Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis

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Interest Rates and Inflation

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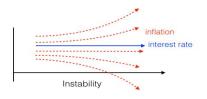
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- This has lead some to suggest that prolonged low nominal interest rates — and central-bank promises to maintain such policy — may bring about lower inflation (rather than higher)
 - some (e.g., Bullard, 2010) even propose that interest rates should be **raised** in order to exit from a deflationary slump

The "Neo-Fisherian" View (Cochrane, 2015b)

(How) will raising rates affect inflation?

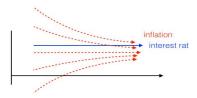
Traditional (Keynesian and Monetarist) view



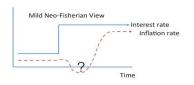


Higher interest rates lead to lower inflation, both short and long run

ZLB lesson and current theory



A credible peg, with fiscal backing is stable



→Raising rates will (eventually) raise inflation

Questions

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Can one maintain the orthodox view — that maintaining a lower nominal rate for longer should cause higher inflation and capacity utilization — while having a view of expectations that implies that central-bank commitments regarding future policy should have any effect?

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- The assumption of **perfect foresight** is nonetheless very strong
 - especially in the context of a novel policy regime, and the anticipated effects of policies announced for many quarters in future
- PFE predictions are relevant only to the extent that
 - the PFE is the limit of an iterative process of belief revision (as in Evans and Ramey, 1992)
 - and this process converges fast enough for the limit to well approximate the outcome from a finite degree of reflection

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- When the iterative process fails to converge, or converges only slowly: no basis to predict definite paths for endogenous variables under a given policy commitment
 - nonetheless, robust conclusions may be possible about how the outcome should be **changed** by choosing one policy rather than another
 - and these can **differ** from what a study of the **PFE** consistent with each policy would suggest

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 - households, firms solve infinite-horizon problems
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[Details of temporary-equilibrium model: Woodford (2013)]

Log-linearization of solution for optimal spending by household i:

$$c_t^i = \sum_{T=t}^{\infty} \beta^{T-t} \hat{\mathcal{E}}_t^i \left\{ (1-\beta) Y_T - \beta \sigma (i_T - \pi_{T+1}) - \beta \Delta \bar{c}_{T+1} \right\}$$

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- generalizes PIH to allow for non-constant desired path of spending owing to
 - (i) real interest-rate variation, or
 - (ii) transitory variation in urgency of spending \bar{c}_t

• Can summarize relevant expectations of i by a single variable:

$$c_t^i = \ldots + \beta \hat{E}_t^i v_{t+1}^i$$

where

$$v_{t}^{i} = \sum_{T=t}^{\infty} \beta^{T-t} \hat{E}_{t}^{i} \left\{ (1-\beta) Y_{T} - \sigma(\beta i_{T} - \pi_{T}) - (1-\beta) \bar{c}_{T} \right\}$$

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 Hence aggregate demand depends on a measure of average subjective expectations

$$e_{1t} \equiv \int \hat{E}_t^i v_{t+1}^i di$$

Aggregate Demand ("IS Equation")

Then defining aggregate demand

$$Y_t = \int_i c_t^i di$$

and output gap $y_t \equiv Y_t - Y_t^n$, individual decision rules aggregate to AD relation

$$y_t = \rho_t - \sigma i_t + e_{1t}$$

where $ho_t \equiv ar{c}_t - Y_t^n$ collects exogenous terms

Price Adjustment

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- Calvo-Yun model of staggered price adjustment [fraction α of prices not reconsidered each period, automatic price increases at target rate π^* between adjustments]
- Log-linear approximation [around steady state with inflation π^*] to optimal price-setting:

$$\rho_t^{*j} = (1 - \alpha \beta) \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \hat{E}_t^j [p_T + \xi y_T - \pi^* (T - t)] \\
- (p_{t-1} + \pi^*)$$

where p_t is (log) price index, p_t^{*j} is j's estimate of optimal (log) price relative to average unadjusted price $p_{t-1} + \pi^*$

Aggregate Supply ("AS Equation")

Implied AS relation:

$$\pi_t = \kappa y_t + (1 - \alpha)\beta \ e_{2t}$$

where π_t is inflation in excess of the target π^* ,

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)\xi}{\alpha} > 0,$$

and

$$e_{2t} \equiv \int \hat{E}_t^j p_{t+1}^{*j} dj$$

measures average expectations of another composite variable

Complete 3-equation TE Model

$$y_t = \rho_t - \sigma i_t + e_{1t}$$

$$\pi_t = \kappa y_t + (1 - \alpha)\beta e_{2t}$$

and a monetary policy rule, such as

$$i_t = \bar{\imath} + \phi_\pi \pi_t + \phi_y y_t$$

- System to determine TE paths of $\{y_t, \pi_t, i_t\}$ given paths for expectations $\{e_{1t}, e_{2t}\}$ and exogenous disturbances $\{\rho_t\}$
 - regardless of how expectations are determined



- Suppose $\{\rho_t\}$ and monetary policy are both deterministic, and we impose the further assumption of **perfect foresight**:
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- Then eq'm relations can equivalently be written

$$y_t = y_{t+1} - \sigma(i_t - \pi_{t+1}) - \Delta \rho_{t+1}$$
 ["NK IS"]
$$\pi_t = \kappa y_t + \beta \pi_{t+1}$$
 ["NKPC"]

- Case of monetary policy that fixes interest rate $i_t = \bar{\imath}$ indefinitely:
 - one-parameter family of solutions to PFE equations (indexed, for example, by value of π_0) with the property that inflation and output gap remain bounded for all t
 - all of these solutions converge asymptotically to the constant- π steady state with nominal interest rate $\bar{\imath}$

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 - all of these solutions converge asymptotically to the constant- π steady state with nominal interest rate $\bar{\imath}$
- This long-run inflation rate is **lower** the lower is $\bar{\iota}$
 - ullet thus one can argue that permanently maintained low i must (at least eventually) bring about correspondingly low π

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 But is this a reasonable view of what should follow from intertemporal optimization, and an ability to reason about the implications of the central bank's policy commitments for future economic outcomes?

- Suppose beliefs are revised through an iterative process:
 - given any sequences $\{e_t\}$ describing the evolution of average expectations, and a specification of monetary policy, the TE relations deliver implied sequences $\{\pi_t, y_t, i_t\}$
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 - these imply sequences $\{e_t^*\}$ of correct expectations, the result of a mapping $e^*=\Psi(e)$
 - define a continuous updating rule for beliefs

$$\dot{e}_t(n) = e_t^*(n) - e_t(n)$$

where the continuous variable n indexes how far along the revision process is; \dot{e}_t is the derivative of e_t with respect to n; for each n, $e^*(n) = \Psi(e(n))$

— starting from some (relatively naive) initial specification $\{e_t(0)\}$ of the evolution of average beliefs

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 selection, left open by the mere conjecture that outcome should
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 - but if the process doesn't converge, the PFE prediction may be quite different from what this model of expectation formation would imply, even if the process of reflection is carried quite far

- Even when the process of belief revision converges, if convergence is slow, the PFE prediction may still not be too relevant
 - unless process of reflection is carried very far, may not lead to beliefs too close to PFE

Bounded Rationality

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 - unless process of reflection is carried very far, may not lead to beliefs too close to PFE
- If slow convergence or non-convergence: the relevant model prediction should be the **set** of possible paths for the economy's evolution corresponding to some not-too-extreme range of possible specifications of e(0), and a range of possible finite values of n

Case of Temporary Fixed Interest Rate

- Suppose that for all $0 \le t < T$, $i_t = \bar{\imath}$ regardless of inflation or output
 - but reversion to "normal" policy (Taylor rule) for all $t \geq T$

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- Case of interest: ZLB prevents any lower interest rate during period of loose policy, so always as low as possible
 - question: to what extent can increasing the length of commitment substitute for possibility of deeper immediate interest-rate cut?

Numerical Illustrations

Parameter values used in numerical illustrations:

 Model parameters are those used in Denes, Eggertsson and Gilbukh (2013), which allow a ZLB episode similar to the US Great Recession, in the case of a suitable exogenous shock:

$$\alpha=0.784,\quad \beta=0.997,\quad \sigma^{-1}=1.22,\quad \xi=0.125$$
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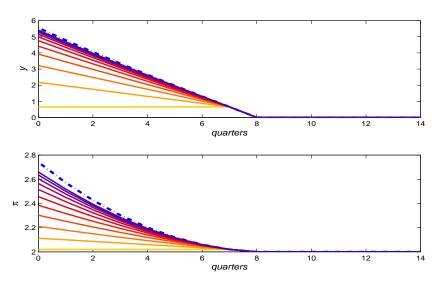
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• Temporary policy: $i_t = \bar{\imath}$ for dates $0 \le t < T$ corresponds to nominal rate of zero (in un-transformed variables)

- **1** TE paths converge to the PFE paths for inflation, output and interest rates, as $n \to \infty$
 - specifically, to the unique PFE paths with property that each of these variables remains **bounded** for $t \to \infty$



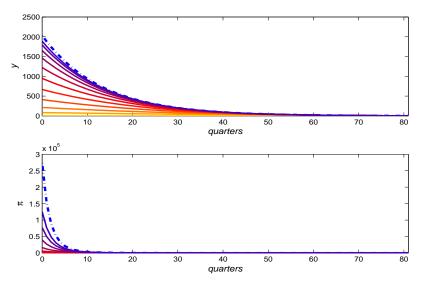
TE paths for n = 0 - 4, if at ZLB for 8 quarters

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 - analysis does not support selection of "backward stable" solution (Cochrane, 2015a) as prediction of the model

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 - hence more reasonable to expect departure from PFE predictions
 - In this case, TE analysis implies forward guidance should be less powerful than the PFE analysis (with conventional equilibrium selection) would imply
 - though still quite powerful (indeed, implausibly so...)
 - and much more powerful than PFE analysis with Cochrane (2015a) equilibrium selection would imply



TE paths for n = 0 - 20, if at ZLB for 20 years

- Moreover, in the fixed-i case, there is no finite n for which TE dynamics and PFE dynamics are similar for all values of T
 - for any finite n, the TE responses are **the same** for all large enough T (and so remain **bounded** as $T \to \infty$)
 - instead, the PFE responses (under the conventional eq'm selection) grow explosively as T is made large

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 - instead, TE analysis for finite *n* avoids the unreasonable prediction
 - but **not** because it's like "backward stable" PFE!
 - the conventional PFE solution is accurate for small T
 - and **no** PFE solution is accurate for large T

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 - in this case, belief revision dynamics don't converge as n grows
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- What if interest rate is expected to be fixed indefinitely?
 - in this case, belief revision dynamics **don't converge** as *n* grows
 - instead, diverge explosively
 - and for no *n* are they similar to **any** of the PFE solutions

• If we let $R^n(T)$ be the vector of responses when commitment is for T periods and belief revision continues to level n, then

$$\lim_{T \to \infty} R^n(T) = R^n(\text{permanent}) \quad \text{for any finite } n$$

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But the limits

$$\lim_{n\to\infty}\lim_{T\to\infty}R^n(T),\qquad \lim_{T\to\infty}\lim_{n\to\infty}R^n(T)$$

both diverge

— neither converges to any of the PFE solutions under assumption of permanently fixed rate

- Hence consideration of the set of PF equilibria is especially misleading in the (somewhat artificial) thought experiment of a permanent interest-rate peg
 - a case in which none of the PFE paths approximate TE paths for finite n, even for very large n

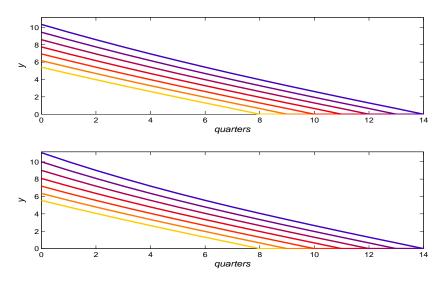
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 - a case in which none of the PFE paths approximate TE paths for finite n, even for very large n
- So we should not necessarily expect the conclusion above to be correct, that a long-enough lasting commitment to remain at the ZLB must eventually make inflation lower, rather than higher
- What would we conclude about the consequences of such a commitment if, instead, we use TE analysis for some finite (though possibly high) value of n?

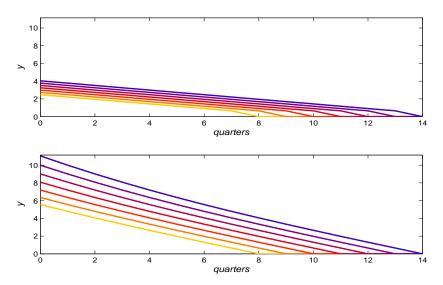
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 - as in the PFE analysis with conventional eq'm selection (not Cochrane's), this should increase output and inflation immediately
 - for large enough n, predictions of TE analysis are similar to PFE predictions (with this eq'm selection)
 - but even for smaller n > 0, predictions are qualitatively like PFE predictions (with this eq'm selection)



varying T from 8-14 quarters; top: n = 4, bottom: PFE



varying T from 8-14 quarters; top: n = 0.5, bottom: PFE

 Where the conventional PFE analysis of effects of forward guidance is less reliable: in its implication that further lengthening of time expected to remain at ZLB should be able to increase output and inflation effects without bound

- Where the conventional PFE analysis of effects of forward guidance is less reliable: in its implication that further lengthening of time expected to remain at ZLB should be able to increase output and inflation effects without bound
 - PFE predictions for larger T are progressively less reliable, as the size of n required for them to be approximately correct grows and grows
 - in fact, for any finite *n*, TE analysis implies **bounded** effects, no matter how large *T* is made

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 - the finite-T PFE analysis, however, represents an upper bound on the size of effect that one can plausibly expect
 - and the larger T is, the more reason to doubt that the effect should be as large as predicted by the PFE analysis
- Especially for larger values of T, the approach recommended here leads only to a set of possible predictions for a given policy
 - but this still allows qualitative conclusions that remain very useful for practical policy analysis
 - and insisting on PFE analysis simply because it makes more sharply defined predictions may lead to large errors