calvo and Guidotti, which are also discussed in Missale (1999).

5 Production Economies with Optimal Monetary and Fiscal Policies

5.1 The Model

Until now our analysis of optimal policy has been based on a simple flexible price endowment economy, where we have captured the costs of inflation and distortionary taxation by adding quadratic terms in these variables to the policymaker's objective function. We now attempt to generalize these results by considering a production economy where households supply labour to imperfectly competitive firms who are subject to quadratic costs in changing prices as in Rotemberg (1982). The government levies a tax on sales to finance exogenous processes

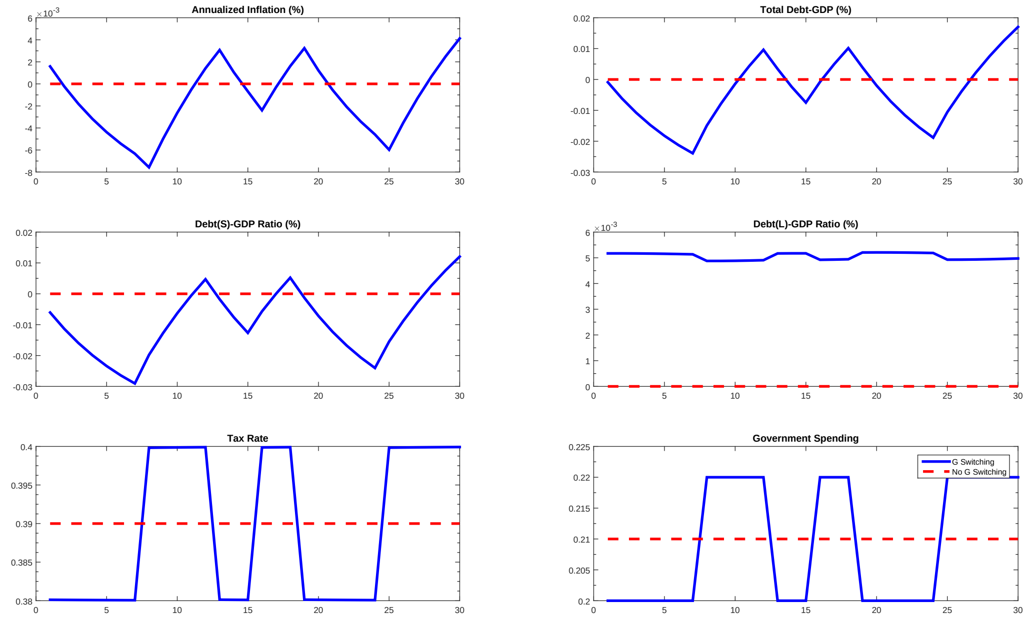

Figure 12: Hedging under discretion.

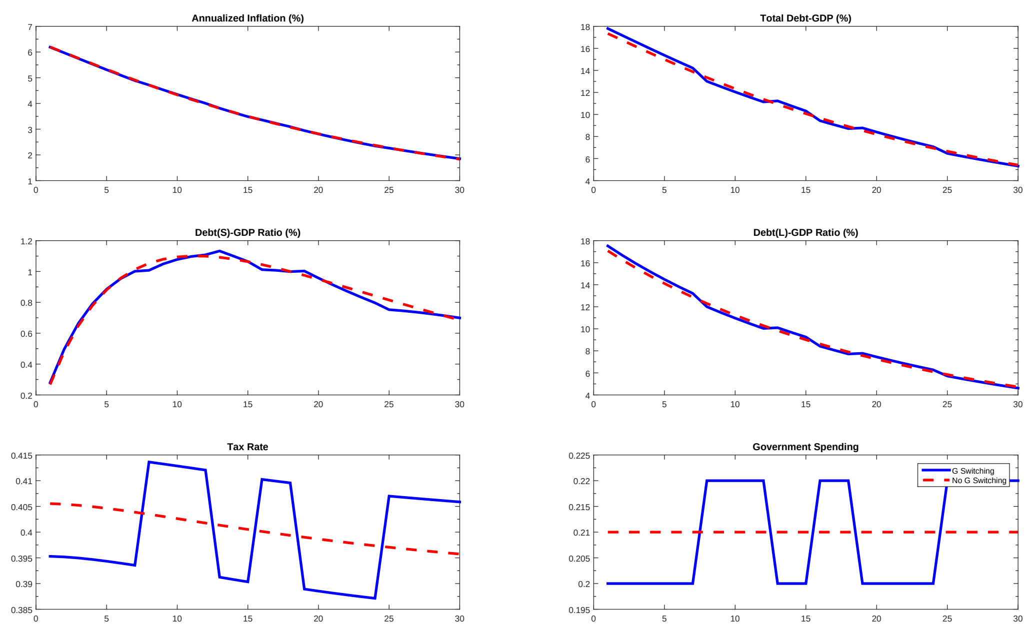

Figure 13: Hedging and time-consistent policy.

for transfers and government consumption. The policymaker aims to maximize the utility of the representative household. This section endogenizes the welfare costs of both inflation and distortionary taxation. We also widen the scope for monetary and fiscal policy interactions because monetary policy not only generates revaluations of government bonds, but also affects real debt service costs and the size of the tax base. Changes in distortionary taxation not only influence the government's budget constraint, they affect production decisions and have a direct cost-push effect on inflation.

This basic set-up is similar to that in Benigno and Woodford (2004) and Schmitt-Grohé and Uribe (2004) but with some differences.[35] We model price stickiness using Rotemberg's (1996) adjustment costs rather than Calvo (1983) pricing because this reduces the number of state variables when solving the model non-linearly. We also consider a richer maturity structure rather than single-period bonds.

5.1.1 Households

There is a continuum of households of size one. We assume complete asset markets, so that through risk sharing households face the same budget constraint. The typical household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \tag{81}$$

where c and N are a consumption aggregate and labor supply respectively. The consumption basket is made up of a continuum of differentiated products, $c_t = (\int_0^1 c(j)_t^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}}$ and the basket of public consumption takes the same form.

The budget constraint at time t is given by

$$\int_0^1 P_t(j) c_t(j) dj + P_t^m B_t^m = \Pi_t + (1+\rho P_t^m) B_{t-1}^m + W_t N_t + Z_t \tag{82}$$

where $P_t(j)$ is the price of variety j , Π is the representative household's share of profits in the imperfectly competitive firms (after tax), W are wages, and Z are lump sum transfers and the bonds the household can invest in are the geometrically declining coupon bonds used above.

We maximize utility subject to the budget constraint (82) to obtain the optimal allocation of consumption across time and the associated pricing of declining coupon bonds

$$\beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) (1+\rho P_{t+1}^m) \right] = P_t^m \tag{83}$$

[35] Leeper and Zhou (2013) study a linear-quadratic version of this setup.

Notice that when these reduce to single-period bonds, $\rho = 0$, the price of these bonds is $P_t^m = R_t^{-1}$.

The second first-order condition relates to the labour supply decision

$$\left(\frac{W_t}{P_t}\right) = N_t^{\varphi} c_t^{\sigma}$$

5.1.2 Firms

Firms produce output using to a linear production function, $y(j)_t = AN(j)_t$, where $a_t = \ln(A_t)$ is time varying and stochastic, such that the real marginal costs of production are $mc_t = \frac{W_t}{P_t A_t}$. Household demand for their product is given by, $y(j)_t = (\frac{P(j)_t}{P_t})^{-\epsilon} y_t$ and firms are also subject to quadratic adjustment costs in changing prices

$$v_t^j P_t = \frac{\phi}{2}\left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1\right)^2 P_t y_t \tag{84}$$

where $\pi^* = 1$ is the steady-state gross inflation rate. In a symmetric equilibrium where $p_t(j) = P_t$ the first-order condition for firms' profit maximization implies

$$(1-\theta)(1-\tau_t) + \theta mc_t - \phi \frac{\pi_t}{\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right) + \phi\beta E_t (\frac{c_t}{c_{t+1}})^{\sigma} \frac{\pi_{t+1}}{\pi^*} \frac{y_{t+1}}{y_t}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right) = 0 \tag{85}$$

which is the non-linear version of the Phillips curve and includes the effects of a distortionary tax on sales revenues, τ_t.

5.1.3 Equilibrium

Goods market clearing requires, for each good j

$$y(j)_t = c(j)_t + g(j)_t + v(j)_t \tag{86}$$

which allows us to write

$$y_t \left[1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right] = c_t + g_t$$

There is also market clearing in the bonds market where the longer term bond portfolio evolves according to the government's budget constraint which we now describe.

5.1.4 Government Budget Constraint

Combining the series of the representative consumer's flow budget constraints, (82), and noting the equivalence between factor incomes and national output, we obtain the government's flow budget constraint

$$P_t^m b_t = (1 + \rho P_t^m)\frac{b_{t-1}}{\pi_t} - y_t\tau_t + g_t - z_t \tag{87}$$

where real debt is defined as $b_t \equiv \frac{B_t^M}{P_t}$.

5.2 Commitment Policy in the New Keynesian Model

Setting up the Lagrangian

$$\begin{aligned} L_t &= E_0\sum_{t=0}^{\infty}\beta^t[(\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}) \\ &+\lambda_{1t}(y_t(1-\frac{\phi}{2}\left(\frac{\pi_t}{\pi^*}-1\right)^2) - c_t - g_t) \\ &+\lambda_{2t}(\beta(\frac{c_t}{c_{t+1}})^\sigma(\frac{P_t}{P_{t+1}})(1+\rho P_{t+1}^m) - P_t^m) \\ &+\lambda_{3t}((1-\theta)(1-\tau_t) + \theta y_t^\varphi c_t^\sigma A_t^{-1-\varphi} - \phi\pi_t(\pi_t - 1) + \phi\beta(\frac{c_t}{c_{t+1}})^\sigma \pi_{t+1}\frac{y_{t+1}}{y_t}(\pi_{t+1}-1)) \\ &+\lambda_{4t}(P_t^M b_t - (1+\rho P_t^M)\frac{b_{t-1}}{\pi_t} + y_t\tau_t - g_t - tr_t)] \end{aligned}$$

and differentiating with respect to $\{c_t, y_t, \tau_t, P_t^m, b_t^m, \pi_t\}$ yields the first-order conditions for the Ramsey program. Those conditions are sufficiently complex to afford little additional insight that was not already gained from the analysis of the comparable problem for our simple endowment economy. But when we solve the model numerically, several interesting results relating to the optimal monetary and fiscal policy mix emerge.

5.3 Numerical Results

The first experiment considers a transfers shock at different initial levels of debt (see figure 14).[36] Transfers start at 18 percent of GDP and then increase with an autocorrelated shock, but do not respond further to GDP. When, as in the first column, the initial debt level is zero the maturity structure of the debt issued after the shock has hit is irrelevant. There is an initial one-period burst in inflation caused by the rise in the tax rate and not fully offset by the tightening of monetary policy. Then a coordinated use of monetary and fiscal policy stabilizes debt at its new steady state level. The tax rate does not jump immediately to its new steady state, but follows a dynamic path which captures the movement in the real

[36] In all cases we solve the model non-linearly under perfect foresight following an initial perturbation from the steady state.

interest rate in the sticky price economy, while monetary policy ensures that inflation is zero outside of the initial period.

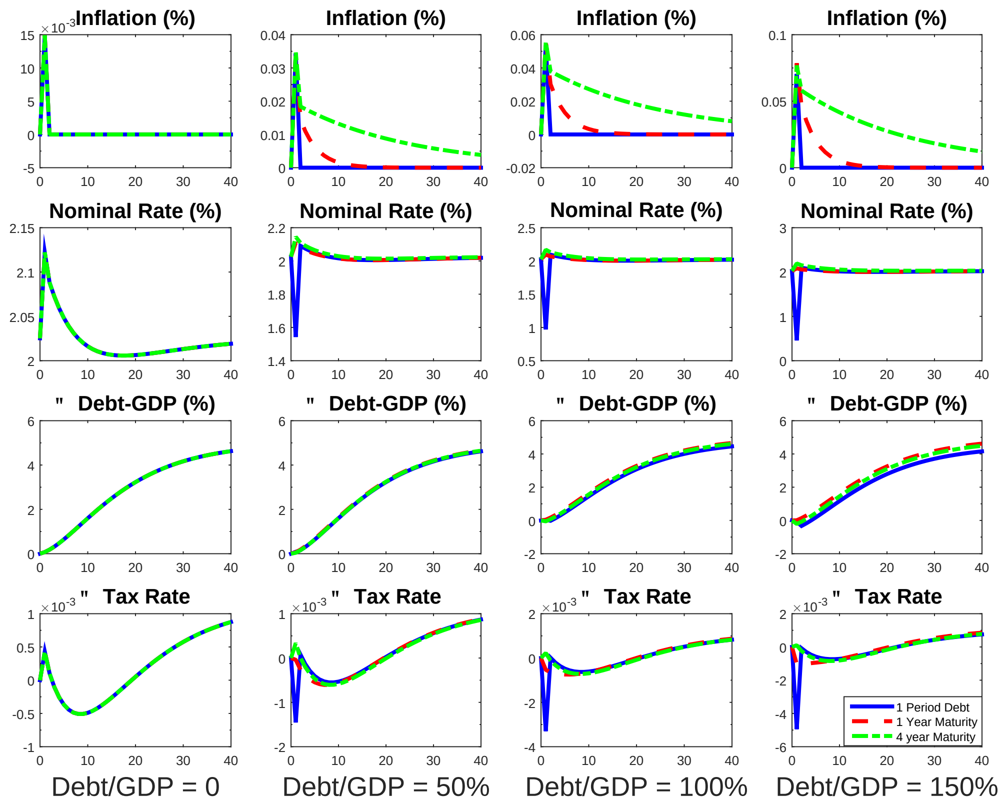

Figure 14: Optimal policy response to higher transfers with different debt levels and maturities.

Moving to column 2, at a higher initial debt level radically different policy responses emerge that depend on debt levels and maturity structures. As in Leith and Wren-Lewis (2013) with single-period debt and a sufficiently high debt stock, the transfers shock results in the policymaker relaxing monetary policy to reduce debt service costs and fuel the initial burst in inflation. Monetary policy stabilizes the debt—just as in the fiscal theory—while tax rates fall to moderate the rise in inflation. Thereafter a combination of monetary and fiscal policy stabilize the debt without generating any further inflation. When the debt is of longer term maturity (1 or 5 years), the initial policy response is quite different, with a tighter monetary policy and higher tax rates. The initial rise in inflation extends beyond the first period to help stabilize debt through reduced bond prices.

We now turn to the government spending shock in figure 15. The first column sets the initial tax rate at $\tau = 0.39$, sufficient to pay for both the initial value of transfers and public consumption, so there is no debt. In this case, as in the simple endowment economy, debt maturity doesn't matter and the policy response is the same regardless of the maturity of the debt. Unlike the endowment economy, there is surprise inflation, but this plays no direct role in stabilizing debt. Here the inflation reflects initial jumps in tax rates and interest rates that deliver the optimal balance between monetary and fiscal policy. There is a tax-smoothing jump in taxation that would fuel inflation, but which is offset by a tighter monetary policy that makes inflation zero after the initial period. As private consumption recovers, the tax rate rises, and ultimately there is a high tax rate to support an increase level of debt.

As we increase the initial level of debt, maturity structure generates differences in policy responses. As before, longer maturity delivers a smaller, but more sustained increase in inflation that stabilizes debt by reducing bond prices. But there are differences in the policy mix behind this result. When initial debt-to-GDP is just under 50 percent, with only single-period debt the policymaker actually cuts taxes to reduce the inflationary consequences of the government spending shock.

At higher initial debt, more radical differences in the policy mix arise across maturities. Sticky prices mean that not only surprises in the path of inflation influence debt dynamics: the policymaker can also influence real ex-ante interest rates and, through the Phillips curve, the size of the tax base. At a debt level near 100 percent, we observe a substantial fall in both tax rates and interest rates when debt is only single period. This amounts to a reversal of the conventional assignment of monetary and fiscal policy:- monetary policy acts to stabilize debt by cutting real interest rates, while fiscal policy mitigates the inflationary consequences of this by reducing tax rates. For an average debt maturity of 5 years we retain the conventional assignment, with tax rates rising and monetary policy tightening to offset the rise in inflation that higher tax rates would generate.

5.4 An Independent Central Bank

Two key features of jointly optimal policy are worth highlighting. First, price level control, which is typically a feature of optimal monetary policy in the new Keynesian model, is absent in the presence of fiscal policy and the associated tax smoothing objective. Typical analyses have policymakers commit not only to return inflation to target after a shock hits, but to return the *price level* back to its pre-shock level. This commitment reduces inflation expectations and improves the trade-off between stabilization of inflation and the real economy. When fiscal policy enters the picture, the initial inflation becomes a desirable means of stabilizing debt through the revaluation effects that are a distinguishing features of the fiscal theory.

Second, the policy mix depends on the size and maturity of government debt. With short

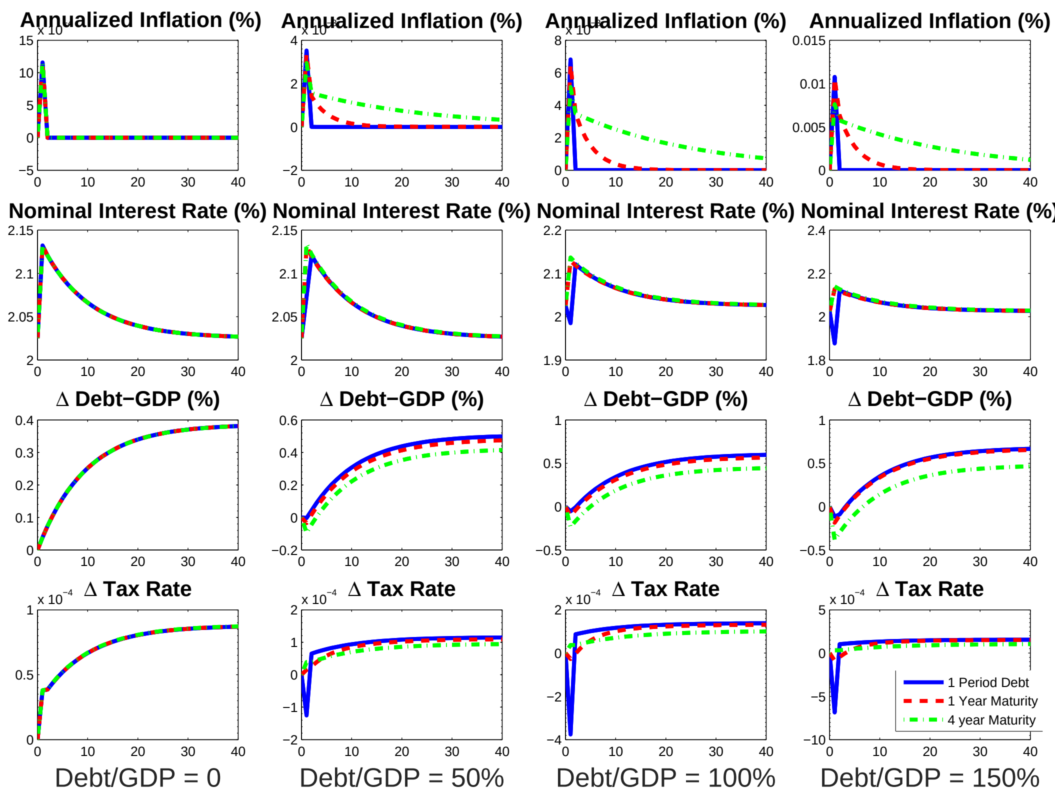

Figure 15: Optimal policy response to an increase in government spending.

maturity and high debt levels, optimal policy reverses the usual policy assignment—raising taxes and interest rates in the face of higher transfers or government consumption—and instead, cuts interest rates to reduce debt interest dynamics and cuts taxes to offset the inflation that the relaxation in monetary policy would otherwise induce. Many economists would be uncomfortable with using monetary policy as a tool of fiscal stabilization in this way and would argue in favour of independent central banks to avoid this policy mix.

We assess the implications of independent monetary policy by deriving the optimal fiscal policy conditional on a given monetary policy rule. We assume that the central bank follows the simple Taylor rule with a coefficient on inflation of $\alpha_\pi = 1.5$. The fiscal authority faces the same optimization described above, but with the additional constraint that monetary policy follows this rule. Figure 16 reports that the policy response to higher government spending exhibits some notable differences from the outcome when monetary and fiscal policies are jointly optimal. Inflation's increase is far more prolonged under an independent central bank. When monetary and fiscal policy operate cooperatively, even for the largest stock of debt we analyzed, inflation is over half that observed when decoupling monetary from fiscal policy. This gives rise to the second surprising result. The active independent monetary policy results in the fiscal policymaker *cutting* rather than raising taxes in response to the government spending shock. The magnitude of the tax cut increases with the stock of debt, but does not vary much across maturities. Optimal fiscal policy counteracts the higher debt service costs that active monetary policy generates by cutting tax rates. This offsets the increase in inflation and under the policy rule mitigates the rise in real interest rates. Because this action is more important the higher the debt, the magnitude of the tax cuts increases with rising debt levels. Similar inflation paths across all debt levels imply that the value of longer maturity debt gets reduced through revaluation effects by more than the other maturities. This also has the implication that the spillovers from monetary policy shocks to the government's finances are likely to be greater at higher and longer maturity debt levels.

These results point to the ubiquity of a central feature of the fiscal theory—debt revaluation through surprise changes in inflation and bond prices. Whether policies are jointly optimal or optimal fiscal policy is constrained by an independent central bank, debt revaluation continues to characterize optimal policy behavior.

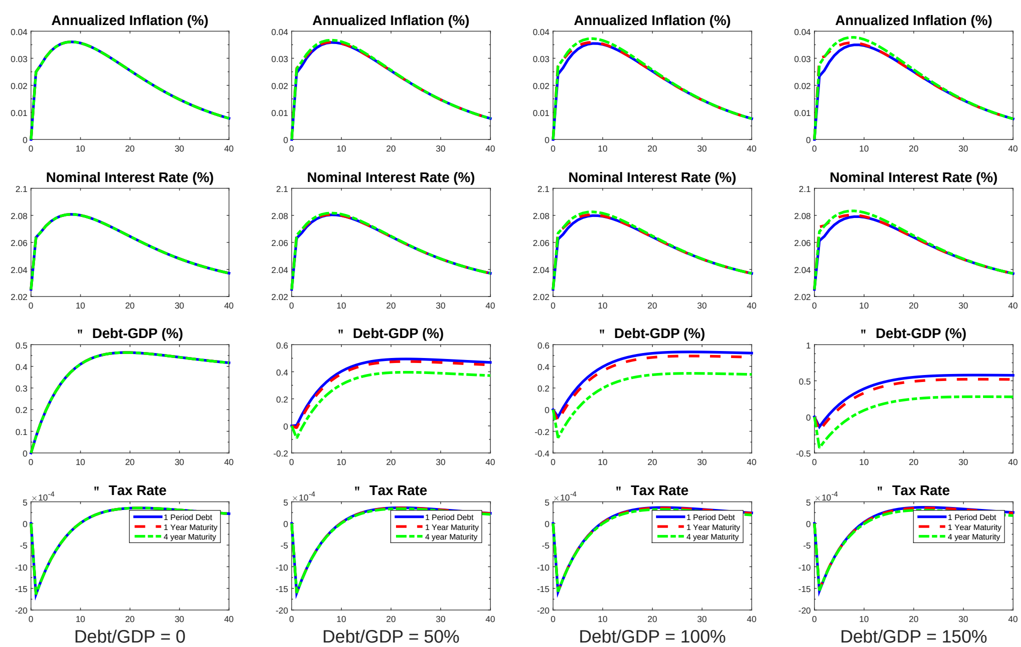

Figure 16: Optimal fiscal policy response to an increase in government spending with an independent central bank.

5.5 Discretion in the New Keynesian Economy

This subsection turns to optimal discretionary policy, following the setup in Leeper, Leith, and Liu (2015). That setup employs a new Keynesian model in which the tax applies to labor income rather than sales revenue and government spending is treated as an endogenous policy instrument rather than an exogenous stream of purchases that need to be financed. There are no transfers. The policy under discretion is a set of decision rules for $\{c_t, y_t, \pi_t, b_t, \tau_t, g_t, P_t^M\}$ that maximize

$$V(b_{t-1}, A_t) = \max\left\{\frac{c_t^{1-\sigma}}{1-\sigma} + \chi\frac{g_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(y_t/A_t)^{1+\varphi}}{1+\varphi} + \beta E_t\left[V(b_t, A_{t+1})\right]\right\} \tag{88}$$

subject to the resource constraint

$$y_t\left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right) - c_t - g_t \tag{89}$$

the Phillips curve

$$\begin{aligned}&(1-\epsilon) + \epsilon(1-\tau_t)^{-1} y_t^{\varphi} c_t^{\sigma} A_t^{-1-\varphi} - \phi\frac{\pi_t}{\pi^*}\left(\frac{\pi_t}{\pi^*} - 1\right)\\ &+ \phi\beta c_t^{\sigma} y_t^{-1} E_t\left[c_{t+1}^{-\sigma}\frac{\pi_{t+1}}{\pi^*}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right)\right]\end{aligned} \tag{90}$$

and the government's budget constraint

$$\begin{aligned}&\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\left(\frac{P_t}{P_{t+1}}\right)\left(1+\rho P_{t+1}^M\right)\right] b_t\\ = &\left\{1 + \rho\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\left(\frac{P_t}{P_{t+1}}\right)(1+\rho P_{t+1}^M)\right]\right\}\frac{b_{t-1}}{\pi_t}\\ &- \left(\frac{\tau_t}{1-\tau_t}\right)\left(\frac{y_t}{A_t}\right)^{1+\varphi} c_t^{\sigma} + g_t\end{aligned} \tag{91}$$

where we have used the bond-pricing equation to eliminate the current value of the portfolio of bonds.

Leeper, Leith, and Liu (2015) solve the nonlinear system consisting of seven first-order conditions and the three constraints to yield the time-consistent optimal policy using the Chebyshev collocation method. In contrast to the case of commitment where steady-state inflation is zero, discretion implies a steady state with a mildly negative debt stock and a mild deflation. Figure 17 shows that starting from high debt levels produces significant policy differences across differing bond maturities. These impulse responses reflect the time-consistent adjustment from a high debt level to the ultimate steady state debt level, which is slightly negative. The most notable element in these dynamic paths is the very high levels of inflation. This inflation does not serve to reduce the real value of debt; instead, it reflects the state-dependent inflationary bias problem generated by high debt levels. When debt levels are raised, the policymaker faces a temptation to use surprise inflation or surprise reductions in bond prices to reduce the real value of government debt. Knowing this, economic agents raise their inflationary expectations until this temptation is no longer present. At empirically plausible debt levels, this temptation is very strong and very high rates of inflation are required ensure the policy remains time consistent. The shorter the debt maturity, the greater the temptation to inflate in the current period rather than delay it, which creates the worse inflationary bias problem. The steady state the economy eventually reaches balances the inflation and debt stabilization biases to generate a small negative long-run optimal value for debt and a slight undershooting of the inflation target. This falls far short of the accumulated level of assets that would be needed to finance government consumption and eliminate tax and other distortions.

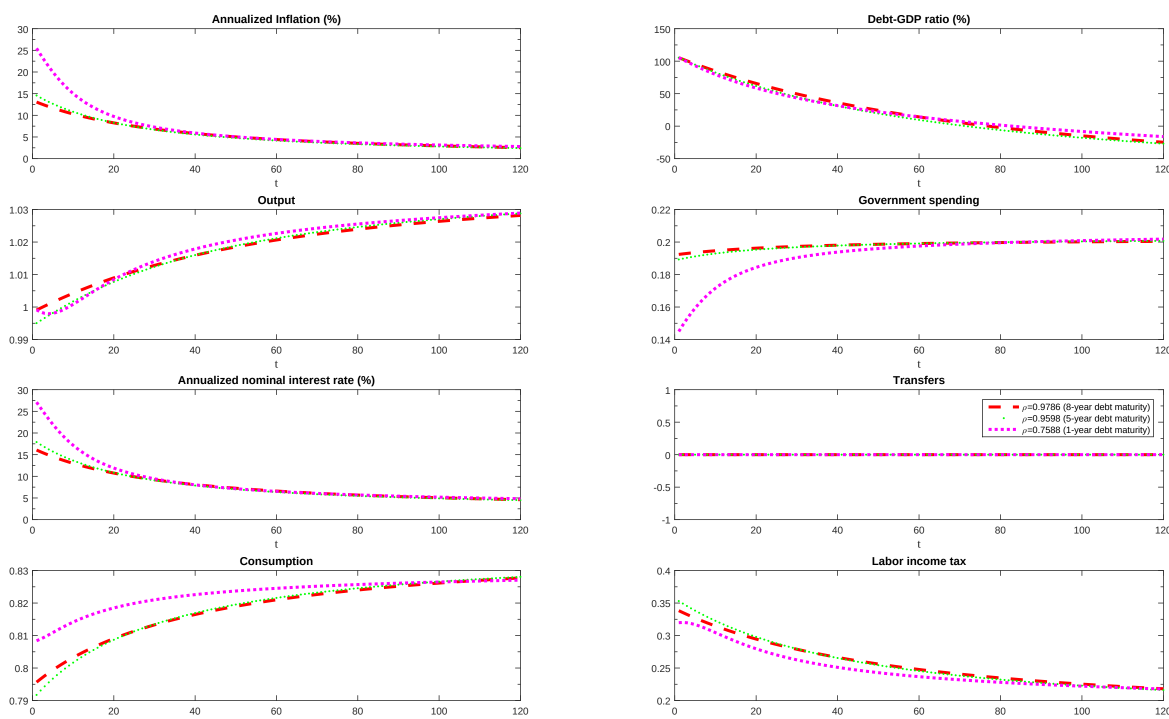

Figure 17: New Keynesian model under discretionary policy.

6 Practical Implications

Viewing practical issues through the lens of the fiscal theory sheds fresh light on policy problems. That new light can also lead to sharply different perspectives on these problems.

6.1 Inflation Targeting

Nearly 30 countries with independent central banks have embraces explicit inflation targeting as the operating principle for monetary policy. Very few of these countries simultaneously adopted fiscal policies that are compatible with the chosen inflation target. This discussion of the policy interactions that are prerequisites for successful inflation targeting does not depend on the prevailing monetary-fiscal regime, so it applies whether policies consist of active monetary/passive fiscal or the reverse.

The derivations rely on a few generic first-order conditions, a government budget constraint, and the condition that optimizing households will not want to over- or under-accumulate assets. For this reason, the results have broad implications that extend well beyond the details of particular models. Consider an economy with a geometrically decaying maturity structure on zero-coupon nominal government bonds. The government's budget constraint is

$$\frac{P_t^m B_t^m}{P_t} = \frac{(1+\rho P_t^m)B_{t-1}^m}{P_t} - s_t \tag{92}$$

Letting $\Lambda_{t,t+k} \equiv \beta^k \frac{u'(c_{t+k})}{u'(c_t)} \frac{P_t}{P_{t+k}}$, asset-pricing conditions yield

$$\frac{1}{R_t} = E_t \Lambda_{t,t+1} \tag{93}$$

$$P_t^m = E_t \Lambda_{t,t+1}(1+\rho P_{t+1}^m) \tag{94}$$

and the term structure relationship is

$$P_t^m = E_t \sum_{k=0}^{\infty} \rho^k \left(\prod_{j=0}^{k} \frac{1}{R_{t+j}} \right) \tag{95}$$

These conditions deliver the usual bond valuation equation

$$\frac{(1+\rho P_t^m)B_{t-1}^m}{P_t} = E_t \sum_{i=0}^{\infty} \beta^i \frac{u'(c_{t+i})}{u'(c_t)} s_{t+i} \tag{96}$$

Rewrite the valuation equation by replacing $(1 + \rho P_t^m)$ using

$$1 + \rho P_t^m = 1 + E_t \sum_{k=1}^{\infty} (\beta\rho)^k \frac{u'(c_{t+k})}{u'(c_t)} \frac{P_t}{P_{t+k}} \tag{97}$$

and, for simplicity, assume a constant-endowment economy, so $\frac{u'(c_{t+i})}{u'(c_t)} = 1$, to generate

$$\left[\sum_{k=0}^{\infty} (\beta\rho)^k \left(\prod_{j=1}^{k} \frac{1}{\pi_{t+j}}\right)\right] \frac{B_{t-1}^m}{P_t} = E_t \sum_{k=0}^{\infty} \beta^k s_{t+k} \tag{98}$$

Imagine an economy that takes as given variables dated $t-1$ and earlier, but commits to hitting an inflation target in all subsequent dates, so $\pi_{t+k} \equiv \pi^*$ for $k \geq 0$. Valuation equation (98) becomes

$$\frac{B_{t-1}^m / P_{t-1}}{EPV_t(s)} = \pi^* - \beta\rho \tag{99}$$

where $EPV_t(s) \equiv E_t \sum_{k=0}^{\infty} \beta^k s_{t+k}$.

This expression imposes stringent conditions on the expected path of primary surpluses if the inflation target is to be achieved. For given initial real debt, if the economy adopts a policy of "too high" surpluses, then the inflation target that is achievable is lower than the desired target, π^*. Another way of seeing the tension between monetary and fiscal policy in this equation is to note that the condition requires the fiscal policymaker to adopt a debt target, which it passively adjusts surpluses to achieve. This means that any period of austerity that raises surpluses must induce a subsequent relaxation of policy to bring $EPV_t(s)$ in line with the outstanding debt stock and the inflation target. An austerity program that never took its foot off the gas would undermine the inflation target just a surely as would a myopic fiscal policymaker prone to runaway deficits.

When the fiscal policymaker does not make surpluses depend on debt, then section 2.3 shows that the monetary authority's ability to control inflation depends crucially on the maturity structure of the outstanding and the nature of their policy response. With a pegged nominal interest rate, inflationary expectations remain consistent with the inflation target with surprise deviations from that target providing the revaluation effects required to satisfy the bond valuation equation. In contrast, when the monetary policy rule attempts to come as close to active as possible by setting $\alpha_\pi = \beta$, the rate of inflation followed a random walk, permanently deviating from the inflation target in the face of fiscal shocks. If the objective is to smooth the inflationary costs of such revaluation effects, then the optimal policy exercises suggest that a persistent deviation from the inflation target is desirable to the extent to which it matches the maturity structure of the government's debt portfolio.

With only single-period debt there is no advantage in having a prolonged increase in inflation following a fiscal shock because only the initial period's inflation helps to reduce the real value of government liabilities. But when debt is of longer maturity, allowing inflation to rise and then gradually decline as the predetermined debt stock matures reduces the discounted value of inflationary costs associated with the required revaluation effects.

The importance of time-consistency was revealed in the analysis of discretionary policy. For conventional parameterizations of the costs of nominal inertia, the reliance on inflationary financing of government debt under the time-inconsistent Ramsey policy was relatively small (although rising in both the size and maturity of the government debt stock). Despite the fact that it appears to be a general result that even a small amount of nominal inertia is sufficiently costly to tilt optimal policy away from the use of inflation surprises as a tool of fiscal stability, when we consider time-consistent policy the inflation bias can be enormous. If policymakers cannot credibly commit, then even a relatively modest desire to inflate away the debt can produce a significant increase in inflation, especially at high levels of debt. These effects can be mitigated by issuing longer term debt because then the urge to pursue an immediate increase inflation to reduce the debt burden is less pressing.

Successful inflation targeting requires more than a resolute central bank that follows "best practice" monetary policy behavior that includes clear objectives, transparency that leads to effective communications, and accountability. Even with all these elements in place, expression (99) implies that the central bank can achieve π^* only if fiscal policy is compatible with that target. If fiscal behavior requires a long-run inflation rate that differs from π^*, even best practice monetary policy cannot succeed in anchoring long-run inflation expectations or inflation outturns on target.

6.2 How Important is Debt Revaluation Through Inflation?

One way to assess the importance of the mechanisms highlighted in this chapter is to look at the magnitude of the revaluation effects in historical data. Sims (2013) calculates that since 1960 the surprise gains and losses on U.S. government debt as a percentage of GDP are similar in magnitude to the fluctuations in the deficit relative to GDP, suggesting that such effects are an important aspect of monetary-fiscal dynamics. Our analysis also found that the efficacy of using revaluation effects as a tool of optimal policy increases in both the size and the maturity of the outstanding debt stock. This suggests that the recent increase in debt-to-GDP ratios in most advanced economies raises the likelihood that such revaluation effects may become an increasingly important feature of policy. The International Monetary Fund (2013) calculates that increasing inflation from a projected average of 1.6 percent to 6 percent for the G7 economies would lead to substantial reductions in the debt-to-GDP ratio that range from 10 percent in the United States to over 20 percent in of Japan. But even if one does not believe that policymakers will resort to utilizing such revaluation effects in responding to high debt levels, the analysis of time-consistent policymaking suggests that even the possibility that they may do so creates a substantial time-inconsistency problem.

While the fiscal theory relies on the use of such revaluation effects to attain the equilibrium value of government bonds, more generally the chapter has shown that such revaluation effects are not solely the preserve of regime F. Even in the simple endowment economy with policy described by simple rules, monetary policy and government spending shocks both induce such effects in both policy regimes. Optimal policy exercises show that it is desirable to use a combination of surprise inflation and tax smoothing to stabilize the economy in the face of fiscal shocks, blurring the lines between the M and F regimes. In richer production economies subject to nominal inertia, the range of monetary and fiscal policy interactions is far wider: monetary and fiscal policy jointly determine the extent to which there are inflation surprises, movements in real interest rates and changes in the tax base.

These considerations point to the need to consider the nature of the policy assignment. Kirsanova, Leith, and Wren-Lewis (2009) describe the conventional policy assignment as tasking monetary policy with targeting inflation and fiscal policy with stabilizing debt. The alternative regime, where monetary policy ensures fiscal solvency, uses revaluation effects as one of the tools that the monetary authority employs to achieve this. Conventional wisdom stresses the desirability of designing policy institutions to support the conventional assignment. For example, analysis of simple rules in the context of new Keynesian models with single-period debt suggests that an interest rate rule that responds aggressively to inflation in combination with a fiscal rule that slowly adjusts the deficit in respond to government debt, achieves welfare levels similar to those achieved under the Ramsey plan [see Schmitt-Grohé and Uribe (2007)]. And Schmitt-Grohé and Uribe (2004) argue that even a small degree of price stickiness results in optimal policy turning away from the Friedman rule and away from inflation surprises as a desirable means of insulating the government's finances from shocks in favour of price stability and tax smoothing.

Despite this apparent consensus surrounding the appropriate assignment of tasks to monetary and fiscal policy, empirically it appears that monetary and fiscal policies do not always fulfill these roles. Bianchi (2012) and Bianchi and Ilut (2014) estimate a simple new Keynesian model augmented with fiscal policy, habits and inflation inertia which allows for switches in monetary and fiscal policy rules. Bianchi (2012) allows for a circular movement across three regimes where policy can transition from the conventional assignment (active monetary policy/passive fiscal policy) through an unconventional assignment of passive monetary /active fiscal policy, to an unstable regime where both monetary and fiscal policy are active.

He finds that the 1960s and 1970s featured a combination of passive monetary and active fiscal policy, before the Volcker disinflation resulted in monetary policy turning active in 1979 while fiscal policy remained active. It is only around 1990 that fiscal policy turned passive in support of the active monetary policy and the conventional assignment as achieved. In Bianchi and Ilut a slightly different set of possible policy transitions is considered allowing the two stable regimes (active monetary/passive fiscal and passive monetary/active fiscal) to briefly transition through the unstable regime of active monetary/active fiscal. Here the story is similar as policy is characterized by a passive monetary/active fiscal policy regime, before monetary policy turns active in 1979 with the Volcker disinflation, and fiscal policy turns passive shortly afterwards.

Chen, Leeper, and Leith (2015) build on this work by allowing both monetary and fiscal policy to be conducted optimally. They find that monetary policy is both optimal and time-consistent, but with switches in the degree of anti-inflation conservatism. Those switches imply that policy was not only less conservative in the 1970s prior to the Volcker disinflation, but also intermittently during the 1960s, briefly following the stock market crash of the late 1980s and the dot-com crash of 2000. At the same time, fiscal policy can rarely be described as optimal, but tends to move between an active and passive rule. For the bulk of the period between 1954 and the 2008 financial crisis, fiscal policy has been predominantly active with the only sustained periods of passive fiscal policy in the late 1950s until the late 1960s and between 1995 and 2000. In other words, the conventional assignment is the exception rather than the norm.

A key question that follows such empirical evidence is whether the failure to observe the conventional assignment in the data represents a clear failure of policy institution design. Following the conjecture of Sims (2013) that the desirability of the conventional assignment was dependent on the assumption of single-period debt, we consider the importance of debt maturity and the level of debt in defining the optimal policy assignment. We find that the Ramsey plan did indeed look like the conventional assignment when debt levels were low and maturity was long: monetary policy was tightened to stabilize inflation in the face of a government spending shock, while tax rates were raised to stabilize debt. But as debt levels rise, especially when maturity was short, we find a reversal of the policy assignment: monetary policy was relaxed in the face of the same shock to reduce debt service costs and stabilize debt, while tax rates were cut to stabilize inflation. When we capture the current institutional design of policy by allowing an independent central bank to follow an active Taylor rule, the Ramsey policy for the fiscal policy instrument actually cut taxes in the face of the same government spending shock in order to reduce inflation and offset the increase in debt service costs induced by the active monetary policy. Despite this anti-inflationary policy on the part of the fiscal policymaker, the equilibrium rate of inflation when the central bank was independent is an order of magnitude higher than when monetary and fiscal policy where jointly optimal.

6.3 Returning to "Normal" Monetary Policy

The financial crisis has seen a substantial increase in debt to GDP ratios in many advanced economies, although the immediate need for fiscal adjustment may have been muted due to the reduced debt service costs as real interest rates have fallen since the financial crisis. To see this consider a small change to our policy problem in the endowment economy, where we allow the households' discount factor to temporarily rise to $\widetilde{\beta} > \beta$, capturing the flight to quality observed in the financial crisis. In this case, if we assume government spending is held constant, the policy problem becomes,

$$\begin{aligned} L_t &= E_0 \frac{1}{2} \sum_{t=0}^{\infty} \beta^t [-\frac{1}{2}(\tau_t^2 + \theta(\nu_t - 1)^2) \\ &\quad + \mu_t(\widetilde{\beta}_t E_t \frac{(1+\rho P_{t+1}^m)}{P_t^m} \nu_{t+1} - 1) \\ &\quad + \lambda_t(b_t P_t^m - (1+\rho P_t^m) b_{t-1} \nu_t - g_t - z_t + \tau_t) \end{aligned}$$

which yields the first order conditions,

$$\begin{aligned} \tau_t &: -\tau_t + \lambda_t = 0 \\ \nu_t &: -\theta(\nu_t - 1) + \mu_{t-1} \frac{(1+\rho P_t^m)}{P_{t-1}^m} \beta^{-1} \widetilde{\beta}_{t-1} - (1+\rho P_t^m)\lambda_t b_{t-1} = 0 \\ P_t^m &: -\frac{\mu_t}{P_t^m} + \mu_{t-1}\rho \frac{\nu_t}{P_{t-1}^m} \beta^{-1} \widetilde{\beta}_{t-1} + \lambda_t(b_t - \rho \nu_t b_{t-1}) = 0 \\ b_t &: \lambda_t P_t^m - \beta E_t (1+\rho P_{t+1}^m) \nu_{t+1} \lambda_{t+1} = 0 \end{aligned}$$

Under a perfect foresight equilibrium this implies the tax smoothing result is recast as,

$$\tau_t = \beta \widetilde{\beta}_t^{-1} \tau_{t+1}$$

which means that the tax rate will be rising during the period in which households have an increased preference for holding government bonds over consumption. Intuitively, the original tax smoothing result balances the short-run costs of raising taxes to reduce debt against the long run benefit of lower debt. These costs are benefits are finely balanced with the interest rate on the debt being exactly offset by the policymaker's rate of time preference so that steady-state debt follows a random walk in the face of shocks. When the interest on debt is less than the policymaker's rate of time preference the policymaker prefers to delay the fiscal adjustment and will allow debt to accumulate, only stabilizing the debt level once the period of increased household preference for debt holdings has passed. Therefore, to the extent that a return to 'normal' monetary policy is associated with a rise in debt service costs optimal policy would suggest that the efforts to stabilize debt are enhanced at this point. However, we also know that, under the Ramsey policy, the use of inflation surprises to revalue debt are only effective if carried out before the predetermined debt stock matures. Therefore the delay in debt stabilization also reduces the efficacy of promising to raise prices in the future placing more of the burden of adjustment on taxation. At the same time, the higher debt stock that emerges at the point of normalization raises the potential time-inconsistency problems inherent in the Ramsey policy such that it is at this point we may start to see increased pressure to inflate away the debt.

6.4 Why Central Banks Need to Know the Prevailing Regime

Davig and Leeper (2006), Bianchi (2012), Bianchi and Ilut (2014) and Chen, Leeper, and Leith (2015) suggest that there have been switches in the conduct of fiscal policy between passive and active, and that these are not always associated with compensating switches in monetary policy which place the economy in either regime M or regime F. If these policy permutations were permanent they would either result in indeterminacy (passive monetary and fiscal policy) or non-existence of equilibrium (active monetary and fiscal policy), according to Leeper (1991). But is policy is expected to return to either the M or F regime sufficiently often, then these policy combinations can still deliver determinate equilibria. Therefore, there are four possible permutations of monetary and fiscal policy which may coexist although only two can be permanent policy configurations. The prevailing policy configuration can have profound implications for the conduct of monetary policy. Using our simple endowment economy and policy rules of section 2, we will now consider each in turn.

Regardless of regime, inflationary dynamics under our monetary policy rule are

$$E_t(\nu_{t+1} - \nu^*) = \frac{\alpha_\pi}{\beta}(\nu_t - \nu^*) \tag{100}$$

Under regime M with an active monetary policy ($\alpha_\pi > \beta$), monetary policy can target inflation in each period, $\nu_t = \nu^*$, while the passive fiscal policy stabilizes debt

$$E_t(\frac{b_{t+1}}{R_{t+1}} - \frac{b^*}{R^*}) = (\beta^{-1} - \gamma)(\frac{b_t}{R_t} - \frac{b^*}{R^*}) - E_t\varepsilon^F_{t+1} \tag{101}$$

provided $\gamma > \beta^{-1} - 1$.

Now suppose we know we are going to enter this regime in period T at which point inflation will be at its target $\nu_T = \nu^*$ and the fiscal rule will stabilize whatever debt is inherited at time T. In this case it does not matter whether or not the monetary policy rule is active or passive prior to period T since T-step-ahead expected inflation is

$$E_t \nu_{t+T} - \nu^* = \left(\frac{\alpha}{\beta}\right)^{T-t} (\nu_t - \nu^*) \tag{102}$$

which implies that inflation will be on target between now and period T. To the extent that fiscal policy is active, debt will be moving off target between now and period T, but the passive fiscal rule will, from that point on, stabilize debt. On the other hand, if fiscal policy was passive prior to T then this would facilitate the debt stabilization prior to T although even if this was not complete by T the targeting of inflation would be uninterrupted by any change of regime at time T.

We now assume that at time T we anticipate entering regime F where monetary policy is passive ($\alpha_\pi < \beta$), and fiscal policy does not respond to debt ($\gamma = 0$). In this case the period T price level will need to be sufficient to satisfy the bond valuation equation at time T given the level of inherited debt B_{T-1}. When $\gamma = 0$, the fiscal rule is $s_t = s^* + \varepsilon_t^F$ and the solution for real debt is

$$E_t \frac{B_{T-1}}{R_{T-1} P_{T-1}} = \frac{b^*}{R^*} + \sum_{j=1}^{\infty} \beta^j E_t \varepsilon_{T-1+j}^F \tag{103}$$

The price level does not jump in period T, but it does in period t when the switch to regime F in period T is first anticipated. The implications for inflation beyond period T depend on how passive the monetary policy rule is. With an interest rate peg, $\alpha = 0$, inflationary expectations remain on target, $E_t \nu_{t+1} = \nu^*$, but there will be innovations to inflation to ensure the bond valuation equation holds in the face of additional fiscal shocks occurring from period T onwards. With some kind of response to inflation, while still ensuring the monetary policy rule is passive, $0 < \alpha_\pi < \beta$, then an initial jump in the price level will result in a temporary, but sustained rise in inflation according to equation (100), which is now a stable difference equation. As discussed in section 4, sustaining the rise in inflation can enhance the revaluation effect, while reducing the discounted sum of distortions caused by higher inflation the longer is debt maturity. While pegging the interest rate when one is in the F regime may seem the obvious thing to do when considering single period debt, allowing for longer debt maturities implies a trade-off between stabilizing inflation expectations and reducing the costs of inflation surprises.

How does anticipating the F regime in period T affect the conduct of policy prior to period T? With fiscal policy following a rule that may or may not be passive, the expected evolution of government debt follows

$$E_t(\frac{B_{t+1}}{R_{t+1}P_{t+1}} - \frac{b^*}{R^*}) = (\beta^{-1} - \gamma)(\frac{B_t}{R_t P_t} - \frac{b^*}{R^*}) - E_t \varepsilon^F_{t+1}$$

We can iterate this forward until period T as,

$$E_t(\frac{B_{T-1}}{R_{T-1}P_{T-1}} - \frac{b^*}{R^*}) = (\beta^{-1} - \gamma)^{T-1-t}(\frac{B_t}{R_t P_t} - \frac{b^*}{R^*}) + \sum_{j=0}^{T-1-t} (\beta^{-1} - \gamma)^j E_t \varepsilon^F_{t+1+j}$$

which defines the initial debt level $\frac{B_t}{R_t P_t}$ required to ensure the economy enters regime F in period T with the appropriate level of debt $\frac{B_{T-1}}{R_{T-1}P_{T-1}}$without any discrete jumps in the price level at that time. This depends upon the extent to which fiscal policy prior to period T acts to stabilize debt as determined by the fiscal feedback parameter, γ, and the expected value of fiscal shocks over that period. If the move to the F regime is sufficiently long in the future and fiscal policy is sufficiently aggressive in stabilizing debt then there will be little need for surprise inflation in the initial period to ensure the appropriate debt level is bequeathed to the future. However, if the switch is more imminent or the fiscal stabilization prior to period T is muted then an initial jump in prices will be required to ensure the bond valuation equation holds. The inflationary implications of this prior to period T then depend on the conduct of monetary policy. If monetary policy is active prior to period T then any initial jump in prices will be explosive until the F regime is established in period T. Effectively the period t price level jump ensures the bond valuation equation holds, while inflation dynamics are determined by equation 100 which is explosive under an active monetary policy. This is a valid equilibrium as the process for inflation stabilizes when the policy regime changes in period T. But before period T the active monetary policy actually destabilizes prices. Postponing the switch to the F regime means the period of explosive inflation dynamics remains in place for longer.

The above analysis captures the game of chicken between the monetary and fiscal policymakers. The monetary authority can stick to an active monetary policy rule and achieve its inflation target provided everyone is sure that policy will eventually be supported by a passive fiscal policy which stabilizes debt. Debt dynamics will be unstable in such a scenario until the fiscal authorities relent and adopt a passive fiscal policy. But when there is the suspicion that monetary policy will eventually turn passive in order to support a fiscal policy which doesn't act to stabilize debt then conventional anti-inflation policies today may actually worsen inflation outcomes even if this monetary policy is supported by a currently passive fiscal policy.

7 Concluding Remarks

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