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# Expectation Driven Business Cycles with Limited Enforcement*

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**Abstract**

We explore the implications of shocks to expected future productivity in a setting with limited enforcement of financial contracts. As in Lorenzoni and Walentin (2007) optimal financial contracts under limited enforcement imply that to obtain external finance firms have to post collateral in terms of liquidation value of the firm. In contrast to previous real one-sector models, we show that a model with this type of "collateral constraint" generates an increase in stock prices in response to positive news about future productivity, as well as the other properties of an expectation driven business cycle, that is, an increase in consumption, investment and hours. Furthermore, our model generates this positive response to news shocks with standard consumption preferences and without investment adjustment costs. The positive response of stock prices to news shocks is a central element of expectation driven booms. Empirically, it is documented in Beaudry and Portier (2006).

*Keywords:* business cycles, news shocks, limited enforcement, stock prices

*JEL codes:* E22, E32, E44, E51

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# 1 Introduction

This paper is part of the growing literature following Beaudry and Portier's (2004) work on expectation driven business cycles. Their basic idea is that there is a time lag between a technological innovation and its broad implementation, and thereby its effect on total factor productivity. A time period where this type of shock seems *prima facie* present was the IT boom in the late 1990's, but it has been shown that this type of shock plays an important part for business cycle dynamics also in general (Beaudry and Portier, 2006, and Schmitt-Grohe and Uribe, 2008).

We explore the implications of shocks to expectations about future total factor productivity ("news shocks") in a real business cycle model with limited enforcement of financial contracts. With this type of financial friction it turns out that a real one-sector model with standard consumption preferences can generate a positive response of investment, consumption, hours worked and stock prices to shocks to expectations about future TFP. The contrast to the literature is mainly the last part - that stock prices increase in response to positive news. This fundamental characteristic of expectation driven booms has not previously been successfully modelled in a real one-sector model with two input factors. Empirically, Beaudry and Portier (2004, 2006) make a strong case that stock prices increase in response to positive news about future TFP. A second, less important, difference compared to the literature is that our model generates this positive response to news shocks without having to assume non-standard consumption preferences.

The present paper shares the focus on stock prices with Christiano, Ilut, Motto and Rostagno's (2008; CIMR) paper on boom-bust cycles. One important limitation of their real model is that it does not generate a positive response of stock prices to news. CIMR solve this problem by adding a monetary dimension with sticky prices and wages to their model and imposing a Taylor rule for the interest rate. We instead address this issue in a purely real model. We thereby provide an alternative to the claim by CIMR that it is inappropriate monetary policy that cause the boom-bust pattern in response to news shocks. This difference in results has important policy implications.

The present paper is also related to work by Jaimovich and Rebelo (2008, 2009). They construct real models, in open and closed economy set-ups respectively, that generate expectation driven business cycles neatly in one-sector settings, but do not get a positive response of stock prices to news. In contrast to our model, they need to introduce special preferences, a variation on the specification in Greenwood, Hercowitz, and Huffman, (1988; GHH), that imply low short-run wealth effects on labor supply to get a positive response of labor. Another related paper is Chen and Song (2008) who explore capital reallocation in a setting with expectation shocks and a collateral constraint on entrepreneurs' financing.[1] The empirical relevance of news shocks has been explored through Bayesian DSGE estimation by Schmitt-Grohe and Uribe (2008) as well as Fujiwara, Hirose and Shintani (2008). Using survey-based measures of expected future economic activity Leduc and Sill (2010) provide VAR evidence on the substantial and positive effects of shocks to these measures for current economic activities and stock prices. Alexopoulos (2010) provide evidence of news shocks from a very different source, book publication in the field of technology, that indicate a lag of roughly a year between when innovations become public knowledge and their effect on TFP. Francois and Lloyd-Ellis (2008) provides an endogenous alternative to expectation driven cycles called "implementation cycles", i.e. that implementation of productive ideas is delayed and clustered. They show that this type of mechanism generates comovement in investment and consumption, and stock prices that lead the cycle.

The model in Beaudry and Portier (2004) implies the same type of comovement between expected future productivity and current stock prices as our model. The main difference is that they use a three sector model with complementarities between capital and the intermediate good, and a shock to the productivity of the intermediate goods sector.[2] Our model, on the other hand, has a simple production structure, but limited enforcement of

[1] There are several recent papers exploring various mechanisms to understand expectation driven business cycles: labor market matching (Den Haan and Kaltenbrunner, 2009), vintage capital (Flodén, 2006), international co-movement in response to news shocks (Beaudry, Dupaigne and Portier, 2009) and collateral constraints for financing wages and intermediate goods (Kobayashi, Nakajima and Inaba, 2007).

[2] A second paper that uses a non-traditional production structure to get comovoment between expected TFP and current stock prices is Gunn and Johri (2010). Their key component to achieve this is human capital that belongs to the firm. To get comovement with this mechanism they also need variable capital utilization.

financial contracts.

The technical contribution to the news shock literature of the present paper is the analysis of limited enforcement of contracts. Two key effects of limited enforcement can be distinguished. First, the *quantity effect* of introducing limited enforcement is that the funds available to a firm, and thereby its investment, become a function of the value of a "collateral" which is the expected discounted next period liquidation value of the firm. This introduces an additional channel through which expectations affect the dynamics. Although the notion of this effect goes back to Minsky and Keynes it has not previously been explored in the recent news shock literature. We conjecture that the quantity effect is present in a wide variety of models with limited enforcement in addition to the model by Lorenzoni and Walentin (2007) used here, e.g. in Albuquerque and Hopenhayn (2004). Second, as shown in Lorenzoni and Walentin (2007), limited enforcement causes a time-varying wedge between marginal $q$ and average $q$, the *price effect*. The wedge reflects the tension between available funds and the future profitability of investment. Accordingly, the wedge will increase with expected future productivity if current funds, and thereby investment, do not increase sufficiently to fully offset the direct effect of the increased future productivity on the return to investment.

We illustrate the above two effects in two model specifications that differ in terms of assumptions on openness, capital adjustment costs and preferences. For comparison purposes we let our main specification be similar to Jaimovich and Rebelo (2008) and the alternative specification be similar to CIMR, except for the presence of limited enforcement. In both these setups the addition of limited enforcement implies that stock prices respond positively to news shocks, correcting the counterfactual implication in the previous literature.

The paper proceeds as follows. In section 2 we set up and solve the model. In section 3 we present impulse response functions and elaborate on the intuition for the key results. Section 4 concludes.

## 2 The model

There are two types of agents: consumers and entrepreneurs, each of unit mass. There are two goods, a perishable consumption good and physical capital. Transformation between consumption good and capital is subject to adjustment costs. All markets are competitive.

Markets are complete, but there is limited enforcement of financial contracts. The modelling of optimal financial contracts are taken from Lorenzoni and Walentin (2007) and we will therefore be slightly brief in the description of that part of the model. The fundamental difference between the present paper and Lorenzoni and Walentin (2007) is that the latter paper does not analyze news shocks. Furthermore, the assumptions regarding household preferences and the labor market are different.

We make use of one key mechanism from the expectation driven business cycle literature - habit formation in labor as in Schmitt-Grohe and Uribe (2008). We consider this a convenient short-cut to fully specified search and hiring frictions in the labor market. It makes hours worked respond positively to news and is similar to labor adjustment costs, as used in Jaimovich and Rebelo (2008, 2009).

Let us briefly mention some mechanisms that have been used in the news shock literature, but that we do not need. Variable capital utilization has been assumed by e.g. Jaimovich and Rebelo (2009). We do not include this mechanism in our model. The reason is that our main interest is in expectation driven booms where the price of capital increases. In that case, the standard variable capacity utilization mechanism - that affects the depreciation rate of capital - will not increase capacity utilization, and thereby output, in the anticipation of a TFP increase. In other words, including this mechanism does not help in generating a news driven expansion. We do not assume habit formation in consumption (as CIMR), nor do we assume GHH/JR preferences that imply low wealth effects on labor supply, as Jaimovich and Rebelo (2008, 2009). GHH/JR preferences would strengthen the response of hours worked to news also in our model, but is not needed to induce comovement.

### 2.1 Setup

*Preferences.* The preferences of a consumer is described by

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{c_t^{1-\sigma_C}}{1-\sigma_C}-\varphi_L\frac{\left(l_t-b_L l_{t-1}\right)^{1-\sigma_L}}{1-\sigma_L}\right)\right]$$

Consumers choose consumption $c$, hours worked $l$, and save in state contingent assets. and $b_L$ denotes the degree of habit formation for labor. The consumer's problem is in other words quite standard, and will be treated very briefly. The only slightly novel aspect is that we allow for habit formation in labor.

Entrepreneurs have finite lives. Each period a fraction $\gamma$ of entrepreneurs dies and is replaced by an equal mass of young entrepreneurs. The first period of their life entrepreneurs are endowed with $l_E$ units of labor. This gives new entrepreneurs positive initial wealth.

The preferences of entrepreneur $i$, born at date $t$, are described by the utility function

$$\mathbb{E}_t\left[\sum_{j=0}^{J_i}\beta_E^j c_{i,t+j}^E\right],$$

where $J_i$ is the random duration of the entrepreneur's life. Entrepreneurs are more impatient than consumers, $\beta_E < \beta$. This assumption, together with the assumption of a finite life for entrepreneurs, guarantees the existence of a steady state where the borrowing constraint is always binding. We will discuss the conditions for this result further below.

*Technology.* Each period $t$ entrepreneurs have access to a constant returns to scale technology described by the production function $A_t F\left(k_{i,t}, l_{i,t}\right) = A_t k_{i,t}^{\alpha} l_{i,t}^{1-\alpha}$, where $k_{i,t}$ is capital installed in period $t-1$. The aggregate productivity parameter $A_t$ follows

$$\log A_t = a_t = \rho a_{t-1} + \varepsilon_t + \eta_{t-p}$$

where $\varepsilon_t$ and $\eta_t$ are Gaussian i.i.d. shocks. Note that $\eta$ is a "news" shock - it is known $p$ periods before it affects the productivity. $\varepsilon$ denotes the "traditional" contemporaneous innovation to TFP. For convenience, as well as comparability to CIMR, we model TFP as a stationary process. We are further motivated in this choice by the empirical finding that news shocks to the stationary part of TFP are the most important shocks in terms of variance decomposition according to Schmitt-Grohe and Uribe's (2008) estimation results.

Aggregate uncertainty is described by the Markov process $s_t$ in the finite state space $\mathcal{S}$, with transition probability $\pi(s_{t+1}|s_t)$. Individual uncertainty is described by the random variable $\chi_{i,t}$, which is equal to 1 in all the periods when entrepreneur $i$ is active, except in the last period of activity, when $\chi_{i,t} = 0$.

We assume convex capital adjustment costs of the form

$$G(I, K) = \frac{\xi}{2}\left(\frac{I_t - \delta K_t}{K_t}\right)^2 K_t$$

and the law of motion for capital is

$$K_{t+1} = (1-\delta) K_t + I_t.$$

The timing of events is as follows. At the beginning of period $t$, production is realized and entrepreneur $i$ learns if period $t$ is his last period of activity. Then, entrepreneurs trade used capital. With this timing assumption entrepreneurs are able to liquidate all their capital on their last period of activity. Furthermore, this assumption also helps in modelling the liquidation proceedings in the event an entrepreneur defaults.

*Financial contracts.* Consider an entrepreneur born at time $t$. The entrepreneur finances his current and future investment by selling a long-term financial contract $\mathcal{C}_{i,t}$. The contract specifies a sequence of state-contingent transfers $\{d_{i,\tau}\}_{\tau=t}^{\infty}$,[3] for all the periods in which the entrepreneur is alive. The transfers are contingent both on the history of aggregate shocks and on the idiosyncratic termination shock of entrepreneur $i$. The choice variable $k_{i,\tau+1}$, and the transfer $d_{i,\tau}$, are set after the idiosyncratic termination shock is realized. Let $q_t^m$ denote the price of capital and $w_t$ the wage rate in period $t$. Feasibility requires that the transfers $\{d_{i,\tau}\}$ satisfy:

$$c_{i,\tau}^E + d_{i,\tau} \leq A_\tau F(k_{i,\tau}, l_{i,\tau}) - w_\tau l_{i,\tau} - q_\tau^m (k_{i,\tau+1} - k_{i,\tau}(1-\delta)), \tag{1}$$

[3]The transfer will typically be negative in the first period (initial investment) and can be positive or negative in the following periods, corresponding to dividend payments minus new investment in the firm.

for all the periods where the entrepreneur is active.[4]

*Limited enforcement.* Financial contracts are subject to limited enforcement. The entrepreneur has full control over the firm's assets. In each period, after production takes place, the entrepreneur can choose to divert part or all of the current profits and the capital stock. In this way he can capture up to a fraction $(1-\theta)$ of the firm's *liquidation value*, $v_{i,t}$, which is equal to current profits plus the resale value of the capital stock:

$$v_{i,t} = A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} + q_t^m k_{i,t}\left(1-\delta\right).$$

The only recourse outside investors have against such behavior is the liquidation of the firm. Upon liquidation, the investors can recover the remaining fraction $\theta$ of the firm's liquidation value. After liquidation the entrepreneur can start anew with initial wealth $(1-\theta)\,v_{i,t}$. That is, the only punishment for a defaulting entrepreneur is the loss of a fraction $\theta$ of the firm's liquidation value.

## 2.2 Optimal financial contracts

Before turning to the competitive equilibrium, we concentrate on the decision problem of a single entrepreneur. We begin by spelling out some results from consumers' optimization and introducing preliminary definitions that will simplify the analysis. Then we give a recursive characterization of the optimal financial contract and show that, under constant returns to scale and given the notion of limited enforcement introduced above, the optimal financial contract is linear.

### 2.2.1 Preliminaries

*Consumers*. We will study equilibria where consumers always have positive consumption, $c_t > 0$. Therefore, the price of a sequence of state-contingent transfers $\{d_{i,t+s}\}_{s=0}^{\infty}$ is dis-

[4] In the first period of activity the constraint is:

$$c_{i,t}^E + d_{i,t} \leq A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} - q_\tau^m\left(k_{i,t+1} - k_{i,t}\left(1-\delta\right)\right) + w_t l_E,$$

with $k_{i,t} = 0$.

counted using the consumer's discount factor, $m\left(X', X\right)$. This factor is defined by

$$m\left(X', X\right) = \beta \frac{\lambda_{C,t+1}}{\lambda_{C,t}}$$

where $\lambda_C$ denotes the marginal utility of consumption and can be written as

$$\lambda_{C,t} = c_t^{-\sigma_c}$$

The consumer's first order condition with respect to labor supply implies:

$$w_t = \frac{\varphi_L \left(l_t - b_L l_{t-1}\right)^{\sigma_L} - \beta \varphi_L b_L E_t \left(l_{t+1} - b_L l_t\right)^{\sigma_L}}{\lambda_{C,t}}$$

*Entrepreneurs.* An entrepreneur born at date $t$ will choose the financial contract $\mathcal{C}_{i,t}$ to maximize his expected utility subject to feasibility, (1), to the intertemporal budget constraint:

$$\sum_{s=0}^{\infty} \mathbb{E}_t \prod_{r=1}^{s} {}_t \left[m(X_{t+r}, X_{t+r-1}) d_{i,t+s}\right] \geq 0,$$

and to the condition that future promised transfers are credible. The last condition is satisfied if, at each date, the entrepreneur prefers repayment to diversion and default. This condition is stated formally below. For a recursive formulation of the problem it is useful to define the net present market value of the firm's liabilities at date $\tau$:

$$b_{i,\tau} = \sum_{s=0}^{\infty} \left( \mathbb{E}_\tau \left[d_{i\tau}\right] + \mathbb{E}_\tau \prod_{r=1}^{s} \left[m(X_{\tau+r}, X_{\tau+r-1}) d_{i\tau+s}\right] \right).$$

The entrepreneur's problem can be simplified by exploiting the assumption of constant returns to scale. Under constant returns to scale the liquidation value of the firm can be written as:

$$v_{i,t} = R_t k_{i,t} = \max_{l_{i,t}} \left\{ A_t F\left(k_{i,t}, l_{i,t}\right) - w_t l_{i,t} + q_t^m k_{i,t} \left(1 - \delta\right) \right\},$$

where $R_t$, the gross return on capital, is taken as given by the single entrepreneur and is a function of the prices $w_t$ and $q_t^m$. Also, constant returns to scale for adjustment costs, and the presence of a competitive market for used capital, imply that there exists a price of capital, $q_t^m$, which is taken as given by the single entrepreneur, such that

$$q_t^m = 1 + \xi \frac{I_t - \delta K_t}{K_t}$$

Combining the definitions above, the feasibility constraint (1) can be written as:

$$c_{i,\tau}^E + d_{i,\tau} + q_\tau^m k_{i,\tau+1} \leq v_{i,\tau}. \tag{2}$$

## 2.3 Recursive characterization of entrepreneur's problem

We study recursive competitive equilibria, where the state of the economy is captured by a vector of aggregate state variables $X_t \in \mathcal{X}$, including the exogenous state $s_t$, with transition probability $H(X_{t+1}|X_t)$. The vector $X_t$ will be defined and discussed in section 2.4. For now, consider an entrepreneur, who takes as given the law of motion for $X_t$. The state $X_t$ determines the wage rate, $w_t$, and the price of capital, $q_t^m$. Therefore, it also determines the gross rate of return, $R_t$. Let this dependence be captured by the functions $R(X_t)$ and $q^m(X_t)$.

Now we can use a recursive approach to characterize the optimal financial contract. The individual state variables for the entrepreneur are given by $v_{i,t}$, $b_{i,t}$, and $\chi_{i,t}$. Define $W(v, b; \chi, X)$ as the expected utility, in state $X$, of an entrepreneur who controls a firm with liquidation value $v$ and outstanding liabilities $b$.[5] The expected utility $W$ is defined at the time when production has already taken place and the idiosyncratic termination shock has been observed. Also, $W$ is defined after the default decision has taken place, assuming that the entrepreneur does not default in the current period. For now, we will assume that the entrepreneur's problem has a solution in each state $X \in \mathcal{X}$, and the expected utility $W$ is finite. This will be the case in the recursive equilibria we study below.

In all periods prior to the last period of activity, i.e. for $\chi = 1$, $W$ satisfies the Bellman

[5]For a newborn entrepreneur, $v$ is the entrepreneur's initial labor income, and $b$ is zero.

equation:

$$W(v,b;1,X) = \max_{\substack{c^E,d \\ k',b'(.)}} c^E + \beta_E \mathbb{E}[W(v',b';\chi',X')|X] \tag{P}$$

$s.t.$

$$c^E + d + q^m(X)k' \le v, \tag{3}$$

$$b = d + \mathbb{E}[m(X',X)b'(\chi',X')|X], \tag{4}$$

$$v'(X') = R(X')k' \quad \forall X', \tag{5}$$

$$W(v'(X'),b'(\chi',X');\chi',X') \ge W((1-\theta)v'(X'),0;\chi',X') \ \forall \chi',X', \tag{6}$$

where the conditional expectation $\mathbb{E}[.|X]$ is computed according to the transition $H(X'|X)$, with $\chi'$ independent of $X'$.

Problem $(P)$ can be interpreted as follows. At each date, an entrepreneur who does not default has to decide how to allocate the firm's resources, $v$, to its potential uses: payments to insiders, $c^E$, payment to outsiders, $d$, or investment in physical capital, $q^m k'$. This is captured by the feasibility constraint (3). Moreover, the entrepreneur has to satisfy the "promise keeping" constraint (4): current and future payments to outsiders have to cover the current liabilities of the firm, $b$. The current payments are $d$, the future payments are captured by the market discounted value of the firm's liabilities in the following period, $b'(\chi',X')$. These liabilities are allowed to be contingent on the realization of the idiosyncratic termination shock $\chi'$ and of the aggregate state $X'$. Constraint (5) simply says that the liquidation value of the firm in the next period will be given by the total returns on the firm's installed capital $k'$. Finally, the no-default constraint (6) ensures that, in all next period states of the world, the future liabilities $b'$ are credible. The no-default constraint take this form, given that the entrepreneur has the option to default and start anew with a fraction $(1-\theta)$ of the firm's liquidation value $v'$ and zero liabilities.

An entrepreneur in his last period of activity will simply liquidate all capital and pay existing liabilities. Therefore, for $\chi = 0$ we have:

$$W(v,b;0,X) = v - b.$$

As shown in Lorenzoni and Walentin (2007), also for surviving entrepreneurs the value function satisfies

$$W(v,b;\chi,X) = W(v-b,0;\chi,X) \tag{7}$$

The no-default condition can accordingly be written as

$$b \leq \theta v. \tag{8}$$

Equation (7) allows us to replace constraint (6) with constraint (8). The latter can be interpreted as a "collateral constraint", where the total value of the entrepreneur's liabilities are bounded from above by a fraction $\theta$ of the liquidation value of the firm. Using this replacement we note that problem $(P)$ is linear and we obtain the following proposition.

**Proposition 1** *The value function $W(.,.;\chi,X)$ is linear in its first two arguments and takes the form:*

$$\begin{aligned} W(v,b;1,X) &= \phi(X)(v-b), \\ W(v,b;0,X) &= v-b. \end{aligned}$$

*There is an optimal policy for $k', c^E, d$ and $b'$ which is linear in $v-b$.*

Entrepreneurial net worth, $n \equiv v-b$, represents the difference between the liquidation value of the firm and the value of the claims issued to outsiders. Proposition 1 shows that the expected utility of the entrepreneur is a linear function of the entrepreneurial net worth. The factor $\phi$, which determines the marginal value of the entrepreneurial net worth, depends on current and future prices, and hence it is dependent on $X$.

The following proposition gives a further characterization of the optimal solution.

**Proposition 2** *For a given law of motion $H(X'|X)$, let $\phi(X)$ be defined by the recursion:*

$$\phi(X) = \max\left\{\frac{\beta_E(1-\theta)\mathbb{E}\left[(\gamma+(1-\gamma)\phi(X'))R(X')|X\right]}{q^m(X)-\theta\mathbb{E}\left[m(X',X)R(X')|X\right]},1\right\}. \tag{9}$$

*Suppose that*

$$m\left(X',X\right)\phi\left(X\right)\geq\beta_E\phi\left(X'\right) \tag{10}$$

*for all pairs* $X, X'$ *such that* $H(X'|X) > 0$. *Then, the optimal policy for the individual entrepreneur involves: (i)* $k' > 0$, *(ii)* $c^E = 0$ *if* $\phi(X) > 1$, *and (iii)* $b(1, X') = \theta v(X')$ *if* $m(X', X)\phi(X) > \beta_E \phi(X')$.

A central result of this proposition is point (iii), which characterizes the state pairs $X, X'$ where it is optimal to borrow as much as possible against the revenue realized in state $X'$ and use the proceeds to invest today.

## 2.4 Equilibrium

We are now in a position to define a recursive competitive equilibrium. The aggregate state is given by

$$X = (K, B, lag(L), s),$$

where $K$ is the aggregate capital stock, $B$ represents the aggregate liabilities of the entrepreneurs who are not in their last period of activity. Lagged labor are part of the state vector because of habit formation in labor supply. Recall that $s$ denotes the aggregate technology.

A recursive competitive equilibrium is given by a transition probability, $H(X'|X)$, such that the optimal behavior of consumers and entrepreneurs is consistent with this transition probability and the goods market, labor market, and capital market clear. The formal definition is given in the Appendix.

A crucial property of this model is that the entrepreneur's problem is linear, and we obtain optimal policies that are linear in entrepreneurial net worth, $v_{i,t} - b_{i,t}$. Given the linearity of the optimal policies it is straightforward to aggregate the behavior of the entrepreneurial sector. We illustrate the aggregation properties of the model in the case where the collateral constraint is always binding. This is the case where the condition (10) holds for every pair $X, X'$ such that $H(X'|X) > 0$. Lorenzoni and Walentin (2007) showed that, in economies with "small" productivity shocks, such an equilibrium exists. This case will be the basis for the numerical analysis in the next section.

Condition (10) implies that, in each state $X$, the state-contingent liabilities are set to their maximum level for each future value of $X'$, i.e. $b'(\chi', X') = \theta v'(X')$. Therefore, the optimal level of investment is given by:

$$k' = \frac{1}{q^m(X) - \theta \mathbb{E}[m(X', X) R(X') | X]} (v - b). \tag{11}$$

Consider an economy that enters period $t$ with an aggregate stock of capital $K_t$, in the hands of old entrepreneurs. The agents who invest in period $t$ are: a mass $(1-\gamma)$ of the old entrepreneurs, who have $v_{i,t} = R_t k_{i,t}$ and $b_{i,t} = \theta R_t k_{i,t}$, and a mass $\gamma$ of newborn entrepreneurs with $v_{i,t} = w_t l_E$. Therefore, the aggregate entrepreneurial net worth of investing entrepreneurs is:

$$N_t = (1-\gamma)(1-\theta) R_t K_t + \gamma w_t l_E.$$

Using the optimal policy (11) and aggregating we obtain:

$$K_{t+1} = \frac{1}{q_t^m - \theta \mathbb{E}_t[m_{t+1} R_{t+1}]} N_t.$$

From these two equations we get the following law of motion for the aggregate capital stock

$$K_{t+1} = \frac{(1-\gamma)(1-\theta) R_t K_t + \gamma w_t l_E}{q_t^m - \theta \mathbb{E}_t[m_{t+1} R_{t+1}]}. \tag{12}$$

The proof of existence of both a deterministic steady state and a recursive competitive equilibrium where the collateral constraint is always binding is given in Lorenzoni and Walentin (2007). The differences in the assumptions on the economic environment between the model in that paper and the ones made here do not induce any major changes in that proof, so it is left out.

### 2.5 Asset prices

We can now define the *financial value* of a representative firm. The value of the firm is simply the sum of all the claims on the firm's future profits, held by insiders and outsiders.

This leads us to the following expression for the ex-dividend value of a continuing firm:

$$p_{i,t} = W\left(v_{i,t}, b_{i,t}; \chi_{i,t}, X_t\right) + b_{i,t} - d_{i,t}.$$

where $W$ corresponds to the net present value of the payments to the insider and $b_{i,t}$ corresponds to the net present value of the payments to outsiders.

Normalizing the financial value of the firm by the total capital invested we obtain our definition of *average q*

$$q_{i,t} \equiv \frac{p_{i,t}}{k_{i,t+1}}.$$

For continuing entrepreneurs, it is possible to show that $q_{i,t}$ is the same for all firms, and we denote it simply by $q_t$.

**Proposition 3** *Average q is greater than or equal to marginal q,* $q_t \geq q_t^m$, *with a strict inequality if the financial constraint is binding.*

**Proof.** Given that $\phi_t \geq 1$ we have

$$p_{i,t} = \phi_t\left(v_{i,t} - b_{i,t}\right) + b_{i,t} - d_{i,t} \geq v_{i,t} - d_{i,t} = q_t^m k_{i,t+1}.$$

■

Notice that, absent financial constraints we have $\phi_t = 1$ and $q_t = q_t^m$. In this case the investment part of the model boils down to the Hayashi (1982) model. On the other hand, in presence of financial frictions there is a wedge between the value of the entrepreneur's claims in case of liquidation $(v_{i,t} - b_{i,t})$ and the value of the claims he holds to future profits. In other words, the fact that $\phi_t > 1$ creates a wedge between $q_t^m$ and $q_t$.

For later analysis it is convenient to define the net risk-free interest rate $r^f$, even if contracts in the model are state contingent and have state contingent interest rates. $r^f$ is the inverse of the probability weighted average of the consumer's state contingent discount factor:

$$r^f(X) = \frac{1}{\mathbb{E}\left[m\left(X', X\right)\right]} - 1$$

Finally, define the external finance premium as

$$f(X) \equiv \frac{\mathbb{E}[m(X', X) R(X')]}{q^m(X)} - 1$$

This positive premium $f(X)$ reflects that consumers ("outsiders") would be willing to pay to be able to invest directly in the physical capital of firms.

### 2.6 Goods Market Clearing and Small Open Economy Aspects

We model a small open economy. In modelling this aspect we follow Jaimovich and Rebelo (2008). The goods market clearing condition is:

$$Y_t = C_t + I_t + C_t^E + NX_t$$

where $NX_t$ denotes net exports. The law of motion for the net foreign asset position $F_t$ is:

$$F_{t+1} = (1 + r_t)F_t + NX_t$$

Finally, we assume the following relation between the interest rate $r_t^*$ that domestic agents face when borrowing abroad and the net foreign asset position of the country:

$$r_t^* = 1/\beta - 1 + \omega \left[\exp\left(\bar{F} - F_t\right) - 1\right]$$

which in steady state implies that $r^* = 1/\beta - 1$. No arbitrage implies that the domestic and foreign interest rates are equalized.

## 3 News Shock Dynamics

### 3.1 Calibration

We calibrate the model to a quarterly time period. To match an annual risk-free rate of 3% implies $\beta = 0.9925$. To satisfy equation (10) we set $\beta_E < \beta$, more specifically, $\beta_E = 0.99$. We let $\alpha = 0.33$, $\delta = 0.0125$ and $\rho = 0.95$ as standard RBC parameter values. We set $\varphi_L$

to get a steady state value of around $L = 0.30$. As in Lorenzoni and Walentin (2007) we let the capital adjustment cost parameter $\varepsilon$ equal 8.5. We use $\sigma_C = 1$, i.e. log utility of consumption, as a natural benchmark. Similarly we assume log disutility of labor, $\sigma_L = 1$. For habit formation in labor we use $b_L = 0.88$.

Regarding the financial side we set $\theta = 0.3$ based on evidence in Fazzari *et al.* (1988) who show that firms finance 30% of their investment using external funds. Matching a 2% annual steady state finance premium, following Bernanke, Gertler and Gilchrist (2000), implies $l_E = 0.05$ and $\gamma = 0.015$.[6]

In the small open economy dimension we calibrate parameters to get a reasonably stable real interest rate, so as to represent developed economies whose borrowing terms do not change dramatically with their net foreign asset position. In particular we set $\bar{F}$ to get steady state net exports equal to 4% of GDP (as in Jaimovich and Rebelo, 2008) and the parameter for international interest rate sensitivity, $\omega$, to 0.001.

## 3.2 Impulse response functions

### 3.2.1 The empirical benchmark

Beaudry and Portier (2006) present VAR evidence in terms of impulse response functions to a news shock to the non-stationary level of TFP showing that stock prices, consumption, investment and hours worked respond positively. This evidence is representative of what is becoming the standard view of an expectation driven business cycle. Qualitatively similar results have been found for Germany by Haertel and Lucke (2007) who also show that news shocks Granger cause patents. For a contrarian view, see Sims (2009). His results differ in that they indicate that hours and GDP respond negatively to news shocks on impact.

Recall that there are papers in the literature, in particular Jaimovich and Rebelo (2008, 2009) and CIMR, that set up models that successfully generate positive responses of the macro variables to news shocks about future TFP. The remaining challenge that we focus on in this paper is how to get stock prices, as well as these macro variables, to respond positively to news shock in a real model.

[6] The model is parametrized so that the labor input of entrepreneurs have negligible impact on aggregate labor supply. It is constant and accounts for one quarter of a percent of the steady state labor supply.

### 3.2.2 Impulse responses

In the figures below we present impulse responses of the key variables to a positive stationary TFP news shock. We set the number of quarters between the news shock and the actual change in productivity to $p = 4$ quarters.[7]

The non-financial part of our model is partly similar to Jaimovich and Rebelo (2008) in that it is an open economy setting with rigidities in labor adjustment. Two differences vs. their model is that we have standard consumption preferences and fixed capital utilization. Instead the quantity effect of limited enforcement is central for the dynamics of our model. To understand the quantity effect, note from equation (12) that investment is an increasing function of $\mathbb{E}_t [m_{t+1} R_{t+1}]$. For a closed economy model with log utility an increase in expected future productivity $\mathbb{E}_t [a_{t+p}]$ would decrease the discount factor $\mathbb{E}_t [m_{t+p}]$ more than it would increase the expected return to capital $\mathbb{E}_t [R_{t+p}]$. To mute the effect of the decrease in the discount factor on investment we use a small open economy setting. To show the strength of the quantity effect we have chosen to have capital adjustment costs, as opposed to resorting to investment adjustment costs.[8]

Impulse responses are presented in Figure 1. Note how consumption, investment, hours and stock prices all increase in response to the positive news shock. In addition, the price of capital, $q^m$, increases, although stock prices, $q$, increase more. To understand the price effect, note from Proposition 3 that the wedge between $q^m$ and $q$ is driven by the marginal value of wealth of entrepreneurs, $\phi$, which in turn depends on the expected future return on investment (see equation (9)) and therefore increases initially. The increase in both stock prices and capital prices is in complete contrast to previous theoretical work on real one-sector models that deliver investment booms in response to positive news. Finally, note that net exports decrease so that consumption and investment can increase faster than output (and hours) in anticipation of the increase in TFP.

[7] To save space we do not display the contemporanous TFP shock IRFs, but simply note that they imply positive response of all key variables.

[8] It is interesting to note that the same qualitative results - i.e. positive responses of key variables to news shocks - go through with investment adjustment costs. Even quantitatively the results are similar. The only notable difference is that a negative price effect obtains as $\phi$ decrease at impact.

Results are also qualitatively unchanged if we assume habit formation in consumption. With consumption habit obviously changes in consumption become more gradual, but this is the only major difference in our setting.

The main limitation to the robustness of the qualitative results of our model is that to get hours worked to respond positively to news we need to assume substantial labor rigidities in the form of strong habit formation in labor: $b_L \geq 0.88$ is required to get hours to increase on impact. But the result that consumption, investment and stock prices increase in response to news shocks is very general in terms of parameter values, see section 3.4 for a detailed robustness analysis. We note that one way to increase the ability of the model to generate positive response of hours worked to news shocks is to change the preference specification to GHH/JR preferences that eliminates or limits the short run wealth effect on labor supply. We prefer to keep the consumption preferences standard.

### 3.3 Alternative model specification

To illustrate the price effect in isolation we present an alternative model specification that is identical to CIMR, with the exception that we have limited enforcement. We use key assumptions that are identical to CIMR: closed economy, investment adjustment costs and habit formation in consumption but not in labor.[9] Preferences are defined by

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{(c_t - b_C c_{t-1})^{1-\sigma_C}}{1-\sigma_C} - \varphi_L \frac{l_t^{1-\sigma_L}}{1-\sigma_L}\right)\right]$$

We assume convex investment adjustment costs of the type introduced by Christiano, Eichenbaum and Evans (2005), where the law of motion for capital is the following:

$$K_{t+1} = (1-\delta)K_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t \quad (13)$$

$$\text{where } S(x) = \frac{g}{2}(x-1)^2 \quad (14)$$

This implies that the price of capital is:

$$q_t^m = \frac{1 - \beta_E E_t\left[\frac{\phi_{t+1}}{\phi_t} q_{t+1}^m\right]\left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)}. \quad (15)$$

We follow CIMR for calibration of the investment adjustment costs setting $g = 15.1$ and

[9] The qualitative results are unchanged if we allow for habit formation in labor.

for the consumption habit parameter setting $b_c = 0.63$. We adjust the value of $\varphi_L$ to keep $L = 0.30$ as in the main specification. All other parameter values are unchanged.

The impulse responses for the alternative specification are presented in Figure 2. The initial increase in consumption, investment and hours are in line with the empirical evidence, as well as with previous models, e.g. CIMR. The key difference versus the latter is that we get an increase in stock prices at the impact of a positive news shock. The mechanism that generates the stock price increase is the price effect, i.e. the increase in the wedge between the cost of capital $q^m$ and the stock price $q$. As can be seen in the figure $\phi$, which determines the wedge, increases following a positive news shock. On the other hand, the price of capital $q^m$ falls because of the investment adjustment costs. The stock price $q$ is affected by these two opposing factors, the price effect dominates, and $q$ therefore increases.

The fact that the law of motion for capital in our model is derived from a financial constraint makes very little difference for the macroeconomic dynamics compared to CIMR's model, a perhaps surprising result. This also means that we get the same problematic size in the interest rate swings as CIMR.

### 3.4 Robustness

In this subsection we document the robustness of the results. We start with an analysis of the sensitivity of the price and quantity effects, and then move on to documenting the robustness of the model's ability to generate expectation driven boom with respect to parameter values.

As we mentioned above, the sign of the quantity effect depend on the volatility of the interest rate (stochastic discount factor), and will turn negative if the interest rate respond too strongly to news shocks. In an open economy setting a high sensitivity of the international interest rate to the country's net foreign asset position, $\omega$, would make the interest rate too responsive. Similarly in a closed economy this would be the case if the degree of intertemporal substitution, $1/\sigma_c$ is low.

The price effect, i.e. that the wedge between the price of capital and the stock price respond positively to news about future productivity, is quite robust to different parametrizations. Three exceptions to this are very high depreciation, $\delta$, high capital adjustment costs, $\xi$, or investment adjustment costs in an open economy setting. Note that even with a positive price effect, it is possible to set up a model with investment adjustment costs and parameterize it so that the price of capital falls sufficiently to get stock prices to respond negatively to news shocks. This can occur if the wedge between the price of capital and stock prices is very small, or the investment adjustment costs very large.

### 3.4.1 Robustness to calibration of main specification

We document the values for various parameters that are consistent with a positive effect of TFP news shocks on all the relevant macro variables as well as stock prices. We also present parameter intervals that imply positive effects on all these variables, except labor. The exercise is performed by varying one parameter at a time, keeping the other parameters at the values in section 3.1.[10] Four parameters that can take any value without affecting the qualitative results are excluded from the table: $\alpha, \beta_e, \delta$ and $L_e$.

In Table 1, the first column shows that if we abstract from hours worked, the restrictions on the parameters needed for a positive response to news shocks are extremely mild. The second column shows that when we include all variables there are reasonably strict restrictions on a couple of parameters, mainly $\rho, \sigma_C$ and $b_L$.[11] Finally, the parameters close to the key mechanism of our model, the quantity effect, are almost unrestricted. By this we mean the fraction of the liquidation value that limits the size of entrepreneur's liabilities, $\theta$, and the size of the capital adjustment costs, $\xi$. It is comforting that our result does not depend on the exact calibration of these parameters given the disagreement in the literature about their values.

## 4 Conclusion

In this paper we have analyzed the effects of shocks to expectations about future productivity in a real business cycle model with limited enforcement of financial contracts. We constructed a real one-sector model with two inputs, capital and labor, that delivers a pos-

[10] The only exception to this approach is $\beta$. For that parameter we also vary $\beta_e$ and set $\beta_e = \beta - 0.025$ to keep the "collateral constraint" binding.

[11] An interesting curiosity, although outside the scope of this paper, is the non-monotone effect of the interest rate sensitivity parameter $\omega$ on hours worked.

| | All variables except hours | All variables |
|---|---|---|
| $p$ | $\leq 10$ | $\leq 4$ |
| $\beta$ | $\geq 0.85$ | $\geq 0.9892$ |
| $\rho$ | any | $\leq 0.955$ |
| $\xi$ | $\geq 0.275$ | $0.275 - 31$ |
| $\sigma_C$ | any | $\leq 1.05$ |
| $\sigma_L$ | any | $\geq 0.35$ |
| $b_L$ | any | $\geq 0.88$ |
| $\theta$ | $\geq 0.025$ | $\geq 0.16$ |
| $\omega$ | $\leq 0.035$ | $0 - 0.00125, 0.025 - 0.035$ |

Table 1: Robustness to parameters of main specification. Values consistent with positive response to news shocks are shown.

itive response of stock prices, as well as the key macro variables, to an expectation shock. This had not been achieved previously in the literature. Furthermore, we showed that this result holds for different assumptions on preferences, capital adjustment costs and open vs. closed economies, as well as for a wide set of parameter values. The fact that a real model can deliver a stock market boom in response to positive news shocks also has important policy implications, as it implies that it is not necessarily the loose monetary policy that causes expectation driven booms in asset prices and the economy in general, as argued by CIMR.

The implications of limited enforcement in the presence of news shock can be considered as two effects. In our main model specification the key effect is the quantity effect, i.e. the additional feedback from expected future productivity to today's investment that limited enforcement implies. Because this effect works through a collateral value, consisting of the expected discounted liquidation value of the firm, it is only present for settings where the price of capital increases and the stochastic discount factor does not change too much with the growth rate of the TFP. One example of such a setting is a small open economy. In that setup our model generates a positive response of consumption, investment, hours and stock prices to shocks to expected future productivity, even with standard consumption preferences (no habit formation, no GHH/JR preferences) and without investment adjustment costs.

Limited enforcement also drives a wedge between the price of physical capital and stock prices. This wedge is increasing in the difference between the available funding and the first best capital stock. We call this the price effect. It can generate an increase in stock prices even in model specifications that imply a decrease in the price of capital in response to positive news, as in our variation of CIMR.

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# Appendix

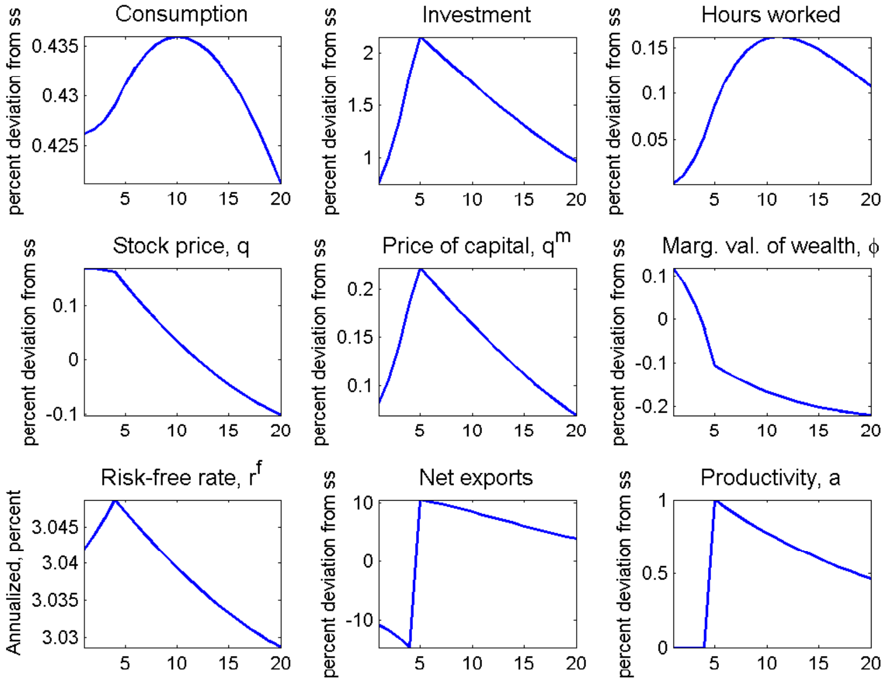


Figure 1. Impulse responses to a $\eta$ shock to future TFP, $E_t\{a_{t+4}\}$. Small open economy, habit formation in labor, capital adjustment costs.

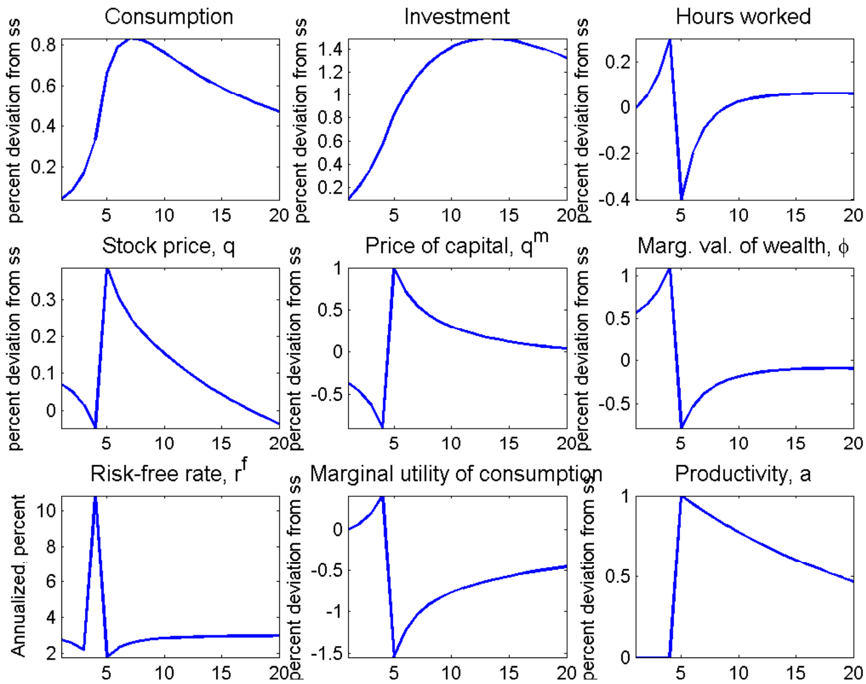


Figure 2. Impulse responses to a $\eta$ shock to future TFP, $E_t\{a_{t+4}\}$. Closed economy, habit formation in consumption, investment adjustment costs.

**Equations determining the equilibrium**

Equilibrium capital stock and factor prices

$$K_{t+1} = \frac{(1-\gamma)(1-\theta) R_t K_t + \gamma w_t l_E}{q_t^m - \theta E_t [m_{t+1} R_{t+1}]}$$
$$q_t^m = 1 + \xi \frac{I_t - \delta K_t}{K_t}$$
$$R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} + q_t^m (1-\delta)$$
$$w_t = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha}$$

Financial variables

$$\phi_t = \frac{\beta_E (1-\theta) E_t \left[ \left( \gamma + (1-\gamma) \phi_{t+1} \right) R_{t+1} \right]}{q_t^m - \theta E_t [m_{t+1} R_{t+1}]}$$
$$q_t = \beta_E (1-\theta) E_t \left[ \left\{ \gamma + (1-\gamma) \phi_{t+1} \right\} R_{t+1} \right] + \theta E_t [m_{t+1} R_{t+1}]$$
$$Wedge_t = q_t - q_t^m$$

Household's marginal utility and the state contingent market discount factor

$$\lambda_{c,t} \equiv u'(c_t) = c_t^{-\sigma_c}$$
$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

Labor market clearing

$$w_t = \frac{\varphi_L (L_t - b_L L_{t-1})^{\sigma_L} - \beta \varphi_L b_L E_t (L_{t+1} - b_L L_t)^{\sigma_L}}{\lambda_{c,t}}$$

Risk-free interest rate

$$r_t^f = \frac{1}{E_t m_{t+1}} - 1$$

Output

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Entrepreneurial consumption

$$C_t^E = \gamma N_t = \gamma (1-\theta) R_t K_t$$

Law of motion for capital

$$K_{t+1} = (1-\delta) K_t + I_t$$

Goods market clearing:

$$Y_t = C_t + I_t + C_t^E + NX_t$$

Net foreign asset position:

$$F_{t+1} = (1 + r_t)F_t + NX_t$$

The international interest rate $r_t$:

$$r_t = 1/\beta - 1 + \omega \left[\exp\left(\bar{F} - F_t\right) - 1\right]$$

Technology

$$a_t = \rho a_{t-1} + \varepsilon_t + \eta_{t-p}$$

**Equations that apply only for the alternative specification**

Habit formation in consumption, but not in labor imply:

$$\lambda_{c,t} = u'(c_t) = (c_t - b_c c_{t-1})^{-\sigma_c} - b_c \beta E_t \left(c_{t+1} - b_c c_t\right)^{-\sigma_c}$$

Investment adjustment costs instead of capital adjustment costs imply:

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t \\ \text{where } S(x) &= \frac{g}{2}(x-1)^2 \end{aligned}$$

and

$$q_t^m = \frac{1 - \beta_E E_t\left[\frac{\phi_{t+1}}{\phi_t} q_{t+1}^m\right]\left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]}{1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)}$$

Goods market clearing, closed economy

$$Y_t = C_t + I_t + C_t^E$$

Finally, the alternative specification is a closed economy so the international interest rate, $r_t$, is not defined. Nor is the net foreign asset position.

**Definition of Recursive Competitive Equilibrium**

A recursive competitive equilibrium, with linear policies for the entrepreneurs, is given by:

(i) a transition probability $H(X'|X)$, where $X = \{K, B, lag(L), s\}$;

(ii) pricing functions $R(X), m(X', X), q^m(X), w(X)$;

(iii) policy functions for the entrepreneur $c^E(v, b, \chi, X), k'(v, b, \chi, X), d(v, b, \chi, X)$ and $b'(\chi', X'; v, b, \chi, X)$, that are linear in $v - b$; and[12]

(iv) policy functions for the consumer $c(X)$ and $l(X)$

which satisfy the following conditions:

(a) the policies in (iii) are optimal for problem $(P)$ in section 2.3, given the transition $H$;

(b) the policies in (iv) are optimal for the consumer's problem outlined in section 2.2.1, given the transition $H$;

(c) the functions $R(X), m(X', X), q^m(X)$ and $w(X)$ satisfy the following equations (these conditions embed market clearing in the used capital market and in the labor market):

$$\begin{aligned}
R(X) &= A(s) F_1(K, L) + q^m(X)(1-\delta), \\
m(X'|X) &= \beta \frac{\lambda_c(X')}{\lambda_c(X)} \\
\text{where } \lambda_c(X) &= \frac{1}{C(X)} \\
q^m(X) &= 1 + \xi \frac{I - \delta K}{K} \\
V &= R(X) K, \\
w(X) &= A(s) F_2(K, L);
\end{aligned}$$

(d) the following inequality is satisfied (this condition ensures market clearing in the consumption goods market, with $c_t > 0$)

$$\begin{aligned}
& A(s) F(K, L) - G(I, K) + \\
- \quad & \gamma c^E(R(X) K, B, 0, X) - (1-\gamma) c^E(R(X) K, B, 1, X) + \\
- \quad & \gamma d(R(X) K, B, 0, X) - (1-\gamma) d(R(X) K, B, 0, X) > 0
\end{aligned}$$

(e) the transition for $s'$ is consistent with $\pi(s'|s)$; the transition probabilities for $K'$ and $B'$ are

[12] The first two arguments of the $b'$ function reflect the state contingent nature of the optimal contract chosen in state $(v, b, \chi, X)$.

The restriction to policy functions that are linear in $v - b$ is justified, given Proposition 1.

consistent with the following:

$$
\begin{aligned}
K' &= k'\left(R\left(X\right)K, B, 1, X\right) \text{ with probability } 1, \\
B' &= \left(1-\gamma\right) b'\left(1, \left\{K', B', s'\right\}; V, B, 1, X\right) - \gamma w\left(X\right) l_E \text{ with probability } \pi\left(s'|s\right).
\end{aligned}
$$

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