Appendix for ‘Introducing Financial Frictions and Unemployment into a Small Open Economy Model’

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B. Appendix

B.1. Scaling of Variables

We adopt the following scaling of variables. The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an investment-specific shock to technology and it is convenient to define the following combination of investment-specific and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{B.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+\Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m, respectively) are scaled by z_t^+. Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods and C_t are final consumption goods) are scaled by z_t^+. Government expenditure, the real wage and real foreign assets are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled z_t^+. Also, υ_t is the shadow value in utility terms to the household of domestic currency and $\upsilon_t P_t$ is the shadow value of one unit of the homogenous domestic good. The latter must be multiplied by z_t^+ to induce stationarity. $\tilde{P}_t$ is the within-sector relative price of a good. w_t denotes the ratio between the (Nash) wage paid to workers $\tilde{W}_t$ and the "shadow wage" W_t paid by intermediate goods producers to the employment agencies in the employment friction model. Thus,

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+\Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+\Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+\Psi_t},\ i_t^m = \frac{I_t^m}{z_t^+} \\ c_t^m &= \frac{C_t^m}{z_t^+}, c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ a_t \equiv \frac{S_t A_{t+1}^*}{P_t z_t^+}, \\ x_t^m &= \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+,\ (y_t =)\, \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t} \\ n_{t+1} &= \frac{\bar{N}_{t+1}}{P_t z_t^+},\ w^e = \frac{W_t^e}{P_t z_t^+}. \end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the domestic homogeneous good.

The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{S_t}{S_{t-1}}.$$

We define the following inflation rates:

$$\pi_t = \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*},$$
$$\pi_t^i = \frac{P_t^i}{P_{t-1}^i},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},$$

for $j = c, x, i$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the 'CPI'), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t. When the price is denominated in foreign currency units, we divide by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i. This grows at a rate slower than P_t, and we therefore scale it by P_t/Ψ_t. Thus,

$$p_t^{m,x} = \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t},$$
$$p_t^x = \frac{P_t^x}{P_t^*},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^i = \frac{\Psi_t P_t^i}{P_t}. \tag{B.2}$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$, and in the production of final investment goods in the case of $j = i$. When there is just a single superscript the underlying good is a final good, with $j = x, c, i$ corresponding to exports, consumption and investment, respectively.

B.2. Functional Forms

We adopt the following functional form for capital utilization a :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b (1 - \sigma_a) u + \sigma_b ((\sigma_a/2) - 1), \tag{B.3}$$

where σ_a and σ_b are the parameters of this function.

The functional form for investment adjustment costs, as well as its derivatives are:

$$\tilde{S}(x) = \frac{1}{2}\left\{\exp\left[\sqrt{\tilde{S}''}(x - \mu_{z^+}\mu_\Psi)\right] + \exp\left[-\sqrt{\tilde{S}''}(x - \mu_{z^+}\mu_\Psi)\right] - 2\right\} = 0,\ x = \mu_{z^+}\mu_\Psi. \tag{B.4}$$

$$\tilde{S}'(x) = \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}(x - \mu_{z^+}\mu_\Psi)\right] - \exp\left[-\sqrt{\tilde{S}''}(x - \mu_{z^+}\mu_\Psi)\right]\right\} = 0,\ x = \mu_{z^+}\mu_\Psi. \tag{B.5}$$

$$\begin{aligned}\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}\left(x-\mu_{z^+}\mu_\Psi\right)\right]+\exp\left[-\sqrt{\tilde{S}''}\left(x-\mu_{z^+}\mu_\Psi\right)\right]\right\}\\ &= \tilde{S}'',\ x=\mu_{z^+}\mu_\Psi.\end{aligned}$$

In the employment friction model we assume a log-normal distribution for idiosyncratic productivities of workers. This implies the following:

$$\mathcal{E}\left(\bar{a}_t^j;\sigma_{a,t}\right)=\int_{\bar{a}_t^j}^{\infty} a d\mathcal{F}\left(a;\sigma_{a,t}\right)=1-prob\left[v<\frac{\log\left(\bar{a}_t^j\right)+\frac{1}{2}\sigma_{a,t}^2}{\sigma_{a,t}}-\sigma_{a,t}\right], \tag{B.6}$$

where $prob$ refers to the standard normal distribution and eq. (B.6) simply is eq. (4.7) spelled out under this distributional assumption. We similarly spell out eq. (4.2):

$$\begin{aligned}\mathcal{F}\left(\bar{a}^j;\sigma_a\right) &= \int_0^{\bar{a}^j} d\mathcal{F}(a;\sigma_a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma}} \exp^{\frac{-v^2}{2}} dv \\ &= prob\left[v<\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma_a^2}{\sigma_a}\right].\end{aligned} \tag{B.7}$$

B.3. Baseline Model

B.3.1. First Order Conditions for Domestic Homogenous Goods Price Setting

Substituting eq. (2.7) into eq. (2.6) to obtain, after rearranging,

$$E_t\sum_{j=0}^{\infty}\beta^j\upsilon_{t+j}P_{t+j}Y_{t+j}\{\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1-\frac{\lambda_d}{\lambda_d-1}}-mc_{t+j}\left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d-1}}\},$$

or,

$$E_t\sum_{j=0}^{\infty}\beta^j\upsilon_{t+j}P_{t+j}Y_{t+j}\{\left(X_{t,j}\tilde{p}_t\right)^{1-\frac{\lambda_d}{\lambda_d-1}}-mc_{t+j}\left(X_{t,j}\tilde{p}_t\right)^{\frac{-\lambda_d}{\lambda_d-1}}\},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}}=X_{t,j}\tilde{p}_t,\ X_{t,j}\equiv\begin{cases}\frac{\tilde{\pi}_{d,t+j}\cdots\tilde{\pi}_{d,t+1}}{\pi_{t+j}\cdots\pi_{t+1}},\ j>0\\ 1,\ j=0.\end{cases}.$$

The i^{th} firm maximizes profits by choice of the within-sector relative price $\tilde{p}_t$. The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d-1}+1}$, rearranging, and scaling we obtain:

$$E_t\sum_{j=0}^{\infty}\left(\beta\xi_d\right)^j A_{t+j}\left[\tilde{p}_tX_{t,j}-\lambda_d mc_{t+j}\right]=0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j}=\psi_{z^+,t+j}\tilde{y}_{t+j}X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t^d = \frac{E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j}\lambda_d mc_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} X_{t,j}} = \frac{K_t^d}{F_t^d},$$

say, where

$$\begin{aligned} K_t^d &\equiv E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j}\lambda_d mc_{t+j} \\ F_t^d &= E_t \sum_{j=0}^{\infty} (\beta\xi_d)^j A_{t+j} X_{t,j}. \end{aligned}$$

These objects have the following convenient recursive representations:

$$\begin{aligned} E_t \left[\psi_{z^+,t}\tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}} \beta\xi_d F_{t+1}^d - F_t^d \right] &= 0 \\ E_t \left[\lambda_d \psi_{z^+,t}\tilde{y}_t mc_t + \beta\xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] &= 0. \end{aligned}$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di \right]^{(1-\lambda_d)} \\ &= \left[\left(1-\xi_p\right) \tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p \left(\tilde{\pi}_{d,t} P_{t-1}\right)^{\frac{1}{1-\lambda_d}} \right]^{(1-\lambda_d)} \end{aligned} \tag{B.8}$$

After dividing by P_t and rearranging:

$$\frac{1-\xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d} = \left(\tilde{p}_t^d\right)^{\frac{1}{1-\lambda_d}}. \tag{B.9}$$

In sum, the equilibrium conditions associated with price setting for producers of the domestic homogenous good are:[39]

$$E_t\left[\psi_{z^+,t}y_t+\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}}\beta\xi_d F_{t+1}^d-F_t^d\right] = 0 \tag{B.10}$$

$$E_t\left[\lambda_d\psi_{z^+,t}y_t mc_t+\beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}}K_{t+1}^d-K_t^d\right] = 0, \tag{B.11}$$

$$\mathring{p}_t=\left[(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right)^{\lambda_d}+\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_d}{1-\lambda_d}}\right]^{\frac{1-\lambda_d}{\lambda_d}} \tag{B.12}$$

$$\left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{(1-\lambda_d)}=\frac{K_t^d}{F_t^d} \tag{B.13}$$

$$\tilde{\pi}_{d,t}\equiv(\pi_{t-1})^{\kappa_d}\left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d}\left(\breve{\pi}\right)^{\varkappa_d} \tag{B.14}$$

B.3.2. Export Demand

Scaling (2.17) we obtain,

$$x_t=\left(p_t^x\right)^{-\eta_f}y_t^* \tag{B.15}$$

B.3.3. First Order Conditions for Export Goods Price Setting

$$E_t\left[\psi_{z^+,t}q_t p_t^c p_t^x x_t+\left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{1}{1-\lambda_x}}\beta\xi_x F_{x,t+1}-F_{x,t}\right] = 0 \tag{B.16}$$

$$E_t\left[\lambda_x\psi_{z^+,t}q_t p_t^c p_t^x x_t mc_t^x+\beta\xi_x\left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{\lambda_x}{1-\lambda_x}}K_{x,t+1}-K_{x,t}\right] = 0, \tag{B.17}$$

$$\mathring{p}_t^x=\left[(1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right)^{\lambda_x}+\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\mathring{p}_{t-1}^x\right)^{\frac{\lambda_x}{1-\lambda_x}}\right]^{\frac{1-\lambda_x}{\lambda_x}} \tag{B.18}$$

[39] When we linearize about steady state and set $\varkappa_d=0$, we obtain,

$$\begin{aligned}\hat{\pi}_t-\hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1}-\hat{\bar{\pi}}_{t+1}^c\right)+\frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1}-\hat{\bar{\pi}}_t^c\right)\\ &-\frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t^c\\ &+\frac{1}{1+\kappa_d\beta}\frac{\left(1-\beta\xi_d\right)\left(1-\xi_d\right)}{\xi_d}\widehat{mc}_t,\end{aligned}$$

where a hat indicates log-deviation from steady state.

$$\left[\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right]^{(1-\lambda_x)} = \frac{K_{x,t}}{F_{x,t}} \tag{B.19}$$

When we linearize around steady state and set $\varkappa_{m,j}=0$, equations (B.16)-(B.19) reduce to:

$$\begin{aligned}\hat{\pi}_t^x &= \frac{\beta}{1+\kappa_x\beta}E_t\hat{\pi}_{t+1}^x+\frac{\kappa_x}{1+\kappa_x\beta}\hat{\pi}_{t-1}^x \\ &\quad +\frac{1}{1+\kappa_x\beta}\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x}\widehat{mc}_t^x,\end{aligned}$$

where a hat over a variable indicates log deviation from steady state.

B.3.4. Demand for Domestic Inputs in Export Production

Integrating eq. (2.24):

$$\begin{aligned}\int_0^1 X_{i,t}^d di &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}(1-\omega_x)\int_0^1 X_{i,t}di \\ &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}(1-\omega_x)X_t\frac{\int_0^1\left(P_{i,t}^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}di}{\left(P_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}}.\end{aligned} \tag{B.20}$$

Define $\mathring{P}_t^x$, a linear homogeneous function of $P_{i,t}^x$:

$$\mathring{P}_t^x=\left[\int_0^1\left(P_{i,t}^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}di\right]^{\frac{\lambda_x-1}{-\lambda_x}}.$$

Then,

$$\left(\mathring{P}_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}=\int_0^1\left(P_{i,t}^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}di,$$

and

$$\int_0^1 X_{i,t}^d di=\left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}(1-\omega_x)X_t\left(\mathring{p}_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}, \tag{B.21}$$

where

$$\mathring{p}_t^x\equiv\frac{\mathring{P}_t^x}{P_t^x},$$

and the law of motion of $\mathring{p}_t^x$ is given in (B.18).

We now simplify (B.21). Rewriting the second equality in (2.20), we obtain:

$$\frac{\lambda}{P_t\tau_t^x R_t^x}=\frac{S_t P_t^x}{P_t q_t p_t^c p_t^x}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+(1-\omega_x)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t\tau_t^x R_t^x}=\frac{S_t P_t^x}{P_t\frac{S_t P_t^*}{P_t^c}\frac{P_t^c}{P_t}\frac{P_t^x}{P_t^*}}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+(1-\omega_x)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t \tau_t^x R_t^x} = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (B.21), we obtain:

$$X_t^d = \int_0^1 X_{i,t}^d di = \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}_t^x\right)^{\frac{-\lambda_x}{\lambda_x - 1}} \left(p_t^x\right)^{-\eta_f} Y_t^*$$

B.3.5. Demand for Imported Inputs in Export Production

Scaling (2.26) we obtain:

$$x_t^m = \omega_x \left(\frac{\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}}{p_t^{m,x}} \right)^{\eta_x} \left(\mathring{p}_t^x\right)^{\frac{-\lambda_x}{\lambda_x - 1}} \left(p_t^x\right)^{-\eta_f} y_t^* \qquad \text{(B.22)}$$

B.3.6. Value of Imports of the Intermediate Consumption Goods Producers

It is of interest to have a measure of the total value of imports of the intermediate consumption good producers:

$$S_t P_t^* R_t^{\nu,*} \int_0^1 C_{i,t}^m di.$$

In order to relate this to C_t^m, we substitute the demand curve into the previous expression:

$$\begin{aligned} & S_t P_t^* R_t^{\nu,*} \int_0^1 C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}}\right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} di \\ = \; & S_t P_t^* R_t^{\nu,*} C_t^m \left(P_t^{m,c}\right)^{\frac{\lambda_{m,c}}{\lambda_{m,c}-1}} \int_0^1 \left(P_{i,t}^{m,c}\right)^{\frac{-\lambda_{m,c}}{\lambda_{m,c}-1}} \\ = \; & S_t P_t^* R_t^{\nu,*} C_t^m \left(\frac{\mathring{P}_t^{m,c}}{P_t^{m,c}}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \end{aligned}$$

where

$$\mathring{P}_t^{m,c} = \left[\int_0^1 \left(P_{i,t}^{m,c}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}\right]^{\frac{1-\lambda_{m,c}}{\lambda_{m,c}}}$$

We conclude that the total value of imports accounted for by the consumption sector is:

$$S_t P_t^* R_t^{\nu,*} C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}, \qquad \text{(B.23)}$$

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}},$$

The derivation for the value imports used by the investment and export production sectors are analogous.

B.3.7. Marginal Costs of Importers

Real marginal cost is

$$\begin{aligned} mc_t^{m,j} &= \tau_t^{m,j}\frac{S_tP_t^*}{P_t^{m,j}}R_t^{\nu,*} = \tau_t^{m,j}\frac{S_tP_t^*P_t^cP_t}{P_t^cP_t^{m,j}P_t}R_t^{\nu,*} \\ &= \tau_t^{m,j}\frac{q_tp_t^c}{p_t^{m,j}}R_t^{\nu,*} \end{aligned} \tag{B.24}$$

for $j = c, i, x$.

B.3.8. First Order Conditions for Import Goods Price Setting

$$E_t\left[\psi_{z^+,t}p_t^{m,j}\Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}\beta\xi_{m,j}F_{m,j,t+1} - F_{m,j,t}\right] = 0 \tag{B.25}$$

$$E_t\left[\lambda_{m,j}\psi_{z^+,t}p_t^{m,j}mc_t^{m,j}\Xi_t^j + \beta\xi_{m,j}\left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}}K_{m,j,t+1} - K_{m,j,t}\right] = 0, \tag{B.26}$$

$$\mathring{p}_t^{m,j} = \left[\left(1-\xi_{m,j}\right)\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda_{m,j}} + \xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}}\right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \tag{B.27}$$

$$\left[\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right]^{(1-\lambda_{m,j})} = \frac{K_{m,j,t}}{F_{m,j,t}}, \tag{B.28}$$

for $j = c, i, x$.[40] Here,

$$\Xi_t^j = \begin{cases} c_t^m & j = c \\ x_t^m & j = x \\ i_t^m & j = i \end{cases}.$$

[40] When we linearize around steady state and $\varkappa_{m,j} = 0$,

$$\begin{aligned} \hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_{m,j}\beta}E_t\left(\hat{\pi}_{t+1}^{m,j} - \hat{\bar{\pi}}_{t+1}^c\right) + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta}\left(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c\right) \\ &\quad - \frac{\kappa_{m,j}\beta\left(1-\rho_\pi\right)}{1+\kappa_{m,j}\beta}\hat{\bar{\pi}}_t^c \\ &\quad + \frac{1}{1+\kappa_{m,j}\beta}\frac{\left(1-\beta\xi_{m,j}\right)\left(1-\xi_{m,j}\right)}{\xi_{m,j}}\widehat{mc}_t^{m,j}, \end{aligned}$$

B.3.9. Wage Setting Conditions in Baseline Model

Substituting eq. (2.37) into the objective function eq. (2.36),

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \, [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+\upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}],$$

It is convenient to recall the scaling of variables:

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+, \; \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \; \tilde{y}_t = \frac{Y_t}{z_t^+}, \; w_t = \tilde{W}_t / W_t, \; z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Then,

$$\begin{aligned}
\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_{t+i}^+ P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} \\
&= \frac{W_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i},
\end{aligned}$$

where

$$\begin{aligned}
X_{t,i} &= \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}, \; i > 0 \\
&= 1, \; i = 0.
\end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i \to \infty$. Thus,

$$X_{t,i} = \frac{\left(\pi_t^c \cdots \pi_{t+i-1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \cdots \bar{\pi}_{t+i}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}$$

In steady state,

$$\begin{aligned}
X_{t,i} &= \frac{\left(\bar{\pi}^i\right)^{\kappa_w} \left(\bar{\pi}^i\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^i\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w}}{\bar{\pi}^i \mu_{z^+}^i} \\
&= \left(\frac{\breve{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w} \left(\mu_{z^+}^i\right)^{\vartheta_w - 1} \\
&\to 0,
\end{aligned}$$

in the no-indexing case, when $\breve{\pi} = 1$, $\varkappa_w = 1$ and $\vartheta_w = 0$.

Simplifying using the scaling notation,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \, [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+\upsilon_{t+i} W_{t+i} \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L}$$

$$+\psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} \left(\frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}],$$

or,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}$$

$$+\psi_{z^+,t+i} w_t^{1+\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}],$$

Differentiating with respect to w_t,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \lambda_w (1+\sigma_L) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)-1}$$

$$+\psi_{z^+,t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}] = 0$$

Dividing and rearranging,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}$$

$$+\frac{\psi_{z^+,t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}] = 0$$

Solving for the wage rate:

$$\begin{aligned}
w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}} \\
&= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}
\end{aligned}$$

where

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w}.
\end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}}\right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w \zeta_{t+1}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\sigma_L} \\
&\quad + (\beta\xi_w)^2 \zeta_{t+2}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \\
&\quad + ...
\end{aligned}$$

or,

$$\begin{aligned}
K_{w,t} &= \zeta_t^h H_t^{1+\sigma_L} + E_t \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \{\zeta_{t+1}^h H_{t+1}^{1+\sigma_L} \\
&\quad + \beta\xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \zeta_{t+2}^h + ...\} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1},
\end{aligned}$$

using,

$$\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1}\mu_{z^+,t+1}\pi_{t+1}}{\bar{w}_t} \tag{B.29}$$

Also,

$$
\begin{aligned}
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} \left(\beta\xi_w\right)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau^y}{1+\tau^w} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau^y}{1+\tau^w} \\
&\quad + \beta\xi_w \frac{\psi_{z^+,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \frac{1-\tau^y}{1+\tau^w} \\
&\quad + \left(\beta\xi_w\right)^2 \frac{\psi_{z^+,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \\
&\quad \times \left(\frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \frac{1-\tau^y}{1+\tau^w} \\
&\quad + \ldots
\end{aligned}
$$

or,

$$
\begin{aligned}
F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau^y}{1+\tau^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \{ \frac{\psi_{z^+,t+1}}{\lambda_w} H_{t+1} \frac{1-\tau^y}{1+\tau^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z^+,t+2}}{\lambda_w} H_{t+2} \frac{1-\tau^y}{1+\tau^w} \\
&\quad + \ldots \} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau^y}{1+\tau^w} + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
\end{aligned}
$$

so that

$$
F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau^y}{1+\tau^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$
W_t = \left[\left(1-\xi_w\right)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.
$$

Dividing both sides by W_t and rearranging,

$$
w_t = \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}.
$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \qquad \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{B.30}$$

where

$$\mathring{w}_t \equiv \frac{\mathring{W}_t}{W_t},\ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\mathring{W}_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\mathring{W}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that,

$$\begin{aligned} \mathring{w}_t &= \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{B.31}$$

In addition to (B.31), we have following equilibrium conditions associated with sticky wages[41]:

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau^y}{1+\tau^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1} \tag{B.33}$$

$$K_{w,t} = \zeta_t^h \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t\right)^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \tag{B.34}$$

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}. \tag{B.35}$$

B.3.10. Scaling Law of Motion of Capital

Using (2.38) we can write the law of motion of capital in scaled terms as:

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{B.36}$$

B.3.11. Output and Aggregate Factors of Production

Below we derive a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production.

[41] Log linearizing these equations about the nonstochastic steady state and under the assumption of $\varkappa_w = 0$, we obtain

$$E_t \begin{bmatrix} \eta_0 \widehat{\bar{w}}_{t-1} + \eta_1 \widehat{\bar{w}}_t + \eta_2 \widehat{\bar{w}}_{t+1} + \eta_3\left(\hat{\pi}_t - \widehat{\bar{\pi}}_t^c\right) + \eta_4\left(\hat{\pi}_{t+1} - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\ +\eta_5\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) + \eta_6\left(\hat{\pi}_t^c - \rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right) \\ +\eta_7\hat{\psi}_{z^+,t} + \eta_8\hat{H}_t + \eta_9\hat{\tau}_t^y + \eta_{10}\hat{\tau}_t^w + \eta_{11}\hat{\zeta}_t^h \\ +\eta_{12}\hat{\mu}_{z^+,t} + \eta_{13}\hat{\mu}_{z^+,t+1} \end{bmatrix} = 0, \tag{B.32}$$

where

$$b_w = \frac{[\lambda_w\sigma_L - (1-\lambda_w)]}{[(1-\beta\xi_w)(1-\xi_w)]}$$

and

$$\begin{pmatrix} \eta_0 \\ \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \\ \eta_6 \\ \eta_7 \\ \eta_8 \\ \eta_9 \\ \eta_{10} \\ \eta_{11} \\ \eta_{12} \\ \eta_{13} \end{pmatrix} = \begin{pmatrix} b_w\xi_w \\ \left(\sigma_L\lambda_w - b_w\left(1+\beta\xi_w^2\right)\right) \\ b_w\beta\xi_w \\ -b_w\xi_w \\ b_w\beta\xi_w \\ b_w\xi_w\kappa_w \\ -b_w\beta\xi_w\kappa_w \\ (1-\lambda_w) \\ -(1-\lambda_w)\sigma_L \\ -(1-\lambda_w)\frac{\tau^y}{(1-\tau^y)} \\ -(1-\lambda_w)\frac{\tau^w}{(1+\tau^w)} \\ -(1-\lambda_w) \\ -b_w\xi_w \\ b_w\beta\xi_w \end{pmatrix}.$$

Consider the unweighted average of the intermediate goods:

$$\begin{aligned} Y_t^{sum} &= \int_0^1 Y_{i,t} di \\ &= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi \right] di \\ &= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{it}} \right)^{\alpha} H_{it} - z_t^+ \phi \right] di \\ &= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^{\alpha} \int_0^1 H_{it} di - z_t^+ \phi \end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

say, where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{B.37}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{B.38}$$

The preceding discussion implies:

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^{\alpha} H_t^{1-\alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{B.39}$$

We need to replace aggregate homogeneous labor, H_t, with aggregate household labor, h_t. From eq. (B.30) we have $H_t = \mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t$. Plugging this is we obtain:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^{\alpha} \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right].$$

which completes the derivation.

B.3.12. Restrictions Across Inflation Rates

We now consider the restrictions across inflation rates implied by our relative price formulas. In terms of the expressions in (B.2) there are the restrictions implied by $p_t^{m,j}/p_{t-1}^{m,j}$, $j = x, c, i$, and p_t^x. The restrictions implied by the other two relative prices in (B.2), p_t^i and p_t^c, have already been exploited in (2.16) and (B.36), respectively. Finally, we also exploit the restriction across inflation rates implied by q_t/q_{t-1} and (2.23). Thus,

$$\frac{p_t^{m,x}}{p_{t-1}^{m,x}} = \frac{\pi_t^{m,x}}{\pi_t} \tag{B.40}$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \tag{B.41}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \tag{B.42}$$

$$\frac{p_t^{x}}{p_{t-1}^{x}} = \frac{\pi_t^{x}}{\pi_t^*} \tag{B.43}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t \pi_t^*}{\pi_t^c}. \tag{B.44}$$

B.3.13. Endogenous Variables of the Baseline Model

In the above sections we derived the following 71 equations,

$$2.3, 2.4, 2.5, B.10, B.11, B.12, B.13, B.14, B.3, 2.10, 2.11, 2.12, 2.15, 2.16, 2.14,$$
$$B.15, 2.21, 2.20, 2.27, B.16, B.17, B.18, B.19, B.22, 2.29, B.25, B.26, B.27, B.28, 2.32,$$
$$B.24, B.4, B.5, B.36, 2.39, 2.41, 2.42, 2.43, 2.44, 2.45, 2.47, B.33, B.34, B.35, B.31,$$
$$2.35, B.29, B.30, \mathbf{??}, B.39, 2.50, 2.52, 2.51, B.40, B.41, B.42, B.43, B.44, 2.48$$

which can be used to solve for the following 71 unknowns:

$$\bar{r}_t^k, \bar{w}_t, R_t^{\nu,*}, R_t^f, R_t^x, R_t, mc_t, mc_t^x, mc_t^{m,c}, mc_t^{m,i}, mc_t^{m,x}, \pi_t, \pi_t^x, \pi_t^c, \pi_t^i, \pi_t^{m,c}, \pi_t^{m,i}, \pi_t^{m,x},$$
$$p_t^c, p_t^x, p_t^i, p_t^{m,x}, p_t^{m,c}, p_t^{m,i}, p_{k',t}, k_{t+1}, \bar{k}_{t+1}, u_t, h_t, H_t, q_t, i_t, c_t, x_t, a_t, s_t, \psi_{z+,t}, y_t$$
$$K_t^d, F_t^d, \tilde{\pi}_{d,t}, \mathring{p}_t, K_{x,t}, F_{x,t}, \tilde{\pi}_t^x, \mathring{p}_t^x, \{K_{m,j,t}, F_{m,j,t}, \tilde{\pi}_t^{m,j}, \mathring{p}_t^{m,j}; j = c, i, x\}, K_{w,t}, F_{w,t}, \tilde{\pi}_t^w, R_t^k$$
$$\Phi_t, \tilde{S}_t, \tilde{S}_t', a(u_t), \mathring{w}_t, c_t^m, i_t^m, x_t^m, \pi_w.$$

B.4. Equilibrium Conditions for Financial Frictions Model

B.4.1. Derivation of optimal contract

As noted in the text, we suppose that the equilibrium debt contract maximizes entrepreneurial welfare, subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt, B_{t+1} and a state $t+1-$contingent rate of interest, Z_{t+1}. We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1}\right] dF(\omega;\sigma_t)}{R_t N_{t+1}}$$
$$= \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[\omega - \bar{\omega}_{t+1}\right] dF(\omega;\sigma_t) R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1}}{R_t N_{t+1}}$$
$$= E_t \left\{ \left[1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)\right] \frac{R_{t+1}^k}{R_t} \right\} \varrho_t,$$

after making use of (3.1), (3.2) and

$$1 = \int_0^{\infty} \omega dF(\omega;\sigma_t) = \int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega;\sigma_t) + G(\bar{\omega}_{t+1};\sigma_t).$$

We can equivalently characterize the contract by a state-$t+1$ contingent set of values for $\bar{\omega}_{t+1}$ and a value of ϱ_t. The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ϱ_t which maximizes entrepreneurial welfare (relative to $R_t N_{t+1}$), subject to the bank zero profits condition. The Lagrangian representation of this problem is:

$$\max_{\varrho_t, \{\bar{\omega}_{t+1}\}} E_t \left\{ \left[1 - \Gamma(\bar{\omega}_{t+1};\sigma_t)\right] \frac{R_{t+1}^k}{R_t} \varrho_t + \lambda_{t+1} \left(\left[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)\right] \frac{R_{t+1}^k}{R_t} \varrho_t - \varrho_t + 1 \right) \right\},$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period $t+1$ state of nature. The first order conditions for this problem are:

$$
\begin{aligned}
E_t \left\{ [1-\Gamma(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} + \lambda_{t+1} \left([\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} - 1 \right) \right\} &= 0 \\
-\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) \frac{R^k_{t+1}}{R_t} + \lambda_{t+1} [\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} &= 0 \\
[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t - \varrho_t + 1 &= 0,
\end{aligned}
$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that we assume $\lambda_{t+1} > 0$ in each period $t+1$ state of nature. Substituting out for λ_{t+1} from the second equation into the first, the first order conditions reduce to:

$$
E_t \left\{ \begin{array}{c} [1-\Gamma(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} + \frac{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)}{\Gamma_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t) - \mu G_{\bar{\omega}}(\bar{\omega}_{t+1};\sigma_t)} \\ \left([\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} - 1 \right) \end{array} \right\} = 0, \tag{B.45}
$$

$$
[\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t)] \frac{R^k_{t+1}}{R_t} \varrho_t - \varrho_t + 1 = 0, \tag{B.46}
$$

for $t = 0, 1, 2, ...\infty$ and for $t = -1, 0, 1, 2, ...$ respectively.

Since N_{t+1} does not appear in the last two equations, we conclude that ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs, regardless of their net worth.

B.4.2. Derivation of Aggregation of Across Entrepreneurs

Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth, N_{t+1}. Then, aggregate average net worth, $\bar{N}_{t+1}$, is

$$
\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1}) \, dN_{t+1}.
$$

We now derive the law of motion of $\bar{N}_{t+1}$. Consider the set of entrepreneurs who in period $t-1$ had net worth N. Their net worth after they have settled with the bank in period t is denoted V_t^N, where

$$
V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma(\bar{\omega}_t;\sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N, \tag{B.47}
$$

where $\bar{K}_t^N$ is the amount of physical capital that entrepreneurs with net worth N_t acquired in period $t-1$. Clearing in the market for capital requires:

$$
\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t) \, dN_t.
$$

Multiplying (B.47) by $f(N_t)$ and integrating over all entrepreneurs,

$$
V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma(\bar{\omega}_t;\sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.
$$

Writing this out more fully:

$$
\begin{aligned}
V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ \left[1 - F(\bar{\omega}_t; \sigma_{t-1})\right] \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\
&= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\
&- \left\{ \left[1 - F(\bar{\omega}_t; \sigma_{t-1})\right] \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.
\end{aligned}
$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$. Substituting:

$$
V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF(\omega; \sigma_{t-1}) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right\} (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t).
$$

which implies eq. (3.5) in the main text.

B.4.3. Adjustment to the Baseline model When Financial Frictions Are Introduced

In this subsection we indicate how the equilibrium conditions of the baseline model must be modified to accommodate financial frictions.

Consider the households. Households no longer accumulate physical capital, and the first order condition, (2.42), must be dropped. No other changes need to be made to the household first order conditions. Equation (2.45) can be interpreted as applying to the household's decision to make bank deposits. The household equations, (B.36) and (2.43), pertaining to the law of motion and first order condition for investment respectively, can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the first order condition of many identical, competitive firms that build capital (note that each has a state variable in the form of lagged investment). We must add the three equations pertaining to the entrepreneur's loan contract: the law of motion of net worth, the bank's zero profit condition and the optimality condition. Finally, we must adjust the resource constraints to reflect the resources used in bank monitoring and in consumption by entrepreneurs.

We adopt the following scaling of variables, noting that W_t^e is set so that its scaled counterpart is constant:

$$
n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+}, \; w^e = \frac{W_t^e}{P_t z_t^+}.
$$

Dividing both sides of (3.5) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$
n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} \left[R_t^k p_{k',t-1} \bar{k}_t - R_{t-1} \left(p_{k',t-1} \bar{k}_t - n_t \right) - \mu G\left(\bar{\omega}_t; \sigma_{t-1}\right) R_t^k p_{k',t-1} \bar{k}_t \right] + w^e, \tag{B.48}
$$

for $t = 0, 1, 2, \ldots$. Equation (B.48) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payments to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, eq. (B.46), can be expressed in terms of the scaled variables as:

$$\Gamma(\bar{\omega}_{t+1};\sigma_t) - \mu G(\bar{\omega}_{t+1};\sigma_t) = \frac{R_t}{R_{t+1}^k}\left(1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}\right), \tag{B.49}$$

for $t = -1, 0, 1, 2, \ldots$. The optimality condition for bank loans is (B.45).

The output equation, (2.50), does not have to be modified. Instead, the resource constraint for domestic homogenous goods (2.51) needs to be adjusted for the monitoring costs:

$$\begin{aligned} y_t - d_t \quad = \quad & g_t + (1-\omega_c)\left(p_t^c\right)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i}\left(i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \\ & + \left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}}(1-\omega_x)\left(\mathring{p}_t^x\right)^{\frac{-\lambda_x}{\lambda_x-1}}\left(p_t^x\right)^{-\eta_f} y_t^*, \end{aligned} \tag{B.50}$$

where

$$d_t = \frac{\mu G(\bar{\omega}_t;\sigma_{t-1}) R_t^k p_{k',t-1}\bar{k}_t}{\pi_t \mu_{z^+,t}}.$$

When we bring the model to the data measured GDP is y_t adjusted for both monitoring costs and, as in the baseline model, capital utilization costs:

$$gdp_t = y_t - d_t - \left(p_t^i\right)^{\eta_i}\left(a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i).$$

Account has to be taken of the consumption by exiting entrepreneurs. The net worth of these entrepreneurs is $(1-\gamma_t)V_t$ and we assume a fraction, $1-\Theta$, is taxed and transferred in lump-sum form to households, while the complementary fraction, Θ, is consumed by the exiting entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta\frac{1-\gamma_t}{\gamma_t}(n_{t+1} - w^e) z_t^+ P_t$$

from the right side of (2.9). In practice we do not make this adjustment because we assume Θ is sufficiently small that the adjustment is negligible.

The financial frictions brings a net increase of 2 equations (we add (B.45), (B.48) and (B.49), and delete (2.42)) and two variables, n_{t+1} and $\bar{\omega}_{t+1}$. This increases the size of our system to 72 equations in 72 unknowns. The financial frictions also introduce the additional shocks, σ_t and γ_t.

B.5. Equilibrium Conditions from the Employment Frictions Model

B.5.1. Labor Hours

Scaling (4.5) by $P_t z_t^+$ yields:

$$\bar{w}_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\psi_{z^+,t} \frac{1-\tau^y}{1+\tau^w}} \tag{B.51}$$

Note, that the ratio

$$\frac{\mathcal{G}_t^i}{\varsigma_{i,t}^{\sigma_L}}$$

will be the same for all cohorts since no other variables in (B.51) are indexed by cohort.

B.5.2. Vacancies and the Employment Agency Problem

An employment agency in the i^{th} cohort which does not renegotiate its wage in period t sets the period t wage, $W_{i,t}$, as in (2.34):

$$W_{i,t} = \tilde{\pi}_{w,t} W_{i-1,t-1},\ \tilde{\pi}_{w,t} \equiv (\pi_{t-1})^{\kappa_w} (\bar{\pi}_t)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}, \tag{B.52}$$

for $i = 1, ..., N-1$ (note that an agency that was in the i^{th} cohort in period t was in cohort $i-1$ in period $t-1$) where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$.

After wages are set, employment agencies in cohort i decide on endogenous separation, post vacancies to attract new workers in the next period and supply labor services, $l_t^i \varsigma_{i,t}$, into competitive labor markets. Simplifying,

$$\begin{aligned} F\left(l_t^0, \omega_t\right) &= \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j} [\left(W_{t+j}\mathcal{E}_{t+j}^j - \Gamma_{t,j}\omega_t \left[1-\mathcal{F}_{t+j}^j\right]\right)\varsigma_{j,t+j} \\ &\quad - P_{t+j}\frac{\kappa z_{t+j}^+}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}\left(1-\mathcal{F}_{t+j}^j\right)] l_{t+j}^j \\ &\quad + \beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right), \end{aligned} \tag{B.53}$$

For convenience, we omit the expectation operator E_t below. Let

Writing out (B.53):

$$
\begin{aligned}
F\left(l_t^0,\omega_t\right) \;=\; & \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{\left[\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_t - P_t\frac{\kappa z_t^+}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left(1-\mathcal{F}_t^0\right)\right] l_t^0 \\
& +\beta E_t\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{t+1} - P_{t+1}\frac{\kappa z_{t+1}^+}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right] \\
& \times\left(\chi_t^0+\rho\right)\left[1-\mathcal{F}_t^0\right] l_t^0 \\
& +\beta^2 E_t\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{t+2} - P_{t+2}\frac{\kappa z_{t+2}^+}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right] \\
& \times\left(\chi_{t+1}^1+\rho\right)\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) l_t^0 \\
& +\ldots+ \\
& +\beta^N E_t\frac{\upsilon_{t+N}}{\upsilon_t}F\left(l_{t+N}^0,\tilde{W}_{t+N}\right)\}.
\end{aligned}
$$

$$
\begin{aligned}
J\left(\omega_t\right) \;=\; & \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t} - P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left[1-\mathcal{F}_t^0\right] \\
& +\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left(1-\mathcal{F}_{t+1}^1\right)\right)\varsigma_{1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right]\times \\
& \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(1-\mathcal{F}_t^0\right) \\
& +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2} - P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right]\times \\
& \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left[1-\mathcal{F}_t^0\right] \\
& +\ldots+ \\
& +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
& \left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left(1-\mathcal{F}_t^0\right)\}.
\end{aligned}
\tag{B.54}
$$

We derive optimal vacancy posting decisions of employment agencies by differentiating (B.54) with respect to $\tilde{v}_t^0$ and multiply the result by $\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}$, to obtain:

$$\begin{aligned}
0 &= -P_t z_t^+ \kappa \left(\tilde{v}_t^0\right)^{\varphi-1}\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&\quad +\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{E}_{t+1}^1-\Gamma_{t,1}\omega_t\left[1-\mathcal{F}_{t+1}^1\right]\right)\varsigma_{1,t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^1\right)^{\varphi}\left(1-\mathcal{F}_{t+1}^1\right)\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left[1-\mathcal{F}_{t+2}^2\right]\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left(1-\mathcal{F}_{t+2}^2\right)\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +...+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
&\quad \left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right]\} \\
&= J\left(\omega_t\right)-\left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}+P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left[1-\mathcal{F}_t^0\right] \\
&\quad -P_t z_t^+\kappa\left(\tilde{v}_t^0\right)^{\varphi-1}\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}
\end{aligned}$$

Since the latter expression must be zero, we conclude:

$$\begin{aligned}
J\left(\omega_t\right) &= \left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}-P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^0\right)^{\varphi}\left[1-\mathcal{F}_t^0\right] \\
&\quad +P_t z_t^+\kappa\left(\tilde{v}_t^0\right)^{\varphi-1}\left[1-\mathcal{F}_t^0\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&= \left(W_t\mathcal{E}_t^0-\omega_t\left(1-\mathcal{F}_t^0\right)\right)\varsigma_{0,t}+P_t z_t^+\kappa\left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_t^0\right)^{\varphi}+\left(\tilde{v}_t^0\right)^{\varphi-1}\frac{\rho}{Q_t^{1-\iota}}\right]\left[1-\mathcal{F}_t^0\right].
\end{aligned}$$

Next, we obtain simple expressions for the vacancy decisions from their first order necessary conditions for optimality. Multiplying the first order condition for $\tilde{v}_{t+1}^1$ by

$$\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}},$$

we obtain:

$$\begin{aligned}
0 &= -\beta\frac{\upsilon_{t+1}}{\upsilon_t}P_{t+1}z_{t+1}^+\kappa\left(\tilde{v}_{t+1}^1\right)^{\varphi-1}\left[1-\mathcal{F}_{t+1}^1\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}}\left[1-\mathcal{F}_t^0\right] \\
&\quad +\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}\mathcal{E}_{t+2}^2-\Gamma_{t,2}\omega_t\left(1-\mathcal{F}_{t+2}^2\right)\right)\varsigma_{2,t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^2\right)^{\varphi}\left[1-\mathcal{F}_{t+2}^2\right]\right]\times \\
&\quad \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\
&\quad +...+ \\
&\quad +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right)\times \\
&\quad \left[1-\mathcal{F}_{t+N-1}^{N-1}\right]\cdots\left[1-\mathcal{F}_t^0\right].
\end{aligned}$$

Substitute out the period $t+2$ and higher terms in this expression using the first order condition for $\tilde{v}_t^0$. After rearranging, we obtain,

$$\frac{P_t z_t^+ \kappa \left(\tilde{v}_t^0\right)^{\varphi-1}}{Q_t^{1-\iota}} = \beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\begin{array}{c} \left(W_{t+1}\mathcal{E}_{t+1}^1 - \Gamma_{t,1}\omega_t \left[1-\mathcal{F}_{t+1}^1\right]\right) \varsigma_{1,t+1} \\ +P_{t+1} z_{t+1}^+ \kappa \left(1-\mathcal{F}_{t+1}^1\right) \left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+1}^1\right)^{\varphi} + \left(\tilde{v}_{t+1}^1\right)^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}}\right] \end{array} \right].$$

Following the pattern set with $\tilde{v}_{t+1}^1$, multiply the first order condition for $\tilde{v}_{t+2}^2$ by

$$\left(\tilde{v}_{t+2}^2 Q_{t+2}^{1-\iota} + \rho\right) \frac{1}{Q_{t+2}^{1-\iota}}.$$

Substitute the period $t+3$ and higher terms in the first order condition for $\tilde{v}_{t+2}^2$ using the first order condition for $\tilde{v}_{t+1}^1$ to obtain, after rearranging,

$$\frac{P_{t+1} z_{t+1}^+ \kappa \left(\tilde{v}_{t+1}^1\right)^{\varphi-1}}{Q_{t+1}^{1-\iota}} = \beta \frac{\upsilon_{t+2}}{\upsilon_{t+1}} \left[\begin{array}{c} \left(W_{t+2}\mathcal{E}_{t+2}^2 - \Gamma_{t,2}\omega_t \left[1-\mathcal{F}_{t+2}^2\right]\right) \varsigma_{2,t+2} \\ +P_{t+2} z_{t+2}^+ \kappa \left(1-\mathcal{F}_{t+2}^2\right) \left[\left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+2}^2\right)^{\varphi} + \left(\tilde{v}_{t+1}^2\right)^{\varphi-1} \frac{\rho}{Q_{t+2}^{1-\iota}}\right] \end{array} \right].$$

Continuing in this way, we obtain,

$$\frac{P_{t+j} z_{t+j}^+ \kappa \left(\tilde{v}_{t+j}^j\right)^{\varphi-1}}{Q_{t+j}^{1-\iota}} = \beta \frac{\upsilon_{t+j+1}}{\upsilon_{t+j}} \left[\begin{array}{c} \left(W_{t+j+1}\mathcal{E}_{t+j+1}^{j+1} - \Gamma_{t,j+1}\omega_t \left[1-\mathcal{F}_{t+j+1}^{j+1}\right]\right) \varsigma_{j+1,t+j+1} \\ +P_{t+j+1} z_{t+j+1}^+ \kappa \left(1-\mathcal{F}_{t+j+1}^{j+1}\right) \left[\begin{array}{c} \left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+j+1}^{j+1}\right)^{\varphi} \\ +\left(\tilde{v}_{t+j+1}^{j+1}\right)^{\varphi-1} \frac{\rho}{Q_{t+j+1}^{1-\iota}} \end{array} \right] \end{array} \right],$$

for $j = 0, 1, ..., N-2$. Now consider the first order necessary condition for the optimality of $\tilde{v}_{t+N-1}^{N-1}$. After multiplying this first order condition by

$$\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \frac{1}{Q_{t+N-1}^{1-\iota}},$$

we obtain,

$$\begin{aligned} 0 \;=\; & -\beta^{N-1} \frac{\upsilon_{t+N-1}}{\upsilon_t} P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \cdots \\ & \cdots \left(\tilde{v}_{t+N-2}^{N-2} Q_{t+N-2}^{1-\iota} + \rho\right) \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \frac{1}{Q_{t+N-1}^{1-\iota}} \left[1-\mathcal{F}_{t+N-2}^{N-2}\right] \cdots \left[1-\mathcal{F}_t^0\right] \\ & +\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right) \times \\ & \left[1-\mathcal{F}_{t+N-1}^{N-1}\right] \cdots \left[1-\mathcal{F}_t^0\right]\} \end{aligned}$$

or,

$$P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \frac{1}{Q_{t+N-1}^{1-\iota}} = \beta \frac{\upsilon_{t+N}}{\upsilon_{t+N-1}} J\left(\tilde{W}_{t+N}\right).$$

Making use of our expression for J, we obtain:

$$P_{t+N-1} z_{t+N-1}^+ \kappa \left(\tilde{v}_{t+N-1}^{N-1}\right)^{\varphi-1} \frac{1}{Q_{t+N-1}^{1-\iota}} = \beta \frac{\upsilon_{t+N}}{\upsilon_{t+N-1}} \left[\begin{array}{c} \left(W_{t+N}\mathcal{E}_{t+N}^0 - \tilde{W}_{t+N}\left(1-\mathcal{F}_{t+N}^0\right)\right) \varsigma_{0,t+N} \\ +P_{t+N} z_{t+N}^+ \kappa \left[\begin{array}{c} \left(1-\frac{1}{\varphi}\right)\left(\tilde{v}_{t+N}^0\right)^{\varphi} \\ +\left(\tilde{v}_{t+N}^0\right)^{\varphi-1} \frac{\rho}{Q_{t+N}^{1-\iota}} \end{array} \right] \left[1-\mathcal{F}_{t+N}^0\right] \end{array} \right].$$

The above first order conditions apply over time to a group of agencies that bargain at date t. We now express the first order conditions for a fixed date and different cohorts:

$$\begin{aligned} P_t z_t^+ \kappa \left(\tilde{v}_t^j\right)^{\varphi-1} \frac{1}{Q_t^{1-\iota}} &= \beta \frac{\upsilon_{t+1}}{\upsilon_t} [\Big(W_{t+1} \mathcal{E}_{t+1}^{j+1} - \Gamma_{t-j,j+1} \tilde{W}_{t-j} \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \Big) \varsigma_{j+1,t+1} \\ &+ P_{t+1} z_{t+1}^+ \kappa \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \left(\left(1 - \frac{1}{\varphi}\right) \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi} + \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}} \right)], \\ &\text{for } j = 0, ..., N-2. \end{aligned}$$

Scaling by $P_t z_t^+$ yields the following scaled first order optimality conditions:

$$\begin{aligned} \kappa \left(\tilde{v}_t^j\right)^{\varphi-1} \frac{1}{Q_t^{1-\iota}} &= \beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [\left(\bar{w}_{t+1} \mathcal{E}_{t+1}^{j+1} - G_{t-j,j+1} w_{t-j} \bar{w}_{t-j} \left(1 - \mathcal{F}_{t+1}^{j+1}\right)\right) \varsigma_{j+1,t+1} \\ &+ \kappa \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \left(\left(1 - \frac{1}{\varphi}\right) \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi} + \left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}} \right)], \\ &\text{for } j = 0, ..., N-2, \end{aligned} \quad \text{(B.55)}$$

where

$$\begin{aligned} G_{t-i,i+1} &= \frac{\tilde{\pi}_{w,t+1} \cdots \tilde{\pi}_{w,t-i+1}}{\pi_{t+1} \cdots \pi_{t-i+1}} \left(\frac{1}{\mu_{z^+,t-i+1}}\right) \cdots \left(\frac{1}{\mu_{z^+,t+1}}\right), \ i \geq 0, \\ w_t &= \frac{\tilde{W}_t}{W_t}, \ \bar{w}_t = \frac{W_t}{z_t^+ P_t}. \end{aligned} \quad \text{(B.56)}$$

Also,

$$G_{t,j} = \begin{cases} \frac{\tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+j} \cdots \pi_{t+1}} \left(\frac{1}{\mu_{z^+,t+1}}\right) \cdots \left(\frac{1}{\mu_{z^+,t+j}}\right) & j > 0 \\ 1 & j = 0 \end{cases}. \quad \text{(B.57)}$$

The scaled vacancy first order condition of agencies that are in the last period of their contract is:

$$\begin{aligned} \kappa \left(\tilde{v}_t^{N-1}\right)^{\varphi-1} \frac{1}{Q_t^{1-\iota}} &= \beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [\left(\bar{w}_{t+1} \mathcal{E}_{t+1}^0 - w_{t+1} \bar{w}_{t+1} \left(1 - \mathcal{F}_{t+1}^0\right)\right) \varsigma_{0,t+1} \\ &+ \kappa \left(1 - \mathcal{F}_{t+1}^0\right) \left(\left(1 - \frac{1}{\varphi}\right) \left(\tilde{v}_{t+1}^0\right)^{\varphi} + \left(\tilde{v}_{t+1}^0\right)^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}} \right)]. \end{aligned} \quad \text{(B.58)}$$

B.5.3. Agency Separation Decisions

This section presents details of the employment agency separation decision. We start by considering the separation decision of a representative agency in the $j = 0$ cohort which renegotiates the wage in the current period. After that, we consider $j > 0$.

The Separation Decision of Agencies that Renegotiate the Wage in the Current Period We start by considering the impact of $\bar{a}_t^0$ on agency and worker surplus, respectively. The aggregate surplus across all the l_t^0 workers in the representative agency is given by (4.18). The object, $\mathcal{F}_t^0$, is a function of $\bar{a}_t^0$ as indicated in (4.2). We denote its derivative by

$$\mathcal{F}_t^{j\prime} \equiv \frac{d\mathcal{F}_t^j}{d\bar{a}_t^j}, \tag{B.59}$$

for $j = 0...N-1$. Where convenient, in this subsection we include expressions that apply to the representative agency in cohort $j > 0$ as well as to those in cohort, $j = 0$. According to (4.5), $\bar{a}_t^0$ affects V_t^0 via its impact on hours worked, $\varsigma_{0,t}$. Hours worked is a function of $\bar{a}_t^0$ because $\mathcal{G}_t^0$ is (see (4.6), (4.5) and (4.13)). These observations about V_t^0 also apply to V_t^j, for $j > 0$. Thus, differentiating (4.13), we obtain:

$$V_t^{j\prime} \equiv \frac{d}{d\bar{a}_t^j} V_t^j = \left[\Gamma_{t-j,j} \tilde{W}_{t-j} \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\zeta_t \varsigma_{j,t}^{\sigma_L}}{\upsilon_t} \right] \varsigma_{j,t}^{\prime}, \tag{B.60}$$

where

$$\varsigma_{j,t}^{\prime} \equiv \frac{d\varsigma_{j,t}}{d\bar{a}_t^j} = \frac{1}{\sigma_L} \left(\varsigma_{j,t}\right)^{1-\sigma_L} \frac{W_t \upsilon_t}{\zeta_t A_L} \frac{1-\tau^y}{1+\tau^w} \mathcal{G}_t^{j\prime}, \tag{B.61}$$

and

$$\mathcal{G}_t^{j\prime} \equiv \frac{d\mathcal{G}_t^j}{d\bar{a}_t^j}. \tag{B.62}$$

The counterpart to (B.61) in terms of scaled variables is:

$$\varsigma_{j,t}^{\prime} \equiv \frac{1}{\sigma_L} \left(\varsigma_{j,t}\right)^{1-\sigma_L} \frac{\bar{w}_t w_t \psi_{z^+,t}}{\zeta_t A_L} \frac{1-\tau^y}{1+\tau^w} \mathcal{G}_t^{j\prime} \tag{B.63}$$

The value of being unemployed, U_t, is not a function of the $\bar{a}_t^0$ chosen by the representative agency because U_t is determined by economy-wide aggregate variables such as the job finding rate (see (4.14)).

According to (4.12) agency surplus per worker in l_t^0 is given by $J(\omega_t)$ and this has the following representation:

$$J(\omega_t) = \max_{\bar{a}_t^0} \tilde{J}\left(\omega_t; \bar{a}_t^0\right) \left(1 - \mathcal{F}_t^0\right).$$

Here, $\tilde{J}(\omega_t; \bar{a}_t^0)$ is given by (4.19) and

$$\begin{aligned} J_{t+1}^{j+1}(\omega_t) = & \max_{\left\{\bar{a}_{t+i}^i, \tilde{v}_{t+i}^i\right\}_{i=j}^{N-1}} \{ \left[\left(W_{t+1} \mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1} \omega_{t-j} \right) \varsigma_{j+1,t+1} - P_{t+1} z_{t+1}^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_{t+1}^{j+1} \right)^{\varphi} \right] \\ & \times \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \\ & + \beta \frac{\upsilon_{t+2}}{\upsilon_{t+1}} \left[\left(W_{t+2} \mathcal{G}_{t+2}^{j+2} - \Gamma_{t-j,j+2} \omega_{t-j} \right) \varsigma_{j+2,t+2} - P_{t+2} z_{t+2}^{+} \frac{\kappa}{\varphi} \left(\tilde{v}_{t+2}^{j+2} \right)^{\varphi} \right] \\ & \times \left(1 - \mathcal{F}_{t+2}^{j+2}\right) \left(\chi_{t+1}^{j+1} + \rho\right) \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \\ & + ... + \\ & + \beta^{N-j-1} \frac{\upsilon_{t+N-j}}{\upsilon_{t+1}} J\left(\tilde{W}_{t+N-j}\right) \left(\chi_{t+N-j-1}^{N-1} + \rho\right) \left(1 - \mathcal{F}_{t+N-j-1}^{N-1}\right) \cdots \left(\chi_{t+1}^{j+1} + \rho\right) \left(1 - \mathcal{F}_{t+1}^{j+1}\right) \}, \end{aligned} \tag{B.64}$$

for $j = 0$.

In (4.19) and (B.64), it is understood that χ^j_{t+j}, $\tilde{v}^j_{t+j}$ are connected by (4.8). Thus, the surplus of the representative agency with workforce, l^0_t, expressed as a function of an arbitrary value of $\bar{a}^0_t$ is given by (4.20). Differentiation of $\tilde{J}$ with respect to $\bar{a}^j_t$ need only be concerned with the impact of $\bar{a}^j_t$ on $\mathcal{G}^j_t$ and $\varsigma_{j,t}$. Generalizing (4.19) to cohort j:

$$\tilde{J}\left(\omega_{t-j};\bar{a}^j_t\right) = \max_{\tilde{v}^j_t}\left\{\left(W_t\mathcal{G}^j_t - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t} - P_t z^+_t \frac{\kappa}{\varphi}\left(\tilde{v}^j_t\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left(\chi^j_t + \rho\right)J^{j+1}_{t+1}\left(\omega_{t-j}\right)\right\}.$$

Then,

$$\tilde{J}_{\bar{a}^j}\left(\omega_{t-j};\bar{a}^j_t\right) \equiv \frac{d\tilde{J}\left(\omega_{t-j};\bar{a}^j_t\right)}{d\bar{a}^j_t} = \left(W_t\mathcal{G}^j_t - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma'_{j,t} + W_t\mathcal{G}^{j\prime}_t\varsigma_{j,t}, \tag{B.65}$$

where $\varsigma'_{j,t}$ and $\mathcal{G}^{j\prime}_t$ are defined in (B.61) and (B.62), respectively.

We now evaluate $\mathcal{F}^{j\prime}_t$ and $\mathcal{G}^{j\prime}_t$ for $j \geq 0$. We assume that productivity, a, is drawn from a log-normal distribution having the properties, $Ea = 1$ and $Var\left(\log a\right) = \sigma^2_a$. This assumption simplifies the analysis because analytic expressions are available for objects such as $\mathcal{F}^{j\prime}_t$, $\mathcal{G}^{j\prime}_t$. Although these expressions are readily available in the literature (see, for example, BGG), we derive them here for completeness. It is easily verified that $\mathcal{F}$ has the following representation:[42]

$$\mathcal{F}\left(\bar{a}^j;\sigma_a\right) = \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{\log\bar{a}^j} e^x e^{\frac{-\left(x+\frac{1}{2}\sigma^2_a\right)^2}{2\sigma^2_a}}dx,$$

where $x = \log a$. Combining the exponential terms,

$$\mathcal{F}\left(\bar{a}^j;\sigma_a\right) = \frac{1}{\sigma_a\sqrt{2\pi}}\int_{-\infty}^{\log\bar{a}^j} \exp^{\frac{-\left(x-\frac{1}{2}\sigma^2_a\right)^2}{2\sigma^2_a}}dx.$$

Now, make the change of variable,

$$v \equiv \frac{x - \frac{1}{2}\sigma^2_a}{\sigma_a}$$

so that

$$dv = \frac{1}{\sigma_a}dx.$$

Substituting into the expression for $\mathcal{F}$:

$$\mathcal{F}\left(\bar{a}^j;\sigma_a\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma^2_a}{\sigma_a}} \exp^{\frac{-v^2}{2}}dv.$$

This is just the standard normal cumulative distribution, evaluated at $\left(\log\left(\bar{a}^j\right) + \frac{1}{2}\sigma^2_a\right)/\sigma_a$. Differentiating $\mathcal{F}$, we obtain an expression for (B.59):

$$\mathcal{F}^{j\prime}_t = \frac{1}{\bar{a}^j\sigma_a\sqrt{2\pi}}\exp^{-\frac{\left(\log\left(\bar{a}^j\right)+\frac{1}{2}\sigma^2_a\right)^2}{2\sigma^2_a}}. \tag{B.66}$$

[42] Note that $Ea = 1$ is imposed by specifying $E\log a = -\sigma^2_a/2$.

The object on the right of the equality is just the normal density with variance σ_a^2 and mean $-\sigma_a^2/2$, evaluated at $\log(\bar{a}^j)$ and divided by $\bar{a}^j$. From (4.7) we obtain:

$$\mathcal{E}_t^{j\prime} = -\bar{a}_t^j \mathcal{F}_t^{j\prime}. \tag{B.67}$$

Differentiating (B.62),

$$\mathcal{G}_t^{j\prime} = \frac{\mathcal{E}_t^{j\prime}\left(1-\mathcal{F}_t^j\right) + \mathcal{E}_t^j \mathcal{F}_t^{j\prime}}{\left[1-\mathcal{F}_t^j\right]^2} \tag{B.68}$$

The surplus criterion governing the choice of $\bar{a}_t^0$ is (4.21). The first order necessary condition for an interior optimum is given by (4.22), which we reproduce here for convenience:

$$s_w V_t^{0\prime} + s_e \tilde{J}_{\bar{a}^0}\left(\tilde{W}_t; \bar{a}_t^0\right) = \left[s_w\left(V_t^0 - U_t\right) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right] \frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0},$$

where we have made use of the fact that the wage paid to workers in the bargaining period is denoted $\tilde{W}_t$. After substituting from (B.60) and (B.65):

$$s_w\left(\tilde{W}_t \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\upsilon_t}\right)\varsigma_{0,t}' + s_e\left[\left(W_t \mathcal{G}_t^0 - \tilde{W}_t\right)\varsigma_{0,t}' + W_t \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] = \tag{B.69}$$
$$\left[s_w\left(V_t^0 - U_t\right) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right]\frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}.$$

In scaled terms this is

$$s_w\left(w_t W_t \frac{1-\tau^y}{1+\tau^w} - A_L P_t z_t^+ \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{0,t}' + s_e\left[\left(W_t \mathcal{G}_t^0 - w_t W_t\right)\varsigma_{0,t}' + W_t \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] =$$
$$\left[s_w\left(P_t z_t^+ V_{z^+,t}^0 - U_{z^+,t} P_t z_t^+\right) + s_e \tilde{J}\left(\tilde{W}_t; \bar{a}_t^0\right)\right]\frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}$$
$$P_t z_t^+ s_w\left(w_t \bar{w}_t \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{0,t}' + P_t z_t^+ \bar{w}_t s_e\left[\left(\mathcal{G}_t^0 - w_t\right)\varsigma_{0,t}' + \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] =$$
$$P_t z_t^+\left[s_w\left(V_{z^+,t}^0 - U_{z^+,t}\right) + s_e \tilde{J}_{z^+,t}^0\right]\frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0}$$

Dividing through by $P_t z_t^+$ yields:

$$s_w\left(w_t \bar{w}_t \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\zeta_t \varsigma_{0,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma_{0,t}' + s_e \bar{w}_t\left[\left(\mathcal{G}_t^0 - w_t\right)\varsigma_{0,t}' + \mathcal{G}_t^{0\prime}\varsigma_{0,t}\right] = \left[\begin{array}{c} s_w\left(V_{z^+,t}^0 - U_{z^+,t}\right) \\ +s_e \tilde{J}_{z^+,t}^0 \end{array}\right]\frac{\mathcal{F}_t^{0\prime}}{1-\mathcal{F}_t^0} \tag{B.70}$$

The Separation Decision of Agencies that Renegotiated in Previous Periods We now turn to the $\bar{a}_t^j$ decision, for $j = 1, ..., N-1$. The representative agency that selects $\bar{a}_t^j$ is a member of the cohort of agencies that bargained j periods in the past. We denote the present discounted value of profits of the representative agency in cohort j by $F_t^j(\omega_{t-j})$:

$$\frac{F_t^j\left(l_t^j,\omega_{t-j}\right)}{l_t^j} \equiv J_t^j\left(\omega_{t-j}\right) = \max_{\left\{\bar{a}_{t+i}^{j+i},\tilde{v}_{t+i}^{j+i}\right\}_{i=0}^{N-j-1}} \{ \left[\left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t} - P_t z_t^+ \frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi}\right]$$
$$\times\left(1-\mathcal{F}_t^j\right)$$
$$+\beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j}\right)\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^+\frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right]$$
$$\times\left(1-\mathcal{F}_{t+1}^{j+1}\right)\left(\chi_t^j+\rho\right)\left(1-\mathcal{F}_t^j\right)$$
$$+ ... +$$
$$+\beta^{N-j}\frac{\upsilon_{t+N-j}}{\upsilon_t}J\left(\tilde{W}_{t+N-j}\right)\left(\chi_{t+N-1-j}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots$$
$$\left(\chi_t^j+\rho\right)\left(1-\mathcal{F}_t^j\right)\}.$$

Here, we exploit that $F_t^j\left(l_t^j,\omega_{t-j}\right)$ is proportional to l_t^j, as in the case $j=0$ considered in (4.12). In particular, $J_t^j\left(\omega_{t-j}\right)$ is not a function of l_t^j and corresponds to the object in (B.64) with the time index, t, replaced by $t-j$. We can write $J_t^j\left(\omega_{t-j}\right)$ in the following form:

$$J_t^j\left(\omega_{t-j}\right) = \tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right)\left(1-\mathcal{F}_t^j\right),$$

where

$$\tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right) = \left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t} - P_t z_t^+\frac{\kappa}{\varphi}\left(\tilde{v}_t^j\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}J_{t+1}^{j+1}\left(\omega_{t-j}\right)\left(\chi_t^j+\rho\right).$$

from a generalization of (4.19) to $j=1...N-1$.

In this way, we obtain an expression for agency surplus for agencies that have not negotiated for j periods which is symmetric to (4.20):

$$F_t^j\left(\omega_{t-j}\right) = \tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right)\left(1-\mathcal{F}_t^j\right)l_t^j. \tag{B.71}$$

Our expression for total surplus is the analog of (4.21):

$$\left[s_w\left(V_t^j - U_t\right) + s_e\tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right)\right]\left(1-\mathcal{F}_t^j\right)l_t^j. \tag{B.72}$$

Differentiating,

$$s_w V_t^{j\prime} + s_e\tilde{J}_{\bar{a}^j}\left(\omega_{t-j};\bar{a}_t^j\right) = \left[s_w\left(V_t^j-U_t\right) + s_e\tilde{J}_t^j\left(\omega_{t-j};\bar{a}_t^j\right)\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}, \tag{B.73}$$

which corresponds to (4.22). Here, $\tilde{J}_{\bar{a}^j}\left(\omega_{t-1};\bar{a}_t^j\right)$ is the analog of (B.65) with index 0 replaced by j. After substituting from the analogs for cohort j of (B.60), (B.65):

$$s_w\left(\Gamma_{t-j,j}\tilde{W}_{t-j}\frac{1-\tau^y}{1+\tau^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\upsilon_t}\right)\varsigma_{j,t}' + s_e\left[\left(W_t\mathcal{G}_t^j - \Gamma_{t-j,j}\tilde{W}_{t-j}\right)\varsigma_{j,t}' + W_t\mathcal{G}_t^{j\prime}\varsigma_{j,t}\right] =$$
$$\left[s_w\left(V_t^j-U_t\right) + s_e\tilde{J}\left(\tilde{W}_{t-j};\bar{a}_t^j\right)\right]\frac{\mathcal{F}_t^{j\prime}}{1-\mathcal{F}_t^j}.$$

Scaling analogously to (B.70) and plugging in $\tilde{W}_{t-j} = w_{t-j}\bar{w}_{t-j}P_{t-j}z^{+}_{t-j}$ and $\bar{w}_t z^{+}_t P_t = W_t$ we obtain:

$$s_w\left(G_{t-j,j}w_{t-j}\bar{w}_{t-j}\frac{1-\tau^y}{1+\tau^w} - A_L\frac{\zeta_t\varsigma_{j,t}^{\sigma_L}}{\psi_{z^+,t}}\right)\varsigma'_{j,t} + s_e\left[\left(\bar{w}_t\mathcal{G}^j_t - G_{t-j,j}\bar{w}_{t-j}w_{t-j}\right)\varsigma'_{j,t} + \bar{w}_t\mathcal{G}^{j\prime}_t\varsigma_{j,t}\right] \tag{B.74}$$
$$\left[s_w\left(V^j_{z^+,t} - U_{z^+,t}\right) + s_e\tilde{J}^j_{z^+,t}\right]\frac{\mathcal{F}^{j\prime}_t}{1-\mathcal{F}^j_t}$$

Finally, we need an explicit expression for $\tilde{J}\left(\tilde{W}_t;\bar{a}^j_t\right)$, or rather its scaled equivalent $\tilde{J}^j_{z^+,t}$. For this we use (B.64) to write out $J^{j+1}_{t+1}\left(\omega_{t-j}\right)$ for $j = 1...N$ and plug into (4.19):

$$\tilde{J}^j_t\left(\omega_{t-j};\bar{a}^j_t\right) = \left(W_t\mathcal{G}^j_t - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t} - P_t z^+_t\frac{\kappa}{\varphi}\left(\tilde{v}^j_t\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}J^{j+1}_{t+1}\left(\omega_{t-j}\right)\left(\chi^j_t + \rho\right)$$

Redisplaying (B.64) for convenience:

$$\begin{aligned}
&J^{j+1}_{t+1}\left(\omega_{t-j}\right) = \max_{\left\{\bar{a}^i_{t+i},\tilde{v}^i_{t+i}\right\}_{i=j}^{N-1}}\{\left[\left(W_{t+1}\mathcal{G}^{j+1}_{t+1} - \Gamma_{t-j,j+1}\omega_{t-j}\right)\varsigma_{j+1,t+1} - P_{t+1}z^+_{t+1}\frac{\kappa}{\varphi}\left(\tilde{v}^{j+1}_{t+1}\right)^{\varphi}\right] \\
&\times\left(1-\mathcal{F}^{j+1}_{t+1}\right) \\
&+\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\left(W_{t+2}\mathcal{G}^{j+2}_{t+2} - \Gamma_{t-j,j+2}\omega_{t-j}\right)\varsigma_{j+2,t+2} - P_{t+2}z^+_{t+2}\frac{\kappa}{\varphi}\left(\tilde{v}^{j+2}_{t+2}\right)^{\varphi}\right] \\
&\times\left(1-\mathcal{F}^{j+2}_{t+2}\right)\left(\chi^{j+1}_{t+1}+\rho\right)\left(1-\mathcal{F}^{j+1}_{t+1}\right) \\
&+ ... + \\
&+\beta^{N-j-1}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}}J\left(\tilde{W}_{t+N-j}\right)\left(\chi^{N-1}_{t+N-j-1}+\rho\right)\left(1-\mathcal{F}^{N-1}_{t+N-j-1}\right)\cdots \\
&\left(\chi^{j+1}_{t+1}+\rho\right)\left(1-\mathcal{F}^{j+1}_{t+1}\right)\},
\end{aligned}$$

Accordingly:

$$\begin{aligned}
\tilde{J}^j_t\left(\omega_{t-j};\bar{a}^j_t\right) = {}& \left(W_t\mathcal{G}^j_t - \Gamma_{t-j,j}\omega_{t-j}\right)\varsigma_{j,t} - P_t z^+_t\frac{\kappa}{\varphi}\left(\tilde{v}^j_t\right)^{\varphi} + \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left(\chi^j_t+\rho\right)\{ \\
&\left[\left(W_{t+1}\mathcal{G}^{j+1}_{t+1} - \Gamma_{t-j,j+1}\omega_{t-j}\right)\varsigma_{j+1,t+1} - P_{t+1}z^+_{t+1}\frac{\kappa}{\varphi}\left(\tilde{v}^{j+1}_{t+1}\right)^{\varphi}\right]\left(1-\mathcal{F}^{j+1}_{t+1}\right) \\
&+\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\left(W_{t+2}\mathcal{G}^{j+2}_{t+2} - \Gamma_{t-j,j+2}\omega_{t-j}\right)\varsigma_{j+2,t+2} - P_{t+2}z^+_{t+2}\frac{\kappa}{\varphi}\left(\tilde{v}^{j+2}_{t+2}\right)^{\varphi}\right] \\
&\times\left(1-\mathcal{F}^{j+2}_{t+2}\right)\left(\chi^{j+1}_{t+1}+\rho\right)\left(1-\mathcal{F}^{j+1}_{t+1}\right) \\
&+ ... + \\
&+\beta^{N-j-1}\frac{\upsilon_{t+N-j}}{\upsilon_{t+1}}J\left(\tilde{W}_{t+N-j}\right)\left(\chi^{N-1}_{t+N-j-1}+\rho\right)\left(1-\mathcal{F}^{N-1}_{t+N-j-1}\right)\cdots \\
&\left(\chi^{j+1}_{t+1}+\rho\right)\left(1-\mathcal{F}^{j+1}_{t+1}\right)\}
\end{aligned}$$

for $j = 0, ..., N-1$. Plugging in for $\omega_{t-j} = \tilde{W}_{t-j} = w_{t-j}\bar{w}_{t-j}P_{t-j}z_{t-j}^{+}$ and scaling one obtains:

$$\begin{aligned}
\tilde{J}_{z^{+},t}^{j}\left(\tilde{W}_{t-j};\bar{a}_{t}^{j}\right) \equiv & \frac{\tilde{J}^{j}\left(\tilde{W}_{t};\bar{a}_{t}^{j}\right)}{P_{t}z_{t}^{+}} = \left(\bar{w}_{t}\mathcal{G}_{t}^{j} - G_{t-j,j}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j,t} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t}^{j}\right)^{\varphi} + \\
& \beta\frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}}\frac{P_{t}z_{t}^{+}}{P_{t+1}z_{t+1}^{+}}\left(\chi_{t}^{j}+\rho\right) \\
& \times\{\frac{P_{t+1}z_{t+1}^{+}}{P_{t}z_{t}^{+}}\left[\left(\bar{w}_{t+1}\mathcal{G}_{t+1}^{j+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+1,t+1} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right]\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
& +\beta\frac{\psi_{z^{+},t+2}}{\psi_{z^{+},t+1}}\frac{P_{t+1}z_{t+1}^{+}}{P_{t+2}z_{t+2}^{+}}\frac{P_{t+2}z_{t+2}^{+}}{P_{t}z_{t}^{+}}\left[\begin{array}{c}\left(\bar{w}_{t+2}\mathcal{G}_{t+2}^{j+2} - G_{t-j,j+2}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+2,t+2} \\ -\frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi}\end{array}\right] \\
& \times\left(1-\mathcal{F}_{t+2}^{j+2}\right)\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
& +...+ \\
& +\beta^{N-j-1}\frac{\psi_{z^{+},t+N-j}}{\psi_{z^{+},t+1}}\frac{P_{t+1}z_{t+1}^{+}}{P_{t+N-j}z_{t+N-j}^{+}}\frac{P_{t+N-j}z_{t+N-j}^{+}}{P_{t}z_{t}^{+}}J_{z^{+},t+N-j} \\
& \times\left(\chi_{t+N-j-1}^{N-1}+\rho\right)\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\}
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
\tilde{J}_{z^{+},t}^{j}\left(\tilde{W}_{t-j};\bar{a}_{t}^{j}\right) = & \left(\bar{w}_{t}\mathcal{G}_{t}^{j} - G_{t-j,j}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j,t} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t}^{j}\right)^{\varphi} + \beta\frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}}\left(\chi_{t}^{j}+\rho\right) \\
& \times\{\left[\left(\bar{w}_{t+1}\mathcal{G}_{t+1}^{j+1} - G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+1,t+1} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t+1}^{j+1}\right)^{\varphi}\right]\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
& +\beta\frac{\psi_{z^{+},t+2}}{\psi_{z^{+},t+1}}\left[\left(\bar{w}_{t+2}\mathcal{G}_{t+2}^{j+2} - G_{t-j,j+2}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{j+2,t+2} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t+2}^{j+2}\right)^{\varphi}\right] \\
& \times\left(1-\mathcal{F}_{t+2}^{j+2}\right)\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right) \\
& +...+ \\
& +\beta^{N-j-1}\frac{\psi_{z^{+},t+N-j}}{\psi_{z^{+},t+1}}J_{z^{+},t+N-j}\left(\chi_{t+N-j-1}^{N-1}+\rho\right) \\
& \times\left(1-\mathcal{F}_{t+N-j-1}^{N-1}\right)\cdots\left(\chi_{t+1}^{j+1}+\rho\right)\left(1-\mathcal{F}_{t+1}^{j+1}\right)\}
\end{aligned} \quad (B.75)$$

B.5.4. Bargaining Problem

The first order condition associated with the Nash bargaining problem is:

$$\eta_{t}V_{w,t}J_{z^{+},t} + (1-\eta_{t})\left[V_{z^{+},t}^{0} - U_{z^{+},t}\right]J_{w,t} = 0, \quad (B.76)$$

after division by $z_{t}^{+}P_{t}$.

The following is an expression for J_t evaluated at $\omega_t = \tilde{W}_t$, in terms of scaled variables:

$$\begin{aligned}
J_{z^{+},t} = & \sum_{j=0}^{N-1}\beta^{j}\frac{\psi_{z^{+},t+j}}{\psi_{z^{+},t}}\left[\left(\bar{w}_{t+j}\mathcal{G}_{t}^{j} - G_{t,j}w_{t}\bar{w}_{t}\right)\varsigma_{j,t+j} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t+j}^{j}\right)^{\varphi}\right]\Omega_{t+j}^{j} \\
& +\beta^{N}\frac{\psi_{z^{+},t+N}}{\psi_{z^{+},t}}J_{z^{+},t+N}\frac{\Omega_{t+N}^{N}}{1-\mathcal{F}_{t+N}^{0}}.
\end{aligned} \quad (B.77)$$

We also require the derivative of J with respect to ω_t, i.e. the marginal surplus of the employment agency with respect to the negotiated wage. By the envelope condition, we can ignore the impact of a change in ω_t on endogenous separations and vacancy decisions, and only be concerned with the direct impact of ω_t on J. Taking the derivative of (B.54):

$$\begin{aligned} J_{w,t} &= -\left(1-\mathcal{F}_t^0\right)\varsigma_{0,t} \\ &\quad -\beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{1,t+1}\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_{t+1}^1\right)\left(1-\mathcal{F}_t^0\right) \\ &\quad -\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{2,t+2}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\left(1-\mathcal{F}_{t+2}^2\right)\left[1-\mathcal{F}_{t+1}^1\right]\left[1-\mathcal{F}_t^0\right] \\ &\quad -...-\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{N-1,t+N-1}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+1}^{N-2}+\rho\right)\times \\ &\quad \left(1-\mathcal{F}_{t+N-1}^{N-1}\right)\cdots\left[1-\mathcal{F}_t^0\right]. \end{aligned}$$

Let,

$$\Omega_{t+j}^j = \begin{cases} \left(1-\mathcal{F}_{t+j}^j\right)\prod_{l=0}^{j-1}\left(\chi_{t+l}^l+\rho\right)\left(1-\mathcal{F}_{t+l}^l\right) & j>0 \\ 1-\mathcal{F}_t^0 & j=0 \end{cases}. \tag{B.78}$$

It is convenient to express this in recursive form:

$$\begin{aligned} \Omega_t^0 &= 1-\mathcal{F}_t^0,\ \Omega_{t+1}^1 = \left(1-\mathcal{F}_{t+1}^1\right)\left(\chi_t^0+\rho\right)\overbrace{\left(1-\mathcal{F}_t^0\right)}^{\Omega_t^0}, \\ \Omega_{t+2}^2 &= \left(1-\mathcal{F}_{t+2}^2\right)\left(\chi_{t+1}^1+\rho\right)\overbrace{\left(\chi_t^0+\rho\right)\left(1-\mathcal{F}_t^0\right)\left(1-\mathcal{F}_{t+1}^1\right)}^{\Omega_{t+1}^1},... \end{aligned}$$

so that

$$\Omega_{t+j}^j = \left(1-\mathcal{F}_{t+j}^j\right)\left(\chi_{t+j-1}^{j-1}+\rho\right)\Omega_{t+j-1}^{j-1},$$

for $j=1,2,....$. It is convenient to define these objects at date t as a function of variables dated t and earlier for the purposes of implementing these equations in Dynare:

$$\begin{aligned} \Omega_t^0 &= 1-\mathcal{F}_t^0,\ \Omega_t^1 = \left(1-\mathcal{F}_t^1\right)\left(\chi_{t-1}^0+\rho\right)\overbrace{\left(1-\mathcal{F}_{t-1}^0\right)}^{\Omega_{t-1}^0}, \\ \Omega_t^2 &= \left(1-\mathcal{F}_t^2\right)\left(\chi_{t-1}^1+\rho\right)\overbrace{\left(\chi_{t-2}^0+\rho\right)\left(1-\mathcal{F}_{t-2}^0\right)\left(1-\mathcal{F}_{t-1}^1\right)}^{\Omega_{t-1}^1} \end{aligned}$$

so that

$$\Omega_t^j = \left(1-\mathcal{F}_t^j\right)\left(\chi_{t-1}^{j-1}+\rho\right)\Omega_{t-1}^{j-1}.$$

Then, in terms of scaled variables we obtain:

$$J_{w,t} = -\sum_{j=0}^{N-1}\beta^j\frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}G_{t,j}\Omega_{t+j}^j\varsigma_{j,t+j}. \tag{B.79}$$

Scaling V_t^i by $P_t z_t^+$, we obtain:

$$V_{z^+,t}^i = G_{t-i,i} w_{t-i} \bar{w}_{t-i} \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+,t}} + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right) V_{z^+,t+1}^{i+1} + \left(1-\rho+\rho\mathcal{F}_{t+1}^{i+1}\right) U_{z^+,t+1}\right], \tag{B.80}$$

for $i = 0, 1, ..., N-1$, where

$$\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i,\ U_{z^+,t+1} = \frac{U_{t+1}}{P_{t+1} z_{t+1}^+}.$$

In our analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. To define this derivative, it is useful to have:

$$\mathcal{M}_{t+j} = \left(1-\mathcal{F}_t^0\right)\cdots\left(1-\mathcal{F}_{t+j}^j\right), \tag{B.81}$$

for $j = 0, ..., N-1$. Then, the derivative of V^0, which we denote by $V_w^0(\omega_t)$, is:

$$\begin{aligned} V_w^0(\omega_t) &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau^y}{1+\tau^w} \Gamma_{t,j} \frac{\upsilon_{t+j}}{\upsilon_t} \\ &= E_t \sum_{j=0}^{N-1} (\beta\rho)^j \mathcal{M}_{t+j} \varsigma_{j,t+j} \frac{1-\tau^y}{1+\tau^w} G_{t,j} \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}. \end{aligned} \tag{B.82}$$

Note that ω_t has no impact on the intensity of labor effort. This is determined by (B.51), independent of the wage rate paid to workers.

Scaling (4.14),

$$U_{z^+,t} = b^u\left(1-\tau^y\right) + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[f_t V_{z^+,t+1}^x + (1-f_t) U_{z^+,t+1}\right]\cdot \tag{B.83}$$

This value function applies to any unemployed worker, whether they got that way because they were unemployed in the previous period and did not find a job, or they arrived into unemployment because of an exogenous separation, or because they arrived because of an endogenous separation.

B.5.5. Resource Constraint in the Employment Frictions Model

We assume that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation, (2.50), unchanged, and we alter the resource constraint:

$$\begin{aligned} y_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} \left(\tilde{v}_t^j\right)^2 \left[1-\mathcal{F}_t^j\right] l_t^j &= g_t + c_t^d + i_t^d \\ &+ \left(R_t^x\right)^{\eta_x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(p_t^x\right)^{-\eta_f} y_t^*. \end{aligned} \tag{B.84}$$

Measured GDP is y_t adjusted for both recruitment (hiring) costs and capital utilization costs:

$$gdp_t = y_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} \left(\tilde{v}_t^j\right)^2 \left[1-\mathcal{F}_t^j\right] l_t^j - \left(p_t^i\right)^{\eta_i} \left(a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i)$$

B.5.6. Final Equilibrium Conditions

Total job matches must also satisfy the following matching function:

$$m_t = \sigma_m (1 - L_t)^{\sigma} v_t^{1-\sigma}, \tag{B.85}$$

where

$$L_t = \sum_{j=0}^{N-1} \left(1 - \mathcal{F}_t^j\right) l_t^j. \tag{B.86}$$

and σ_m is the productivity of the matching technology.

In our environment, there is a distinction between effective hours and measured hours. Effective hours is the hours of each person, adjusted by their productivity, a. Recall that the average productivity of a worker in working in cohort j (i.e., who has survived the endogenous productivity cut) is $\mathcal{E}_t^j / \left(1 - \mathcal{F}_t^j\right)$. The number of workers who survive the productivity cut in cohort j is $\left(1 - \mathcal{F}_t^j\right) l_t^j$, so that our measure of total effective hours is:

$$H_t = \sum_{j=0}^{N-1} \varsigma_{j,t} \mathcal{E}_t^j l_t^j, \tag{B.87}$$

In contrast, total measured hours is:

$$H_t^{meas} = \sum_{j=0}^{N-1} \varsigma_{j,t} \left(1 - \mathcal{F}_t^j\right) l_t^j.$$

The job finding rate is:

$$f_t = \frac{m_t}{1 - L_t}. \tag{B.88}$$

The probability of filling a vacancy is:

$$Q_t = \frac{m_t}{v_t}. \tag{B.89}$$

Total vacancies v_t are related to vacancies posted by the individual cohorts as follows:

$$v_t = \frac{1}{Q_t^{\iota}} \sum_{j=0}^{N-1} \tilde{v}_t^j \left(1 - \mathcal{F}_t^j\right) l_t^j.$$

Note however, that this equation does not add a constraint to the model equilibrium. In fact, it can be derived from the equilibrium equations (B.89), (4.16) and (4.8).

B.5.7. Characterization of the Bargaining Set

Implicitly, we assumed that the scaled wage,

$$w_t^i = \frac{W_t^i}{z_t^+ P_t},$$

paid by an employment agency which has renegotiated most recently i periods in the past is always inside the bargaining set, $[\underline{w}_t^i, \bar{w}_t^i]$, $i = 0, 1, ..., N-1$. Here, $\bar{w}_t^i$ has the property that if $w_t^i > \bar{w}_t^i$ then the agency prefers not to employ the worker and $\underline{w}_t^i$ has the property that if $w_t^i < \underline{w}_t^i$ then the worker prefers to be unemployed. We now describe our strategy for computing $\underline{w}_t^i$ and $\bar{w}_t^i$.

The lower bound, $\underline{w}_t^i$, sets the surplus of a worker, $(1 - \mathcal{F}_t^i)\left(V_{z^+,t}^i - U_{z^+,t}\right)$, in an agency in cohort i to zero. By (B.80):

$$\begin{aligned} U_{z^+,t} &= \underline{w}_t^i \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+,t}} \\ &\quad + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\rho\left(1-\mathcal{F}_{t+1}^{i+1}\right) V_{z^+,t+1}^{i+1} + \left(1-\rho+\rho\mathcal{F}_{t+1}^{i+1}\right) U_{z^+,t+1}\right], \end{aligned}$$

for i = 0, ..., N - 1. In steady state, this is

$$\underline{w}^i = \frac{U_{z^+} + \zeta^h A_L \frac{\varsigma_i^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+}} - \beta\left[\rho\left(1-\mathcal{F}^{i+1}\right) V_{z^+}^{i+1} + \left(1-\rho+\rho\mathcal{F}^{i+1}\right) U_{z^+}\right]}{\varsigma_i \frac{1-\tau^y}{1+\tau^w}}$$

where a variable without time subscript denotes its steady state value. We now consider the upper bound, $\bar{w}_t^i$, which sets the surplus $J_{z^+,t}$ of an agency in cohort i to zero, i = 0, ..., N - 1. From (B.77)

$$\begin{aligned} 0 &= \sum_{j=0}^{N-1-i} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+j} \frac{\mathcal{E}_{t+j}^j}{1-\mathcal{F}_{t+j}^j} - G_{t,j}\bar{w}_t^i\right) \varsigma_{j,t+j} - \frac{\kappa}{\varphi}\left(\tilde{v}_{t+j}^j\right)^{\varphi}\right] \Omega_{t+j}^j \\ &\quad + \beta^{N-i} \frac{\psi_{z^+,t+N-i}}{\psi_{z^+,t}} J_{z^+,t+N-i} \frac{\Omega_{t+N-i}^{N-i}}{1-\mathcal{F}_{t+N-i}^0} \end{aligned}$$

for i = 0, ..., N - 1. In steady state:

$$\begin{aligned} 0 &= \sum_{j=0}^{N-1-i} \beta^j \left[\left(\bar{w} \frac{\mathcal{E}^j}{1-\mathcal{F}^j} - G_j \bar{w}^i\right) \varsigma_j - \frac{\kappa}{\varphi}\left(\tilde{v}^j\right)^{\varphi}\right] \Omega^j \\ &\quad + \beta^{N-i} J_{z^+} \frac{\Omega^{N-i}}{1-\mathcal{F}^0}. \end{aligned}$$

For the dynamic economy, the additional unknowns are the $2N$ variables composed of $\underline{w}_t^i$ and $\bar{w}_t^i$ for i = 0, 1, ..., N - 1. We have an equal number of equations to solve for them.

B.5.8. Summary of Equilibrium Conditions for Employment Friction Model

This subsection summarizes the equations of the labor market that define the equilibrium and how they are integrated with the baseline model. The equations include the N efficiency conditions that determines hours worked, (B.51); the law of motion of the workforce in each cohort, (4.9); the first order conditions associated with the vacancy decision, (B.55), (B.58), $j = 0, ..., N-1$; the derivative of the employment agency surplus with respect to the wage rate, (B.79); scaled agency surplus, (B.77); the value function of a worker, $V_{z^+,t}^i$, (B.80); the derivative of the worker value function with respect to the wage rate, (B.82); the growth adjustment term, $G_{t,j}$ (B.57); the scaled value function for unemployed workers, (B.83); first order condition associated with the Nash bargaining problem, (B.76); the (suitably modified) resource constraint, (B.84); the equations that characterize the productivity cutoff for job separations, (B.70) and (B.74); the equations that characterize $\tilde{J}_{z^+,t}^j$ (B.75); the value of finding a job, (4.15); the job finding rate, (B.88); the probability of filling a vacancy, (B.89); the matching function, (4.16); the wage updating equation for cohorts that do not optimize, (B.52); the equation determining total employment, (B.86); the equation determining Ω_{t+j}^j, (B.78); the equation determining the hiring rate, χ_t^i (4.8); the equation determining the number of matches (the matching function), (B.85); the definition of total effective hours (B.87); the equations defining $\mathcal{M}_t^j$, (B.81); the equations defining $\mathcal{F}_t^j$, (B.7); the equations defining $\mathcal{E}_t^i$, (B.6); the equations defining $\mathcal{G}_t^{j\prime}$ (B.68); the equations defining $\mathcal{F}_t^{j\prime}$ (B.66)

The following additional endogenous variables are added to the list of endogenous variables in the baseline model:

$$l_t^j, \mathcal{E}_t^j, \mathcal{F}_t^j, \varsigma_{j,t}, \mathcal{M}_t^j, \bar{a}_t^j, \tilde{v}_t^j, G_{t,j}, Q_t, \Omega_{t+j}^j, J_{w,t}, w_t, J_{z^+,t}, V_{z^+,t}^j, U_{z^+,t}, V_{w,t}^0,$$
$$V_{z^+,t}^x, f_t, m_t, v_t, \chi_t^j, \tilde{\pi}_{w,t}, L_t, \mathcal{G}_t^{j\prime}, \mathcal{F}_t^{j\prime} \text{ and } \tilde{J}_{z^+,t}^j$$

We drop the equations from the baseline model that determines wages, eq. (B.33), (B.34), (B.35),(B.31) and (2.35).

B.6. Summary of equilibrium conditions of the Full Model

In this subsection, we integrate financial frictions and labor market frictions together into what we call the full model.

The equations which describe the dynamic behavior of the model are those of the baseline model discussed in section B.3.13 and section B.3 plus those discussed in the financial frictions model specified in section B.4 plus those discussed in the employment friction model presented in section B.5.8. Finally, the resource constraint needs to be adjusted to include monitoring as well as recruitment (hiring) costs. Similarly measured GDP is adjusted to exclude both monitoring costs and recruitment costs (and, as in the baseline model, capital utilization costs).

B.7. Endogenous Priors

We select our model priors endogenously, using a strategy similar to the one suggested by Del Negro and Schorfheide (2008). We use a sequential-learning interpretation of the data. We begin with an initial set of priors. These have the form that is typical in Bayesian analyses, with the priors on different parameters being independent. Then, we suppose we are made aware of several statistics (i.e., second moments, or impulse response functions) that have been estimated in a sample of data that is independent of the data currently under analysis (we refer to these data as the 'pre-sample'). We use classical large sample theory to form a large-sample approximation to the likelihood for the pre-sample statistics.[43] The product of the initial priors and the likelihood of the pre-sample statistics form the 'endogenous priors' we take to the sample of data currently under analysis. Although our initial priors are independent across parameters, the procedure results in a set of priors for the actual sample that are not independent across parameters.

Our method for endogenizing priors differs from the one suggested by Del Negro and Schorfheide (2008), which is based on fitting a $p-$th vector autoregression (VAR) to a presample. As is well known, this likelihood is only a function of the sample variance covariance matrix and $p-$lagged covariances. The likelihood of these second moments, conditional on the DSGE model parameters, is known exactly, and requires no large-sample approximation. This is an important advantage of the Del Negro and Schorfheide (2008) approach over the one proposed here. The advantage of our approach is that it provides the user greater flexibility in which statistics to target. For example, a researcher may have prior information on the variances of 10 variables and suppose there are 20 model parameters. To apply this prior information in the Del Negro-Schorfheide (2008) approach, one sets $p = 0$ and works with the variance covariance of the 10 variables. But, herein lies a difficulty. This variance-covariance matrix represents 55 statistics, far more than the 10 over which the researcher actually has priors. Given there are only 20 model parameters, even if the priors are imposed dogmatically (by imagining that the pre-sample is large) the researcher may still not be able to 'hit' his/her true priors - the 10 variances. Because our approach focuses only on the statistics over which the researcher actually has prior information, the example just described is not a problem. With the approach described here, the assumption that the pre-sample is large will in general imply that the 10 variances are 'hit' exactly, with 10 degrees of freedom to spare.

Denote our presample observations by $\{\tilde{y}_t; t = 1, ..., T\}$. The column vector, w_t, is related to the vector of observed variables, y_t, by

$$\tilde{y}_t = H w_t.$$

The stochastic process, w_t, is assumed to have mean zero (say, because the sample mean has been removed). The column vector composed of the variances in w_t is denoted F. We estimate the elements of F by standard GMM methods. In particular, define the following GMM euler error:

$$h_t(F) = diag(\tilde{y}_t \tilde{y}_t{}') - F,$$

where $diag(X)$ denotes the column vector formed from the diagonal elements of the matrix, X. Note that

$$Eh_t(F^0) = 0,$$

[43] In this respect, our approach resembles the approach taken in Chernozhukov and Hong (2003).

where F^0 denotes the true value of F. Note too, that F need not specifically be a set of variances, it could include any sort of second moment, or even impulse responses from a VAR. We suppose that F is a set of variances, only for specificity.

The GMM estimator of F is $\hat{F}$ such that

$$g_T\left(\hat{F}\right) \equiv \frac{1}{T}\sum_{t=1}^{T} h_t\left(\hat{F}\right) = 0.$$

This corresponds to the standard variance estimator. The sampling uncertainty in $\hat{F}$ can be estimated using standard GMM formulas. Thus, define

$$\begin{aligned} D &= \frac{\partial g_T(F)}{\partial F'}|_{F=F^0} = -I \\ S^0 &= \sum_{j=-\infty}^{\infty} E h_t\left(F^0\right) h_{t-j}\left(F^0\right)'. \end{aligned}$$

Then, for T large, $\sqrt{T}\left(\hat{F}-F^0\right) \sim N\left(0, D^{-1}SD^{-1}\right)$, or,

$$\hat{F} \sim N\left(F^0, \frac{S^0}{T}\right). \tag{B.90}$$

The zero-frequency spectral density, S^0, is estimated using $\hat{S}$, where

$$\begin{aligned} \hat{S} &= \hat{C}(0) + \left(1-\frac{1}{l+1}\right)^{\theta}\left[\hat{C}(1)+\hat{C}(1)'\right] + \left(1-\frac{2}{l+1}\right)^{\theta}\left[\hat{C}(2)+\hat{C}(2)'\right] \\ &\quad + \cdots + \left(1-\frac{l}{l+1}\right)^{\theta}\left[\hat{C}(l)+\hat{C}(l)'\right] \end{aligned}$$

Here,

$$\hat{C}(j) = \begin{cases} \frac{1}{T-j}\sum_{t=j+1}^{T} h_t\left(\hat{F}\right) h_{t-j}\left(\hat{F}\right)' & j = 0,...,l \\ \hat{C}(-j)' & j = -1,...,l \end{cases}.$$

In practice, we set $\theta = 1$ and $l = 2$. We conclude that, for large T,

$$\hat{F} \sim N\left(F^0, \frac{\hat{S}}{T}\right) \tag{B.91}$$

We suppose that the data are generated by our DSGE model with parameters, θ. The mapping from θ to F is denoted by $F(\theta)$. Results (B.91) implies the following. Conditional on a set of parameter values, θ, and for sufficiently large T, the density of $\hat{F}$ is

$$f\left(\hat{F}|\theta\right) = \left(\frac{T}{2\pi}\right)^{\frac{n}{2}} \left|\hat{S}\right|^{-\frac{1}{2}} \exp\left[-\frac{T}{2}\left(\hat{F}-F(\theta)\right)' \hat{S}^{-1}\left(\hat{F}-F(\theta)\right)\right].$$

Here, $|\cdot|$ denotes the determinant operator and n is the number of elements in w_t. The object, $f\left(\hat{F}|\theta\right)$, is the likelihood of $\hat{F}$, given θ. Let $p(\theta)$ denote our primitive priors, before observing $\hat{F}$. According to Bayes rule, after observing $\hat{F}$ the posterior on those priors is:

$$p\left(\theta|\hat{F}\right) = \frac{f\left(\hat{F}|\theta\right) p(\theta)}{\int_{\tilde{\theta}} f\left(\hat{F}|\tilde{\theta}\right) p\left(\tilde{\theta}\right) d\tilde{\theta}}. \tag{B.92}$$

The denominator of this expression is the marginal density of $\hat{F}$. Since this marginal density is not a function of θ, we need not be concerned with it when we compute the mode of the posterior distribution after observing our actual sample of data (denote this by Y). The marginal density of $\hat{F}$ is also not of direct concern to us when we apply the MCMC algorithm to approximate the posterior distribution of θ after observing both $\hat{F}$ and Y (we denote this by $p\left(\theta|Y, \hat{F}\right)$). However, the marginal density of $\hat{F}$ does need to be computed if we wish to compare $p\left(\theta|\hat{F}\right)$ with the final posterior distribution. We turn to a brief discussion of this now.

One strategy for computing $p\left(\theta|\hat{F}\right)$ is to use the MCMC algorithm. This would be computed using a jump distribution whose variance is proportional to minus the inverse of the second derivative of $f\left(\hat{F}|\theta\right) p(\theta)$, evaluated at θ^*, where

$$\theta^* = \arg\max_{\theta} f\left(\hat{F}|\theta\right) p(\theta). \tag{B.93}$$

We assume there is unique interior solution to the maximization problem in (B.93). An alternative strategy for computing $p\left(\theta|\hat{F}\right)$ uses the Laplace approximation. That approximation is only justified when T is large, but then our basic $p\left(\theta|\hat{F}\right)$ only has an asymptotic justification anyway. A disadvantage of the Laplace approximation is that, by construction, it is symmetric around θ^*, and thus it may hide asymmetries induced by possible asymmetries in $p(\theta)$.

To derive the Laplace approximation to $p\left(\theta|\hat{F}\right)$, define

$$g(\theta) \equiv \log p\left(\theta|\hat{F}\right) = \log f\left(\hat{F}|\theta\right) + \log p(\theta) - c,$$

where c is not a function of θ. Define

$$g_{\theta\theta} = -\frac{\partial^2 g(\theta)}{\partial\theta\partial\theta'}|_{\theta=\theta^*}.$$

The second order Taylor series expansion of g about $\theta = \theta^*$ is:

$$g(\theta) = g(\theta^*) - \frac{1}{2}(\theta - \theta^*)' g_{\theta\theta}(\theta - \theta^*),$$

where the slope term is zero because of our assumption that the solution to (B.93) is interior. Then,

$$f\left(\hat{F}|\theta\right) p(\theta) \approx f\left(\hat{F}|\theta^*\right) p(\theta^*) \exp\left[-\frac{1}{2}(\theta - \theta^*)' g_{\theta\theta}(\theta - \theta^*)\right].$$

Note that this approximation is exact for large enough T since in this case p is dominated by f, in which case p is Normal. Note also that

$$\frac{1}{(2\pi)^{\frac{n}{2}}} |g_{\theta\theta}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}(\theta-\theta^*)' g_{\theta\theta}(\theta-\theta^*)\right]$$

is a multivariate normal distribution, so that

$$\int \frac{1}{(2\pi)^{\frac{n}{2}}} |g_{\theta\theta}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}(\theta-\theta^*)' g_{\theta\theta}(\theta-\theta^*)\right] d\theta = 1.$$

Bringing together the previous results, we obtain:

$$\begin{aligned}
& \int f\left(\hat{F}|\theta\right) p(\theta)\, d\theta \\
\approx\ & \int f\left(\hat{F}|\theta^*\right) p(\theta^*) \exp\left[-\frac{1}{2}(\theta-\theta^*)' g_{\theta\theta}(\theta-\theta^*)\right] d\theta \\
=\ & \frac{f\left(\hat{F}|\theta^*\right) p(\theta^*)}{\frac{1}{(2\pi)^{\frac{n}{2}}} |g_{\theta\theta}|^{\frac{1}{2}}} \int \frac{1}{(2\pi)^{\frac{n}{2}}} |g_{\theta\theta}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}(\theta-\theta^*)' g_{\theta\theta}(\theta-\theta^*)\right] d\theta \\
=\ & \frac{f\left(\hat{F}|\theta^*\right) p(\theta^*)}{\frac{1}{(2\pi)^{\frac{n}{2}}} |g_{\theta\theta}|^{\frac{1}{2}}},
\end{aligned}$$

by the integral property of the normal distribution. We now have the Laplace approximation to the denominator in (B.92). We therefore have a simple expression for $p\left(\theta|\hat{F}\right)$.

B.8. Calibration of Tax Rates

We briefly discuss the treatment of the tax rates. The data discussed below refers to the sample period 1995q1-2010q3, or a subset of that period when data availability is limited. In the versions of our model without financial frictions, capital is accumulated and capital income accrues directly to the household. However, an observationally equivalent representation of the model has these activities occurring in the firm. This latter interpretation is the convenient one, when thinking about the data and, in particular, the measurement of τ^k. We set the tax rate on capital income, τ^k, to 0.25. We arrived at this number as follows. The statutory rate on household capital income is 30 percent and the statutory rate on corporate income is 28 percent. Combining these two numbers we conclude that the statutory rate on corporate and household income is 50 percent. Indirect evidence from Devereux, Griffith and Klemm (2002) suggests to us that the effective tax on capital income may be one half this amount, and this is why we set $\tau^k = 0.25$ in the model. We reach this conclusion because of the Devereux, Griffith and Klemm observation that the effective corporate income tax is roughly 1/2 of the statutory rate and we adopt the rough approximation that the same applies to the household tax rate. Our assumption that τ^k is constant is also motivated by Devereux, Griffith and Klemm. Their measure of the corporate component of the effective capital income tax rate exhibits very little variation over the part that overlaps with our sample, i.e. 1995-2005.

Now we turn to the tax rate on bonds, τ^b. We set $\tau^b = 0$ to be able to match the pre-tax real rate on bonds of 2.25%. Setting $\tau^b = 0$ is required to get the interest rate on bonds to be this low, given the high GDP growth rate, log utility of consumption and β below 1.

For evidence on τ^w we use the data collected by ALLV. Based on these data, we set the payroll tax rate, τ^w, to 0.35. Data on the value-added tax on consumption, τ^c, and the personal income tax rate that applies to labor, τ^y, are available from Statistics Sweden and indicate $\tau^c = 0.25$ and $\tau^y = 0.3$. We keep these tax rates constant because they exhibit very little variability over this period.

B.9. Measurement Equations

Below we report the measurement equations we use to link the model to the data. Our data series for inflation and interest rates are annualized in percentage terms, so we make the same transformation for the model variables i.e. multiplying by 400:[44]

$$
\begin{aligned}
R_t^{data} &= 400(R_t - 1) - \vartheta_1 400(R-1) \\
R_t^{*,data} &= 400(R_t^* - 1) - \vartheta_1 400(R^*-1)
\end{aligned}
$$

$$
\begin{aligned}
\pi_t^{d,data} &= 400 \log \pi_t - \vartheta_1 400 \log \pi + \varepsilon_{\pi,t}^{me} \\
\pi_t^{c,data} &= 400 \log \pi_t^c - \vartheta_1 400 \log \pi^c + \varepsilon_{\pi^c,t}^{me} \\
\pi_t^{i,data} &= 400 \log \pi_t^i - \vartheta_1 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \\
\pi_t^{*,data} &= 400 \log \pi_t^* - \vartheta_1 400 \log \pi^*,
\end{aligned}
$$

where $\varepsilon_{i,t}^{me}$ denote the measurement errors for the respective variables. In addition, we introduce the parameters $\vartheta_1 \in \{0,1\}$ and $\vartheta_2 \in \{0,1\}$ which allows us to handle demeaned and non-demeaned data. In particular, our data for inflation and interest rates is not demeaned, and we therefore set $\vartheta_1 = 0$. An alternative specification would be that we use demeaned inflation and interest rates which would require to set $\vartheta_1 = 1$ in order to correctly match the data with the model.

We match hours worked per capita in terms of deviation from steady state. First differences and deviations from steady state are written in percentages so model variables are multiplied by 100 accordingly:

$$
\hat{H}_t^{data} = 100 \left(\frac{H_t^{meas} - H^{meas}}{H^{meas}} \right) + \varepsilon_{H,t}^{me}
$$

[44] Note that in the data we measure $\pi_t^{data} = 400(\ln P_t^{data} - \ln P_{t-1}^{data})$. In the model, we have defined $\pi_t = \frac{P_t}{P_{t-1}}$. Matching data with the model results in the above measurement equations for inflation.

We use demeaned first-differenced data for the remaining variables. This implies setting the second indicator variable $\vartheta_2 = 1$.

$$
\begin{aligned}
\Delta \log Y_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log \left(y_t - p_t^i a\left(u_t\right) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} - d_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} \left(\tilde{v}_t^j\right)^2 \left[1 - \mathcal{F}_t^j\right] l_t^j \right)) - \\
&\quad \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{y,t}^{me} \\
\Delta \log Y_t^{*,data} &= 100(\log \mu_{z^+,t} + \Delta \log y_t^*) - \vartheta_2 100(\log \mu_{z^+}) \\
\Delta \log C_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log c_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{c,t}^{me} \\
\Delta \log X_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log x_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{x,t}^{me} \\
\Delta \log q_t^{data} &= 100 \Delta \log q_t + \varepsilon_{q,t}^{me} \\
\Delta \log M_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log \ \text{Imports}_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
&= 100 \left[\log \mu_{z^+,t} + \Delta \log \left(\begin{array}{c} c_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}} \\ +i_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda_{m,i}}{1-\lambda_{m,i}}} \\ +x_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda_{m,x}}{1-\lambda_{m,x}}} \end{array} \right) \right] - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{M,t}^{me} \\
\Delta \log I_t^{data} &= 100 \left[\log \mu_{z^+,t} + \log \mu_{\psi,t} + \Delta \log i_t\right] - \vartheta_2 100(\log \mu_{z^+} + \log \mu_{\psi}) + \varepsilon_{I,t}^{me} \\
\Delta \log G_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log g_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{g,t}^{me}
\end{aligned}
$$

Note that neither measured GDP nor measured investment include investment goods used for capital maintenance. The reason is that the documentation for calculation of the Swedish National Accounts (SOU (2002)) indicate that these are not included in the investment definition (and the national accounts are primarily based on the expenditure side). To calculate measured GDP we also exclude monitoring costs and recruitment costs. Note that it is measured GDP that enters the Taylor rule.

The real wage is measured by the employment-weighted average Nash bargaining wage in the model:

$$
w_t^{avg} = \frac{1}{L} \sum_{j=0}^{N-1} l_t^j G_{t-j,j} w_{t-j} \bar{w}_{t-j}
$$

Given this definition the measurement equation for demeaned first-differenced wages is:

$$
\Delta \log(W_t/P_t)^{data} = 100 \Delta \log \frac{\tilde{W}_t}{z_t^+ P_t} = 100(\log \mu_{z^+,t} + \Delta \log w_t^{avg}) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{W/P,t}^{me}
$$

Finally, we measure demeaned first-differenced net worth, interest rate spread and unemployment as follows:

$$
\begin{aligned}
\Delta \log N_t^{data} &= 100(\log \mu_{z^+,t} + \Delta \log n_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{N,t}^{me} \\
\Delta \log Spread_t^{data} &= 100 \Delta \log(Z_{t+1} - R_t) = 100 \Delta \log \left(\frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t \right) + \varepsilon_{Spread,t}^{me} \\
\Delta \log Unemp_t^{data} &= 100 \Delta \log(1 - L_t) + \varepsilon_{Unemp,t}^{me}.
\end{aligned}
$$