Asset pricing implications of two financial accelerator models*

Karl Walentin†
New York University

April 2005

Abstract

This paper compares two financial accelerator models by analyzing their respective cyclical characteristics of the external finance premium and its implications for the equity premium. We answer the question posed by Gomes, Yaron and Zhang (2003) - Can the desirable business cycle implications of financial accelerator models be reconciled with the empirically observed cyclical characteristics of the finance premium and an "amplified" equity premium? We show that the answer is yes. We thereby contradict the claim of Gomes *et al* and note that their result is not general.

We constrast the result of Gomes *et al* with a different financial accelerator model by Bernanke, Gertler and Gilchrist (2000). In that model two key assumptions are different and the asset pricing results are in line with the facts. The two key assumptions are (i) firms have self-financing (leverage) ratios that are sensitive to changes in capital prices and (ii) a broad group of firms are potentially credit constrained.

*Thanks to Mark Gertler, Sydney Ludvigson, Guido Lorenzoni, Amir Yaron, Pierpaolo Benigno, John Leahy and Jinyong Kim for advice and helpful comments.

†Contact: New York University, Department of Economics, 269 Mercer Street 7th floor, New York, NY 10003. Phone: 212-998 3746. E-mail karl.walentin@nyu.edu.

1 Introduction

This paper was inspired by "Asset Prices and Business Cycles with Costly External Finance" by Gomes, Yaron and Zhang (2003) (hereafter GYZ) and the issues they investigate. Firstly, are models of the financial accelerator consistent with the external finance premium that we observe empirically? Secondly, are the implications for the equity premium reasonable? Their answer is no on both these questions, and they thereby cast some doubts on the existing financial accelerator literature.

The main claims of their paper are the following: In financial accelerator models, the external finance premium shoots off and peaks at the impact of a positive technology shock, driven by the increase in marginal product of capital. By using this procyclical characteristic of the finance premium their model can explain a small part (less than one hundreth (1/100)) of the equity premium puzzle. But, GYZ then note that the empirically observed finance premium is countercyclical (or at best acyclical), so the class of models that includes a financial accelerator are incorrect in that they predict a counterfactual (procyclical) finance premium.

In the present paper we show that the GYZ result, that financial accelerator models generate a procyclical finance premium, is not robust. Nor is the link they claim between a procyclical finance premium and an "amplified" equity premium robust. GYZ's results depends crucially on strong assumptions made only in models in the tradition of Carlstrom and Fuerst (1997) (hereafter CF).[1]

In fact, there is a large strand of financial accelerator models that are consistent with the empirics in the two dimensions that GYZ explore. Any model where (i) firms have self-financing (leverage) ratios that are sensitive to changes in capital prices and (ii) a broad group of firms are potentially credit constrained, will generate a countercyclical finance premium that in turn amplifies the equity premium. This strand of financial accelerator literature builds on Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

It should be noted that the present paper is very much in the spirit of GYZ: We use two asset pricing dimensions (finance premium cyclicality and the implied equity premium) to evaluate the empirical success of two business cycle models with financing frictions.

The present paper is also related to the more general literature on equity premium in production economies. Two papers in this genre are Jermann (1998) and Boldrin, Christiano and Fisher (2001). The aim of both of these papers is to explain the equity premium puzzle by introducing habit formation in the utility of consumption. We note an important insight from Jermann (1998): An equity premium in the neighborhood of the empirically observed

[1] GYZ used the CF model without any changes, so we consider these two papers to be the same model, denoted the CF/GYZ model.

value (roughly 6 percent), can not be generated in a general equilibrium production economy unless we construct the model such that consumers have a strong preference for smoothing consumption *and* this smoothing is prevented by some friction. In the present paper we use standard time-separable preferences and low values of risk aversion, so we know from the start that the first of these conditions is not satisfied. Accordingly, we are not able to quantitatively match the observed equity premium. Our quantitative goal is therefore more modest - simply to get a equity premium generated in part by a countercyclical finance premium.

We contrast the CF/GYZ model with the model by Bernanke, Gertler and Gilchrist (2000) (BGG). The aspect of the two models that we treat first is the cyclical characteristics of the finance premium. We do this in section 2. Secondly, we study the implications of these cyclical characteristics for generating an "amplified" equity premium in section 3. Section 4 presents a simulation of the BGG model where we quantify the results for the equity premium. Finally, section 5 concludes.

2 A tale of two models - the cyclicality of the finance premium

In this section, we contrast the BGG and CF/GYZ models regarding their implications for the cyclicality of the finance premium. The basic fact is presented in Figure 1: The BGG model implies a countercyclical finance premium, while the CF/GYZ model implies a procyclical premium.[2] In the figure, we also plot the empirically observed finance premium. This figure is equivalent to figure 5 in GYZ, but with the BGG simulation result added. GYZ presented "figure 5" as their key evidence against models of costly external finance. This seems to be an overstatement - the figure only shows major discrepancy between the specific model they studied (CF/GYZ) and the data. The BGG model fit the data reasonably well.[3]

[2] That the BGG model generates a countercyclical finance premium is not a new observation, with some work it can be seen from the original article.

[3] This result is robust to alternative measurements of the finance premium.

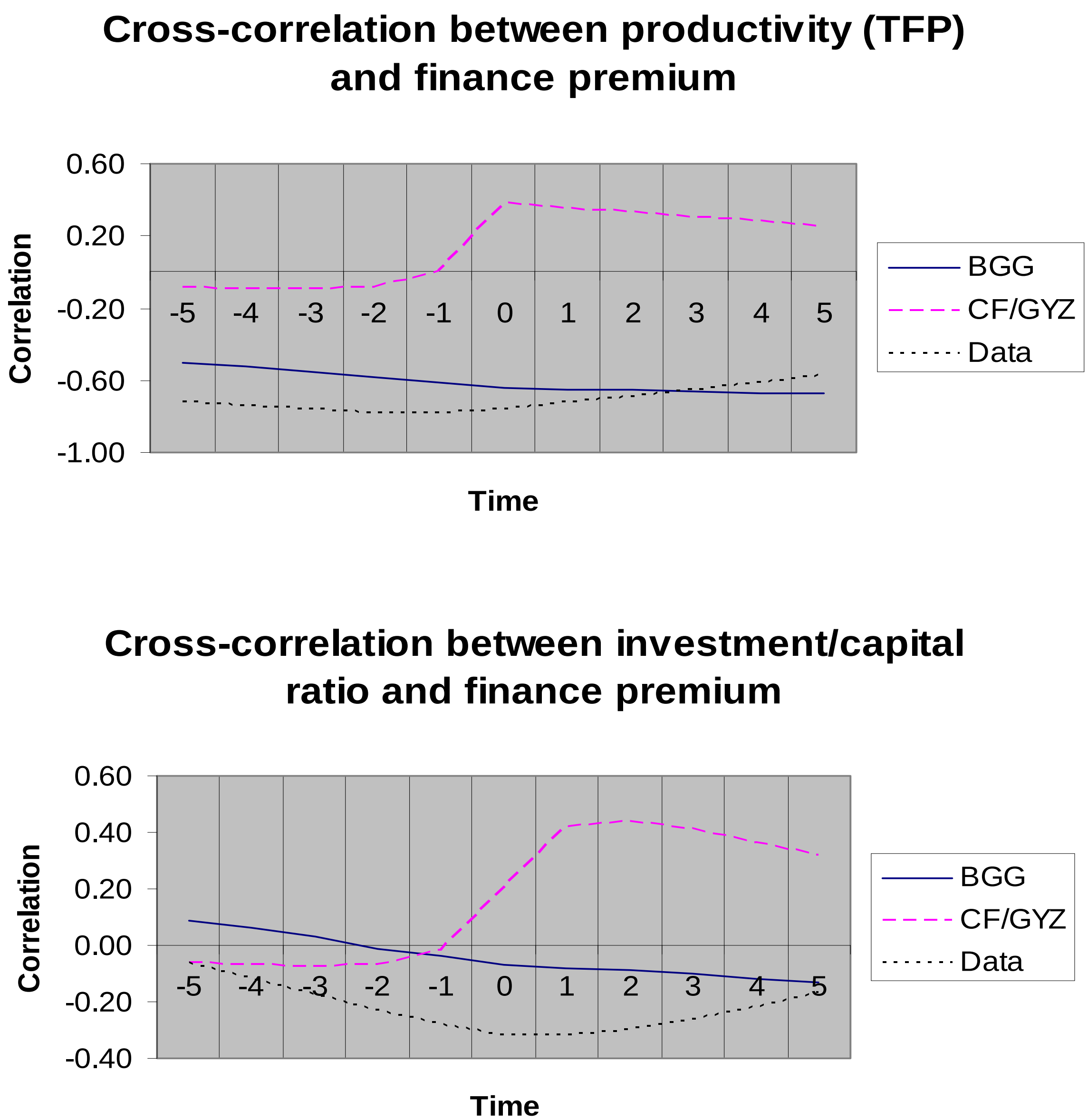

Figure 1. Source: For BGG, author's simulation. See section 4 and the Appendix for details. For CF/GYZ and data, GYZ. Technology shocks where used in both models. The empirical finance premium (labeled "Data") is measured as the spread between Aaa and Baa corporate bonds.

The remaining part of this section explains why the two models have opposite results in this respect. Informally, the main point is that in the CF/GYZ model, the self-financing ratio of the firm is unaffected by capital price changes, so in an economic upturn increased investment must correspond to increased borrowing and thereby increased risk of default, which leads to an increased finance premium. In BGG, this link between investment and the finance premium is modified by the effect that cyclical variation in the price of capital has on the self-financing ratio (and thereby default risk) of firms. Secondly, as we will see, it is pivotal which firms are credit constrained.

The notation is as follows. Capital letters refer to BGG, and lower case letters to CF/GYZ.[4] Equations will be denoted by the letters a (BGG) and b (CF/GYZ) respectively. All prices are in units of the final good.

2.1 The Bernanke, Gertler and Gilchrist model

We start by analyzing the BGG model. The model has three types of agents: households, intermediate goods producers and retailers. Retailers are merely a modeling device to allow price rigidities. They bundle together intermediate goods and transform them into final goods. The model revolves around the intermediate goods producers who are considered to be the potentially credit constrained entrepreneurs (throughout the paper we will call the credit constrained firms' owners "entrepreneurs"). The credit constraints and associated agency problems accordingly have a very broad reach in BGG.

Final goods can be used either for consumption or as capital in production, although there are adjustment costs related to installing capital. The household's problem is very standard and we will therefore skip the derivation of the standard Euler equation.

The remaining part of this section will focus on the intermediate goods producing entrepreneurs. Entrepreneurs are risk-neutral and maximize:

$$E_o\left[\sum_{t=0}^{\infty}(\beta\gamma)^t C_t^e\right] \tag{1a}$$

where β is the normal discount factor and γ the extra discounting made by entrepreneurs due to a constant risk of dying. C_t^e denotes entrepreneurial consumption.

The entrepreneurs produce a homogenous wholesale good in a competitive market using the production function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{2a}$$

where Y_t, A_t, K_t and L_t denote output, technology level, capital and labor input respectively.

The law of motion for capital includes standard adjustment costs that depends on the investment/capital $\left(\frac{I}{K}\right)$ ratio:

$$K_{t+1} = \phi\left(\frac{I_t}{K_t}\right)K_t + (1-\delta)K_t \tag{3a}$$

[4] With the exception of the finance premium (gross risky within-period interest rate), R^d, in CF/GYZ.

where δ is the depreciation rate of capital.

The price of capital, Q, is pinned down by the replacement cost:

$$Q_t = \left[\phi'\left(\frac{I_t}{K_t}\right)\right]^{-1} \tag{4a}$$

The marginal product of capital, MPK, is derived in the standard way from the production function, equation (2a). The return to capital, R^k, consists of the MPK and the capital gain:

$$E_t\left\{R^k_{t+1}\right\} = E_t\left\{\frac{MPK_{t+1} + Q_{t+1}(1-\delta)}{Q_t}\right\} \tag{5a}$$

We can get the full expression for the demand for capital by substituting in for Q and MPK. Note that because of constant returns to scale (CRS) we can easily aggregate all entrepreneurs and treat them as one unit (e.g. regarding the demand for capital). Individual entrepreneurial return is also affected by idiosyncratic risk, so that ex post return is $\omega^j R^k_{t+1}$.

To analyze supply of investment capital consider firm j with capital stock K^j_{t+1} and net worth N^j_{t+1}. The supply of capital can be described by the required return when lending to firm j, which is $s\left(\frac{N^j_{t+1}}{Q_t K^j_{t+1}}\right) R^f_{t+1}$. s is a decreasing function that maps the self-financing ratio, $\frac{N^j_{t+1}}{Q_t K^j_{t+1}}$, into the external finance premium. The intuition for the finance premium, s, is that the higher the self-financing ratio is, the lower is the probability of default and therefore the lower the finance premium (more on this below). R^f_{t+1} is the risk-free interest rate between period t and $t+1$ and is determined by the household's problem.

Combining demand and supply of capital we get the equation for equilibrium in the capital market:

$$E_t\left\{R^{k,j}_{t+1}\right\} = s\left(\frac{N^j_{t+1}}{Q_t K^j_{t+1}}\right) R^f_{t+1} \tag{6a}$$

where $R^{k,j}_{t+1}$ is the return to capital of firm j, and the firm's net worth is N^j_{t+1}.[5]

Net worth is equal to the gross return to capital times the value of the capital used, minus the gross interest rate times the amount borrowed[6]:

$$N_{t+1} = R^k_t Q_{t-1} K_t - \left(R^f_t + \frac{\mu \int_0^{\bar{\omega}} \omega dF(\omega) R^k_t Q_{t-1} K_t}{Q_{t-1} K_t - N_t}\right)(Q_{t-1} K_t - N_t) \tag{7a}$$

[5] The production technology is CRS, so it is actually the increasing finance premium (note that $\frac{\partial s}{\partial K_{t+1}} > 0$) that makes equation (6a) yield a unique K^j_{t+1} for firm j, given the firm's net worth N^j_{t+1}.

[6] Here we abstract from the exogenous entreprenuerial death that is part of the BGG model.

where μ is the fraction of the capital (gross value) of the firm that is lost in case of bankruptcy and the integral over ω, the idiosyncratic productivity, represents the probability of default.[7] $\bar{\omega}$ is the default threshold (the lower bound for solvency) for the firm. The positive probability of costly bankruptcy is the ultimate cause of the finance premium. The fraction $\frac{\mu \int_0^{\bar{\omega}} \omega dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1}K_t - N_{t-1}}$ is the expected default costs divided by the amount borrowed, i.e. the mechanism that drives s.[8] We note that bankruptcy costs are proportional, so it is straight forward to aggregate also the supply of capital.

We will not derive the optimal contract that gives rise to the finance premium, but simply note than in an environment with costly monitoring the optimal type of contract is standard risky debt for which monitoring only takes place in case of default.[9] We note that the BGG and CF/GYZ models are very similar in this respect - the finance premium in both models are caused by the fact that a fraction of a firm's value is lost in monitoring costs in case of default. The mapping from net worth to finance premium is therefore basically the same in both models.

Let us now study the effects of a technology shock on the finance premium in the BGG model using equilibrium condition (6a) and definitions (5a) and (7a). Equation (6a) holds at the end of period t when investment, I_t, (or, equivalently K_{t+1}) is chosen at the capital price Q_t, net worth is N_{t+1} and the values for MPK_{t+1} and Q_{t+1} has not yet been realized (see the Appendix for an illustration of the timing protocol).

Proposition 1. *Technology shocks imply a countercyclical external finance premium in the BGG model.*

Intuition: A positive technology shock at the beginning of period t makes MPK_t increase and thereby increases Q_t and, by a lagged version of (5a), R_t^k. The increase of R_t^k results in an increase of N_{t+1}, by (7a). The impact on investment occurs at the end of period t when equation (6a) is applicable: There have been changes on the LHS (increase in MPK_{t+1}), but the dominating force is the increase in the self-financing ratio $\frac{N_{t+1}}{Q_t K_{t+1}}$ due to leverage, i.e. the value of the loans does not change with the price of capital, so net worth increases more than proportionally with Q. We call this the *leverage effect.* The increase in the self-financing ratio $\frac{N_{t+1}}{Q_t K_{t+1}}$ results in a decrease of $s\,(-)$, i.e. a countercyclical finance premium. Because of the decreased cost of borrowing, investment I_t (or equivalently K_{t+1}) increases to make (6a)

[7] Clearly $\bar{\omega}$ is dependent on the firm's self-financing (leverage) ratio.

[8] To be explicit: $R_t^f + \frac{\mu \int_0^{\bar{\omega}} \omega dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1}K_t - N_{t-1}} = sR_t^f$ or equivalently,
$s = 1 + \frac{\frac{\mu \int_0^{\bar{\omega}} \omega dF(\omega) R_t^k Q_{t-1} K_t}{Q_{t-1}K_t - N_{t-1}}}{R_t^f}$

[9] See Gale and Hellwig (1985) and Townsend (1979) for the general case and BGG, GYZ or CF for specific details.

hold. Adjustment costs will then push up Q_t (and down R^k_{t+1}) and the equality is restored.[10]

The above result, a countercyclical finance premium, always holds in the BGG model, regardless of the type of shock that drives the economy. This property is illustrated in the following table of contemporaneous correlations between the finance premium and GDP.

Shock	Correlation
Technology	-0.79
Monetary Policy	-0.81
Government Expenditure	-0.95

Table 1. Correlation between the finance premium and GDP.

Source: Author's simulation. See section 4 and the Appendix for details.

2.2 The Carlstrom and Fuerst / Gomes, Yaron and Zhang model

In the CF/GYZ model there are also three type of agents: households, final goods producers and capital goods producers. In this model, it is the capital goods producers that are credit constrained and thereby play the key role.[11] We note that here the credit constraints apply only to this small and rather special group. The problems of the households and final goods producers are standard and will not be described here.

Instead, this section describes the problem of the capital goods producers (we call them entrepreneurs). Each such producer maximizes:

$$E_o\left[\sum_{t=0}^{\infty}(\beta\gamma)^t c^e_t\right]$$

where the notation is identical to BGG described above.

Net wealth of entrepreneurs evolves in two steps (subperiods), see the Appendix for an illustration of the timing protocol. In subperiod 1 the aggregate productivity shock is realized, capital prices determined and final goods productions takes place using capital and labor as inputs. For the entrepreneurs this subperiod merely consists of collecting factor incomes from 1 unit of labor and a^e_t units of capital:

$$n_t = w^e_t + r_t a^e_t + q_t(1-\delta)a^e_t \tag{2b}$$

[10]Note that neither Proposition 1 or 2 (which is stated below) depend on the assumption of constant returns to scale (CRS) technology - both results remain qualitatively unchanged with decreasing returns. But CRS is required for simple aggregation.

[11]To compare the two models, note that GYZ's capital producers are a strict subset of BGG's intermediate goods producers (if we classify firms by type of output) as they do not produce consumption goods.

i.e. net wealth n_t is equal to labor income w_t^e plus net capital income $r_t a_t^e$ and the value of undepreciated capital $q_t(1-\delta)a_t^e$.

In subperiod 2 all of n_t is invested in the entrepreneur's own firm. The entrepreneur then supplements his net worth with external debt, $i_t - n_t$, and make a total investment of i_t (corresponding to $Q_t K_{t+1}^j$ in BGG). The role of the capital goods producers is to "convert" final goods to capital goods. The production technology yields $\omega_t i_t$ units of capital goods output with i_t units of final goods as input. ω_t is an idiosyncratic productivity shock with mean 1. The within-period external gross risky interest rate is $R_t^d \equiv q_t\left(1+r_t^l\right)$, where r_t^l (and thereby R_t^d) is endogenously determined by the contracting problem (see below). In equilibrium, R_t^d will also be the internal return to entrepreneurial within-period investment.[12] The resulting budget constraint for subperiod 2 is:

$$q_t a_{t+1}^e + c_t^e = q_t\left[\omega_t i_t - \left(1+r_t^l\right)(i_t - n_t)\right] \tag{3b}$$

where the LHS represents expenditures and the RHS is the net income. CF assume that capital is not an input in the entrepreneurial production process and that investment and loans are within-period (so there is no need for any capital stock). This results in q being constant ($=q_t$) during the time period that entrepreneurs are exposed to leveraged swings in the price of capital.[13] The result of this characteristic is that capital gains are separated from entrepreneurial activity and, as opposed to in BGG, there is no (positive) effect on the self-financing ratio, $\frac{n}{i}$, from capital gains as q increases (because the financing capacity of the entrepreneur is unchanged). In other words, there is no *leverage effect*.

We are now ready to derive the capital market equilibrium equation in CF/GYZ. By substituting the above two budget constraints, (2b) and (3b), into the maximization problem (1b) we arrive at the capital market equation, which is the Euler equation for the entrepreneurs:

$$1 = \beta\gamma E_t\left\{\frac{r_{t+1} + q_{t+1}(1-\delta)}{q_t} R_{t+1}^d\right\} \tag{4b}$$

where the within-period gross risky interest rate is R_{t+1}^d and the fraction inside the parenthesis is the gross between-period return to capital. r is the net return to capital (i.e. MPK) and δ is the depreciation rate. q is pinned down by the replacement cost, just as in BGG.[14] To intuitively understand (4b) we note that entrepreneurs receive two returns from postponing

[12] This is equivalent to BGG where the 1-period return, $E_t\left\{R_{t+1}^{k,j}\right\}$, is determined (in equilibrium) by the finance premium $s()$ in equation (6a).

[13] Thanks to Amir Yaron for pointing out this timing difference between the two models.

[14] Although the replacement cost of capital here, but not in BGG, is a function of financial variables because the capital producers are financially constrained entrepreneurs.

consumption one period: first the between-period return to capital and then, in period $t+1$, the within-period internal return to entrepreneurial investment which is equal to R^d_{t+1}.[15]

An expression for the endogenous finance premium R^d_{t+1} can be derived yielding $R^d_{t+1} = \frac{\bar{\omega}}{g(\bar{\omega})}$, where $\bar{\omega}$ is the default threshold and $g(\bar{\omega})$ is the fraction of net output that goes to the lender. Note that the ratio $\frac{\bar{\omega}}{g(\bar{\omega})}$ must be increasing in $\bar{\omega}$, as $g(\bar{\omega})$ increases less than proportionally with $\bar{\omega}$ because of increasing monitoring costs. So, the lower the self-financing ratio is, and thereby the higher the default threshold $\bar{\omega}$, the higher is the default premium.

We are now ready to draw the conclusion from the above setup. The proposition below in itself is not new. The important contribution of this section is the analysis of which assumptions, or characteristics, that are needed to get the procyclical finance premium of the CF/GYZ model.

Proposition 2. *Technology shocks imply a procyclical external finance premium in the CF/GYZ model.*

Intuition: Following a positive productivity shock, investment demand increases and drives up q_t. As we noted above in the discussion of (3b), because of the absence of entrepreneurial capital stock, the increase in q_t has no direct impact on the self-financing ratio $\frac{n}{i}$ (no *leverage effect*). Secondly, the increased price of capital, which results in increased within-period internal return to investment for entrepreneurs leads to increased entrepreneurial investment, i_t.

The above two factors: (i) the absence of any *leverage effect* and (ii) the increase in i_t due to increased within-period internal return, jointly cause a decrease in the self-financing ratio $\frac{n}{i}$ which implies a higher probability of default and accordingly a higher external finance premium. In conclusion, the finance premium increases following a positive productivity shock in CF/GYZ.

Over time, the higher internal return yields increased n so that the self-financing ratio, $\frac{n}{i}$, increases and the agency costs decreases.[16] This will result in the finance premium R^d falling back to its original level.

2.3 Comparison of the two models and the cyclicality of their finance premia

In this section we have seen that the two models yield opposite results regarding the cyclicality of the finance premium: In BGG the premium is countercyclical and in CF/GYZ the

[15] We can see the rough equivalency between the two models' capital market equilibrium equations (6a) and (4b). This requires some work because (4b) is from the perspective of the entrepreneurs, while (6a) is from the "outside" investor perspective.

[16] A decrease of entrepreneurial consumption due to the increased return to (entrepreneurial) investment also contributes to the increase in n.

premium is procyclical. Intuition for both these results have been given and the BGG result has been confirmed in simulation as reported in Figure 1a, Figure 1b and Table 1.

The result of the CF/GYZ model hinges on two characteristics: i) entrepreneurs' self-financing ratios are not directly affected by shocks to the price of capital, because they have no capital stock, and ii) procyclical investment induces countercyclical self-financing ratios and thereby a procyclical finance premium.

BGG instead assumes that entrepreneurs own capital and issue debt, and therefore their self-financing ratios are affected by the price of capital (the *leverage effect*) and this effect will, for any reasonable parameterization, dominate and create procyclical self-financing ratios. For illustration purposes we can use (5a) from the BGG setup (substituting in for R^k_{t+1} using (6a)) to compare the two models following a shock.

$$E_t \left\{ \frac{MPK_{t+1} + Q_{t+1}(1-\delta)}{Q_t} \right\} = s \left(\frac{N^j_{t+1}}{Q_t K^j_{t+1}} \right) R^f_{t+1}$$

In the context of this equation, in BGG the dominant mechanism is the change in net worth N^j_{t+1} and thereby the self-financing ratio $\frac{N^j_{t+1}}{Q_t K^j_{t+1}}$ on the RHS. In CF/GYZ the driving force is instead the change in MPK_{t+1} that will induce the finance premium s to change in the same direction.

The CF/GYZ prediction of a procyclical finance premium is counterfactual. That the finance premium is countercyclical, or at best acyclical, has been widely documented by, among others, GYZ and House (2002). Given this problem with the CF/GYZ model, we must ask ourselves if there is any other advantage with this model compared to BGG. Maybe the underlying assumptions are more reasonable in CF/GYZ? Are there strong reasons to believe that entrepreneurial production does not require any capital stock and instead yield instantaneous output from investment?

3 Implications for the equity premium

In this section we establish that both the CF/GYZ and the BGG models yield larger equity premia than the corresponding models without financial frictions, in spite of having opposite results regarding the cyclicality of the finance premium. We start with the definition of the equity premium. Then we move on to analyze the implications of the cyclical finance premium for the equity premium in the two models, taking Propositions 1 and 2 as starting points.

The *equity premium* is the extra return, above the risk-free interest rate, that households require to hold equity. It is generated because risk-averse households need to be compensated for the covariance between equity returns and the stochastic discount factor. We use the covariances from the CF/GYZ and BGG models, respectively, to determine the *ex ante* equity premium, in each model. In this respect we broadly follow GYZ.

Households, which are not subject to any credit constraints, will price equity returns using the standard consumption-based stochastic discount factor,

$$M_{t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)} \tag{8}$$

In both the CF/GYZ model and the BGG model, households (as opposed to entrepreneurs) are risk-averse, i.e. they have decreasing marginal utility of consumption. So M_{t+1} is decreasing in C_{t+1}. Therefore, using the definition of M_{t+1}, a positive equity premium will be generated if the covariance between equity returns and consumption is positive.

It is straightforward to derive the unconditional equity premium and then use the unconditional moments to quantify it. Let R^e_{t+1} denote the return to "outside" equity, i.e. equity held by households (as opposed to by the entrepreneur running the firm). The arbitrage condition for the households is:

$$E_t \left\{ M_{t+1} R^e_{t+1} \right\} = E_t \left\{ M_{t+1} R^f_{t+1} \right\} \tag{9}$$

i.e. the discounted return must be the same for equity and risk-free bonds.[17] From this arbitrage condition we can derive the equity premium in the standard way. In our model consumption is stationary, so we have $E\left\{M_{t+1}\right\} = \beta$. Define $R^f_{ss} = \frac{1}{E\{M_{t+1}\}}$ as the steady state value of the risk-free interest rate. The resulting expression for the equity premium is:

$$\text{Equity premium} \equiv E\left\{R^e_{t+1}\right\} - R^f_{ss} = -\frac{Cov(M_{t+1}, R^e_{t+1})}{\beta} \tag{10}$$

At this point it should be noted that R^e_{t+1} and R^k_{t+1} are perfectly correlated in both models that we study (details below). We will therefore use the cyclicality (and covariance) of R^k_{t+1} to determine the equity premium.

To get an amplified equity premium we need the finance premium to generate extra covariance between the discount factor and the returns to equity, i.e. increase the (negative) magnitude of $Cov(M_{t+1}, R^k_{t+1})$,[18] which is roughly speaking equivalent to requiring R^k_{t+1} to be procyclical, given that M_{t+1} is countercyclical. As we will see, the finance premium will increase the volatility of both the stochastic discount factor, M_{t+1}, and the equity return , R^e_{t+1}, and thus generate a higher equity premium.

[17] We implicitly assume that households hold a positive amount of the risk-free bond, or alternatively that households do not have to pay the finance premium $s()$ on a negative bond holding.

[18] or equivalently $Cov(C_{t+1}, R^k_{t+1}) > 0$.

3.1 Equity premium in BGG

Let us now look at the equity premium in the BGG framework. Note that BGG did not consider traded equity in their model - this is an extension made in the present paper. In this framework, the model equivalent to equity is the capital of the intermediate goods producers. Recall that these firms are the potentially credit constrained ones. This makes introduction of "outside" equity held by households somewhat particular. There are two quantities we need to determine to price equity: the equity returns and the appropriate discount factor. Regarding the returns, we assume that, because of agency problems, households holding equity receive a fixed fraction (less than one) of the entrepreneurial return to capital, R_{t+1}^k. So the return of insiders (entrepreneurs) and outsiders (households) are perfectly correlated, but at different levels. The fraction will be determined by the households' no-arbitrage condition, equation (9). We make the assumption that only households (and not entrepreneurs) are allowed to trade outside equity.[19] Accordingly we use the stochastic discount factor of the households to price equity.[20]

We will use the market for capital to analyze the effects from the finance premium on the equity premium. Our aim is to determine the sign of the *change* generated by the cyclicality of the equity premium on the covariance between the stochastic discount factor, M_{t+1}, and equity returns, R_{t+1}^k, $\Delta Cov(M_{t+1,}R_{t+1}^k)$.[21] In the capital market diagram presented in Figure 2 the initial equilibrium is at point E_1. With adjustment costs, but without financial frictions, the new equilibrium right after a positive technology shock would be E_2.

From Proposition 1 and Table 1 we know that in BGG the finance premium will fall when GDP (denoted Y) increases. The decrease in the finance premium affects the intermediate goods producers positively and yields an extra rightward *demand* shift in the market for capital, and Q accordingly increases (see Figure 2, equilibrium point $\mathrm{E}_{\mathrm{BGG}}$).

We have now established that $\Delta Cov\left(Y_t, Q_t\right) > 0$. The return to capital holdings, R_t^k, is mainly driven by capital gains,[22] so it follows that $\Delta Cov\left(Y_t, R_t^k\right) > 0$. Intuitively C_t covaries positively with Y_t. Technically the requirements for this to happen is that either the adjustment costs of investment are high enough or the utility function is curved enough. So we can plausibly establish that $\Delta Cov\left(Y_t, R_t^k\right) > 0$ implies $\Delta Cov\left(C_t, R_t^k\right) > 0$ and then, because of decreasing marginal utility of consumption, $\Delta Cov(M_{t,}R_t^k) < 0$ and an amplified equity premium follows. In section 4 we will confirm the above intuition numerically and quantify the equity premium numerically.

[19] GYZ make the same assumption.

[20] In settings with limited participation the stochastic discount factor of any agent trading the asset can be used, see e.g. Brav, Constantinides and Geczy (2002).

[21] Note that it is not trivial that $\Delta Cov(M_{t+1,}R_{t+1}^k) < 0$ in both BGG and CF/GYZ as is required for a positive equity premium. The reason for the non-triviality is that the two models have opposite results for the cyclicality of the finance premium.

[22] You can see the relationship in equation (5a), but we had to confirm the quantitative result by simulation.

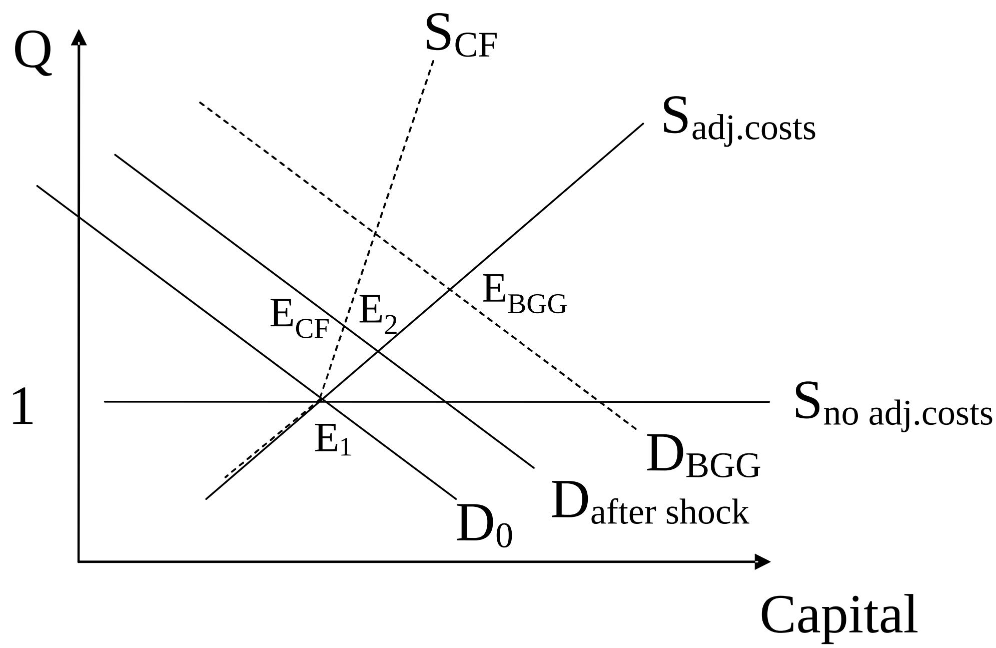

Figure 2. Capital market equilibrium following a positive technology shock.

3.2 Equity premium in CF/GYZ

In the CF/GYZ model, by Proposition 2, we get a procyclical finance premium if we follow GYZ and assume that technology shocks is the only type of shocks. Let us use this procyclical finance premium to analyze the impact of a technology shock that affects the demand for capital on the price of capital, Q and the equity premium.

GYZ introduce equity in a simple and elegant way. In the CF/GYZ model they note the equivalence between the original setup, where final goods producing firms rent capital from households period by period, and the "equity setup" where these firms own the capital they use and households own equity in the firms.

As the credit constrained entrepreneurs are the producers of capital goods, an increase in the finance premium caused (by Proposition 2) by the increased investment that follows a positive technology shock results in a steeper, kinked *supply* curve in the market for capital.[23] Therefore, following an increase in the demand for K, Q increases and the increase in investment is dampened (see Figure 2, equilibrium point E_{CF}) compared to a model

[23] The kink corresponds to the point where entrepreneurs have fully used their internal funds and beyond which they must start using more expensive external funds.

without financial frictions (point E_2). In short, driven by a positive technology shock, an increase in Y will coincide with an extra increase in Q.

Our aim is now to determine the sign of the covariance between equity returns, R_t^k, and consumption, C_t, and in that way see if the finance premium generates an amplified equity premium in CF/GYZ. Note that the model equivalent to equity is capital holdings in the *final goods producers* (not in the credit constrained capital goods producers).[24] Therefore equity returns are given by the standard expression in equation (5a). Above we established that $\Delta Cov(Y_t, Q_t) > 0$, so we have the same result as in the BGG model, but for the "opposite" reason: a procyclical finance premium instead of a countercyclical finance premium. All remaining steps in deriving $\Delta Cov\left(M_t, R_t^k\right) < 0$, and thereby an amplified equity premium, are identical to the reasoning above for the BGG model. GYZ confirmed this intuition numerically.

4 Simulation

In this section we will quantify the equity premium generated by the BGG model. The focus of the simulation will be on the asset returns. For a more complete presentation of the business cycle properties, see BGG.

The benchmark to measure our results against is GYZ with an equity premium (EP) of 0.022 percent, i.e. 2.2 basis points. They use a standard RBC model with only technology shocks, but augmented by adjustment costs and the finance premium. GYZ's value of the EP is surprisingly low and almost identical to what Jermann (1998) gets for a standard frictionless RBC model. At the other end are the empirical estimates of the equity premium of roughly 6 percent.[25]

The log-linear system of equations that we use to run the simulation is straight from BGG (with some typos fixed), and is listed in the Appendix.

4.1 Parametrization

All our parameter values are standard and follow BGG closely, so we will only mention some key parameters in this section. We will work with two values of the coefficient of relative risk aversion / the (inverse of) intertemporal elasticity of substitution, σ; $\sigma = 1$ and $\sigma = 5$. The former value of σ is normally used for business cycle modelling, and the latter value is closer to micro evidence on risk aversion. The subjective discount factor is $\beta = 0.99$.

The steady state finance premium is set to 2 percent annually. Monitoring cost, μ, is set to 0.12, implying that 12 percent of a firm's value is destroyed at bankruptcy. In

[24] This is not a crucial assumption - it is mainly made for simplicity. See GYZ for details.

[25] See Jermann (1998) and Boldrin, Christiano and Fisher (2001)

combination with a lognormal productivity distribution with variance 0.28 this value of μ yields an elasticity of the finance premium with respect to the self-financing ratio, v, of 0.05. Both government and technology shocks follow AR(1) processes with $\rho_a = \rho_g = 0.95$. These shocks and monetary policy shocks all have a standard deviation of 0.01. Monetary policy is governed by a Taylor rule with $\gamma_\pi = 1.5, \gamma_y = 0$ and a interest rate smoothing parameter, ρ, of 0.9.

4.2 Results

All simulation results are summarized in Table 2 below. Impulse response functions (IRFs) are plotted in the Appendix. The returns reported have been annualized from the quarterly output of our model. With only technology shocks we get a very low equity premium, $EP = 0.0093$ percent, roughly one basis point. (With $\sigma = 5$ we get $EP = 0.14$ percent, i.e. 14 basis points). If we turn off the variation in the finance premium (with $\sigma = 1$) the generated equity premium falls to 0.0088, a small decrease. Note that the finance premium has an amplifying effect on the equity premium.

With monetary policy as the driving shock, the equity premium increases substantially, $EP = 0.084$ percent (8.4 basis points). We note that 1.4 basis points (or 1/6) of this equity premium is caused by the countercyclical movement in the finance premium. See Figure A2 for IRFs. The IRFs are another way to see that the finance premium is countercyclical.

If we instead use government expenditure shocks the result changes drastically, $EP = -0.00021$ percent. The reason for the negative equity premium in this case, which occurs independently of the finance premium, is that *private* consumption will move in opposite direction to equity returns following a government shock, see Figure A1 in the Appendix. In other words, equity has good hedging value in a world of only government expenditure shocks. If we disable the variation in the finance premium, the result is $EP = -0.00012$, a less negative equity premium than with the finance premium turned on.

Parameter / Type of Shock	Technology	Monetary Policy	Government
$\sigma = 1$	0.0093	0.084	-0.00021
$\sigma = 1$, no variation in fin. premium	0.0088	0.070	-0.00012
$\sigma = 5$	0.14	0.12	-0.0017

Table 2. Theoretical equity premium (in percent).

5 Summary

In this paper we have shown that the BGG financial accelerator model implies a countercyclical finance premium, which is what we observe in the data. This observation is not new, but is mainly emphasized because of the confusion that seem to exist in the literature.

We showed that the BGG result follows from one key assumption: that firms have self-financing (leverage) ratios that are sensitive to changes in capital prices. The CF/GYZ model generates the opposite result, i.e. a procyclical finance premium. The reason is that in that model no positive link between capital prices and the self-financing ratio of entrepreneurs exists. CF create this characteristic by letting entrepreneurial production be instantaneous and not requiring any capital stock. Given the absence of any direct affect on the self-financing ratio, in the CF/GYZ model procyclical investment induces a countercyclical self-financing ratio which implies a procyclical finance premium.

Secondly, we showed that, through a countercyclical finance premium, the BGG model generates a higher equity premium than the corresponding model without financial frictions. The necessary assumption for this result is that a broad[26] group of firms are potentially credit constrained. CF/GYZ instead assume that only capital goods producers are credit constrained. This puts the credit constrained entrepreneurs on the supply side in the market for capital, and makes it possible for a procyclical finance premium to generate an amplified equity premium.

We note that the critique of financial accelerator models articulated in GYZ only applies to models with these two particular assumptions. In other words, within the literature of costly external finance, only models in the Carlstrom and Fuerst (1997) tradition exhibit the counterfactual procyclical finance premium. Furthermore, only for this type of model is an amplified equity premium associated with a procyclical finance premium.

Finally we quantified the equity premium generated by the BGG model for different shocks and risk aversion values. The resulting equity premium varied from 1 basis point (half the size of the GYZ result) up to 14 basis points. Only a small part of the equity premium was driven by the extra volatility created by the countercyclical finance premium.

To sum up, our main conclusion is that financial accelerator models in the tradition of Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (2000) display reasonable asset pricing implications. The BGG model matches the cyclicality of the empirical finance premium and makes a small contribution to increasing the equity premium generated by business cycle models.

[26] I.e. not limited to capital producers, but instead primarily including producers who actually use capital as an input in their production.

6 References

Bernanke, B. and M. Gertler, "Agency Costs, Net Worth, and Business Fluctuations", American Economic Review, Vol 79, 1989.

Bernanke, B., M. Gertler and S. Gilchrist, "The Financial Accelerator in a Quantitative Business Cycle Framework", Handbook of Macroeconomics, North Holland, 2000.

Boldrin, M. L. Christiano and J. Fisher, "Habit Persistence, Asset Returns and the Business Cycle", American Economic Review, Vol 91, 2001.

Brav, A., G.M. Constantinides and C.C. Geczy, "Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence", Journal of Political Economy, 2002.

Carlstrom, C. and T. S. Fuerst, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis", American Economic Review, Vol 87, 1997.

Gale, D. and M. Hellwig, "Incentive-Compatible Debt Contracts: The One-Period Problem", Review of Economic Studies, LII, 1985.

Galí, J., M. Gertler and D. Lopez-Salido, "Markups, Gaps and the Welfare Costs of Business Fluctuations", NBER WP 8850, 2002

Gomes, J., A. Yaron and L. Zhang, "Asset Prices and Business Cycles with Costly External Finance", Review of Economic Dynamics, Vol 6, Issue 4, 2003.

Jermann, U., "Asset Pricing in Production Economies", Journal of Monetary Economics, April 1998.

Hall, R., "Macroeconomic Fluctuations and the Allocation of Time", Journal of Labor Economics, 1997.

House, Christopher, "Adverse Selection and the Accelerator", mimeo University of Michigan, 2002.

Kiyotaki, N. and J. Moore, "Credit Cycles", Journal of Political Economy, 1997.

Townsend, R. M., "Optimal Contracts and Competitive Markets with Costly State Verification", Journal of Economic Theory, 1979.

7 Appendix

7.1 The loglinearized equation system used in the simulations

Capital letters denote steady state values and lower case letters log deviations.[27]

Aggregate demand:

$$Y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t + \phi_t^y$$

(abstracting from entrepreneurial consumption [In the simulations monitoring costs, ϕ^y, are also ignored])

$$\sigma c_t = -r_{t+1} + \sigma E_t \left\{c_{t+1}\right\}$$

$$E_t \left\{r_{t+1}^k\right\} - r_{t+1} = -v \left[n_{t+1} - (q_t + k_{t+1})\right]$$

$$r_{t+1}^k = (1-\epsilon)\left(y_{t+1} - k_{t+1} - x_{t+1}\right) + \epsilon q_{t+1} - q_t$$

$$q_t = \varphi(i_t - k_t)$$

Aggregate Supply:

$$y_t = a_t + \alpha k_t + (1-\alpha)\, l_t$$

$$y_t = \gamma_l l_t + \gamma_c c_t - x_t$$

$$\pi_t = -\kappa x_t + \beta E_t \left\{\pi_{t+1}\right\}$$

State variables:

$$k_{t+1} = \delta i_t + (1-\delta) k_t$$

$$n_{t+1} = \frac{\gamma RK}{N}\left(r_t^k - r_t\right) + \gamma R(r_t + n_t) + \phi_t^n$$

Monetary Policy Rule and Shock Processes:

$$r_t^n = \rho r_{t-1}^n + (1-\rho)\left[\gamma_\pi \pi_{t-1} + \gamma_y y_{t-1}\right] + \varepsilon_t^{rn}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

$$g_t = \rho_a g_{t-1} + \varepsilon_t^g$$

$$\mu_{w,t} = \rho_w \mu_{w,t-1} + \varepsilon_t^w$$

where $\epsilon = \frac{(1-\delta)}{1-\delta+\alpha Y/(XK)}$, $\gamma_c = 1/\sigma, \gamma_l = 1 + 1/5$ (where 5 is the inverse wage elasticity), $v = \varphi(R^k/R)/\varphi'(R^k/R)$, $\kappa = \frac{1-\theta}{\theta}(1-\theta\beta)$,

$\phi_t^y = \frac{DK}{Y}\left[\log\left(\mu \int_0^{\bar{\omega}_t} \omega dF(\omega) R_t^k Q_{t-1} K_t/(DK)\right)\right]$ with $D \equiv \mu \int_0^{\bar{\omega}} \omega dF(\omega) R^k$

$\phi_t^n = \frac{(R^k/R-1)K}{N}\left(r_t^k + q_{t-1} + k_t\right) + \frac{(1-\alpha)(1-\Omega)Y}{N} y_t$

[27] Note that a couple of typos from BGG have been corrected. Thanks to Daria Finocchiario for pointing out one of the typos.

7.2 Impulse response functions

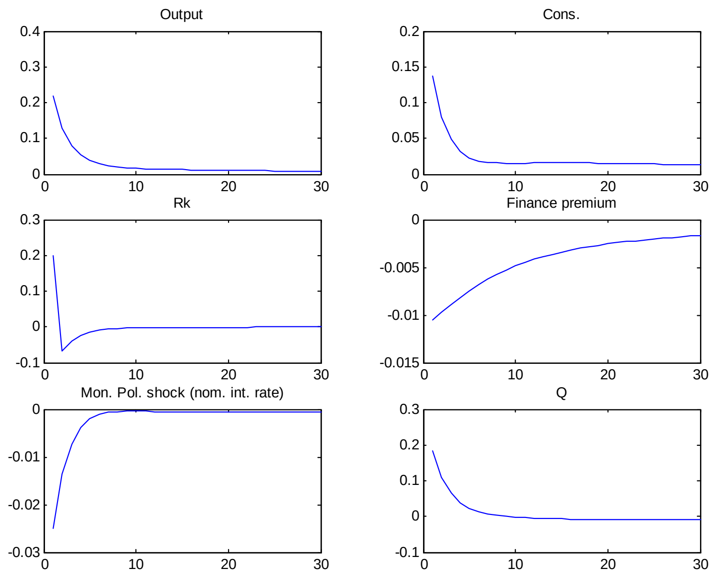

Figure A1. Monetary policy shock.

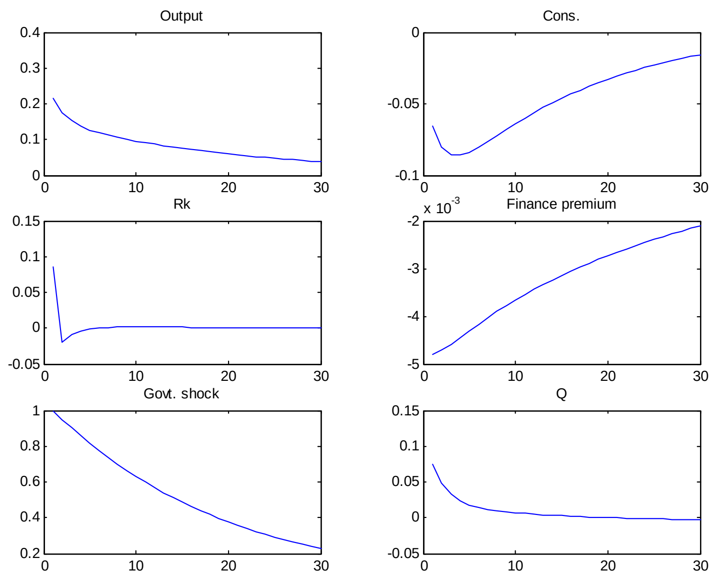

Figure A2. Government expenditure shocks.

7.3 Illustration of timing protocol

7.3.1 BGG timing

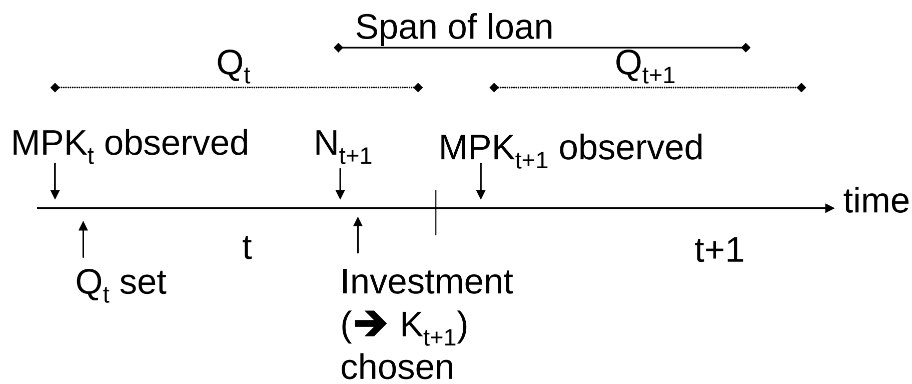

Figure A3. BGG timing.

7.3.2 CF timing

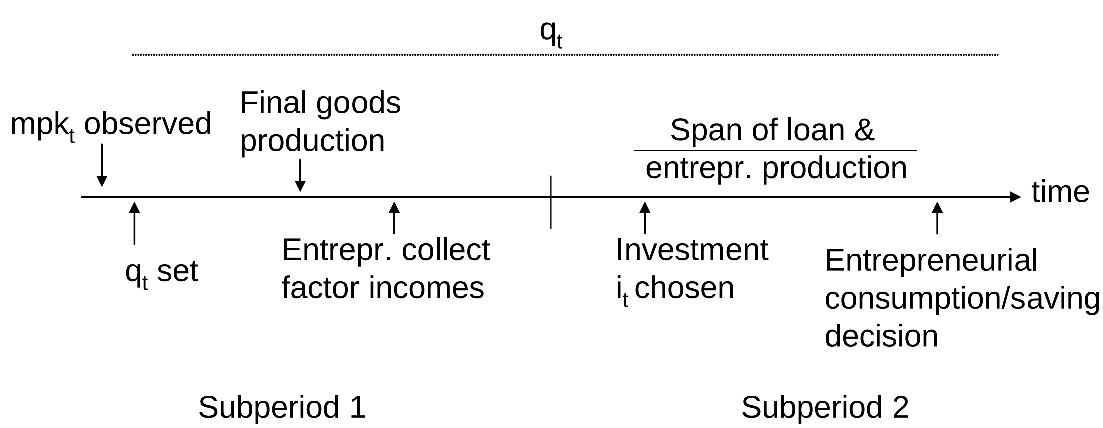

Figure A4. CF timing.