Housing Prices and Consumer Spending: The Bank Balance-Sheet Channel

Nuno Paixão

Bank of Canada

Housing, Credit and Heterogeneity: New Challenges for Stabilization Policies

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Contributions and Findings

Theoretical Contribution

- Introduce a *Banking Sector* with Balance Sheet Frictions in a model of collateralized debt with default
- Credit supply depends on the capitalization of the entire banking sector.
- Mortgage spreads and endogenous down payments increase in periods when banks are poorly capitalized
- Quantify the Bank Balance Sheet Channel
 - Bank Balance Sheet explains 13% of the change in house prices, 9% change in foreclosures and 22% change in consumption

Empirical Contribution

- Document the Bank Balance Sheet Channel using an instrumental variable approach
 - Banks located in areas exposed to higher house price drop faced larger declines in their capital ratio
 - An 1p.p. decrease in the capital ratio induced by exogenous variation in housing prices leads to a decrease of supply of Home Purchase loans by 10.5% and Refinance by 15.2%

Related Work

Consumption response to Housing Price Shocks

- Mian et al. (2013), Kaplan et al. (2016), Mian and Sufi (2011, 2014)
- Berger et al. (2016), Carrol and Dunn (1998)
- Huo and Rios-Rull (2013), Kaplan et al. (2015), Garriga and Hedlund (2016)

Lending Channel

- ▶ Stein (1998), Kashyap and Stein (2000) , Jimenez et al. (2012)
- Chakraborty et al. (2016), Greenstone and Alexandre (2012), Chodow-Reich (2014)
- Gertler and Keradi (2011), Gertler and Kiyotaki (2009)

Credit Crunch and Financial Crisis

 Guerrieri and Lorenzoni (2015), Jermann and Quadrini (2012), Favilukis et al. (2015)

MODEL

Model Overview

Time is discrete and infinite

Households

- Agents live forever
- Homeowners or Renters
- Long-term mortgages

Banks

- Issue and price individual mortgages
- Bank balance sheet frictions
- Credit supply depends on the banks' capitalization

Housing Sector

- Determine housing prices and rental rates
- Endogenous House Prices

Households

Income endowment (y) subject to temporary uninsured shocks

$$y_{it} = w.exp(z_{it}), \quad z_{it} = \rho z_{it-1} + \epsilon_{it}, \quad \epsilon_{it} \sim \mathcal{N}(0, \sigma_z)$$

• Utility over non-durable goods (c) and housing services (s)

- Rented: s = h
- Owned: s = vh, v > 1
- Housing (h):
 - Rental Housing p^r_t
 - Owned Housing- pt
 - Transaction Costs
 - Random maintenance costs

Long-Term Mortgages

Long Term Collateralized Mortgages

- Mortgage face value (principal) originated at time τ : $m_{\tau} = m$
- Borrower receives $q_{\tau}(y, a, h, m, r_{\tau}^m) m$
- Payments
 - **Contract terminates** (house sold or refinance): $X_t^s = m_{t-1}$
 - **Default** (Bank takes the house): $X_t^d = \min\{(1 \chi_d) p_t h_t, m_{t-1}\}$
 - Mortgage payment:

•
$$X_t = \frac{\mu + r_{\tau}^m}{1 + r_{\tau}^m} m_{t-1}$$

• μ amortization term, r_t^m the coupon (or interest) part

•
$$m_t = (1 - \mu)m_{t-1} = (1 - \mu)^t m_{t-1}$$

Households Decisions

• Homeowners $\Lambda_h = (y, a, h, \delta_h, m, r_{\tau}^m)$

- ► Stays Home-owner: Pays Mortgage, Refinances or Changes House
- Default becomes a renter with no access to credit market
- Sells house and becomes a Renter
- Renter $\Lambda_r = (y, a)$
 - Rents
 - Buys a house
 - If have Defaulted before may be restricted of mortgage market
- ► All decide Consumption (*c*) and Savings (*a*)

Value Functions Housing Sector

Representative Bank that behaves competitively

$$Q_t M_t = B_t + N_t$$

 $Q_t M_t = \int q_{it}(m_{it}) m_{it} di$

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- Frictions:
- Low Capital ratio is costly

$$\Phi\left(\frac{N}{QM}\right) = \begin{cases} \kappa_0 + \kappa_1 \left(\tilde{K} - \frac{N}{QM}\right)^2 & \text{if } \frac{N}{QM} < \tilde{K} \\ 0 & \text{otherwise} \end{cases}$$

► Net worth is accumulated through retained earnings $N_{t+1} = (1 - \omega) [N_t + \Pi_{t+1}]$

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$$\Pi_{t+1} = r_{t+1}^m Q_t M_t - rB_t - \Phi\left(\frac{N_t}{Q_t M_t}\right)$$

- Maximize the present discounted value of future dividends Bank's Problem
 - Given N_t , decides M_t and B_t

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 - Given N_t , decides M_t and B_t
- If No frictions

$$r_{t+1}^m - r = 0$$

With Frictions

$$\left\{ r_{t+1}^{m} - \underbrace{r - \Phi\left(\frac{N_{t}}{Q_{t}M_{t}}\right) - \Phi'\left(\frac{N_{t}}{Q_{t}M_{t}}\right)\frac{N_{t}}{Q_{t}M_{t}}}_{r_{t+1}^{c}} \right\} = 0$$

- High Leverage
 - Cost of funding increases r_{t+1}^c \uparrow

Individual Mortgage

Competition: zero expected discounted profit

$$q_{t}(y, a', h', m', r_{t}^{m})m' = \frac{1}{(1 + r_{t+1}^{c})}E_{t}^{i}\{z_{t+1}m' + (1 - d_{it+1} - s_{it+1})q_{t+1}(y', a'', h', (1 - \mu)m', r_{t}^{m})(1 - \mu)m'\}$$

Mortgages price decrease when banks are constraint (higher leverage ratio)

• Cost of funding increases r_{t+1}^c \uparrow

HS

Housing Prices





















Calibration - Target Moments

Moments	Data	Model	Parameters	Value
Homeownership	68%	68.1%	Own-house add utility	<i>v</i> = 1.06
$LTV \ge 90\%$	7.02%	7.51%	Discount Factor	eta= 0.945
Average Equity	62%	63.7%	Mortgage amortization	$\mu=$ 0.018
Default Rate	1.5%	1.45%	High Depreciation shock	$\delta = 0.22$
Depreciation rate	1.06%	1.06%	Prob High Maintenance	$p_{\delta}=0.048$
Refinance Rate	24%	25.7%	Refinance Cost	$\chi_r = 5.1\%$
Mortgage Spread	165b.p.	160b.p.	Capital ratio target	$ ilde{K}=15\%$
Increase in spread	128b.p.		Leverage Cost Param.	$\kappa_0 = 0.0103, \ \kappa_1 = 3.37$

Exogenous Calibration

Calibration



Non Target Moments

Moments	Data	Model
Mortgage Holder Rate	66%	67%
Avg. Income Homeowners / renters	2.05	3.34
Avg. Housing Wealth /Avg. Income	1.69	2.54
Cash Buyers	19	19.41
% Homeowners with 0% equity	1.81	0.39
$\%$ Homeowners with \leq 10% equity	7.02	6.5
$\%$ Homeowners with \leq 20% equity	14.07	13.04
$\%$ Homeowners with \leq 30% equity	22.4	21.05
% Homeowners with 100% equity	28.75	34.05

Home Equity



Quantification of Bank Balance Sheet

- Unanticipated Decrease in Demand for Housing
- Negative Productivity shock (4.7% cumulative over 3 periods)
- Delays in foreclosure process

riangleCumulative	Data	Model (a)	No Fric (b)	(a-b)/a
House prices	-18%	-18%	-16.6%	13%
Default Rate	13p.p.	11.2p.p.	10.2p.p.	9%
Consumption	-11.5%	-10.6%	-8.2 %	22%
Refinancing	-43%	-38.5%	-24.9%	35%
Bank Capital	-1.4p.p.	-1.15p.p.	-0.72p.p.	38%
Mortgage spread	133b.p.	109b.p.	0	

Heterogeneity

Results



Heterogeneity



EMPIRICAL EVIDENCE

Empirical Evidence

- Goal: Estimate how changes in Housing Prices affect Mortgage Supply through Banks'Balance Sheets
- > Part I: Impact of decline in house prices on Capital Ratio

$$\Delta K_{k,t} = \beta_1 + \beta_2 RES_{k,t} + \beta_3 X_{k,05} + \epsilon_{k,t}$$

- Challenge: Reverse Causality
- Solutions:
 - Exploit variation in banks' exposure to different housing markets
 - Instrumental variable approach structural breaks in house prices evolution 2000-2006 (Charles, Hurst and Notowidigdo (2017))
- Part II: Impact of decline in Capital Ratio (induced by house price drop) on Credit Supply
 - Control for Demand characteristics at county level

$$\Delta VolOrig_{j,t} = \beta_1 + \beta_2 \Delta Y_{j,t} + \beta_3 \Delta H_{j,t} + \beta_4 X_{j,05} + \epsilon_{j,t}$$

$$\Delta Y_{j,t} = \sum_k \alpha_{k,j} \Delta \widehat{K_{k,t,-j}}$$

Results - Part I: $\Delta K_{k,t} = \beta_1 + \beta_2 RES_{k,t} + \beta_3 X_{k,05} + \epsilon_{k,t}$

	OLS	IV	OLS	IV
RES(t)	0.088***	0.091***	0.061***	0.082**
	(0.009)	(0.022)	(0.009)	(0.026)
Observations	4908	4908	4888	4888
Adjusted R ²	0.031	0.031	0.117	0.116
SD	robust	robust	robust	robust
Bank controls	No	No	Yes	Yes
Year FE	No	No	Yes	Yes

- If a bank faces an average shock (-4.6p.p. per year), capital decreases by -0.38p.p..
- From 90th to 10th percentile of change in RES implies that Capital Ratio decreases 0.85p.p. more

Results - Part II: $\Delta VolOrig_{j,t} = \beta_1 + \beta_2 \Delta Y_{j,t} + \beta_3 \Delta H_{j,t} + \beta_4 X_{j,05} + \epsilon_{j,t}$

	Banks in sample		All Originations	
	(1a)	(2a)	(1b)	(2a)
Home Purchase				
$\Delta Y_{i,t}$	141.031***	47.090**	37.701***	10.489^{*}
<u>,</u>	(21.241)	(17.293)	(4.514)	(4.352)
Refinance				
$\Delta Y_{j,t}$	60.902***	78.385***	24.908***	15.184^{*}
	(13.507)	(12.809)	(6.453)	(6.038)
Observations	2850	2850	3010	3010
cluster	State	State	State	State
Year FE	No	Yes	No	Yes
State FE	No	Yes	No	Yes

Going from the 90th to the 10th percentile of change in capital ratio induced by a real estate shock distribution (-0.57p.p.) in the cross-section implies a decrease in **Refinance of 8.55% and Home Purchases of 5.98%.**

Conclusion

- Model of long-term collateralized debt with risk of default with a Banking Sector with balance sheet frictions
 - Endogenous Credit Supply
- Bank Balance Sheet Channel is important to explain changes in house prices, foreclosures and consumption between 2006-2009
- Empirical Evidence that Bank's balance sheet are affected by change in house prices
 - More constrained banks contracted credit supply by more

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Payments

- Contract terminates (house sold or refinance): $X_t^s = m_{t-1}$
- **Default** (Bank takes the house): $X_t^d = \min\{(1 \chi_d) p_t h_t, (1 + x) m_{t-1}\}$
- Mortgage payment: $X_t = \frac{\mu + r_{\tau}^m}{1 + r_{\tau}^m} m_{t-1}$
 - μ amortization term, r_t^m the coupon (or interest) part
 - $m_t = (1 \mu)m_{t-1} = (1 \mu)^t m$

HH

Homeowners

Keeps House (Refinance or not)

$$V^{HH}\left(\Lambda_{h},\Lambda_{at}\right) = \max_{\{c,a',h',m'\}} U(c,h') + \beta \mathsf{E}_{\left(y',\delta_{h}'\right)|y}\left[V^{H}(\Lambda_{h}',\Lambda_{at+1})\right]$$

$$\begin{aligned} c + a' + \delta_h p_t h &= w.y + a(1+r) + \left[q_t(y, a', m', h', \Lambda_{at})m' - m - \chi_m \right]_{m' \neq (1-\mu)m, h'=h} \\ &+ \left[(1-\chi_s) \, p_t h - (1+\chi_b) \, p_t h' + q_t(y, a', m', h', \Lambda_{at})m' - m - \chi_m \right]_{h' \neq h} \\ &- \left[x_\tau m \right]_{m'=(1-\mu)m, h'=h} - T(y, h', m, r_\tau^m) \end{aligned}$$

Defaults

$$V^{D}(\Lambda_{h},\Lambda_{at}) = \max_{\{c,h',a'\}} U(c,h') + \beta E_{y'|y} \left[(1-\theta) V^{M}(\Lambda'_{r},\Lambda_{at+1}) + \theta V^{NM}(\Lambda'_{r},\Lambda_{at+1}) \right]$$

s.t. $c + p_{t}^{r}h' + a' = y + a(1+r) + \max\{(1-\chi_{d}-\tau_{h}) p_{t}h - m, 0\} - T(y,0,0,0)$

Becomes a Renter

$$V^{HS}(\Lambda_h,\Lambda_{at}) = \max_{\{c,h',a'\}} U(c,h') + \beta E_{y'|y} V^{GR}(\Lambda'_r,\Lambda_{at+1})$$

s.t.
$$c + p_t^r h' + a' = y + a(1+r) + (1 - \delta_h - \chi_s) p_t h - m$$

$$\boldsymbol{V}^{H}\left(\boldsymbol{\Lambda}_{h},\boldsymbol{\Lambda}_{at}\right)=\max\left\{\boldsymbol{V}^{HH}\left(\boldsymbol{\Lambda}_{h},\boldsymbol{\Lambda}_{at}\right),\boldsymbol{V}^{HD}\left(\boldsymbol{\Lambda}_{h},\boldsymbol{\Lambda}_{at}\right),\boldsymbol{V}^{HS}\left(\boldsymbol{\Lambda}_{h},\boldsymbol{\Lambda}_{at}\right)\right\}$$

HH Decisions

Renters (m' = 0 if w = NM)

Buys a House

$$V^{RHw}(\Lambda_{r},\Lambda_{at}) = max_{\{c,a',h',m'\}}U(c,h') + \beta E_{y'|y}\left[V^{HH}(\Lambda'_{h},\Lambda_{at+1})\right]$$

s.t. $c + a' + (1 + \chi_{b})p_{t}h' = y + a(1 + r) + q(y,a',h',m',r_{t}^{m})m' - T(y,0,h',0)$
 $m' = 0 \text{ if } w = NM$

Rents

$$\begin{split} V^{RRw}\left(\Lambda_{r},\Lambda_{at}\right) &= \max_{\{c,h',a'\}} U(c,h') + \beta E_{y'|y}\left[V^{Rw}(\Lambda_{r}',\Lambda_{at+1})\right] \\ & c + p_{t}^{r}h' + a' = y + a(1+r) \end{split}$$

where
$$V^{RM}(\Lambda_r, \Lambda_{at}) = max \left\{ V^{RHM}(\Lambda_r, \Lambda_{at}), V^{RRM}(\Lambda_r, \Lambda_{at}) \right\}$$
 and $V^{RNM}(\Lambda_r, \Lambda_{at}) = max \left\{ V^{RHNM}(\Lambda_r, \Lambda_{at}), V^{RRNM}(\Lambda_r, \Lambda_{at}) \right\}$
HH Decisions

Housing Sector

Composite Consumption

$$Y_c = AN_c \qquad w = A$$

Construction sector

$$Y_h = Y_c^{\alpha_h} L^{1-\alpha_h} \qquad S_t^h = (\alpha_h p_t)^{\frac{\alpha_h}{1-\alpha_h}} L_t$$

Rental Sector:

- Every period faces a maintenance cost $\delta_r.p_t^h h$
- Can buy/sell housing at the equilibrium price
- ▶ No transaction cost: Arbitrage Condition determines equilibrium rents (p^r)

$$p_t^r - (\delta_r + \tau_h)p_t^h + E_t \left[\frac{p_{t+1}^h}{1+r}\right] = p_t^h$$



Calibration - Exogenous Parameters

Parameters	Value		
Housing share	lpha=0.15		
Elasticity substituition c and h	$rac{1}{\gamma} = 1.25$		
Intertemporal elasticity	$\sigma = 2$		
House sizes	$\mathcal{H}^{h} = \{1.43, 1.79, 2.3, 2.9, 3.6, 4.2\}$		
Rental sizes	$\mathcal{H}^r = \{1.1, 1.43, 1.79\}$		
Autocorrelation earning shocks	$ ho_z=0.97$		
S.D. of earning shocks	$\sigma_z = 0.2$		
Buying Costs	$\chi_b=0.01$		
Selling Costs	$\chi_s=0.06$		
Liquidation cost	$\chi_d=0.25$		
Rental Maintenance cost	$\delta_r = 0.0165$		
World Interest Rate	<i>r</i> = 0.03		
Probability of reentering credit mkt	heta=0.25		
Dividend	$\omega=$ 0.115		

Empirical Evidence

- ▶ Part 1: Fluctuations in housing prices impact banks' balance sheets
- Part 2: banks react to losses induced by changes in housing prices by contracting mortgage loan supply

Data

- 2007-2010 period
- Housing Prices: Zillow Median Home Value Index for All Homes
- Mortgages: Home Mortgage Disclosure Act (HMDA)
- Banks' balance sheets:
 - Report of Condition and Income (Call Reports)
 - Summary of Deposits (SOD)
- County level Unemployment (BLS) and Income (IRS)

Empirical Strategy - Part I

Change in house prices and banks balance sheets

$$\Delta K_{k,t} = \beta_1 + \beta_2 RES_{k,t} + \beta_3 X_{k,05} + \epsilon_{k,t}$$
$$RES_{k,t} = \sum_j \omega_{kj05} \Delta P_{jt}$$

- $\Delta K_{k,t}$ change of Capital Ratio of bank k
- *RES_{kt}*: Real Estate Shock to bank k at time t

Instrumental variable approach

- Estimated structural breaks in the house price evolution between 2000 and 2006, Charles, Hurst and Notowidigdo (2017)
- Assumption: variation in housing prices during the boom and bust derived from a speculative âbubbleâ and not from changes in standard determinants of housing values.
- Boom is strongly correlated with the size of its later housing bust, this structural breaks are strongly correlated with house demand in the bust period

1st Stage

Deposits as proxy

$$RES_{k,t} = \sum_{j} \omega_{kj05} \Delta P_{jt}$$

- ΔP_{jt}: change in House Prices in county j
 - ω_{kj05} share of bank k deposits in county j in 2005
- Two major concerns:
 - 1. Weights are based on deposits rather than loans.
 - 2. Rise of mortgage-backed securities may have allowed banks to diversify away from their physical locations.
- Section 109 of the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994 prohibits a bank from establishing or acquiring branches outside of its home state primarily for the purpose of deposit production.
- Aguirregabiria et. al. (2016): evidence of a strong home bias for 1998-2010 period local deposits are mostly used to fund local loans
- Chakraborty et. al. (2016):
 - when loans are sold, banks are likely to remain as servicers of the mortgage and maintain exposure to the local market.
 - MBS: often maintain a certain share of the security as a signal of its quality

Real Estate Shock - Summary Statistics

	Mean	SD	Median	Perc10	Perc90
RES					
2006-2009	0468	.0547	0445	1085	.0203
2006-2010	0458	.0502	0454	0999	.0049
Δ2006-2009	1267	.1007	1352	2197	.0019
△2006-2010	1573	.1024	1487	2708	0437
∆ House Prices - Unweighted					
2006-2009	0426	.0702	0468	1078	.0293
2006-2010	0482	.0704	0513	1239	.0222
△2006-2009	1182	.144	1142	2786	.0518
∆2006-2010	173	.1557	1815	3554	.0003
Δ House Prices - Weighted					
2006-2009	0674	.0756	0603	1743	.0117
2006-2010	064	.0731	0554	1634	.0109
Δ_2006-2009	182	.1564	1751	396	0082
Δ_2006-2010	2228	.1684	2171	4865	0208

Source: Call Reports. Capital to Assets Ratio weighted by total assets in 2005

- The average Real Estate shock relevant for each bank is similar in size to the house price change in the US.
- Large variation across banks.

Instrument - Housing supply elasticity, Saiz (2010)

Strong 1st Stage: Breaks in House Price evolution explains a large portion of the real estate shocks faced by the banks

	(1)	(2)	(3)	(4)
RES (HP break)	-0.307***	-0.308***	-0.254***	-0.254***
	(0.012)	(0.011)	(0.012)	(0.011)
Observations	7554	7554	7515	7515
Adjusted R ²	0.144	0.227	0.198	0.281
F	630.2	716.7	68.40	81.11
SD	robust	robust	robust	robust
Year FE	No	Yes	No	Yes

Empirical Strategy - Part II

- Estimate the impact of predicted changes in banks' capital ratio on Credit Supply
- Change in mortgages originations at the county level (j)

$$\Delta VolOrig_{j,t} = \beta_1 + \beta_2 \Delta Y_{j,t} + \beta_3 \Delta H_{j,t} + \beta_4 X_{j,05} + \epsilon_{j,t}$$
$$\Delta Y_{j,t} = \sum_k \alpha_{k,j} \Delta \widehat{K_{k,t,-j}}$$

- $\Delta \widehat{Y_{k,t}}$ predicted change in Bank's Capital Ratio (regression part I)
- $\Delta H_{j,t}$ change in House prices, Unemployment Rate and Income at county level
- X_{j,06} bank's controls at county level

- $Q_t M_t$ can be seen as "representative" mortgage.
- Principal Evolution:

$$\tilde{M}_{t+1} = \left(1 - \mathbf{d}_{t+1} - \mathbf{s}_{t+1}\right) \left(1 - \mu\right) M_t$$

•
$$\mathbf{d}_{t+1}M_t = \int \mathbf{1}_{\{\mathbf{d}_{it+1}=1\}} m_{it} di$$
, $s_{t+1}M_t = \int \mathbf{1}_{\{\mathbf{s}_{it+1}=1\}} m_{it} di$

Earnings:

$$\Pi_{t+1} = \underbrace{Z_{t+1}M_t + \left(\tilde{Q}_{t+1}\tilde{M}_{t+1} - Q_tM_t\right)}_{r_{t+1}^m Q_tM_t} - rB_t - \Phi\left(\frac{Q_tM_t}{N_t}\right)$$
$$Z_{t+1}M_t = (1 - \mathbf{d}_{kt+1} - \mathbf{s}_{kt+1}) \left(\mu + x\right)M_t + \mathbf{d}_{t+1}x_{t+1}^d M_t + \mathbf{s}_{t+1}(1 + x)M_t$$
$$r_{t+1}^m = \frac{Z_{t+1} + \tilde{Q}_{t+1} \left(1 - \mathbf{d}_{t+1} - \mathbf{s}_{t+1}\right) \left(1 - \mu\right)}{Q_t} - 1$$

Bank's Problem

$$V_{t-1}(M_{t-1}, N_{t-1}) = \max_{\{M_{t+\tau}, B_{t+\tau}\}} E_t \sum_{\tau=0}^{\infty} \beta_b^{\tau+1} \omega \left[N_{t-1+\tau} + \Pi_{t+\tau} \right]$$
$$= \max_{\{M_t, B_t\}} E_t \left[\omega \left[N_{t-1} + \Pi_t \right] + V_t \left(M_t, N_t \right) \right]$$

s.t.

$$Q_t M_t = B_t + N_t$$

$$N_{t+1} = (1 - \omega) [N_t + \Pi_{t+1}]$$
$$\Pi_t = r_t^m Q_{t-1} M_{t-1} - r B_{t-1} - \Phi \left(\frac{Q_{t-1} M_{t-1}}{N_{t-1}}\right)$$
$$r_t^m = \frac{Z_t + \tilde{Q}_t (1 - \mathbf{d}_t - \mathbf{s}_t) (1 - \mu)}{Q_{t-1}} - 1$$
$$Z_t = (1 - \mathbf{d}_{kt} - \mathbf{s}_{kt}) (\mu + x) + \mathbf{d}_t x_t^d + \mathbf{s}_t (1 + x)$$

Bank's Problem

$$N = (1 - \omega) [(1 + r) N + (r^{m} - r - \Phi(L))QM]$$
$$r^{m} - r - \Phi(L) - \Phi'(L)L = 0$$

$$1 = (1 - \omega) \left[1 + r + \Phi'(L) L^2 \right]$$

• If $(1 - \omega)(1 + r) = 1$

$$L \leq \tilde{L}$$
 $r^m - r = 0$

▶ If $(1-\omega)(1+r) > 1$

 $L > \tilde{L}$ $r^m - r > 0$

Bank's Problem

Equilibrium

Given the **initial** distributions $\Gamma_H(\Lambda_h, 0)$, $\Gamma_M(\Lambda_r, 0)$ and $\Gamma_{NM}(\Lambda_r, 0)$ over $\Lambda_h = (y, a, h, m, \delta_h)$ and $\Lambda_r = (y, a)$; net worth N_0 and asset composition Q_0M_0 ; initial stock of own-occupied H_O and rental H_R houses and an exogenous r, the equilibrium is defined as

- ▶ sequence of house prices $\{p_t^h\}$, rents $\{p_t'\}$, mortgage price function $\{q_t(y, a', m', h')\}$ and funding cost of banks $\{r_t^c\}$ for $t \ge 1$
- ► sequence of decision rules and distributions of homeowners $\Gamma_H(\Lambda_h, t)$, renters $\Gamma_j(\Lambda_r, t), j \in \{M, NM\}\}$ for $t \ge 1$
- Evolution of N_t and asset composition $Q_t M_t$ for $t \ge 1$

such that:

- Decision rules are optimal given prices sequences
- Rents satisfy zero profit condition
- Cost of funding and individual mortgage prices satisfy the bank's problem
- Demand for owner-occupied house equals supply
- Distributions are implied by the sequence of optimal decision rules and initial distributions Individual Mortgage