Appendix To:
Bayesian Estimation of an Open Economy Model with Incomplete Pass-Through

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Abstract

The open economy DSGE model of the euro area developed and estimated in Adolfson et al. (2005) assumes the absence of feedback effects from the rest of the world to the Euro area. This note sheds some light on the reasonableness of this simplifying assumption. It also provides information about convergence in our estimations.

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1. Overview

In this appendix, we do two things. First, we outline and report the results of an analysis where we allow for spillover effects from the Euro area to the rest-of-the-world, i.e. the small open economy assumption maintained in Adolfson et al. (2005) is relaxed. The main focus of this analysis is on the robustness of the results in the main paper. For comparison, we therefore also report estimates from the baseline specification in Adolfson et al. (2005). As will be clear, the analysis in this appendix will provide support for the small open economy approach adopted in the main paper.

Second, we provide details on the convergence properties of the baseline model in Adolfson et al. (2005). We present standard diagnostics for the single MCMC chains on which we base our results in the main paper, but also multiple chain diagnostics based on several parallel MCMC runs with different initial values are reported. Finally, we analyze the robustness of the results with respect to the priors, and provide estimation results when doubling the prior standard deviation on the model's structural parameters and the persistence of shocks.

2. Allowing for feedback effects from the Euro area

Given that the Euro area trades with many countries for which the Euro area is perhaps the main trading partner, the assumption that foreign (i.e., the rest of the world) variables are exogenous in our analysis may not appear to be a satisfactory assumption. Here we provide results from a specification of the DSGE model where we allow some of the Euro area variables to impact the development in the rest of the world (i.e., we allow the rest-of-the-world variables in the DSGE model to respond to Euro area variables). Specifically, we augment the VAR for the foreign variables with a set of contemporaneous and lagged Euro area variables. An alternative approach would have been to use a full-blown two-country model. There is no need for us to go down that route, however, since very recent and competent work of de Walque, Smets and Wouters (2005) find small spillover effects in a joint (structural) analysis of the business cycles in the Euro area, the U.S. and the rest of the world. We therefore settled for a more "reduced form"-approach, in an attempt to examine if the results in de Walque, Smets and Wouters (2005) are driven by some structural misspecification of the international linkages.

When allowing for feedback effects from the Euro area to the rest-of-the-world variables, we thus consider the following VAR

$$\hat{x}_t^* = \Phi_c + \sum_{i=1}^{p} \Phi^i \hat{x}_{t-i}^* + \sum_{j=0}^{q} \Psi^j \hat{x}_{t-j} + \varepsilon_t, \tag{1}$$

where $\hat{x}_t^*$ is a three-dimensional vector of CPI inflation, Output and the short run interest rate for the 'rest of the world', $\hat{x}_t$ is a four-dimensional vector of the Euro area variables that we consider: CPI inflation, Output, the short run interest rate and the real exchange rate. The reason for choosing these four variables is that they are arguably the most important source of fluctuations in the rest-of-the-world economy from the point of view of the adopted model framework. We refer to Adolfson et al. (2005) for exact variable definitions.

2.1. VAR evidence

Before analyzing the effects in the DSGE model of including feedback effects in the foreign VAR, we start out by analyzing the VAR in isolation. Our interest here is to twofold. First, we want to determine if we may reasonably assume the absence of feedback effects from the Euro area to the rest of the world, i.e. that $\Psi^0 = \Psi^1 = ... = \Psi^q = 0$, in equation (1). Second, we are interested in finding out how many lags are sufficient. We consider the two cases $p = q = 2$ and $p = q = 4$. Four lags are standard when using quarterly data, but since we want to estimate the parameters in the VAR (1) jointly with the other parameters in the DSGE, we want to examine if two lags are sufficient, in order to make the analysis parsimonious.

In order to investigate the hypotheses of interest by means of marginal likelihood analysis we need a proper prior on the parameters. We use a rather standard Minnesota prior (Litterman, 1986) on $\Phi_c, \Phi^1, ..., \Phi^p$, with a prior mean equal to zero on all coefficients except the first lag of the foreign interest rate in the foreign interest rate equation, which is set equal to 0.9. We use standard values for the prior overall tightness equal to 0.2, cross-equation tightness equal to 0.5 and geometric lag decay. The estimated residual variance from univariate AR processes are used to scale the priors in the usual way. We use a similar prior on $\Psi^0, ..., \Psi^q$, with zero mean on all coefficients. The tightness around zero is modelled with an overall tightness prior hyperparameter which is here denoted by λ_{EXO}. The priors are scaled to account for the differing scales of the variables in the same way as for the Φ^i-coefficients. The prior tightness on Ψ^0 and Ψ^1 are set to be equal. Furthermore, there is no cross-equation shrinkage on the coefficients in $\Psi^0, ..., \Psi^q$.

Figure A1 depicts the log marginal likelihood of the model as a function of λ_{EXO}. Looking first at the model with two lags, we see that the model without feedback is preferred if $\lambda_{EXO} > 0.46$. The maximal marginal likelihood is obtained for $\lambda_{EXO} = 0.15$, which suggests that if a feedback effect is present, it should be rather small. Turning to the model with four lags, we see from Figure A1 that the set of λ_{EXO}'s where the model with feedback is preferred has shrunk somewhat to $\lambda_{EXO} < 0.37$, and the peak in the marginal likelihood curve occurs again at roughly $\lambda_{EXO} = 0.15$. Tables A1 and A2 presents information criteria and likelihood-ratio tests of the no-feedback restrictions. In both the two and four lags cases we see that the Schwarz (SBC) criterion favors the model with no feedback, whereas Akaike's criterion prefers the model with feedback effects. The likelihood ratio tests strongly rejects the hypothesis of no feedback.

Comparing the log marginal likelihood of the two lag lengths in Figure A1, we see that $p = q = 4$ is preferred in the model without feedback effects. This is the lag length used in the main paper. When feedback effects are present, however, the choice of lag length varies quite a bit with λ_{EXO}. The overall picture, however, is that here the evidence in favor of four lags is weaker, and for $\lambda_{EXO} > 0.6$ the log marginal likelihood is actually higher for the model with two lags. This finding is encouraging for the subsequent analysis with an extended DSGE model with feedback effects in the foreign VAR, where we would like to include as few lags as possible to keep the dimensionality of the parameter space manageable (the model with $p = q = 1$ gave a substantial fall in the marginal likelihood). We will only consider the case $p = q = 2$ in the following.

Table A.3 displays the posterior mean and standard deviations of the coefficients in the VAR(2) model, both when using an uninformative prior and when using the Minnesota prior. Coefficients with Bayesian t-ratios larger than 2 in absolute values are bolded. From the table we see that only a few of the coefficients are significant, and in particular it is the individual first lag that has an effect on each foreign variable. Turning to the feedback effect from the domestic variables we see that the Euro area interest rates appear to have an effect on the rest-of-the world interest rates (i.e., the Fed funds rate). This coefficient (Ψ^0_{33}) turns out significant in the interest rate equation in all three estimations of the VAR parameters, ranging from 0.36 in the joint estimation to 0.65 in the pre-estimation using the Minnesota prior. Also Euro area output seems to be important in the equation for the rest-of-the world output, and its coefficient turns out significantly both contemporaneously and with a lag. However, the collected effect seem to be very small (it is positive contemporaneously and negative at the first lag).

2.2. An extended DSGE model with feedback effects

Although the VAR exercise in the previous section did not present strong evidence in favor of including any feedback effects in the foreign VAR, we nevertheless reestimate the DSGE model when using the most preferred specification of the endogenous VAR in (1), in order to examine the robustness of the results in the main paper.

Table A.4 shows the posterior mode estimates and approximative standard deviation of the DSGE parameters when using the endogenous VAR model with feedback effects. For comparison we also include the estimation results from the model specification using the exogenous VAR model. In the table we show results when the foreign VAR is separately estimated from the DSGE parameters ("Pre-estimated") using either an uninformative prior or the Minnesota prior, as well as results when estimating the DSGE and VAR parameters simultaneously ("Jointly estimated") using the Minnesota prior on the VAR coefficients. From Table A.4 we see that the estimates are broadly similar, irrespective of whether a feedback effect is included in the VAR or not. Compared to the benchmark the persistence of the unit-root shock is somewhat lower for the specifications using the pre-estimated VARs, and higher for the specification where the VAR and DSGE parameters are jointly estimated. An additional difference in the latter specification is that the persistence coefficient for the asymmetric technology shock is substantially lower than for the benchmark. The standard deviation is, however, considerably higher suggesting that it is more difficult to pin down this parameter when the VAR coefficients are free to adjust. Turning to the log marginal likelihoods for the various specifications, it can be seen from the last row that the marginal densities are clearly indicative that the model specification using the exogenous VAR model is preferred by the data. The log marginal likelihoods are about the same for the two versions of the pre-estimated VARs (Minnesota and uninformative prior, respectively), which is not surprising given that the parameters are so similar. For the specification where the coefficients are jointly estimated, there is a notable drop in marginal density, suggesting that the coefficients in the feedback VAR differ from the priors.[1]

To further assess the quantitative implications the endogenous VAR has on the DSGE model we report the impulse response functions to a monetary policy shock in Figure A.2. The solid line show responses in the DSGE model using the exogenous VAR while the dotted and dashed lines show the responses in the specification using the endogenous VAR, pre-estimated and jointly estimated, respectively.[2] As could have been expected from the estimates in Table A.4 the impulses are very similar, both quantitatively and qualitatively for most of the variables. Apart from the foreign variables, export, import and the real exchange rate responses differ somewhat. For the foreign variables, we find the interest rate response to be generally low, whereas the foreign output and inflation responses are about the same as in the Euro area economy, which is a surprising finding. Given that the foreign interest rate response is so low, there is obviously a rather strong exchange rate channel at work in the feedback VAR, particularly for the jointly estimated specification.

In Table A.5 we report the variance decomposition (using the posterior mode estimates) for the DSGE model including the endogenous VAR (pre-estimated and jointly estimated) as well

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[1] It should, however, be acknowledged that we had problems in obtaining satisfactory convergence when the VAR and DSGE parameters were jointly analyzed (recall that a joint estimation approach requires 111 parameters to be estimated). Nevertheless, we do not believe that the log marginal density can improve by very much in any case.

[2] For the pre-estimated VAR, we only report results using the Minnesota prior henceforth, since the results with the uninformative prior are so similar.

as the DSGE model including the exogenous VAR. As expected, the same shocks that were found to matter the most in the benchmark specification are also the ones that are found to matter the most in the alternative specifications. In particular this is the case for the pre-estimated VAR. The numbers for the jointly estimated feedback VAR differ somewhat more. In addition, although not shown it should also be kept in mind that the uncertainty bands for the variance decompositions are generally rather large, in particular so for the jointly estimated feedback VAR. There is consequently not many discrepancies between the specifications that are significant.

3. Convergence diagnostics

In this section we report some diagnostics to verify that the posterior draws have converged to the true target posterior density. We present both convergence diagnostics from the single MCMC chain used for the results of the baseline model in the main paper, and results from multiple chains with widely different initial values.

3.1. Single chain diagnostics

Figures A3a to A3c report the prior density and the posterior density estimates for all estimated parameters in the model.[3] In accordance with the results in Table 1 in the main text, the figures show that the data are generally informative about the parameters, since the posterior distributions differ from the priors, with a few exceptions (i.e., ξ_w).

In Figures A4a to A4c we report the 500,000 post burn-in Metropolis draws for each parameter that our results in the main paper are based on. The MCMC chains in Figure A4 indicate that the chain is mixing well and that there are no trends in the sequential draws. The behavior of the sample paths for b and ρ_{ς^c} displays a few jumps (upward for b and downward for ρ_{ς^c}), in particular shortly after the first 50,000 draws. This is not an artifact of the posterior sampling[4]; there is indeed a genuine bimodality in this dimension of the parameter space which reflects that the internal propagation generated by habit in consumption is to a certain extent exchangeable with the persistence of the exogenous consumption preference shock.

3.2. Multiple chain diagnostics

To check convergence we generated $1,000,000$ post burn-in draws run from four very different initial values. The first MCMC chain (benchmark) is initiated using the posterior mode, another using the prior mode, and the third and fourth using the upper and lower tails of the marginal prior distributions (roughly two standard deviations above and below the mode, respectively). Note that the latter two vectors of initial values ignores the correlation structure in the posterior and are therefore far from the bulk of the posterior density. The sequential posterior mean estimates from the Metropolis draws (CUSUM profiles) in these four chains are depicted in Figures A5a to A5c. The posterior means of most of the model parameters from the first three of the multiple chains are very close after half a million draws. The chain initialized in the lower tail of the prior requires more draws for it to give the same results as the other three

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[3] The posterior density is estimated with a kernel density estimator with a normal kernel and a slightly smaller bandwidth than the optimal value for a normal target density.

[4] We generated several parallel MCMC chains, all starting in the posterior mode, and they all show essentially the same behavior in these two parameters. The relative density in the two modes came out very similar across the parallel MCMC runs.

chains, but not even this chain is very extreme when the scale of the CUSUM plots are taken into consideration.

In Figure A6, we plot the log marginal likelihood sequentially for each $50,000$th draw in the four different chains, using all the preceding iterations in the computation of the marginal likelihood with the modified harmonic estimator in Geweke (1999).[5] The top figure shows the sequential log marginal likelihoods using the entire chains of $1,000,000$ draws, while the bottom figure only uses the last $500,000$ draws. All the chains seem to converge to the same marginal likelihood, but it is clear that more draws are needed to obtain convergence when the two most demanding initializations of the MCMC chain (lower and upper tail of the prior) are used. Starting out the Metropolis algorithm in the posterior mode is, as expected, a good idea. For this choice of initial value convergence is fast and stable.

In Figure A7, we report the so called multivariate potential scale reduction factor (MPSRF) based on the four independent Metropolis chains with different starting values. The MPSRF is a measure of how much we can expect the precision of the posterior mean estimates to improve (in a multivariate sense) if we were to continue the MCMC runs indefinitely. It declines toward unity as the number of MCMC draws approach infinity. A rule of thumb is that a value below 1.2 is a good indication of convergence (Gelman et al. (2004) state that values below 1.1 for the square root of the MPSRF are acceptable). To handle storage demands, the analysis is based on subsampling every 5th draw. As can be seen from Figure A7, the MPSRF dips below the rule of thumb value after approximately $700,000$ draws.

We also generated several parallel MCMC chains which all had the posterior mode as initial value. All chains gave very similar CUSUM plots and sequential log marginal likelihood profiles, and the MPSRF converged much faster below 1.2 than in the case with over-dispersed initial values, verifying that our results are very robust to the inherent randomness in the posterior sampling.

3.3. Prior sensitivity analysis

In Table A.6, we report results when doubling the prior standard deviation on the model's structural parameters and the persistence of shocks, together with the benchmark results. For the exogenous shock persistence parameters, where we use the beta distribution, we also changed the location of the prior mode from 0.85 to 0.70. By comparing the two sets of results in Table A.6, we see that the posterior distributions are similar in most cases, with a few exceptions. These exceptions are the two Calvo probabilities (ξ_d and $\xi_{m,i}$) and the habit formation (b), which are found to be considerably higher. The higher values of $\xi_{m,i}$ and b, which induce more intrinsic propagation in the model, imply that the persistence coefficients for the consumption preference (ρ_{ζ_c}) and importing investment markup ($\rho_{\lambda_{m,i}}$) shocks turn out considerably lower. So as we weaken the sharpness in the prior for these parameters, the posterior estimates change. However, given that we think our priors for the three parameters (ξ_d, $\xi_{m,i}$ and b) to be well grounded based on previous evidence, our overall impression is that the robustness with respect to our choice of priors is satisfactory.

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[5] The marginal likelihood of a model i is defined as $m_i = \int L_i(\theta_i; x)p_i(\theta_i)d\theta_i$, where $L_i(\theta_i; x)$ is the usual likelihood function of the model's parameter vector conditional on the observed data x. $p_i(\theta_i)$ is the prior distribution of the model's parameters. m_i is the unconditional probability of the observed data, under the assumed prior distribution, and is therefore a measure of model fit. The marginal likelihood is a relative measure and should be compared across competing models. The Bayes factor comparing two models i and j is defined as $B_{ij} = m_i/m_j$.

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	Schwarz	Akaike	Log L	LR-test	df	p-val
No Feedback	3.056	2.590	−146.061	114.926	36	≈ 0
Feedback	3.521	2.257	−88.598			

Table A.1: Information criteria and likelihood-ratio (LR) test of no feedback. p=q=2.

	Schwarz	Akaike	Log L	LR-test	df	p-val
No Feedback	3.289	2.416	−114.420	153.626	60	≈ 0
Feedback	4.368	2.151	−37.607			

Table A.2: Information criteria and likelihood-ratio (LR) test of no feedback. p=q=4.

Table A.3: Posterior distribution of the endogenous foreign VAR parameters

Parameter	Pre-estimated (Uninformative prior)		Pre-estimated (Minnesota prior)		Jointly estimated (Minnesota prior)	
	posterior mean	posterior std	posterior mean	posterior std	posterior mean	posterior std
ϕ_{11}^{1}	**0.238**	0.081	**0.312**	0.073	0.186	0.113
ϕ_{12}^{1}	0.130	0.068	**0.095**	0.042	0.038	0.065
ϕ_{13}^{1}	-0.189	0.127	-0.069	0.075	0.110	0.083
ϕ_{21}^{1}	-0.115	0.111	-0.066	0.079	-0.095	0.105
ϕ_{22}^{1}	**0.752**	0.093	**0.745**	0.073	**0.675**	0.109
ϕ_{23}^{1}	0.272	0.174	0.034	0.101	0.060	0.127
ϕ_{31}^{1}	-0.025	0.059	0.009	0.042	**0.117**	0.054
ϕ_{32}^{1}	0.010	0.049	0.045	0.031	-0.009	0.034
ϕ_{33}^{1}	**0.886**	0.092	**0.856**	0.072	**0.771**	0.070
ϕ_{11}^{2}	**0.333**	0.074	**0.233**	0.062	0.071	0.082
ϕ_{12}^{2}	0.067	0.070	0.030	0.032	-0.009	0.038
ϕ_{13}^{2}	0.074	0.123	0.015	0.058	0.032	0.064
ϕ_{21}^{2}	-0.024	0.102	-0.028	0.054	0.023	0.059
ϕ_{22}^{2}	0.131	0.097	0.056	0.066	0.128	0.083
ϕ_{23}^{2}	**-0.435**	0.169	-0.103	0.078	-0.093	0.081
ϕ_{31}^{2}	0.004	0.054	0.003	0.030	0.037	0.031
ϕ_{32}^{2}	**0.115**	0.051	0.024	0.023	0.001	0.023
ϕ_{33}^{2}	-0.113	0.089	-0.025	0.065	-0.007	0.064
ψ_{11}^{0}	0.099	0.113	0.168	0.099	0.023	0.144
ψ_{12}^{0}	-0.121	0.071	-0.084	0.063	-0.031	0.074
ψ_{13}^{0}	-0.298	0.219	-0.291	0.179	-0.172	0.302
ψ_{14}^{0}	0.012	0.012	0.002	0.011	0.007	0.011
ψ_{21}^{0}	0.009	0.155	0.058	0.135	0.195	0.199
ψ_{22}^{0}	**0.442**	0.097	**0.362**	0.085	0.079	0.108
ψ_{23}^{0}	0.329	0.301	0.282	0.244	-0.343	0.361
ψ_{24}^{0}	-0.008	0.017	-0.012	0.015	-0.004	0.015
ψ_{31}^{0}	-0.005	0.082	0.037	0.070	0.064	0.101
ψ_{32}^{0}	0.078	0.051	0.061	0.045	-0.006	0.056
ψ_{33}^{0}	**0.645**	0.159	**0.402**	0.131	**0.363**	0.177
ψ_{34}^{0}	**0.022**	0.009	**0.016**	0.008	0.012	0.009
ψ_{11}^{1}	0.230	0.120	**0.233**	0.103	0.140	0.143
ψ_{12}^{1}	0.114	0.095	0.087	0.072	0.130	0.092
ψ_{13}^{1}	0.229	0.319	0.113	0.222	0.059	0.294
ψ_{14}^{1}	-0.023	0.019	-0.011	0.014	-0.013	0.014
ψ_{21}^{1}	0.108	0.164	0.057	0.139	-0.163	0.193
ψ_{22}^{1}	**-0.374**	0.130	**-0.242**	0.098	-0.010	0.113
ψ_{23}^{1}	-0.463	0.438	-0.226	0.298	0.022	0.354
ψ_{24}^{1}	0.005	0.026	0.011	0.018	0.014	0.017
ψ_{31}^{1}	-0.020	0.087	-0.016	0.074	-0.031	0.106
ψ_{32}^{1}	-0.113	0.069	-0.071	0.052	0.007	0.062
ψ_{33}^{1}	**-0.714**	0.232	**-0.336**	0.158	-0.335	0.204
ψ_{34}^{1}	-0.013	0.014	-0.009	0.010	-0.007	0.011
ψ_{11}^{2}	0.191	0.119	0.122	0.082	0.004	0.103
ψ_{12}^{2}	-0.045	0.073	-0.001	0.049	-0.016	0.067
ψ_{13}^{2}	-0.045	0.216	0.028	0.143	0.124	0.181
ψ_{14}^{2}	0.010	0.012	0.007	0.009	-0.005	0.009
ψ_{21}^{2}	0.000	0.164	-0.049	0.108	-0.088	0.132
ψ_{22}^{2}	-0.113	0.101	-0.105	0.066	-0.069	0.079
ψ_{23}^{2}	0.122	0.297	-0.146	0.191	-0.373	0.231
ψ_{24}^{2}	0.012	0.017	0.008	0.012	0.006	0.012
ψ_{31}^{2}	0.092	0.087	0.013	0.058	0.039	0.068
ψ_{32}^{2}	-0.051	0.053	-0.029	0.035	-0.024	0.040
ψ_{33}^{2}	0.194	0.157	0.015	0.102	-0.075	0.120
ψ_{34}^{2}	-0.007	0.009	-0.005	0.006	-0.001	0.007
Σ_{11}	**0.339**	0.023	**0.341**	0.023	**0.278**	0.026
Σ_{21}	0.063	0.045	0.064	0.045	0.005	0.050
Σ_{22}	**0.461**	0.032	**0.468**	0.032	**0.249**	0.035
Σ_{31}	-0.012	0.024	-0.009	0.024	-0.003	0.034
Σ_{32}	**0.080**	0.023	**0.083**	0.023	**0.140**	0.049
Σ_{33}	**0.233**	0.016	**0.236**	0.016	**0.195**	0.030

Note: Bold numbers indicate significance on the 5% level.

Table A.4: Posterior distributions, with exogenous or endogenous foreign VAR

Parameter		Exogenous VAR		Endogenous VAR with feedback effects					
				Pre-estimated Uninformative prior		Pre-estimated Minnesota prior		Jointly estimated Minnesota prior	
		mode	std. dev. (Hessian)	mode	std. dev. (Hessian)	mode	std. dev. (Hessian)	mode	std. dev. (Hessian)
Calvo wages	ξ_w	0.697	0.047	0.674	0.046	0.678	0.046	0.657	0.047
Calvo domestic prices	ξ_d	0.883	0.015	0.882	0.015	0.882	0.015	0.861	0.019
Calvo import cons. prices	$\xi_{m,c}$	0.463	0.059	0.507	0.062	0.500	0.060	0.499	0.057
Calvo import inv. prices	$\xi_{m,i}$	0.740	0.040	0.735	0.038	0.745	0.037	0.715	0.037
Calvo export prices	ξ_x	0.639	0.059	0.637	0.055	0.632	0.055	0.619	0.056
Calvo employment	ξ_e	0.792	0.022	0.785	0.022	0.792	0.021	0.811	0.023
Indexation wages	κ_w	0.516	0.160	0.511	0.152	0.507	0.150	0.403	0.128
Indexation domestic prices	κ_d	0.212	0.066	0.205	0.085	0.210	0.056	0.193	0.075
Index. import cons. prices	$\kappa_{m,c}$	0.161	0.074	0.168	0.083	0.176	0.084	0.186	0.072
Index. import inv. prices	$\kappa_{m,i}$	0.187	0.079	0.209	0.084	0.204	0.077	0.210	0.061
Indexation export prices	κ_x	0.139	0.072	0.151	0.068	0.149	0.069	0.142	0.068
Markup domestic	λ_d	1.168	0.053	1.158	0.045	1.160	0.048	1.161	0.048
Markup imported cons.	$\lambda_{m,c}$	1.619	0.063	1.622	0.065	1.618	0.062	1.623	0.058
Markup.imported invest.	$\lambda_{m,i}$	1.226	0.088	1.232	0.093	1.221	0.082	1.207	0.065
Investment adj. cost	$\tilde{S}''$	8.732	1.370	9.086	1.340	9.027	1.371	8.192	1.422
Habit formation	b	0.690	0.048	0.673	0.050	0.687	0.048	0.670	0.044
Subst. elasticity invest.	η_i	1.669	0.273	1.623	0.234	1.617	0.229	1.603	0.215
Subst. elasticity foreign	η_f	1.460	0.098	1.505	0.113	1.523	0.118	1.460	0.092
Technology growth	μ_z	1.005	0.000	1.005	0.000	1.005	0.000	1.006	0.000
Capital income tax	τ_k	0.137	0.042	0.144	0.038	0.133	0.037	0.166	0.030
Labour pay-roll tax	τ_w	0.186	0.050	0.186	0.048	0.186	0.050	0.185	0.048
Risk premium	$\tilde{\phi}$	0.145	0.047	0.179	0.058	0.189	0.063	0.085	0.034
Unit root tech. shock	ρ_{μ_z}	0.723	0.106	0.562	0.106	0.570	0.089	0.850	0.040
Stationary tech. shock	ρ_ε	0.909	0.030	0.912	0.039	0.904	0.033	0.933	0.027
Invest. spec. tech shock	ρ_Υ	0.750	0.041	0.774	0.049	0.750	0.044	0.819	0.048
Asymmetric tech. shock	$\rho_{\tilde{z}*}$	0.993	0.002	0.993	0.003	0.993	0.003	0.919	0.037
Consumption pref. shock	ρ_{ζ_c}	0.935	0.029	0.944	0.028	0.940	0.035	0.988	0.005
Labour supply shock	ρ_{ζ_h}	0.675	0.062	0.660	0.065	0.657	0.063	0.557	0.070
Risk premium shock	$\rho_{\tilde{\phi}}$	0.991	0.008	0.990	0.009	0.990	0.009	0.990	0.007
Imp. cons. markup shock	$\rho_{\lambda_{m,c}}$	0.978	0.016	0.961	0.017	0.962	0.017	0.954	0.015
Imp. invest. markup shock	$\rho_{\lambda_{m,i}}$	0.974	0.015	0.978	0.016	0.974	0.017	0.975	0.013
Export markup shock	ρ_{λ_x}	0.894	0.045	0.900	0.046	0.907	0.044	0.905	0.041
Unit root tech. shock	σ_z	0.130	0.025	0.111	0.021	0.112	0.021	0.176	0.034
Stationary tech. shock	σ_ε	0.452	0.082	0.443	0.085	0.447	0.056	0.471	0.068
Invest. spec. tech. shock	σ_Υ	0.424	0.046	0.390	0.056	0.423	0.053	0.358	0.043
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	0.203	0.031	0.248	0.037	0.248	0.039	0.223	0.032
Consumption pref. shock	σ_{ζ_c}	0.151	0.031	0.165	0.036	0.161	0.035	0.186	0.043
Labour supply shock	σ_{ζ_h}	0.095	0.015	0.098	0.015	0.098	0.015	0.094	0.014
Risk premium shock	$\sigma_{\tilde{\phi}}$	0.130	0.023	0.164	0.028	0.174	0.029	0.069	0.019
Domestic markup shock	σ_λ	0.130	0.012	0.127	0.013	0.128	0.013	0.127	0.013
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	2.548	0.710	2.086	0.634	2.170	0.648	2.174	0.600
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	0.292	0.079	0.300	0.078	0.283	0.070	0.332	0.079
Export markup shock	σ_{λ_x}	0.977	0.214	0.961	0.194	0.966	0.200	1.051	0.219
Monetary policy shock	σ_R	0.133	0.013	0.134	0.013	0.133	0.013	0.144	0.014
Inflation target shock	$\sigma_{\bar{\pi}^c}$	0.044	0.012	0.038	0.010	0.040	0.011	0.029	0.006
Interest rate smoothing	ρ_R	0.874	0.021	0.860	0.023	0.868	0.022	0.843	0.021
Inflation response	r_π	1.710	0.067	1.712	0.088	1.714	0.087	1.707	0.071
Diff. infl response	$r_{\Delta\pi}$	0.317	0.059	0.308	0.058	0.304	0.058	0.313	0.059
Real exch. rate response	r_x	-0.009	0.008	-0.013	0.007	-0.013	0.008	-0.011	0.006
Output response	r_y	0.078	0.028	0.069	0.028	0.083	0.028	0.067	0.017
Diff. output response	$r_{\Delta y}$	0.116	0.028	0.104	0.028	0.108	0.027	0.076	0.031
Log marginal likelihood		-1909.3		-1955.0		-1950.7		-2000.1*	

* Note: Problematic convergence.

Table A.5: Variance decomposition, with exogenous or endogenous foreign VAR

4 quarters	Domestic inflation			Output			Interest rate			Real exchange rate			Exports			Imports		
	ExoVAR	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim	Exports	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim
Stationary technology	0.128	0.158	0.239	0.018	0.019	0.041	0.045	0.053	0.136	0.002	0.001	0.001	0.001	0.000	0.002	0.000	0.000	0.002
Unit root technology	0.041	0.016	0.318	0.100	0.036	0.279	0.013	0.002	0.223	0.000	0.000	0.002	0.006	0.002	0.015	0.016	0.004	0.023
Investment specific technology	0.000	0.002	0.004	0.480	0.578	0.401	0.342	0.330	0.206	0.101	0.106	0.095	0.046	0.069	0.035	0.466	0.460	0.577
Asymmetric technology	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.001	0.002	0.001	0.000	0.000	0.000	0.003	0.004	0.002
Consumtion preference	0.032	0.023	0.152	0.123	0.132	0.106	0.154	0.128	0.026	0.013	0.019	0.019	0.006	0.006	0.007	0.004	0.008	0.028
Labour supply	0.564	0.596	0.164	0.067	0.065	0.027	0.169	0.166	0.090	0.005	0.003	0.001	0.002	0.002	0.002	0.001	0.002	0.001
Risk premium	0.006	0.007	0.004	0.014	0.018	0.005	0.027	0.027	0.011	0.026	0.026	0.015	0.011	0.013	0.006	0.090	0.071	0.051
Domestic markup	0.007	0.006	0.007	0.005	0.003	0.004	0.007	0.013	0.009	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.000
Import consumption markup	0.049	0.030	0.004	0.005	0.003	0.001	0.000	0.015	0.043	0.548	0.504	0.521	0.182	0.175	0.195	0.227	0.229	0.170
Import investment markup	0.000	0.001	0.022	0.010	0.021	0.025	0.025	0.036	0.074	0.219	0.235	0.264	0.106	0.125	0.119	0.040	0.032	0.017
Export markup	0.003	0.000	0.003	0.066	0.052	0.045	0.045	0.021	0.020	0.062	0.096	0.071	0.627	0.598	0.605	0.121	0.161	0.120
Monetary policy	0.031	0.024	0.018	0.094	0.056	0.060	0.101	0.158	0.131	0.012	0.001	0.005	0.005	0.001	0.009	0.000	0.013	0.002
Inflation target	0.137	0.134	0.064	0.006	0.003	0.002	0.061	0.043	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000
Fiscal variables	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Foreign variables	0.001	0.000	0.000	0.010	0.012	0.004	0.010	0.008	0.002	0.010	0.007	0.003	0.008	0.008	0.003	0.031	0.015	0.008

20 quarters	Domestic inflation			Output			Interest rate			Real exchange rate			Exports			Imports		
	ExoVAR	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim	Exports	PreEstim	JointEstim	ExoVAR	PreEstim	JointEstim
Stationary technology	0.002	0.003	0.034	0.023	0.027	0.022	0.024	0.024	0.070	0.001	0.002	0.000	0.002	0.002	0.016	0.004	0.004	0.042
Unit root technology	0.045	0.017	0.287	0.345	0.131	0.563	0.081	0.030	0.471	0.000	0.000	0.019	0.041	0.013	0.147	0.198	0.046	0.286
Investment specific technology	0.249	0.387	0.211	0.156	0.215	0.093	0.180	0.284	0.128	0.015	0.014	0.093	0.017	0.014	0.060	0.222	0.069	0.425
Asymmetric technology	0.002	0.004	0.000	0.000	0.000	0.000	0.001	0.002	0.000	0.001	0.002	0.000	0.001	0.002	0.000	0.019	0.024	0.001
Consumtion preference	0.041	0.038	0.335	0.016	0.026	0.202	0.191	0.222	0.144	0.005	0.006	0.000	0.005	0.008	0.004	0.016	0.019	0.001
Labour supply	0.003	0.005	0.002	0.101	0.096	0.005	0.098	0.080	0.013	0.007	0.007	0.000	0.007	0.008	0.004	0.018	0.015	0.019
Risk premium	0.002	0.002	0.002	0.001	0.001	0.001	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000	0.001
Domestic markup	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
Import consumption markup	0.303	0.196	0.028	0.067	0.073	0.022	0.056	0.018	0.000	0.406	0.307	0.164	0.277	0.183	0.142	0.055	0.002	0.037
Import investment markup	0.012	0.027	0.002	0.279	0.411	0.089	0.091	0.071	0.078	0.557	0.647	0.716	0.551	0.610	0.498	0.355	0.684	0.139
Export markup	0.003	0.008	0.001	0.007	0.013	0.002	0.011	0.012	0.008	0.006	0.012	0.003	0.098	0.158	0.125	0.096	0.117	0.017
Monetary policy	0.000	0.000	0.001	0.004	0.004	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.003	0.007
Inflation target	0.336	0.308	0.096	0.000	0.000	0.000	0.265	0.252	0.084	0.000	0.000	0.002	0.000	0.000	0.002	0.000	0.001	0.018
Fiscal variables	0.002	0.002	0.001	0.001	0.001	0.000	0.001	0.002	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Foreign variables	0.000	0.001	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.001	0.001	0.000	0.001	0.001	0.000	0.016	0.016	0.006

Table A.6: Prior sensitivity

Parameter		Prior type	Benchmark prior						Vague prior					
			Prior distribution		Posterior distribution				Prior distribution		Posterior distribution			
			mean*	std /df	5%	mean	95%	std	mean*	std /df	5%	mean	95%	std
Calvo wages	ξ_w	beta	0.675	0.050	0.607	0.690	0.766	0.048	0.675	0.100	0.579	0.711	0.848	0.082
Calvo domestic prices	ξ_d	beta	0.675	0.050	0.862	0.891	0.921	0.018	0.675	0.100	0.934	0.961	0.981	0.015
Calvo import cons. prices	$\xi_{m,c}$	beta	0.500	0.100	0.345	0.444	0.540	0.059	0.500	0.200	0.260	0.366	0.477	0.066
Calvo import inv. prices	$\xi_{m,i}$	beta	0.500	0.100	0.641	0.721	0.792	0.046	0.500	0.200	0.965	0.985	0.996	0.011
Calvo export prices	ξ_x	beta	0.500	0.100	0.506	0.612	0.717	0.065	0.500	0.200	0.492	0.585	0.679	0.057
Calvo employment	ξ_e	beta	0.675	0.100	0.741	0.787	0.827	0.027	0.675	0.200	0.771	0.828	0.892	0.036
Indexation wages	κ_w	beta	0.500	0.150	0.258	0.497	0.739	0.145	0.500	0.200	0.118	0.378	0.689	0.173
Index. domestic prices	κ_d	beta	0.500	0.150	0.095	0.217	0.362	0.081	0.500	0.200	0.048	0.177	0.357	0.097
Index. import cons. prices	$\kappa_{m,c}$	beta	0.500	0.150	0.084	0.220	0.418	0.104	0.500	0.200	0.054	0.219	0.465	0.129
Index. import inv. prices	$\kappa_{m,i}$	beta	0.500	0.150	0.098	0.231	0.405	0.095	0.500	0.200	0.049	0.194	0.458	0.125
Indexation export prices	κ_x	beta	0.500	0.150	0.069	0.185	0.347	0.088	0.500	0.200	0.026	0.106	0.228	0.064
Markup domestic	λ_d	inv. gamma	1.200	2	1.122	1.222	1.383	0.084	1.200	2	1.126	1.248	1.463	0.109
Markup imported cons.	$\lambda_{m,c}$	inv. gamma	1.200	2	1.526	1.633	1.751	0.068	1.200	2	1.518	1.631	1.752	0.071
Markup.imported invest.	$\lambda_{m,i}$	inv. gamma	1.200	2	1.146	1.275	1.467	0.100	1.200	2	1.111	1.183	1.292	0.057
Investment adj. cost	$\tilde{S}''$	normal	7.694	1.500	6.368	8.670	10.958	1.396	7.694	3.000	2.793	7.047	11.488	2.644
Habit formation	b	beta	0.650	0.100	0.608	0.708	0.842	0.068	0.650	0.200	0.948	0.976	0.995	0.015
Subst. elasticity invest.	η_i	inv. gamma	1.500	4	1.393	1.696	2.142	0.235	1.500	4	1.315	1.477	1.699	0.121
Subst. elasticity foreign	η_f	inv. gamma	1.500	4	1.340	1.486	1.674	0.104	1.500	4	1.308	1.441	1.616	0.095
Technology growth	μ_z	trunc. normal	1.006	0.0005	1.004	1.005	1.006	0.000	1.006	0.001	1.004	1.005	1.005	0.001
Capital income tax	τ_k	beta	0.120	0.050	0.072	0.135	0.200	0.039	0.120	0.100	0.120	0.205	0.283	0.049
Labour pay-roll tax	τ_w	beta	0.200	0.050	0.118	0.197	0.286	0.051	0.200	0.100	0.060	0.194	0.379	0.098
Risk premium	$\tilde{\phi}$	inv. gamma	0.010	2	0.139	0.252	0.407	0.084	0.010	2	0.138	0.246	0.404	0.081
Unit root tech. shock	ρ_{μ_z}	beta	0.850	0.100	0.526	0.698	0.852	0.099	0.700	0.200	0.415	0.669	0.875	0.141
Stationary tech. shock	ρ_ε	beta	0.850	0.100	0.810	0.886	0.939	0.041	0.700	0.200	0.881	0.931	0.963	0.025
Invest. spec. tech shock	ρ_Υ	beta	0.850	0.100	0.638	0.720	0.796	0.048	0.700	0.200	0.481	0.634	0.776	0.091
Asymmetric tech. shock	$\rho_{\tilde{z}*}$	beta	0.850	0.100	0.986	0.992	0.995	0.003	0.700	0.200	0.987	0.992	0.996	0.003
Consumption pref. shock	ρ_{ζ_c}	beta	0.850	0.100	0.722	0.892	0.964	0.079	0.700	0.200	0.114	0.269	0.429	0.097
Labour supply shock	ρ_{ζ_h}	beta	0.850	0.100	0.565	0.676	0.774	0.064	0.700	0.200	0.443	0.619	0.774	0.105
Risk premium shock	$\rho_{\tilde{\phi}}$	beta	0.850	0.100	0.922	0.955	0.991	0.021	0.700	0.200	0.925	0.958	0.994	0.021
Imp. cons. markup shock	$\rho_{\lambda_{m,c}}$	beta	0.850	0.100	0.943	0.970	0.991	0.015	0.700	0.200	0.957	0.981	0.997	0.013
Imp. invest. markup shock	$\rho_{\lambda_{m,i}}$	beta	0.850	0.100	0.931	0.963	0.989	0.018	0.700	0.200	0.228	0.492	0.655	0.123
Export markup shock	ρ_{λ_x}	beta	0.850	0.100	0.789	0.886	0.961	0.054	0.700	0.200	0.871	0.940	0.987	0.038
Unit root tech. shock	σ_z	inv. gamma	0.200	2	0.099	0.137	0.185	0.027	0.200	2	0.103	0.143	0.190	0.027
Stationary tech. shock	σ_ε	inv. gamma	0.700	2	0.361	0.519	0.756	0.130	0.700	2	0.384	0.664	1.470	0.370
Invest. spec. tech. shock	σ_Υ	inv. gamma	0.200	2	0.389	0.469	0.561	0.052	0.200	2	0.423	0.514	0.623	0.062
Asymmetric tech. shock	$\sigma_{\tilde{z}*}$	inv. gamma	0.400	2	0.166	0.217	0.276	0.034	0.400	2	0.158	0.205	0.260	0.031
Consumption pref. shock	σ_{ζ_c}	inv. gamma	0.200	2	0.108	0.157	0.224	0.036	0.200	2	0.130	0.174	0.221	0.028
Labour supply shock	σ_{ζ_h}	inv. gamma	0.200	2	0.075	0.098	0.128	0.016	0.200	2	0.073	0.097	0.128	0.017
Risk premium shock	$\sigma_{\tilde{\phi}}$	inv. gamma	0.050	2	0.128	0.183	0.246	0.036	0.050	2	0.117	0.172	0.236	0.037
Domestic markup shock	σ_λ	inv. gamma	0.300	2	0.111	0.132	0.157	0.014	0.300	2	0.113	0.133	0.155	0.013
Imp. cons. markup shock	$\sigma_{\lambda_{m,c}}$	inv. gamma	0.300	2	1.737	2.882	4.463	0.831	0.300	2	2.198	4.004	6.338	1.269
Imp. invest. markup shock	$\sigma_{\lambda_{m,i}}$	inv. gamma	0.300	2	0.218	0.354	0.550	0.106	0.300	2	0.184	0.252	0.335	0.046
Export markup shock	σ_{λ_x}	inv. gamma	0.300	2	0.772	1.124	1.604	0.270	0.300	2	0.858	1.252	1.795	0.286
Monetary policy shock	σ_R	inv. gamma	0.150	2	0.113	0.135	0.160	0.014	0.150	2	0.111	0.132	0.156	0.014
Inflation target shock	$\sigma_{\bar{\pi}^c}$	inv. gamma	0.050	2	0.032	0.053	0.081	0.015	0.050	2	0.051	0.083	0.116	0.020
Interest rate smoothing	ρ_R	beta	0.800	0.050	0.844	0.881	0.915	0.022	0.800	0.050	0.846	0.882	0.915	0.021
Inflation response	r_π	normal	1.700	0.100	1.577	1.730	1.876	0.090	1.700	0.100	1.500	1.667	1.832	0.101
Diff. infl response	$r_{\Delta\pi}$	normal	0.300	0.100	0.212	0.310	0.411	0.060	0.300	0.100	0.201	0.302	0.405	0.061
Real exch. rate response	r_x	normal	0.000	0.050	-0.024	-0.009	0.006	0.009	0.000	0.050	-0.005	0.011	0.028	0.010
Output response	r_y	normal	0.125	0.050	0.051	0.104	0.168	0.036	0.125	0.050	0.135	0.191	0.252	0.036
Diff. output response	$r_{\Delta y}$	normal	0.0625	0.050	0.081	0.128	0.177	0.029	0.0625	0.050	0.050	0.102	0.156	0.032

*Note: For the inverse gamma distribution, the mode and the degrees of freedom are reported. Also, for the parameters $\lambda_d, \eta_i, \eta_f, \lambda_{m,c}, \lambda_{m,i}$ and μ_z the prior densities are translated so that values below unity are excluded.

Figure A1: Marginal likelihoods as a function of the shrinkage prior hyperparameter for the feedback coefficients in the VAR. With (solid) and without (dashed) feedback from the Euro area variables.

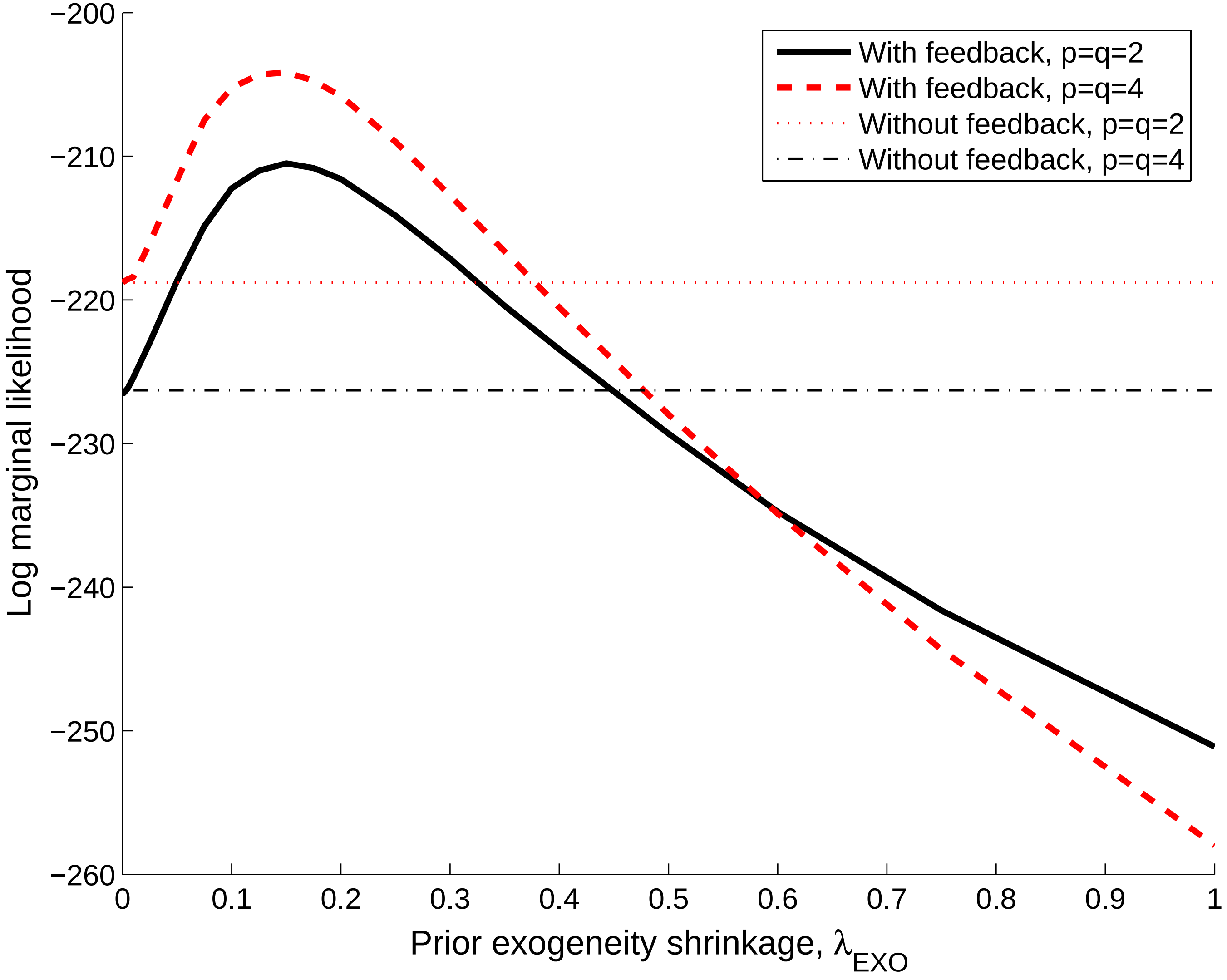


Figure A2: Impulse responses to a monetary policy shock, with exogenous and endogenous foreign VAR

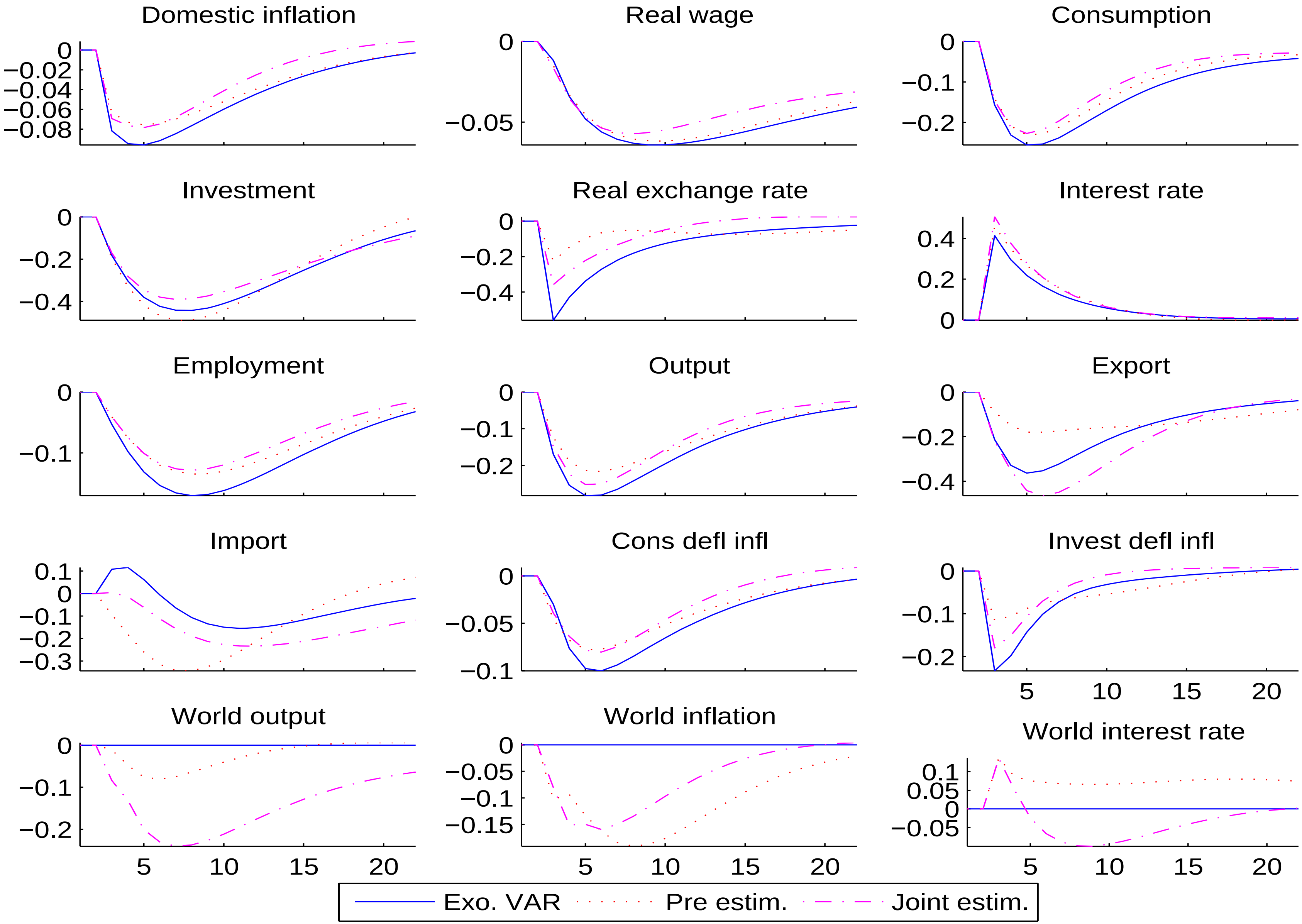


Figure A3a: Prior and posterior distributions, friction parameters

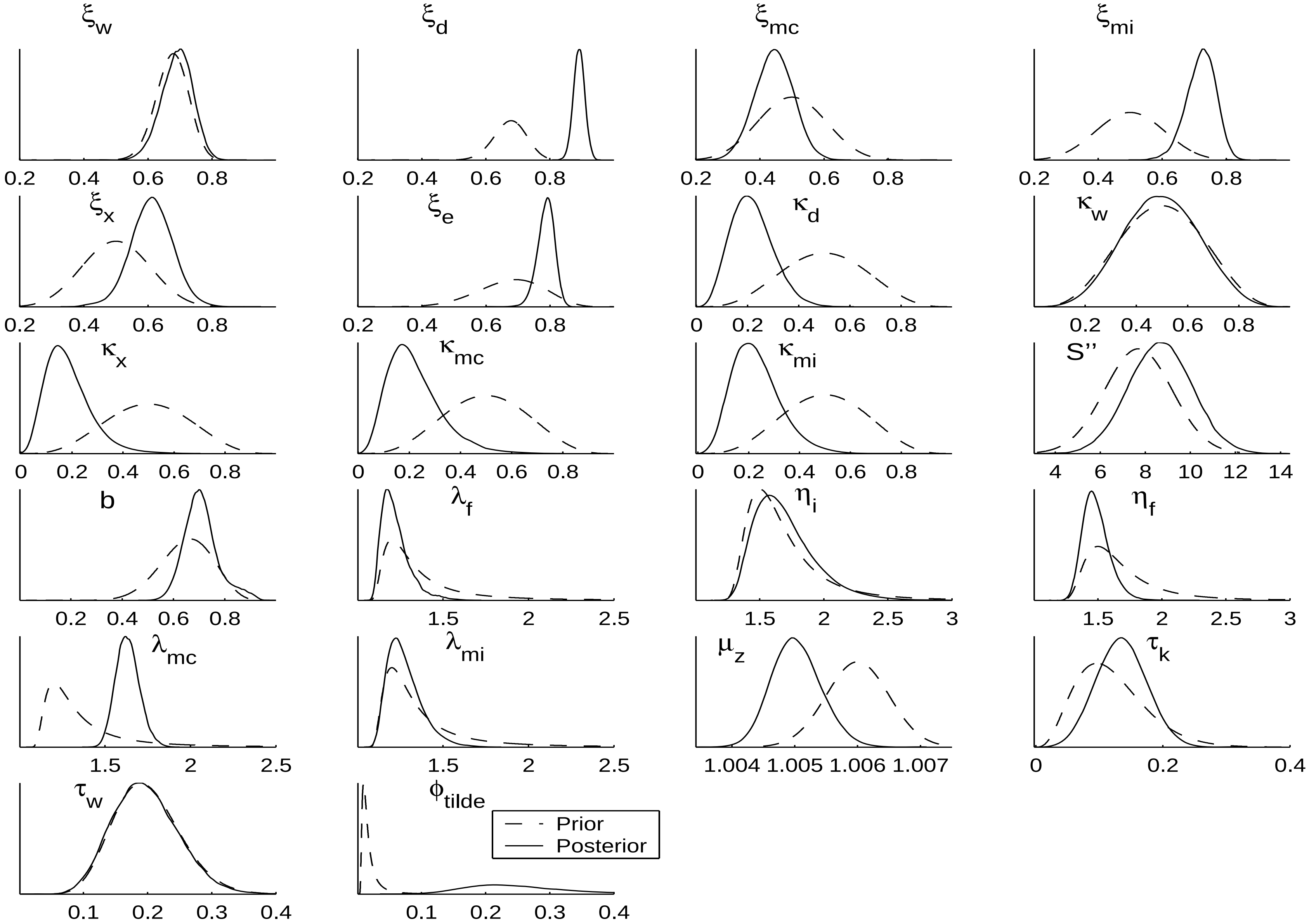


Figure A3b: Prior and posterior distributions, shock parameters

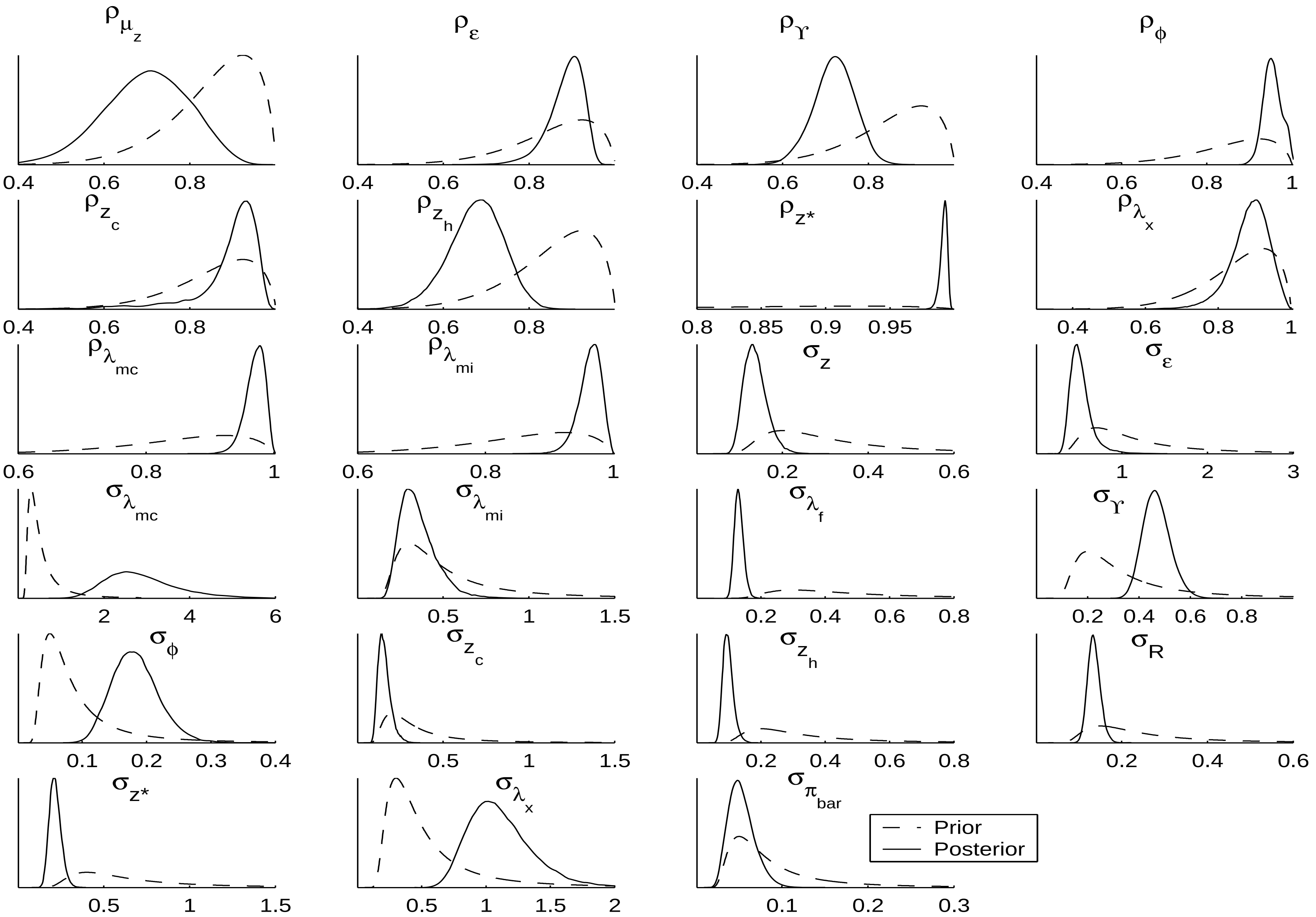


Figure A3c: Prior and posterior distributions, policy parameters

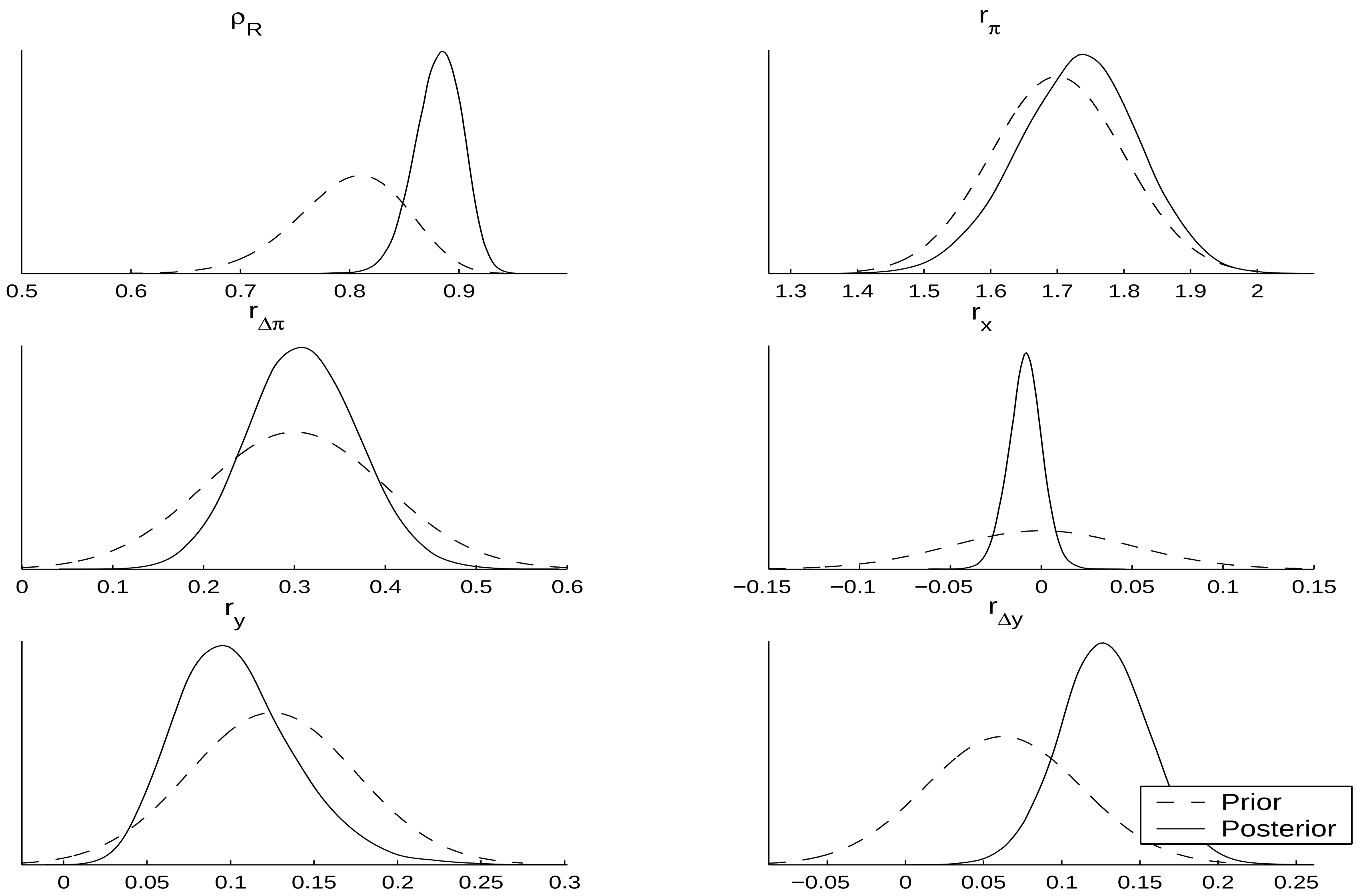


Figure A4a: Plots of the raw Metropolis draws, friction parameters

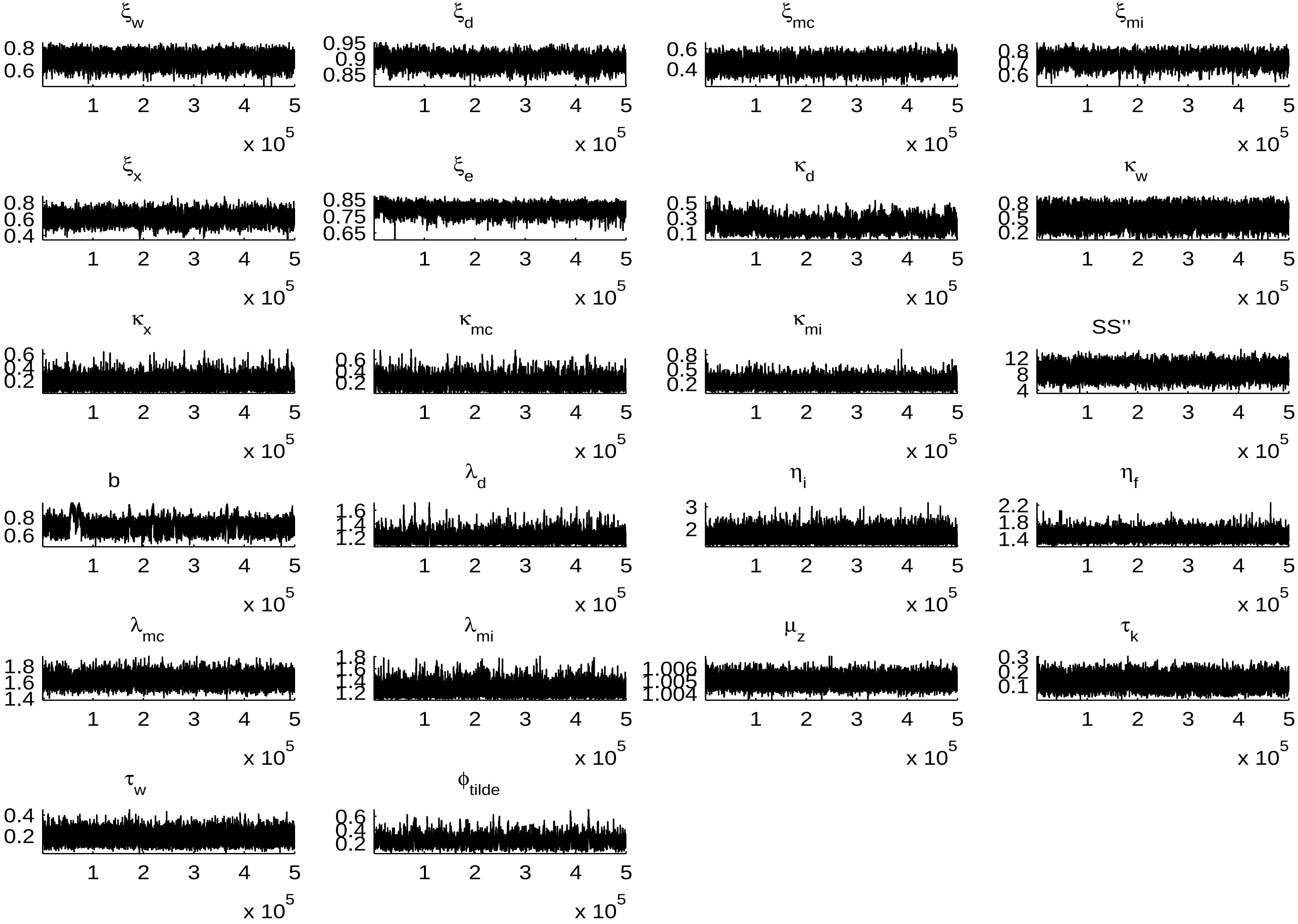
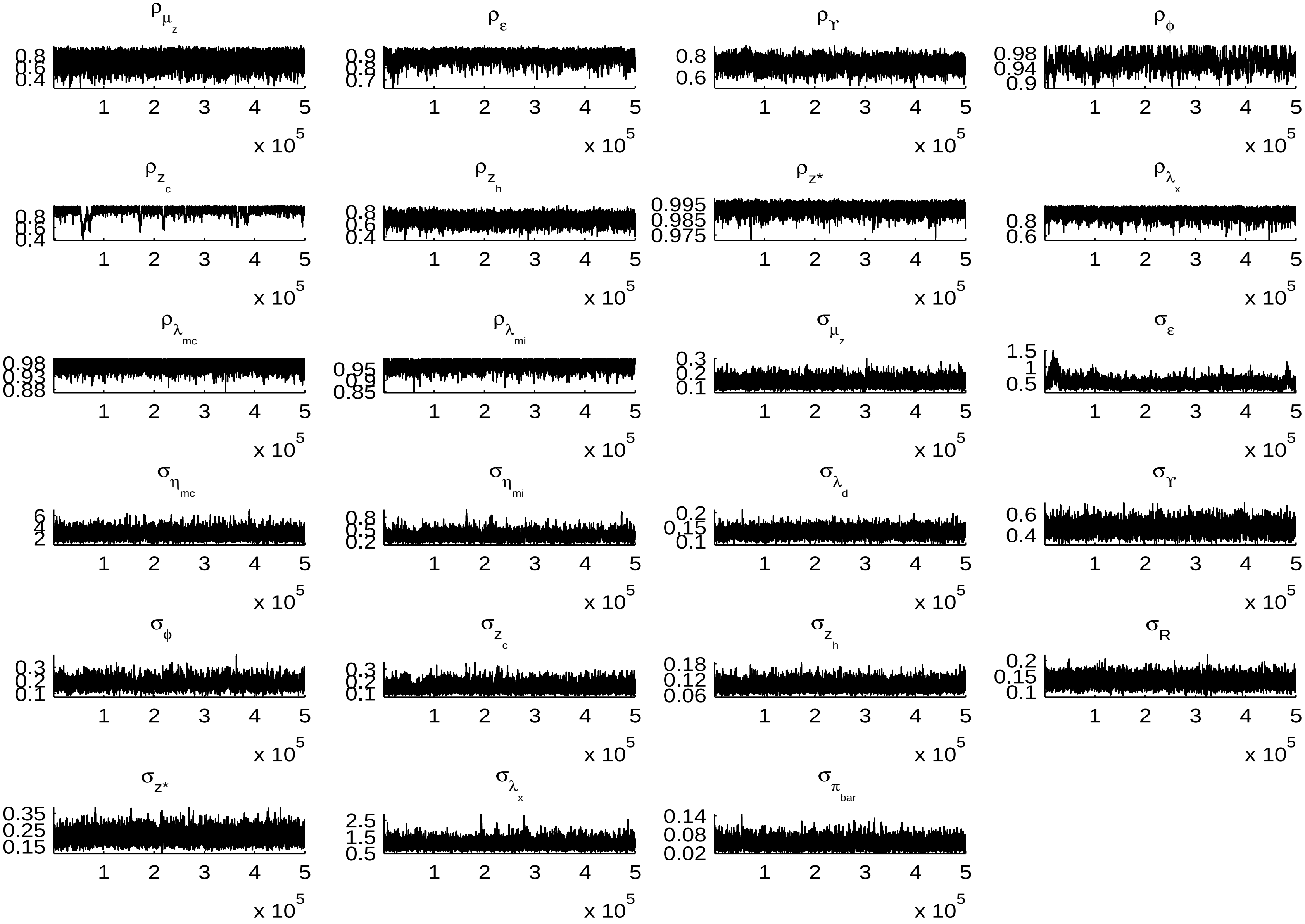


Figure A4b: Plots of the raw Metropolis draws, shock parameters



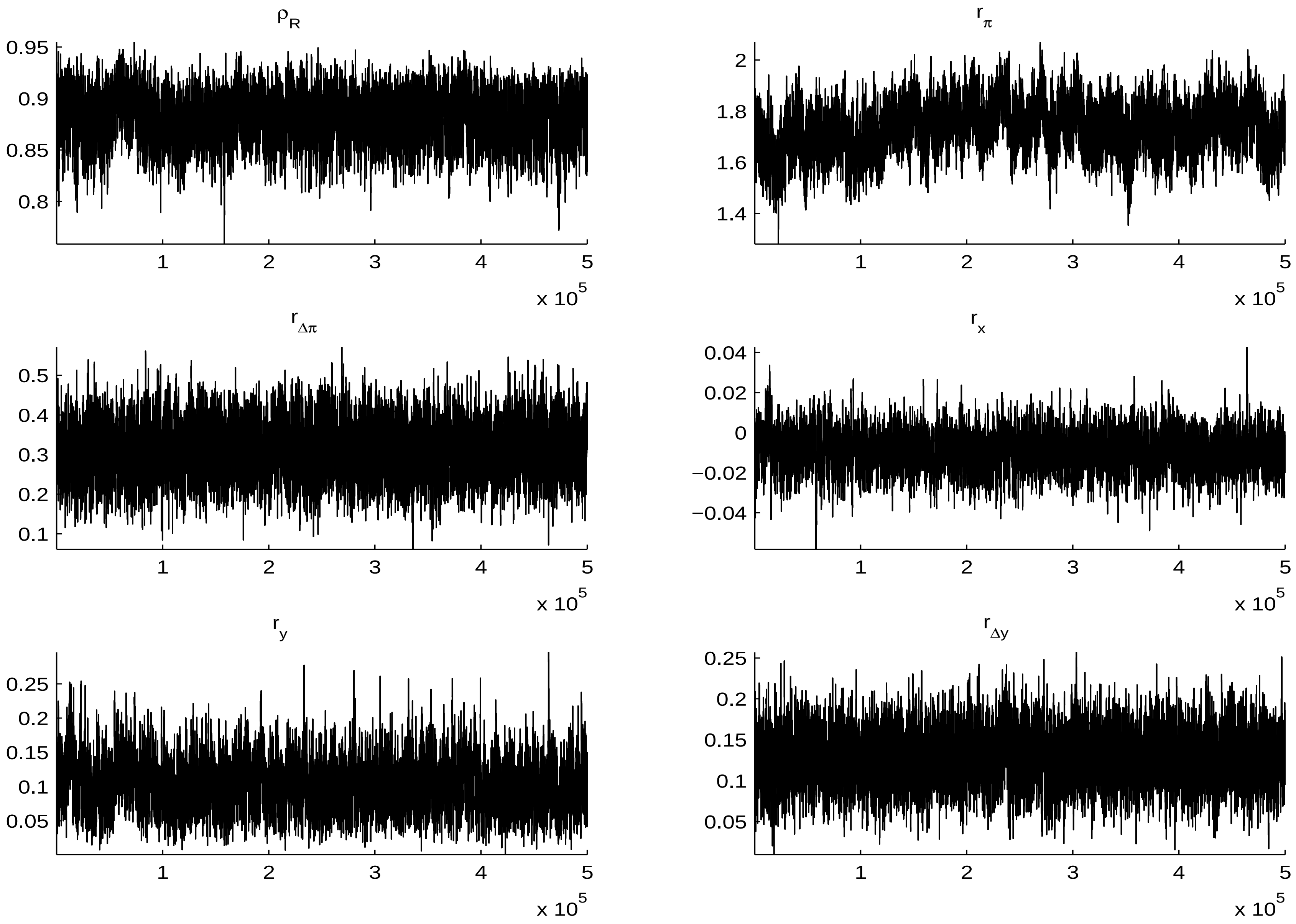


Figure A4c: Plots of the raw Metropolis draws, policy parameters

Figure A5a: CUSUM plots of the Metropolis draws, friction parameters

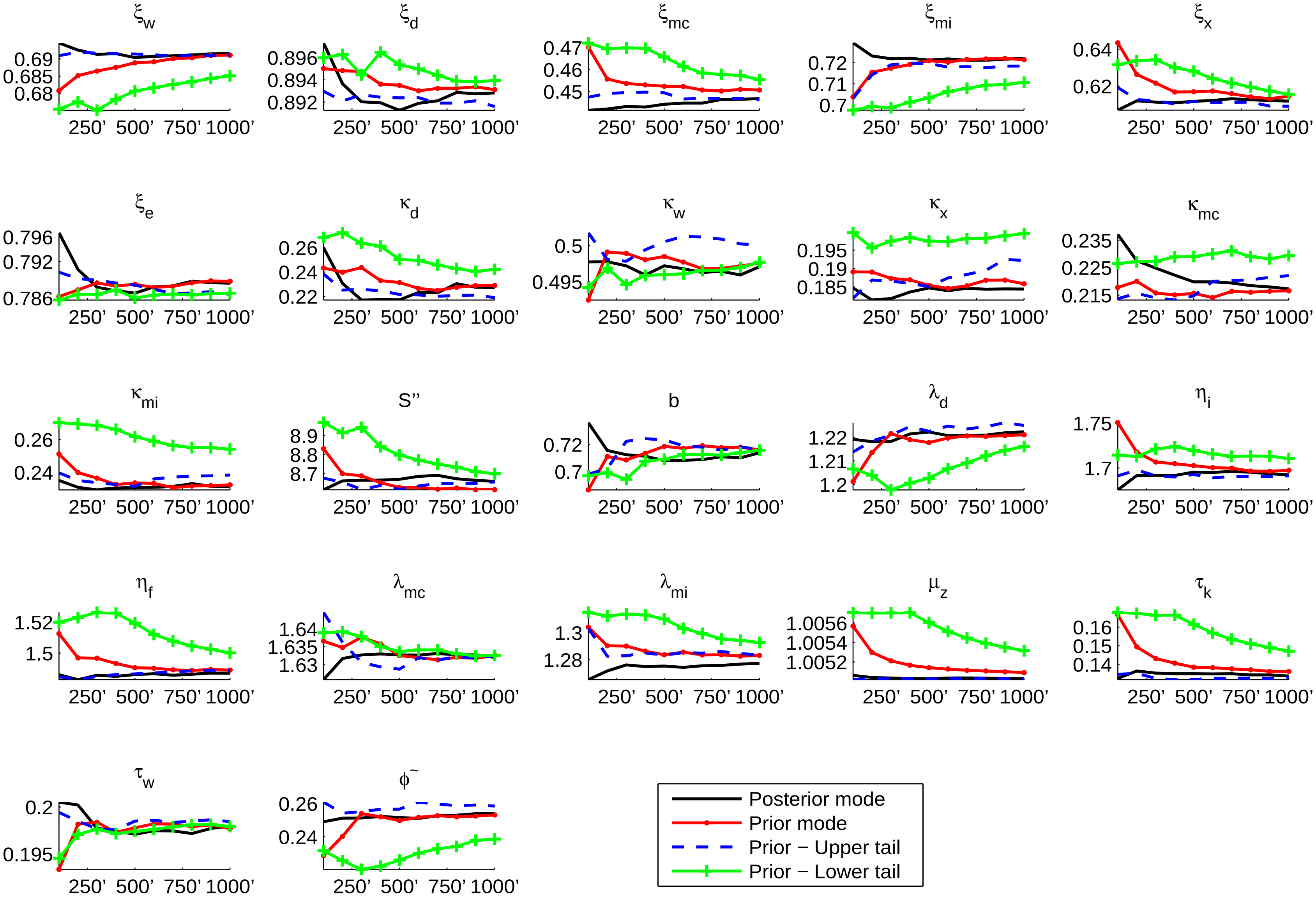


Figure A5b: CUSUM plots of the Metropolis draws, shock parameters

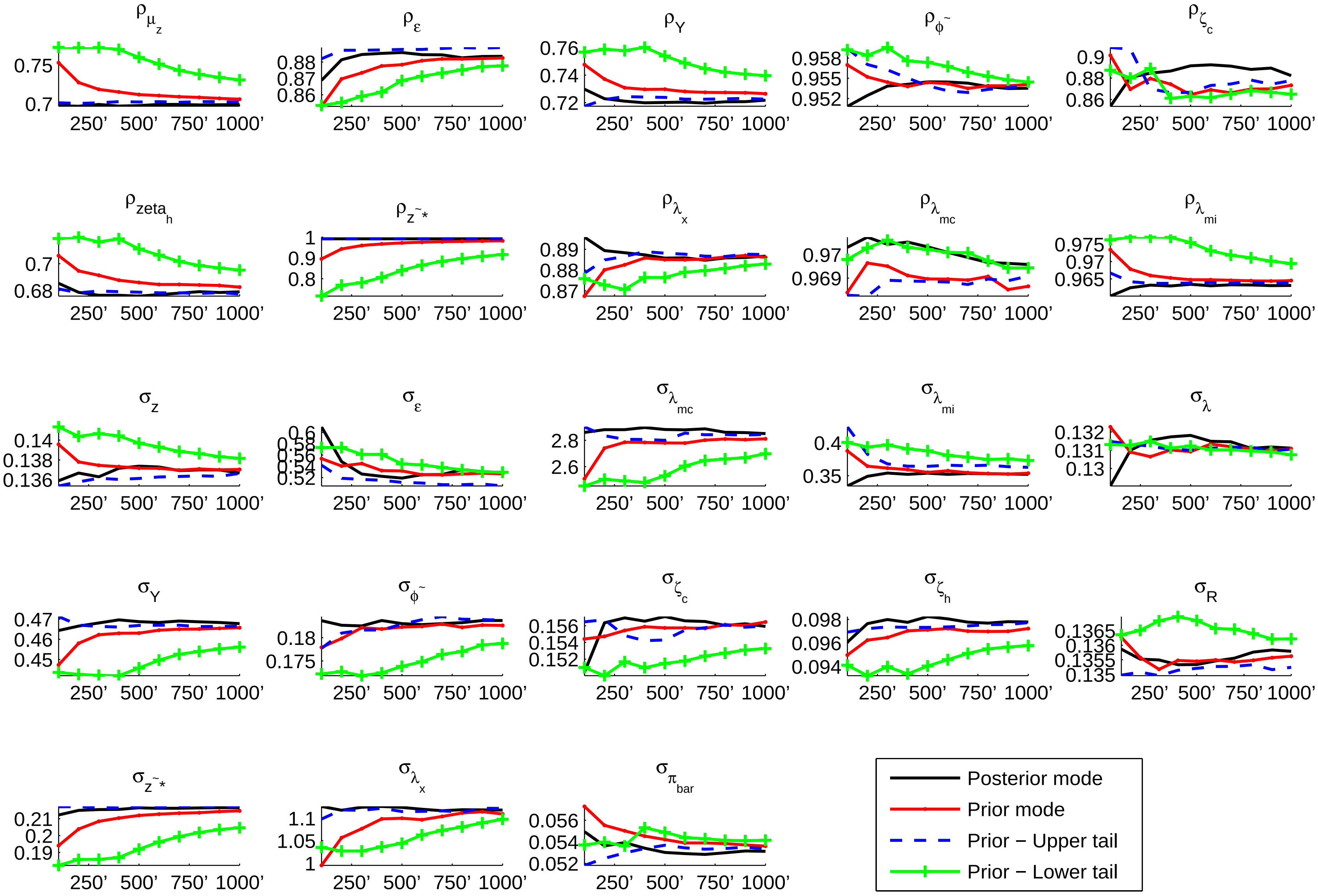


Figure A5c: CUSUM plots of the Metropolis draws, policy parameters

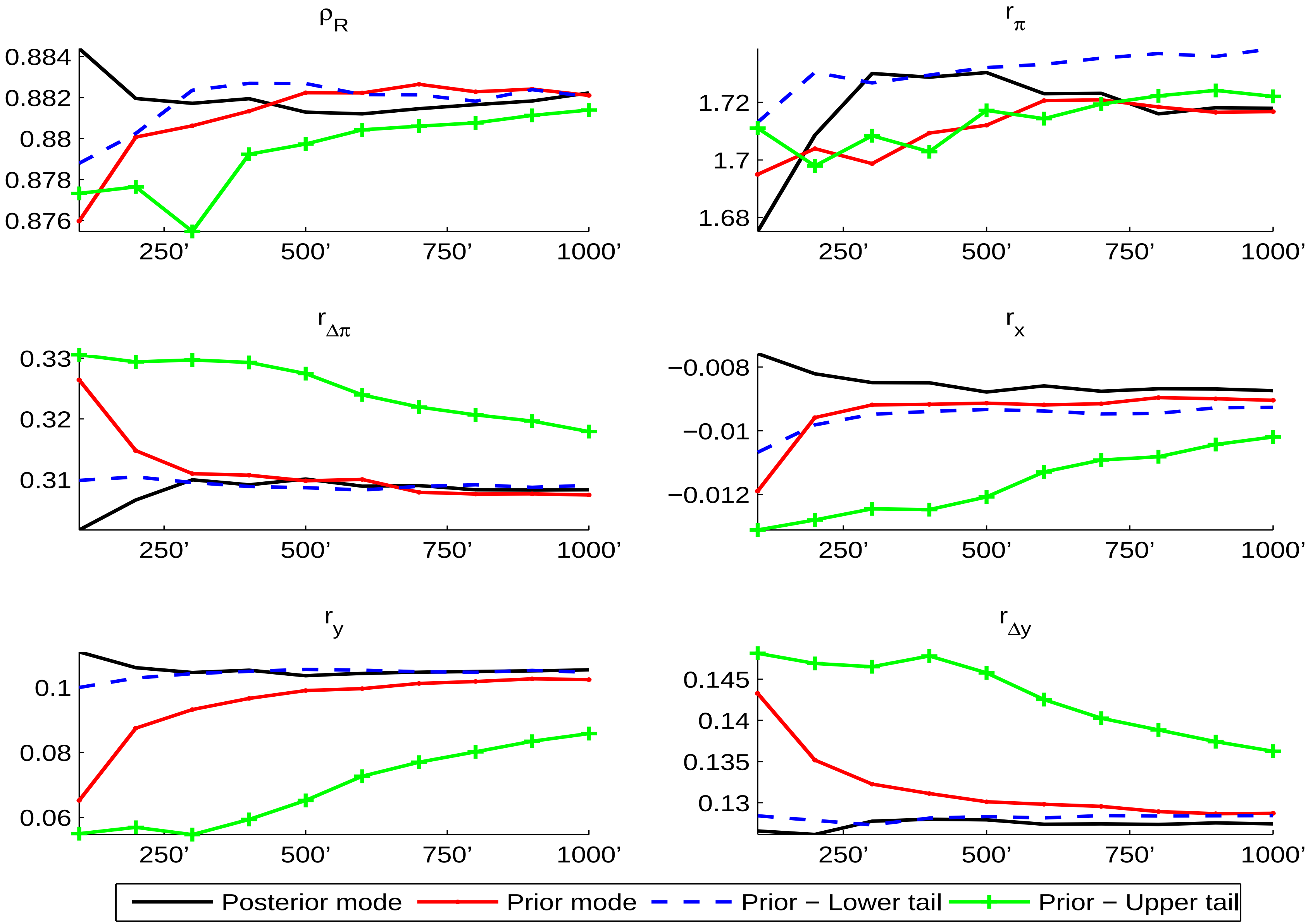


Figure A6: Sequential marginal likelihoods

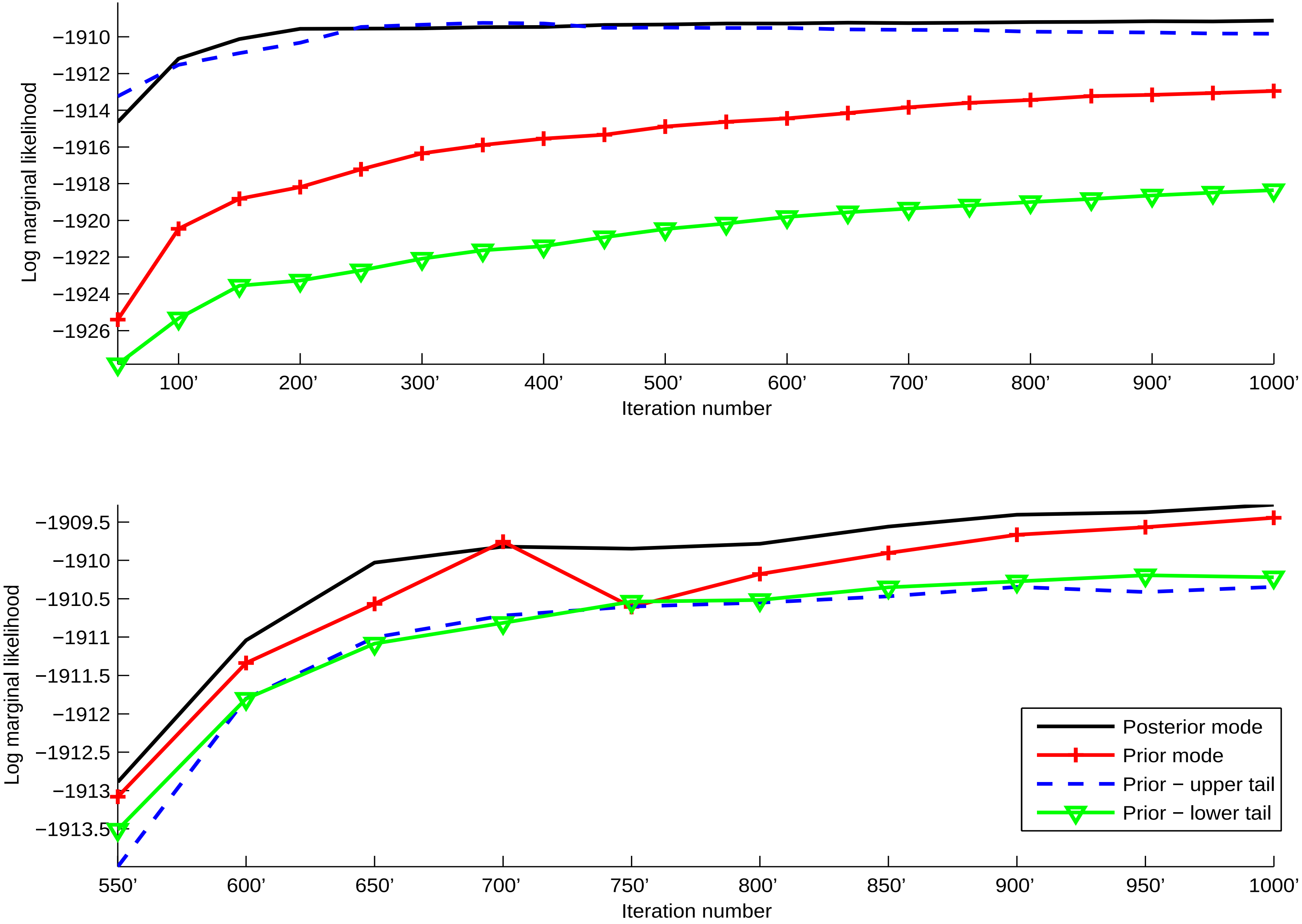


Figure A7: Multivariate ANOVA