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# Technical Appendix
## "Involuntary Unemployment and the Business Cycle"

by

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Apart from sections B and D below, the derivations in this appendix are standard and can be found in the technical appendices in ACEL and CEE, for example. We display them here so that the technical results are all available in one place and in a consistent notation. The material in sections B and D involve straightforward (but, sometimes tedious) extensions of the results in the manuscript. Equation numbers refer to equation numbers in the text of the paper.

## A. Equilibrium Conditions in the Model Without Capital

We first derive the equilibrium conditions associated with price setting. We then derive the other conditions.

### A.1. Price Setting Equilibrium Conditions

The discounted profits of the

$$E_t \sum_{j=0}^{\infty} \beta^j \ \upsilon_{t+j} \overbrace{\left[ \overbrace{P_{i,t+j} Y_{i,t+j}}^{\text{revenues}} - \overbrace{P_{t+j} s_{t+j} Y_{i,t+j}}^{\text{total cost}} \right]}^{\text{period } t+j \text{ profits sent to household}},$$

where $\upsilon_{t+j}$ denotes the period $t+j$ Lagrange multiplier on household budget constraint. Let $\tilde{P}_t$ denote the price selected by each of the $1 - \xi_p$ firms that have opportunity to reoptimize price in period $t$. Because firms have no state variables, they are only concerned about future histories in which they cannot reoptimize price. This leads to the following objective function for a firm that can reoptimize price in period $t$ :

$$E_t \sum_{j=0}^{\infty} \left( \beta \xi_p \right)^j \upsilon_{t+j} \left[ \tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j} \right].$$

Substitute out for intermediate good firm output using the demand curve:

$$E_t \sum_{j=0}^{\infty} \left( \beta \xi_p \right)^j \upsilon_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[ \tilde{P}_t^{1-\varepsilon} - P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon} \right],$$

where

$$\varepsilon \equiv \frac{\lambda_f}{\lambda_f - 1}$$

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Differentiate with respect to $\tilde{P}_t$ :

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon} \left[(1-\varepsilon)\left(\tilde{P}_t\right)^{-\varepsilon} + \varepsilon P_{t+j} s_{t+j} \tilde{P}_t^{-\varepsilon-1}\right] = 0,$$

or,

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j v_{t+j} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j}\right] = 0.$$

Note that when $\xi_p = 0$, one obtains the standard result, that price is fixed markup over marginal cost.

Now, substitute out the multiplier:

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \overbrace{\frac{u'\left(C_{t+j}\right)}{P_{t+j}}}^{\text{marginal utility of currency } = v_{t+j}} Y_{t+j} P_{t+j}^{\varepsilon+1} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j}\right] = 0,$$

or, given our assumption about log utility,

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} P_{t+j}^{\varepsilon} \left[\frac{\tilde{P}_t}{P_{t+j}} - \lambda_f s_{t+j}\right] = 0.$$

or,

$$E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \left[\tilde{p}_t X_{t,j} - \lambda_f s_{t+j}\right] = 0,$$

where

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t}, X_{t,j} = \begin{cases} \frac{1}{\pi_{t+j}\pi_{t+j-1}\cdots\pi_{t+1}}, & j \geq 1 \\ 1, & j = 0. \end{cases}, X_{t,j} = X_{t+1,j-1}\frac{1}{\pi_{t+1}}, j > 0$$

Solving for $\tilde{p}_t$ :

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\varepsilon} \lambda_f s_{t+j}}{E_t \sum_{j=0}^{\infty} \left(\beta \xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{1-\varepsilon}} = \frac{K_{p,t}}{F_{p,t}},$$

We now obtain simple recursive expressions for $K_{p,t}$ and $F_{p,t}$.

Consider $K_{p,t}$ first. Accordingly,

$$
\begin{aligned}
K_{p,t} &= E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \sum_{j=1}^{\infty} \left(\beta\xi_p\right)^{j-1} \frac{Y_{t+j}}{C_{t+j}} \left(\frac{1}{\pi_{t+1}} X_{t+1,j-1}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f s_{t+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p \overbrace{E_t E_{t+1}}^{=E_t} \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} E_{t+1} \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j X_{t+1,j}^{-\frac{\lambda_f}{\lambda_f-1}} \lambda_f \frac{Y_{t+1+j}}{C_{t+1+j}} s_{t+1+j} \\
&= \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{-\frac{\lambda_f}{\lambda_f-1}} K_{p,t+1}
\end{aligned}
$$

so that

$$
K_{p,t} = \lambda_f \frac{Y_t}{C_t} s_t + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1}. \tag{A.1}
$$

Similarly,

$$
F_{p,t} \equiv E_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \frac{Y_{t+j}}{C_{t+j}} \left(X_{t,j}\right)^{\frac{\lambda_f}{1-\lambda_f}} = \frac{Y_t}{C_t} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} F_{p,t+1} \tag{A.2}
$$

In (A.1), marginal cost is defined in (3.8) and (??), repeated here for convenience:

$$
s_t = \frac{1}{\lambda_f} \frac{C_t z_h\left(h_t, \varsigma_t\right)}{A_t}.
$$

Evaluating (3.7)

$$
P_t = \left[\left(1-\xi_p\right) \tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p P_{t-1}^{\frac{1}{1-\lambda_f}}\right]^{1-\lambda_f}.
$$

Dividing by $P_t$ and rearranging, we obtain:

$$
\tilde{p}_t = \left[\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right]^{1-\lambda_f} \tag{A.3}
$$

We conclude that the equilibrium conditions associated with price setting are (A.1), (A.2) and:

$$
\left[\frac{1-\xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1-\xi_p}\right]^{1-\lambda_f} = \frac{K_{p,t}}{F_{p,t}}. \tag{A.4}
$$

3

In a zero inflation steady state without price distortions,

$$K_p = F_p \tag{A.5}$$

according to (A.4). It then follows from (A.1) and (A.2) that

$$s = 1/\lambda_f, \quad F_p = \frac{\frac{1}{1-\eta_g}}{1 - \beta\xi_p}, \tag{A.6}$$

where $1/\left(1 - \eta_g\right)$ is the steady state output to consumption ratio, $Y_t/C_t$, and $\eta_g$ is the steady state $G_t/Y_t$ ratio. the object, $\eta_g$, is something we fix in the calculations. See below for further discussion.

In the analysis of the linearized equilibrium conditions in the manuscript, we set $g_t, \eta_g \equiv 0$. Differentiating (A.1), (A.2) and (A.4) in steady state:

$$\hat{K}_{p,t} = \left(1 - \beta\xi_p\right)\hat{s}_t + \beta\xi_p \left[\frac{\lambda_f}{\lambda_f - 1}\hat{\pi}_{t+1} + \hat{K}_{p,t+1}\right]$$

$$\hat{F}_{p,t} = \beta\xi_p \left[\frac{1}{\lambda_f - 1}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right]$$

$$\hat{K}_{p,t} = \frac{\xi_p}{1 - \xi_p}\hat{\pi}_t + \hat{F}_{p,t}$$

Substitute the third equation into the first:

$$\frac{\xi_p}{1 - \xi_p}\hat{\pi}_t + \hat{F}_{p,t} = \left(1 - \beta\xi_p\right)\hat{s}_t + \beta\xi_p \left[\frac{\lambda_f}{\lambda_f - 1}\hat{\pi}_{t+1} + \frac{\xi_p}{1 - \xi_p}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right]$$

Substitute the recursive expression for $\hat{F}_{p,t}$ :

$$\frac{\xi_p}{1 - \xi_p}\hat{\pi}_t + \beta\xi_p \left[\frac{1}{\lambda_f - 1}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right] = \left(1 - \beta\xi_p\right)\hat{s}_t + \beta\xi_p \left[\frac{\lambda_f}{\lambda_f - 1}\hat{\pi}_{t+1} + \frac{\xi_p}{1 - \xi_p}\hat{\pi}_{t+1} + \hat{F}_{p,t+1}\right],$$

and rearrange, to obtain:

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{\left(1 - \beta\xi_p\right)\left(1 - \xi_p\right)}{\xi_p}\hat{s}_t.$$

This is the linearized Phillips curve used in the text.

## A.2. Other Private Sector Equilibrium Conditions

We now derive the equilibrium relationship between aggregate consumption and aggregate inputs, (3.11), using the approach described in Yun (1996). Let $Y_t^*$ denote the unweighted integral of intermediate inputs:

$$Y_t^* \equiv \int_0^1 Y_{i,t}di = \int_0^1 A_t h_{i,t}di = A_t h_t.$$

Using the demand curve, (3.6),

$$Y_t^* = \int_0^1 Y_{i,t} di = Y_t \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\frac{\lambda_f}{\lambda_f - 1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f - 1}} \int_0^1 P_{i,t}^{-\frac{\lambda_f}{\lambda_f - 1}} di = Y_t P_t^{\frac{\lambda_f}{\lambda_f - 1}} (P_t^*)^{-\frac{\lambda_f}{\lambda_f - 1}},$$

say, where

$$P_t^* \equiv \left[\int_0^1 P_{i,t}^{-\frac{\lambda_f}{\lambda_f - 1}} di\right]^{\frac{-(\lambda_f - 1)}{\lambda_f}} = \left[(1 - \xi_p) \tilde{P}_t^{-\frac{\lambda_f}{\lambda_f - 1}} + \xi_p \left(P_{t-1}^*\right)^{-\frac{\lambda_f}{\lambda_f - 1}}\right]^{\frac{-(\lambda_f - 1)}{\lambda_f}}.$$

Combining the preceding three equations, we obtain (3.11), which we reproduce here for convenience:

$$
\begin{aligned}
\frac{C_t}{A_t} &= p_t^* h_t - g_t n_t \\
\log \frac{C_t}{C_{t-1}} &= \log \left(p_t^* h_t - g_t n_t\right) - \log \left(p_{t-1}^* h_{t-1} - g_{t-1} n_{t-1}\right) + g_{A,t}
\end{aligned}
$$

$$G_t + C_t = p_t^* A_t h_t. \tag{A.7}$$

In (A.7), include government consumption expenditures, which we model as follows:

$$G_t = g_t N_t,$$

where $\log g_t$ is potentially a stationary stochastic process independent of any other shocks in the system, such as $A_t$. Also,

$$N_t = A_t^\gamma N_{t-1}^{1-\gamma}, \ 0 < \gamma \leq 1. \tag{A.8}$$

In the extreme case, $\gamma = 1$, this reduces to the model of $G_t$ studied in Christiano and Eichenbaum (1992). A problem with the latter model, however, is that it implies $G_t$ moves immediately with shocks to $A_t$, an implication that seems implausible. With $\gamma$ close to zero, the immediate impact of $A_t$ on $G_t$ is virtually nil. Yet, regardless of the value of $\gamma$, $G_t/A_t$ converges to a constant in nonstochastic steady state. This is necessary if we are to have balanced growth in the case that $A_t$ follows a growth path in the steady state. To see that $G_t/A_t$ converges in steady state, note

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}}\right)^{1-\gamma}, \ n_t \equiv \frac{N_t}{A_t}, \tag{A.9}$$

so that the steady state value of $n_t$ is:

$$n = \left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}. \tag{A.10}$$

From this we conclude that

$$\frac{G_t}{A_t} = g_t n_t, \tag{A.11}$$

is constant in a steady state too. In our steady state analysis, we are interested in fixing $\eta_g$, the steady state ratio of $G_t$ to $Y_t$ :

$$\eta_g = \frac{G}{Y} = \frac{G/A}{Y/A} = \frac{g\left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}}{h}.$$

In practice, we fix $\eta_g$, $g_A$ and $h$ at their empirically relevant values and so the above equation can be thought of as determining a value for $g$.

In (A.7), we have used the goods clearing condition, (3.10), and $p_t^*$ captures the distortions to output due to the price setting frictions:

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}.$$

The law of motion of the distortions is, using (A.3) and (A.4):

$$p_t^* = \left[ (1 - \xi_p) \left( \frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right]^{-1}. \tag{A.12}$$

By (A.1), we require an expression for $\lambda_f Y_t s_t / C_t$. After substituting out for $C_t$ from (A.7) into the expression for marginal cost, (3.8), and using (??), we obtain:

$$\lambda_f \frac{Y_t}{C_t} s_t = \frac{1}{\lambda_f} \frac{C_t z_h \left( h_t, \varsigma_t \right)}{A_t} \lambda_f \frac{Y_t}{C_t} = \frac{Y_t z_h \left( h_t, \varsigma_t \right)}{A_t}.$$

Then,

$$\lambda_f \frac{Y_t}{C_t} s_t = p_t^* h_t z_h \left( h_t, \varsigma_t \right), \tag{A.13}$$

where $z_h$ denotes the marginal disutility of labor and $z$ is defined in (2.19). Equation (A.13) combines the marginal cost of intermediate good firms with the optimal employment decision by the family, (3.4). By (A.2) we require an expression for $Y_t / C_t$ :

$$\frac{Y_t}{C_t} = \frac{p_t^* A_t h_t}{p_t^* A_t h_t - g_t n_t A_t} = \frac{p_t^* h_t}{p_t^* h_t - g_t n_t} \tag{A.14}$$

The family's intertemporal Euler equation is, using (3.3):

$$1 = \beta E_t \frac{p_t^* h_t - g_t n_t}{\left[ p_{t+1}^* h_{t+1} - g_{t+1} n_{t+1} \right] g_{A,t+1}} \frac{R_t}{\pi_{t+1}}, \tag{A.15}$$

6

where we have used (A.7) to substitute out for $C_t$ and $C_{t+1}$ and

$$g_{A,t+1} \equiv \frac{A_{t+1}}{A_t}.$$

Writing the equilibrium conditions, (A.1) and (A.2), after using (A.13) and (A.14), we obtain:

$$K_{p,t} = p_t^* h_t z_h (h_t, \varsigma_t) + \beta \xi_p E_t \left( \frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} \qquad (A.16)$$

$$F_{p,t} = \frac{p_t^* h_t}{p_t^* h_t - g_t n_t} + \beta \xi_p E_t \left( \frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} F_{p,t+1} \qquad (A.17)$$

The 6 private sector equilibrium conditions are (A.16), (A.17), (A.4), (A.9), (A.12), and (A.15). There are 7 endogenous variables:

$$h_t, p_t^*, R_t, \pi_t, K_{p,t}, F_{p,t}, n_t.$$

When government spending is zero, then (A.9) and $n_t$ drop from the system.

## A.3. Closing the Model

The previous subsection enumerated 6 equilibrium conditions for determining 7 endogenous variables. One way to close the model is to consider the Ramsey-optimal allocations. Another is to add a Taylor rule equation.

## A.3.1. Ramsey-optimal Allocations

We set government spending to zero here. Substituting out for $C_t$ using (A.7), and for $s_t$ using (A.13), the Lagrangian representation of the Ramsey problem is:

$$\max_{p_t^*, h_t, R_t, \pi_t, F_{p,t}, K_{p,t}} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log h_t + \log p_t^* - z\left(h_t, \varsigma_t\right)$$

$$+ \lambda_{1t} \left[ \frac{1}{p_t^* h_t} - E_t \frac{\beta}{p_{t+1}^* h_{t+1} g_{A,t+1}} \frac{R_t}{\pi_{t+1}} \right]$$

$$+ \lambda_{2t} \left[ \frac{1}{p_t^*} - \left( (1 - \xi_p) \left( \frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right) \right]$$

$$+ \lambda_{3t} \left[ 1 + \beta \xi_p E_t \left( \frac{1}{\pi_{t+1}} \right)^{\frac{1}{1 - \lambda_f}} F_{p,t+1} - F_{p,t} \right]$$

$$+ \lambda_{4t} p_t^* \left[ h_t z_h\left(h_t, \varsigma_t\right) + \beta \xi_p E_t \left( \frac{1}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1 - \lambda_f}} \frac{p_{t+1}^*}{p_t^*} \frac{K_{p,t+1}}{p_{t+1}^*} - \frac{K_{p,t}}{p_t^*} \right]$$

$$+ \lambda_{5t} \left[ F_{p,t} \left( \frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{1 - \lambda_f} - K_{p,t} \right] \}.$$

In the fourth Lagrangian constraint, it is convenient to factor out $p_t^*$. We conjecture (and can later verify) that the first, third, fourth and fifth constraints are not binding on the problem. In particular, we can simply select $R_t$ to satisfy the first constraint, $F_{p,t}$ to satisfy the third, $K_{p,t}$ to satisfy the fourth, and we then need to verify that the fifth constraint is satisfied, as well as $R_t \geq 1$.

Implementing the conjecture, the problem, with $g_t \equiv 0$, reduces to:

$$\max_{p_t^*, h_t, \pi_t} E_0 \sum_{t=0}^{\infty} \beta^t \{ \log p_t^* + \log h_t - z\left(h_t, \varsigma_t\right)$$

$$+ \lambda_{2t} \left[ \frac{1}{p_t^*} - \left( (1 - \xi_p) \left( \frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p} \right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f - 1}}}{p_{t-1}^*} \right) \right] \},$$

where $\log A_t$ in the utility function is ignored because it cannot be controled. This leads to the efficiency condition for hours and $p_t^*$:

$$h_t z_h\left(h_t, \varsigma_t\right) = 1. \tag{A.18}$$

Interestingly, this coincides with the first-best setting for $h_t$. The efficiency conditions for $p_t^*$

and $\pi_t$ are, respectively,

$$p_t^* = \lambda_{2t} - \beta\lambda_{2t+1}\xi_p\pi_{t+1}^{\frac{\lambda_f}{\lambda_f-1}} = 0$$

$$\left(\frac{1 - \xi_p\pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p}\right)^{\lambda_f-1} = \frac{\pi_t}{p_{t-1}^*}. \tag{A.19}$$

Rearranging, and substituting into (A.12), we obtain:

$$p_t^* = \left[\left(p_{t-1}^*\right)^{\frac{1}{\lambda_f-1}}\xi_p + \left(1 - \xi_p\right)\right]^{\lambda_f-1} \tag{A.20}$$

$$\pi_t = \frac{p_{t-1}^*}{p_t^*}. \tag{A.21}$$

We now verify that all the constraints on the Ramsey problem assumed to be non-binding are in fact satisfied. Consider the fourth Lagrangian constraint, after substituting out (A.18), (A.20), and (A.21):

$$1 + \beta\xi_p E_t\left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}}\frac{K_{p,t+1}}{p_{t+1}^*} = \frac{K_{p,t}}{p_t^*}.$$

We let this equation define $K_{p,t}$, so that the fourth Lagrangian is satisfied. The requirement that the third Lagrangian constraint also be satisfied implies

$$\frac{K_{p,t}}{p_t^*} = F_{p,t}. \tag{A.22}$$

Letting (A.22) define $F_{p,t}$ we have that the third Lagrangian is satisfied. From (A.19) and (A.21),

$$\left(\frac{1 - \xi_p\pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p}\right)^{\lambda_f-1} = p_t^*. \tag{A.23}$$

It follows from (A.22) and (A.23) that the fifth Lagrangian constraint is satisfied. The first Lagrangian constraint is trivially satisfy if we use it to define the nominal rate of interest, $R_t$. As long as the shocks are not too big, $R_t \geq 1$.

We have established that allocations in which hours worked, the price distortions and inflation satisfy (A.18), (A.20), and (A.21), respectively, solve the Ramsey problem.

### A.3.2. Taylor Rule Equilibrium

Consider the following policy rule:

$$R_t = R^{1-\rho_R}R_{t-1}^{\rho_R}\pi_t^{(1-\rho_R)r_\pi}x_t^{(1-\rho_R)r_y}, \tag{A.24}$$

where
$$x_t = \frac{Y_t}{\tilde{Y}_t},$$

and $\tilde{Y}_t$ is the first-best level of consumption. We can write

$$x_t = \frac{p_t^* A_t h_t}{A_t \tilde{h}_t} = \frac{p_t^* h_t}{\tilde{h}_t},$$

where $h_t$ is equilibrium hours worked and $\tilde{h}_t$ is the value of hours worked in the efficient equilibrium, which is define by:

$$\frac{A_t}{\tilde{C}_t} = z_h\left(\tilde{h}_t, \varsigma_t\right).$$

Here, $\tilde{C}_t$ denotes consumption in the efficient equilibrium. Using the resource constraint with no price distortions, (A.7) with $p_t^* = 1$, and rewriting:

$$\frac{A_t}{A_t \tilde{h}_t - G_t} = \frac{1}{\tilde{h}_t - g_t n_t} = z_h\left(\tilde{h}_t, \varsigma_t\right),$$

or,

$$\left(\tilde{h}_t - g_t n_t\right) z_h\left(\tilde{h}_t, \varsigma_t\right) = 1.$$

Equilibrium is characterized by the requirement that (A.24) as well as the above equation be satisfied, and also (A.16), (A.17), (A.4), (A.9), (A.12), and (A.15). Summarizing:

$$0 = E_t\left[p_t^* h_t z_h\left(h_t, \varsigma_t\right) + \beta \xi_p \left(\frac{1}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} - K_{p,t}\right] \tag{A.25}$$

$$0 = E_t\left[\frac{p_t^* h_t}{p_t^* h_t - g_t n_t} + \beta \xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} F_{p,t+1} - F_{p,t}\right] \tag{A.26}$$

$$K_{p,t} = \left[\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f - 1}}}{1 - \xi_p}\right]^{1-\lambda_f} F_{p,t} \tag{A.27}$$

$$p_t^* = \left[(1-\xi_p)\left(\frac{1 - \xi_p \pi_t^{\frac{1}{\lambda_f-1}}}{1 - \xi_p}\right)^{\lambda_f} + \frac{\xi_p \pi_t^{\frac{\lambda_f}{\lambda_f-1}}}{p_{t-1}^*}\right]^{-1} \tag{A.28}$$

$$1 = \beta E_t \frac{p_t^* h_t - g_t n_t}{\left(p_{t+1}^* h_{t+1} - g_{t+1} n_{t+1}\right) g_{A,t+1}} \frac{R_t}{\pi_{t+1}} \tag{A.29}$$

$$R_t = R^{1-\rho_R} R_{t-1}^{\rho_R} \pi_t^{(1-\rho_R)r_\pi} \left(\frac{p_t^* h_t}{\tilde{h}_t}\right)^{(1-\rho_R)r_y} \tag{A.30}$$

$$1 = \left(\tilde{h}_t - g_t n_t\right) z_h\left(\tilde{h}_t, \varsigma_t\right) \tag{A.31}$$

$$n_t = \left(\frac{n_{t-1}}{g_{A,t}}\right)^{1-\gamma} \tag{A.32}$$

The 8 variables to be determined by these 8 equations are:

$$K_{p,t}, F_{p,t}, h_t, \tilde{h}_t, p_t^*, \pi_t, R_t, n_t \tag{A.33}$$

The present discounted value of utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - z\left(h_t, \varsigma_t\right) \right] \tag{A.34}$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right) \right] + \frac{\log A_0}{1 - \beta} + \frac{1}{1 - \beta} E_0 \sum_{t=1}^{\infty} \beta^t \log g_{A,t}$$

The piece that is exogenous and additive is not interesting. Define

$$w\left(g_{A,0}, p_{-1}^*, \varsigma_0\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right) \right]$$

$$= \log\left(p_0^* h_0 - g_0 n_0\right) - z\left(h_0, \varsigma_0\right) + \beta E_0 w\left(g_{A,1}, p_0^*, \varsigma_1\right).$$

We can add this equation to the list, (A.25)-(A.30), above,

$$E_t \left[ \log\left(p_t^* h_t - g_t n_t\right) - z\left(h_t, \varsigma_t\right) + \beta w_{t+1} - w_t \right] = 0, \tag{A.35}$$

giving us one additional variable, $w_t$, and one additional equation.

This system can be solved in Dynare. Simply type in equations (A.25)-(A.32) and (A.35). We have three stochastic processes, $g_t$, $g_{A,t}$ and $\varsigma_t$. Let $x_t$ denote one of these. Then, the law of motion of $x_t$ is:

$$\log x_t = \left(1 - \rho_x\right) \log x + \rho_x \log x_{t-1} + \varepsilon_t^x,$$

for $x_t = g_t$, $g_{A,t}$, $\varsigma_t$.

## A.4. Steady State

From (A.5) and (A.6),

$$K_p = F_p, \ s = 1/\lambda_f, \ F_p = \frac{\frac{1}{1 - \eta_g}}{1 - \beta \xi_p}.$$

From (A.12), in a zero inflation steady state,

$$p^* = 1.$$

We want to impose that government spending is a given proportion, $\eta_g$, of total output in the steady state:

$$\eta_g = \frac{G}{C + G} = \frac{gnA}{Ah} = \frac{gn}{h} = \frac{g\left(\frac{1}{g_A}\right)^{\frac{1-\gamma}{\gamma}}}{h},$$

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by (A.10). Thus, the steady state value of $g$ is:

$$g = \eta_g h g_A^{\frac{1-\gamma}{\gamma}}, \quad gn = \eta_g h \tag{A.36}$$

Combining (A.13), the result for the steady state value of $s$ and (A.7), (A.11) to obtain:

$$\frac{h}{h - gn} = h z_h(h, \varsigma),$$

or, using (A.36),

$$1 = [1 - \eta_g] h z_h(h, \varsigma). \tag{A.37}$$

We now proceed to develop the formulas necessary to compute $z_h$. Recall, the disutility of labor function, $z(h_t)$ in (2.19). Write this in terms of $m_t$:

$$Z(m_t) = \log\left[Q(m_t)\left(e^{F + \varsigma_t(1 + \sigma_L)m_t^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2\varsigma_t^2(1 + \sigma_L)\sigma_L^2}{2\sigma_L + 1}m_t^{2\sigma_L + 1} - \eta\varsigma_t\sigma_L m_t^{\sigma_L + 1}, \tag{A.38}$$

where

$$\begin{aligned}
h_t &= m_t\eta + a^2\varsigma_t\sigma_L m_t^{\sigma_L + 1} \equiv Q(m_t), \tag{A.39}\\
m_t &= Q^{-1}(h_t),
\end{aligned}$$

where $Q^{-1}$ is inverse function of $Q$, defined by:

$$h_t = Q\left(Q^{-1}(h_t)\right). \tag{A.40}$$

Then,

$$z(h_t) = Z\left(Q^{-1}(h_t)\right). \tag{A.41}$$

We require the first and second derivatives of $z$. Thus,

$$z_h(h_t) = Z_m\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]'(h_t), \tag{A.42}$$

where $\left[Q^{-1}\right]'$ denotes the derivative of the function, $Q^{-1}$. To obtain an expression for $\left[Q^{-1}\right]'(h_t)$, differentiate (A.40) with respect to $h_t$:

$$1 = Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]'(h_t),$$

so that:

$$\left[Q^{-1}\right]'(h_t) = \frac{1}{Q'\left(Q^{-1}(h_t)\right)}. \tag{A.43}$$

We also require the second derivative of $Q^{-1}$. Differentiating (A.40) a second time with respect to $h_t$:

$$0 = Q''\left(Q^{-1}(h_t)\right)\left(\left[Q^{-1}\right]'(h_t)\right)^2 + Q'\left(Q^{-1}(h_t)\right)\left[Q^{-1}\right]''(h_t).$$

Substituting from the first derivative:

$$0 = Q'' \left( Q^{-1} \left( h_t \right) \right) \left( \frac{1}{Q' \left( Q^{-1} \left( h_t \right) \right)} \right)^2 + Q' \left( Q^{-1} \left( h_t \right) \right) \left[ Q^{-1} \right]'' \left( h_t \right),$$

so that

$$\left[ Q^{-1} \right]'' \left( h_t \right) = -Q'' \left( Q^{-1} \left( h_t \right) \right) \left( \frac{1}{Q' \left( Q^{-1} \left( h_t \right) \right)} \right)^3. \tag{A.44}$$

Substituting (A.43) into (A.42),

$$z_h \left( h_t \right) = \frac{Z_m \left( Q^{-1} \left( h_t \right) \right)}{Q' \left( Q^{-1} \left( h_t \right) \right)}.$$

Differentiating $z$ in (A.41) a second time,

$$z_{hh} \left( h_t \right) = Z_{mm} \left( Q^{-1} \left( h_t \right) \right) \left( \left[ Q^{-1} \right]' \left( h_t \right) \right)^2 + Z_m \left( Q^{-1} \left( h_t \right) \right) \left[ Q^{-1} \right]'' \left( h_t \right),$$

which, after substituting from (A.43) and (A.44), is:

$$
\begin{aligned}
z_{hh} \left( h_t \right) &= Z_{mm} \left( Q^{-1} \left( h_t \right) \right) \left( \frac{1}{Q' \left( Q^{-1} \left( h_t \right) \right)} \right)^2 - Z_m \left( Q^{-1} \left( h_t \right) \right) Q'' \left( Q^{-1} \left( h_t \right) \right) \left( \frac{1}{Q' \left( Q^{-1} \left( h_t \right) \right)} \right)^3 \\
&= \left( \frac{1}{Q' \left( Q^{-1} \left( h_t \right) \right)} \right)^2 Z_m \left( Q^{-1} \left( h_t \right) \right) \left[ \frac{Z_{mm} \left( Q^{-1} \left( h_t \right) \right)}{Z_m \left( Q^{-1} \left( h_t \right) \right)} - \frac{Q'' \left( Q^{-1} \left( h_t \right) \right)}{Q' \left( Q^{-1} \left( h_t \right) \right)} \right] \\
&= \left( \frac{1}{Q' \left( m_t \right)} \right)^2 Z_m \left( m_t \right) \left[ \frac{Z_{mm} \left( m_t \right)}{Z_m \left( m_t \right)} - \frac{Q'' \left( m_t \right)}{Q' \left( m_t \right)} \right],
\end{aligned}
$$

where the expressions for $Q'$, $Q''$, $Z_m$, $Z_{mm}$ can be obtained by symbolic differentiation of the underlying functions, (A.38) and (A.39).

The endogenous variables are

$$h, m, \sigma_z, u, \bar{p},$$

and the equations are:

$$
\begin{aligned}
(1) h &= m\eta + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \tag{A.45} \\
(2) \bar{p} &= \eta + \varsigma a^2 \left( 1 + \sigma_L \right) m^{\sigma_L} \\
(3) 1 &= \left[ 1 - \eta_g \right] h z_h \left( h, \varsigma \right) \\
(4) \sigma_z &= \frac{z_{hh} h}{z_h} \\
(5) \kappa^{okun} &= \frac{a^2 \varsigma \sigma_L^2 m^{\sigma_L} \left( 1 - u \right)}{1 - u + a^2 \varsigma \sigma_L^2 m^{\sigma_L}} \\
u &= \frac{m - h}{m}.
\end{aligned}
$$

The parameters are:

$$F, \varsigma, a, \eta, \sigma_L.$$

We solve the steady state using two different strategies, depending on our purposes. The first strategy, the straightforward one, takes the parameters as given and computes the steady values of the endogenous variables. We implement this strategy by placing a grid of values of $m$ on the unit interval. For each value of $m$ on the grid we compute $h$ using (1) and evaluate (3). We count the number of places on the grid where there is a switch in the sign of (3), *i.e.*, in $1 - \left[1 - \eta_g\right] h z_h\left(h, \varsigma\right)$. For each sign switch, we then narrowed down the corresponding value of $m$ that implements (3) exactly using a nonlinear equation solver. After this, we computed $\bar{p}$ using (2) as well as steady state unemployment, $u$. In practice, we found two sets of solutions to the steady state equations, but one was inadmissible because it implied $u < 0$. We applied this 'first strategy' to compare the steady states of the involuntary unemployment model studied in this section with the steady state of the full information model in section B holding model parameters fixed. Section B shows that the steady state of the full information model is also characterized by (A.45). The difference between the two models lies in the details of the function, $z_h$. With this approach to computing the steady state we are able to evaluate the impact on welfare and other variables of the assumption of limited information. We compute the value of information in consumption units as follows. According to (A.34), steady state utility is, apart from an additive constant, as follows:

$$U = \frac{1}{1 - \beta}\left[\log\left(h\right) - z\right].$$

Here, $h$ denotes steady state employment and $z$ denotes the steady state disutility of labor (the fact, $h - gn = \left(1 - \eta_g\right)h$, in steady state has been used here). Let utility in the steady state of the full information model be denoted:

$$U^{fi} = \frac{1}{1 - \beta}\left[\log\left(h^{fi}\right) - z^{fi}\right]$$

Let $\lambda$ denote the percent increase in consumption in the involuntary unemployment model that makes households in that model indifferent between staying in that environment, or converting to the environment with full information. Let $U\left(\lambda\right)$ denote the level of utility in the involuntary unemployment equilibrium when consumption is raised from $h$ to $h\left(1 + \lambda/100\right)$. We seek $\lambda$ such that

$$U\left(\lambda\right) = U^{fi}.$$

Note:

$$
\begin{aligned}
U\left(\lambda\right) &= \frac{1}{1 - \beta}\left[\log\left(h\right) + \log\left(1 + \lambda/100\right) - z\right] \\
&= U + \frac{1}{1 - \beta}\log\left(1 + \lambda/100\right),
\end{aligned}
$$

so that

$$\lambda = 100 \left[ e^{(1-\beta)\left(U^{fi}-U\right)} - 1 \right].$$

Our second strategy for computing the steady state takes the values of $\bar{p}$, $m, u$ and $\sigma_z$ as given. Thus, the 'parameters' become:

$$\bar{p}, m, u, \sigma_z, \sigma_L \qquad\qquad (A.46)$$

and the 'endogenous' variables become:

$$F, \varsigma, a, \eta, h.$$

We wish to solve the system for the endogenous variables, for a given set of values for the parameters. To do so, note first that $h$ is determined by

$$h = m\left(1-u\right).$$

We then proceed to solve equations (1)-(4) with respect to $F$, $\varsigma$, $a$, $\eta$ using a nonlinear multipler equation solver. This solution strategy is useful when we want to compute observationally equivalent parameterizations for the models considered in this paper. When we do this, we take observed data as given. Variables like $m$ and $u$ are measured from time averages of unemployment and the labor force, and $\sigma_z$ can be estimated from applying time series techniques to the reduced form of the model using macroeconomic data that do not include unemployment and the labor force.

In our analysis, we find it convenient to compare the model with involuntary unemployment with two other models. This includes the CGG model and the model in which the family problem is based on full information. The lattter is treated in the section devoted to that model. Here, we develop the steady state equations for the CGG model.

The only change required for computing the steady state lies in the specification of $z_h$ in (A.37). We define the disutility of labor in the CGG model as the one implicit in the 'standard model', (4.20):

$$z\left(h_t, \varsigma_t\right) = \varsigma h_t^{1+\sigma_L}.$$

Then, (A.37) reduces to:

$$1 = \left[1 - \eta_g\right] h z_h\left(h, \varsigma\right) = \left[1 - \eta_g\right] \varsigma\left(1 + \sigma_L\right) h^{\sigma_L + 1}.$$

Note that as $\eta_g$ increases, $h$ does too. In the CGG model $m = h$ because there is no job search.

We follow the approach taken in the involuntary unemployment model in calibrating $m$, and thus $h$ too (see (A.46)). We thus define the preference shock in this model as

$$\varsigma = \frac{1}{\left[1 - \eta_g\right]\left(1 + \sigma_L\right) h^{1+\sigma_L}}.$$

Of course the steady state unemployment rate is zero (as it is outside of steady state too) because there is no monopoly power in the labor market. In addition, we parameterizer the curvature, $\sigma_z$, of the disutility of labor. This is simply

$$\sigma_L = \sigma_z$$

in the CGG model. Then, labor supply is simply:

$$z_h\left(h_t, \varsigma_t\right) = \varsigma\left(1 + \sigma_L\right) h_t^{\sigma_L},$$

## B. The Family Utility Function Under Full Information

Our model of involuntary unemployment makes two sorts of assumptions: (i) households have different aversions to work and must make an effort to find work and (ii) their type and effort levels are private information. The fact that there is unemployment in the BLS sense follows from (i) only and (ii) is required for unemployment to be 'involuntary'. This section allows us to determine the impact on the analysis of (ii), by deriving the family utility function that applies when (ii) is not satisfied.

The first subsection derives the family utility function. The second subsection discusses the steady state when this model of unemployment is introduced into CGG.

### B.1. Family Utility Function

Thus, we assume that the family observes everything about the individual household: the effort it exerts to find a job, if any, and its aversion to work. The family selects a consumption allocation and level of search effort, conditional on a household's type. It does so by optimizing the ex ante utility of an arbitrary household or, equivalently, by optimizing the average utility of all households ex post. Households with $0 \le l \le m$ participate in the labor force and those with $1 \ge l \ge m$ do not, where $m$ is a choice variable. We drop subscripts to simplify the notation. The family optimization problem is:

$$\max_{m,\{e_l\},c^w,c^{nw}} \int_0^m \left( p\left(e_l\right)\left[\log\left(c^w\right) - F - \varsigma\left(1 + \sigma_L\right) l^{\sigma_L}\right] + \left(1 - p\left(e_l\right)\right)\log\left(c^{nw}\right) - \frac{1}{2}e_l^2 \right) dl$$
$$+ \left(1 - m\right)\log\left(c^{nw}\right)$$

subject to the resource constraint, (2.14), and the link between the number employed, $h$, and the labor force, $m$, (2.9). We reproduce these constraints here for convenience:

$$C = hc^w + (1-h) c^{nw}$$

$$h = \int_0^m p(e_l) \, dl = m\eta + a \int_0^m e_l dl.$$

Rewrite the objective:

$$\max_{m,\{e_l\},c^w,c^{nw}} \int_0^m \left( p(e_l) \left[ \log\left(\frac{c^w}{c^{nw}}\right) - F - \varsigma(1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(c^{nw}).$$

From the resource constraint:

$$\frac{c^w}{c^{nw}} = \frac{\frac{C}{c^{nw}} - (1-h)}{h}.$$

So that, after substituting out the resource constraint, we get:

$$\max_{m,\{e_l\},c^{nw}} \int_0^m \left( p(e_l) \left[ \log\left(\frac{\frac{C}{c^{nw}} - (1-h)}{h}\right) - F - \varsigma(1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(c^{nw})$$

$$+ \lambda \left[ \int_0^m p(e_l) \, dl - h \right],$$

where $\lambda$ is the multiplier on the restriction linking $m$ and $h$. The first order condition for $c^{nw}$ is:

$$c^{nw} = C,$$

so that the objective reduces to:

$$\max_{m,\{e_l\}} \int_0^m \left( -p(e_l) \left[ F + \varsigma(1+\sigma_L) l^{\sigma_L} \right] - \frac{1}{2} e_l^2 \right) dl + \log(C) \tag{B.1}$$

$$+ \lambda \left[ \int_0^m p(e_l) \, dl - h \right].$$

Optimization with respect to $e_l$ implies:

$$e_l = \lambda a - a \left[ F + \varsigma(1+\sigma_L) l^{\sigma_L} \right],$$

so that, using (2.2),

$$p(e_l) = \eta + \lambda a^2 - a^2 \left[ F + \varsigma(1+\sigma_L) l^{\sigma_L} \right].$$

Also,

$$h = \int_0^m p(e_l) \, dl = m \left( \eta + \lambda a^2 - a^2 \left[ F + \varsigma m^{\sigma_L} \right] \right), \tag{B.2}$$

or,

$$\lambda = \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L}. \tag{B.3}$$

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Optimality of the choice of $m$ in (B.1) implies, by Leibniz's rule:

$$-p\left(e_m\right)\left[F + \varsigma\left(1+\sigma_L\right)m^{\sigma_L}\right] - \frac{1}{2}e_m^2 + \lambda p\left(e_m\right) = 0,$$

or,

$$-\left[\eta + \lambda a^2 - a^2 x\right]x - \frac{1}{2}\left(\lambda a - ax\right)^2 + \lambda\left(\eta + \lambda a^2 - a^2 x\right) = 0,$$

where

$$x \equiv F + \varsigma\left(1+\sigma_L\right)m^{\sigma_L}. \tag{B.4}$$

Simplifying,

$$-\left(\eta + \lambda a^2\right)x + a^2 x^2 - \frac{1}{2}\left(\lambda a\right)^2 + \lambda a^2 x - \frac{1}{2}a^2 x^2 + \lambda\left(\eta + \lambda a^2\right) - \lambda a^2 x = 0,$$

or,

$$f\left(x\right) \equiv x^2 - 2\left(\frac{\eta + \lambda a^2}{a^2}\right)x + \lambda\left[\lambda + 2\frac{\eta}{a^2}\right] = 0. \tag{B.5}$$

It is easy to verify that there are two values of $x$ with the property, $f\left(x\right) = 0$:

$$\frac{1}{a^2}\left(\lambda a^2 + 2\eta\right), \ \lambda.$$

Substituting out for $\lambda$ from (B.3) these solutions reduce to:

$$\frac{\frac{h}{m} + \eta}{a^2} + F + \varsigma m^{\sigma_L}, \ \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L}.$$

Consider the first solution:

$$F + \varsigma\left(1+\sigma_L\right)m^{\sigma_L} = \frac{\frac{h}{m} + \eta}{a^2} + F + \varsigma m^{\sigma_L}$$

or,

$$m^{\sigma_L} = \frac{\frac{h}{m} + \eta}{a^2 \varsigma \sigma_L} \tag{B.6}$$

There is a unique value of $m$, $m \geq 0$, that satisfies (B.6). To see this, note that the left side of (B.6) starts at zero and increases without bound as $m$ increases. The right side starts at plus infinity (thus, greater than the left size) with $m = 0$ and declines monotonically to a finite number as $m$ increases (thus, the right side is eventually below the left side). By continuity and monotonicity, there is a unique value of $m$ that satisfies the equality in (B.6). Of course, the only admissible solution satisfies $m \in (h, 1)$. The second solution to $f\left(x\right) = 0$ implies

$$F + \varsigma\left(1+\sigma_L\right)m^{\sigma_L} = \frac{\frac{h}{m} - \eta}{a^2} + F + \varsigma m^{\sigma_L},$$

or

$$m^{\sigma_L} = \frac{\frac{h}{m} - \eta}{a^2 \varsigma \sigma_L}. \tag{B.7}$$

Interestingly, this also reduces to the expression in (A.39):

$$h = \eta m + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \equiv Q(m). \tag{B.8}$$

There is a unique value of $m$, $m \geq 0$, that satisfies (B.7) for any $h \geq 0$. This is because $m$ is monotone increasing for $m \geq 0$ and $Q(0) = 0$. Whether one or both of (B.6) and (B.7) correspond to local maxima requires examining the relevant second order condition. We do so now, graphically. Of course, we can anticipate that the smaller of the two solutions, the one associated with (B.7), is likely to correspond to the maximum sought in (B.1).

The existence of more than one solution to (B.5) implies that we must investigate second order conditions. Differentiate (B.5) with respect to $m$ :

$$f'(x)\frac{dx}{dm} = \left[ x - \frac{\eta + \lambda a^2}{a^2} \right] 2\frac{dx}{dm}.$$

Since $2dx/dm > 0$ the sign of the above expression corresponds to the sign of the object in square brackets, which is, after substituting out for $x$ from (B.4) and for $\lambda$ from (B.3):

$$\varsigma \sigma_L m^{\sigma_L} - \frac{\frac{h}{m}}{a^2}$$

It is easy to verify that (B.6) implies the above expression is positive, while (B.7) implies the above expression is negative. Thus, (B.7) satisfies the first and second order conditions necessary for an optimum. Thus, (B.7) is a local optimum, while (B.6) is a local minimum.

It is interesting to investigate whether (B.9) satisfies the usual bounds for a probability, given (B.3) and (B.7). Using (B.3) to substitute out for $\lambda$ in (B.9) and rearranging,

$$p(e_l) = \frac{h}{m} + a^2 \varsigma \left[ m^{\sigma_L} - (1 + \sigma_L) l^{\sigma_L} \right].$$

Using (B.7) to substitute out for $h/m$ in the previous expression, we obtain:

$$p(e_l) = \eta + (1 + \sigma_L) a^2 \varsigma \left[ m^{\sigma_L} - l^{\sigma_L} \right]. \tag{B.9}$$

Interestingly, this is the same function obtained for the involuntary unemployment model (to see this, substitute the incentive constraint, (2.8), into the probability function for that model, (2.6)).

We would like to have an expression for the family utility function, the function that

attains the optimum in (B.1) for given $C$ and $h$. Consider the object under the integral in the objective function, (B.1):

$$-p\left(e_l\right)\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]-\frac{1}{2}e_l^2$$

$$= -\left[\eta+\lambda a^2-a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]\right]\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]-\frac{1}{2}\left[\lambda a-a\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]\right]^2$$

$$= -\left(\eta+\lambda a^2\right)\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2-\frac{1}{2}\left[\lambda a-a\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]\right]^2$$

$$= -\left(\eta+\lambda a^2\right)\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2$$
$$-\frac{1}{2}\left[\left(\lambda a\right)^2-2\lambda a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2\right]$$

$$= -\left(\eta+\lambda a^2\right)\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]+a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2$$
$$-\frac{1}{2}\left(\lambda a\right)^2+\lambda a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]-\frac{1}{2}a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2$$

$$= -\eta\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]+\frac{1}{2}a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2-\frac{1}{2}\left(\lambda a\right)^2$$

Note,

$$\int_0^m\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]dl = Fm+\varsigma m^{\sigma_L+1}$$

$$\int_0^m\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2 dl = \int_0^m\left[F^2+2\varsigma\left(1+\sigma_L\right)l^{\sigma_L}F+\varsigma^2\left(1+\sigma_L\right)^2 l^{2\sigma_L}\right]dl$$

$$= mF^2+2\varsigma m^{\sigma_L+1}F+\varsigma^2\left(1+\sigma_L\right)^2\frac{m^{2\sigma_L+1}}{2\sigma_L+1}$$

Then, the integral in (B.1) is

$$\int_0^m\left\{-\eta\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]+\frac{1}{2}a^2\left[F+\varsigma\left(1+\sigma_L\right)l^{\sigma_L}\right]^2-\frac{1}{2}\left(\lambda a\right)^2\right\}dl$$

$$= -\eta m\left[F+\varsigma m^{\sigma_L}\right]+\frac{1}{2}a^2 m\left[F^2+2\varsigma m^{\sigma_L}F+\varsigma^2\left(1+\sigma_L\right)^2\frac{m^{2\sigma_L}}{2\sigma_L+1}\right]-\frac{1}{2}\left(\lambda a\right)^2 m,$$

with $\lambda$ given by (B.3) and $m$ given by (B.6). The graph of this object, for $m\in(h,1)$ takes the form of a partial sine wave. Note that there are two values of $m$ where this function is flat. They correspond to (B.6) and (B.7). Consistent with the algebra above, the second flat point is a local minimum, while the first is a local maximum.

To construct the utility function of the representative agent, write the following function of $m$:

$$Z\left(m,h\right)=\eta m\left[F+\varsigma m^{\sigma_L}\right]-\frac{1}{2}a^2 m\left[F^2+2\varsigma m^{\sigma_L}F+\varsigma^2\left(1+\sigma_L\right)^2\frac{m^{2\sigma_L}}{2\sigma_L+1}\right]+\frac{1}{2}\left(\lambda\left(m,h\right)a\right)^2 m,$$

$$(\text{B.10})$$

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where $\lambda(m, h)$ is the particular function of $m$ and $h$ given by (B.3). Express (B.8) as

$$m = Q^{-1}(h). \tag{B.11}$$

Then, the utility function of the representative agent is:

$$u(C, h) = \log(C) - z(h), \tag{B.12}$$

where

$$z(h) = Z\left(Q^{-1}(h), h\right), \tag{B.13}$$

where $Q^{-1}$ is defined as the inverser function of $Q$ (see (A.40)).

Our calculations require the derivative of $z$, $z_h(h)$. According to (B.13),

$$z_h(h) = Z_1\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) + Z_2\left(Q^{-1}(h), h\right),$$

where $\left[Q^{-1}\right]'(h)$ is the derivative of the inverse function of $Q$, defined in terms of $Q'$ in (A.43). Using (A.43):

$$z_h(h) = \frac{Z_1\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} + Z_2\left(Q^{-1}(h), h\right). \tag{B.14}$$

We also require the second derivative of $z$, $z_{hh}$ :

$$
\begin{aligned}
z_{hh}(h) = \ & Z_{11}\left(Q^{-1}(h), h\right)\left(\left[Q^{-1}\right]'(h)\right)^2 + Z_{12}\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) \\
& + Z_1\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]''(h) + Z_{21}\left(Q^{-1}(h), h\right)\left[Q^{-1}\right]'(h) \\
& + Z_{22}\left(Q^{-1}(h), h\right),
\end{aligned}
$$

where the second derivative of the inverse function, $\left[Q^{-1}\right]''(h)$, can be computed using (A.44). Using (A.43) and (A.44):

$$
\begin{aligned}
z_{hh}(h) = \ & Z_{11}\left(Q^{-1}(h), h\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^2 + \frac{Z_{12}\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} \\
& - Z_1\left(Q^{-1}(h), h\right) Q''\left(Q^{-1}(h_t)\right)\left(\frac{1}{Q'\left(Q^{-1}(h_t)\right)}\right)^3 \\
& + \frac{Z_{21}\left(Q^{-1}(h), h\right)}{Q'\left(Q^{-1}(h_t)\right)} + Z_{22}\left(Q^{-1}(h), h\right),
\end{aligned}
\tag{B.15}
$$

Given (B.14) and (B.14), we can compute the curvature of utility:

$$\sigma_z = \frac{z_{hh}h}{z_h}.$$

All these formulas can be computed symbolically given the definitions of the $Z$ and $Q$ functions in (B.10) and (B.8), respectively.

## B.2. Steady State

We discuss two ways of computing the steady state of the model. The first is the 'natural' one. This takes the parameters of the model as given and computes the implied exogenous variables. This is useful for doing one type of comparison of the full information version of the model with our involuntary unemployment model. For example, we can ask how the level of unemployment is affected by the absence of full information. The second approach allows us to impose certain endogenous variables exogenously, and back out parameters.

### B.2.1. A First Approach to Computing the Steady State

The structural parameters of the model are

$$F, \varsigma, a, \eta, \sigma_L.$$

The endogenous variables of interest are

$$\bar{p}, h, m, u, \sigma_z.$$

The relevant equations are (A.45):

$$
\begin{aligned}
(1)h &= m\eta + a^2\varsigma\sigma_L m^{\sigma_L+1}\\
(2)\bar{p} &= \eta + \varsigma a^2 (1 + \sigma_L) m^{\sigma_L}\\
(3)1 &= \left[1 - \eta_g\right] h z_h (h, \varsigma)\\
(4)\sigma_z &= \frac{z_{hh} h}{z_h}\\
u &= \frac{m - h}{m}.
\end{aligned}
$$

Notably, equations (1) and (2) in (A.45) are literally unchanged from what they are in the full information model because the expressions for $p(e_l)$ and the mapping from $m$ to $h$ are unchanged. In addition, equations (3) and (4) are formally the same, although the details of $z_h$ and $z_{hh}$ are different for the partial information model because $z$ is different. The steady state is found by solving (3) for $h$. Then, (1) is solved for $m$ and (2) is solved for $\bar{p}$. Finally, the last equation is solved for $u$.

### B.2.2. A Second Approach to Solving the Steady State

We proceed here in a way that is parallel to the approach taken in the involuntary unemployment model, in section A.4. In particular, the exogenous 'parameters' are (A.46):

$$\bar{p}, m, u, \sigma_z, \sigma_L.$$

treated asand the 'endogenous' variables are:

$$F, \varsigma, a, \eta, h.$$

As in A.4, $h$ is derived trivially from the parameters:

$$h = (1 - u)\, m.$$

There now remain four endogenous variables to be solved for, $F, \varsigma, a, \eta$, using (1)-(4). As before, we select $F, \varsigma, a, \eta$ to set these equations to zero for given values of the exogenous parameters.

## C. Quantitative Properties of the Model Without Capital

This section explores various quantitative properties of our simple model without capital. The assumption of limited information lies at the heart of our model of unemployment, and we quantity the impact of this assumption by comparing the properties of the model with and without limited information.

### C.1. Parameter Values

To do the numerical calculations, we must assign values to our involuntary unemployment model parameters. Table A1 reports the values of a subset of parameters that are relatively uncontroversial. For example, we set values for the Taylor rule that imply the Taylor principle is satisfied (i.e., $r_\pi > 1$), we assume substantial interest rate smoothing ($\rho_R$ is large) and the feedback on the output gap is modest ($r_y$ is small). The steady state share, $\eta_g$, of government consumption to gross output is 20 percent. Government consumption is a long weighted average of current and past technology (i.e., $\gamma$ is close to zero). The autocorrelation of the three shocks in the model is large. Finally, the innovation standard deviation of the three shocks was chosen so that each shock, when operative in isolation, causes the standard deviation of quarterly output growth in the model to be around 1 percent, the corresponding post WWII average in US data. The exception is the standard deviation of the preference shock which is set to zero throughout the analysis in this section.

We treated the structural parameters associated with households' job finding function, $p(e)$, and their disutility of labor differently. These parameters are:

$$F, \ \varsigma, \ a, \ \eta, \ \sigma_L. \tag{C.1}$$

We do not have direct observations on these parameters, nor are we aware of any estimates

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of these parameters in the literature. So, we chose values for them so that, conditional on the value of $\eta_g$ given in Table A1, the steady state of the involuntary unemployment model implies the values of

$$m, \ u, \ 1/\sigma_z, \ 1/\kappa^{okun}, \ \bar{p} \qquad (C.2)$$

that are reported in the top left panel of Table A2. The indicated steady state value of the labor force participation rate, $m = 2/3$, corresponds to the average value of this variable in recent years. The steady state value of unemployment, $u = 0.056$, corresponds to the average value of unemployment in the post war US data. The compensated family labor supply elasticity, $1/\sigma_z = 2$, is roughly the corresponding object used in the real business cycle literature.[56] This is where the distinction between the compensated family labor supply elasticity and the Frisch elasticity is important. If $1/\sigma_z$ were interpretable as a Frisch elasticity, then the labor literature implies a value of $1/\sigma_z$ well below unity. However, as noted above, $1/\sigma_z$ has no connection to any individual agent's willingness to vary hours worked in response to a wage change in our model. The value of $\kappa^{okun}$ was selected so that the model is consistent with a standard estimate of Okun's law. Finally, we chose a value for $\bar{p}$ to ensure that in the stochastic version of the model, the likelihood of violating the upper bound constraint on the job finding probability is small. The mapping from (C.2) to (C.1) using the steady state equilibrium conditions is described in detail in section A.4 of this technical appendix.

Table A2 contains an in-depth comparison of the involuntary unemployment model (imperfect information) against the full information model and the standard model.

## D. Non-Separability in Utility

In the manuscript, we work with a household utility function in which consumption and leisure are additively separable. In this appendix, we show that the analysis can also easily be done with two non-separable utility functions that have been used extensively in the literature.

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[56] A standard real business cycle model (see, e.g., the 'divisible labor' model in Christiano and Eichenbaum, 1992) uses preferences, $\sum_{t=0}^{\infty} \beta^t \left[ \log\left(C_t\right) + \psi \left(1 - h_t\right) \right]$, where $h_t$ denotes time worked of the representative agent, as a fraction of available time. The labor first order condition associated with the agent's optimal labor choice is $\psi C_t / \left(1 - h_t\right) = w_t$, where $w_t$ denotes the real wage. The consumption compensated (actually 'consumption constant', with these preferences) labor supply function is (apart from a constant), $log\left(1 - h_t\right) = \log w_t$. This implies the following steady state elasticity of employment with respect to the wage:

$$\frac{d \log h_t}{d \log w_t} = -\frac{1 - h}{h} = -2.$$

Here, we assume that in steady state, $1/3$ of available time is devoted to market work, i.e., $h = 1/3$.

### D.1. King-Plosser-Rebelo Preferences

We replace the preferences in (2.3) with:

$$p\left(e_t\right)\frac{\left(c_t^w\right)^{1-\gamma}}{1-\gamma}v\left(l\right)+\left(1-p\left(e_t\right)\right)\frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma}v\left(0\right)-\frac{1}{2}\frac{e_t^2}{1-\gamma},\quad\gamma\ >\ 1$$

$$v\left(l\right)=F+\varsigma_t\left(1+\sigma_L\right)l^{\sigma_L}.$$

With these preferences, utility is decreasing in $l$, and the marginal utility of consumption is increasing in $l$. In this case, the household with aversion to work, $l$, sets

$$e_{l,t}=\max\left\{a\left[\left(c_t^w\right)^{1-\gamma}v\left(l\right)-\left(c_t^{nw}\right)^{1-\gamma}v\left(0\right)\right],0\right\}.$$

In this case, households that participate in the labor market receive utility

$$\frac{1}{2}a^2\left[\frac{\left(c_t^w\right)^{1-\gamma}}{1-\gamma}v\left(l\right)-\frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma}v\left(0\right)\right]^2+\frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma}v\left(0\right),$$

while households that do not participate receive

$$\frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma}v\left(0\right).$$

The incentive constraint (i.e., the analog of (2.8)) requires:

$$\left(c_t^w\right)^{1-\gamma}v\left(m\right)=\left(c_t^{nw}\right)^{1-\gamma}v\left(0\right)\tag{D.1}$$

The mapping from the labor force, $m_t$, to the number of people working, $h_t$, is given by:

$$h_t=\int_0^{m_t}p\left(e_{l,t}\right)dl=a^2\frac{\left(c_t^w\right)^{1-\gamma}}{\gamma-1}\varsigma_t\left(1+\sigma_L\right)\int_0^{m_t}\left[m_t^{\sigma_L}-l^{\sigma_L}\right]dl=a^2\frac{\left(c_t^w\right)^{1-\gamma}}{\gamma-1}\varsigma_t\sigma_Lm_t^{\sigma_L+1}.\tag{D.2}$$

Combining the resource constraint, (2.14), with the incentive constraint, (D.1), we obtain:

$$c_t^w\ =\ \frac{C_t}{h_t+\left(1-h_t\right)\left[\frac{v\left(m_t\right)}{v\left(0\right)}\right]^{\frac{1}{1-\gamma}}}\tag{D.3}$$

$$c_t^{nw}\ =\ \frac{C_t\left[\frac{v\left(m_t\right)}{v\left(0\right)}\right]^{\frac{1}{1-\gamma}}}{h_t+\left(1-h_t\right)\left[\frac{v\left(m_t\right)}{v\left(0\right)}\right]^{\frac{1}{1-\gamma}}}.\tag{D.4}$$

Integrating utility over all the households in the family, the analog of (2.17) is:

$$u\left(c_t^w,c_t^{nw},m_t\right)=a^2\frac{\left(c_t^w\right)^{2\left(1-\gamma\right)}}{\left(1-\gamma\right)^2}\left(1+\sigma_L\right)\frac{\sigma_L^2m_t^{2\sigma_L+1}}{\left(2\sigma_L+1\right)}+\frac{\left(c_t^{nw}\right)^{1-\gamma}}{1-\gamma}v\left(0\right).\tag{D.5}$$

Equations (D.2), (D.3) and (D.4) provide a mapping from $C_t$ and $h_t$ to $c_t^w$, $c_t^{nw}$ and $m_t$. Utility is then given by (D.5). Thus, we have family utility in terms of $C_t$ and $h_t$ only.

## D.2. Greenwood-Hercowitz-Huffman Preferences

We now replace the preferences in (2.3) with:

$$p\left(e_t\right) \frac{\left(c_t^w + F - \left(1 + \sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} - 1}{1 - \gamma} + \left(1 - p\left(e_t\right)\right) \frac{\left(c_t^{nw} + F\right)^{1-\gamma} - 1}{1 - \gamma} - \frac{1}{2} e_t^2, \ \gamma > 0.$$

In this case,

$$e_{l,t} = \max \left\{ a \left[ \frac{\left(c_t^w + F - \left(1 + \sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw} + F\right)^{1-\gamma}}{1 - \gamma} \right], 0 \right\},$$

so that the utility of a household that participates in the labor force is:

$$\frac{1}{2} a^2 \left[ \frac{\left(c_t^w + F - \left(1 + \sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} - \left(c_t^{nw} + F\right)^{1-\gamma}}{1 - \gamma} \right]^2 + \frac{\left(c_t^{nw} + F\right)^{1-\gamma} - 1}{1 - \gamma}.$$

Comparing this with the utility of households that choose to be out of the labor force, we obtain the incentive constraint:

$$c_t^w - \left(1 + \sigma_L\right) m_t^{1+\sigma_L} = c_t^{nw} \tag{D.6}$$

The mapping between the labor force and the number of people working is provided by:

$$h_t = \int_0^{m_t} p\left(e_{l,t}\right) dl = \frac{a^2}{1 - \gamma} \int_0^{m_t} \left[ \begin{array}{c} \left(c_t^w + F - \left(1 + \sigma_L\right) l^{1+\sigma_L}\right)^{1-\gamma} \\ - \left(c_t^w + F - \left(1 + \sigma_L\right) m_t^{1+\sigma_L}\right)^{1-\gamma} \end{array} \right] dl. \tag{D.7}$$

Now,

$$\int_0^{m_t} \left[c_t^w + F - \left(1 + \sigma_L\right) l^{1+\sigma_L}\right]^{1-\gamma} dl$$

$$= \ m_t \left[c_t^w + F - \left(1 + \sigma_L\right) m_t^{1+\sigma_L}\right]^{1-\gamma} \left( \frac{c_t^w + F - \left(1 + \sigma_L\right) m_t^{(1+\sigma_L)}}{c_t^w + F} \right)^{-(1-\gamma)}$$

$$\times \mathcal{F}\left( \left[ -\left(1 - \gamma\right) \ \frac{1}{1+\sigma_L} \right]; 1 + \frac{1}{1 + \sigma_L}; \frac{\left(1 + \sigma_L\right) m_t^{1+\sigma_L}}{c_t^w + F} \right),$$

where

$$\mathcal{F}\left(x; a; b\right)$$

denotes the hypergeometric function where $x$ is a $1 \times 2$ row vector and $a$ and $b$ are scalars.[57] In this way, (D.7) defines a mapping from $c_t^w, c_t^{nw}$ and $m_t$ to $h_t$ :

$$h_t = f\left(c_t^w, c_t^{nw}, m_t\right). \tag{D.8}$$

---

[57] This formula may be found at http://integrals.wolfram.com/index.jsp. In MATLAB, the hypergeometric function is evaluated using hypergeom(x,a,b).

Combining the resource constraint, (2.14), with the incentive constraint, (D.6), we obtain:

$$c_t^w = C_t + (1 - h_t)(1 + \sigma_L) m_t^{1+\sigma_L} \tag{D.9}$$

$$c_t^{nw} = C_t - h_t (1 + \sigma_L) m_t^{1+\sigma_L} \tag{D.10}$$

Ex ante utility of all the households in the family, the analog of (2.17), is (after using the incentive constraint):

$$u(c_t^w, c_t^{nw}, m_t) = \frac{1}{2} \frac{a^2}{(1-\gamma)^2} \int_0^{m_t} \left[ \begin{array}{c} (c_t^w + F - (1+\sigma_L) l^{1+\sigma_L})^{1-\gamma} \\ - (c_t^{nw} + F)^{1-\gamma} \end{array} \right]^2 dl \tag{D.11}$$
$$+ \frac{(c_t^{nw} + F)^{1-\gamma} - 1}{1 - \gamma}$$

Expanding the square term:

$$\int_0^{m_t} \left[ (c_t^w + F - (1+\sigma_L) l^{1+\sigma_L})^{1-\gamma} - (c_t^{nw} + F)^{1-\gamma} \right]^2 dl$$
$$= \int_0^{m_t} (c_t^w + F - (1+\sigma_L) l^{1+\sigma_L})^{2(1-\gamma)} dl$$
$$- 2 (c_t^{nw} + F)^{1-\gamma} \int_0^{m_t} (c_t^w + F - (1+\sigma_L) l^{1+\sigma_L})^{2(1-\gamma)} dl$$
$$+ m_t (c_t^{nw} + F)^{2(1-\gamma)},$$

which can be evaluated using formulas analogous to the one after (D.7). Equations (D.8), (D.9) and (D.10) provide a mapping from $C_t$ and $h_t$ to $c_t^w$, $c_t^{nw}$ and $m_t$. Utility is then given by (D.11). Thus, we have family utility in terms of $C_t$ and $h_t$ only.

## E. Solving the Model Used in the Empirical Analysis

We first derive the equilibrium conditions associated with optimal wage setting. We then indicate the remaining equilibrium conditions of the model. Finally, we describe a strategy for solving the model's steady state.

### E.1. Scaling of Variables

We adopt the following scaling of variables. The neutral shock to technology is $z_t$ and its growth rate is $\mu_{z,t}$ :

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, $\Psi_t$, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$
\begin{aligned}
z_t^+ &\equiv \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\
\mu_{z^+,t} &\equiv \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}.
\end{aligned} \tag{E.1}
$$

Capital, $\bar{K}_t$, and investment, $I_t$, are scaled by $z_t^+ \Psi_t$. Consumption goods $C_t$, government consumption $G_t$ and the real wage, $W_t/P_t$ are scaled by $z_t^+$. Also, $\upsilon_t$ is the multiplier on the nominal household budget constraint in the Lagrangian version of the household problem. That is, $\upsilon_t$ is the marginal utility of one unit of currency. The marginal utility of a unit of consumption is $\upsilon_t P_t$. The latter must be multiplied by $z_t^+$ to induce stationarity. Thus, our scaled variables are:

$$
k_{t+1} = \frac{K_{t+1}}{z_t^+ \Psi_t}, \; \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t}, \; i_t = \frac{I_t}{z_t^+ \Psi_t}, \; c_t = \frac{C_t}{z_t^+}, \; g_t = \frac{G_t}{z_t^+}, \; \bar{w}_t = \frac{W_t}{z_t^+ P_t} \tag{E.2}
$$

$$
\psi_{z^+,t} = \upsilon_t P_t z_t^+, \; \tilde{y}_t = \frac{Y_t}{z_t^+}, \; \tilde{p}_t = \frac{\tilde{P}_t}{P_t}, \; w_t = \frac{\tilde{W}_t}{W_t}, \; \ddot{w}_t \equiv \frac{\dddot{W}_t}{W_t}, \; \overset{\infty}{\tilde{w}}_t = \frac{\overset{\infty}{\tilde{W}}_t}{W_t}.
$$

We define the scaled date $t$ price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$ :

$$
p_{k',t} = \Psi_t P_{k',t}, \; \bar{r}_t^k = \Psi_t r_t^k.
$$

where $P_{k',t}$ is in units of the homogeneous good. We define the following inflation rates:

$$
\pi_t = \frac{P_t}{P_{t-1}}, \; \pi_t^i = \frac{P_t^i}{P_{t-1}^i}.
$$

Here, $P_t$ is the price of the homogeneous output good and $P_t^i$ is the price of the domestic final investment good.

## E.2. Wage Setting by the Family

We consider the problem of a monopolist who represents households that supply the type $j$ labor service. That monopolist optimizes the utility function of $j-$type households, (2.18), subject to Calvo frictions. With probability $1 - \xi_w$ the monopolist reoptimizes the wage and with probability $\xi_w$ the monopolist sets the current wage rate according to (4.9). In each period, type $j$ households supply the quantity of labor dictated by demand, (4.5). Because the $j-$type family has perfect consumption insurance, the monopolist can take the $j-$type family's consumption as given. However, the monopolist does assign a weight to the revenues from $j-$type labor that corresponds to the value, $\upsilon_t$, assigned to income by

the family. Ignoring terms beyond the control of the monopolist the monopolist seeks to maximize:

$$E_t^j \sum_{i=0}^{\infty} \beta^i \left[ -z \left( h_{t+i,j}, \zeta_{t+i} \right) + \upsilon_{t+i} W_{t+i,j} h_{t+i,j} \right].$$

Here, $\upsilon_t$ denotes the Lagrange multiplier on the type $j$ family's time $t$ flow budget constraint, (3.2). The function, $z$, is defined in (2.18), but we reproduce it here for convenience:

$$z \left( h_{j,t}, \zeta_t \right) = \log \left[ h_{j,t} \left( e^{F + \varsigma (1+\sigma_L) f(h_t)^{\sigma_L}} - 1 \right) + 1 \right] - \frac{a^2 \varsigma^2 (1 + \sigma_L) \sigma_L^2}{2\sigma_L + 1} f \left( h_{j,t} \right)^{2\sigma_L + 1} - \eta \varsigma \sigma_L f \left( h_{j,t} \right)^{\sigma_L + 1}.$$

Here, $f \left( h_{j,t}, \zeta_t \right)$ is the unique value of $m_t$ that satisfies

$$h_{j,t} = Q \left( m_{j,t}, \zeta_t \right) \equiv m_{j,t} \eta + a^2 \varsigma_t \sigma_L m_{j,t}^{\sigma_L + 1}, \tag{E.3}$$

for a given value of $m_t$. That is,

$$m_{j,t} = f \left( h_{j,t}, \zeta_t \right) \equiv Q^{-1} \left( h_{j,t}, \zeta_t \right),$$

where $Q^{-1}$ is the inverse function of $Q$. In the case of the standard model, $z$ is implicitly defined in (4.20).

### E.2.1. Differentiating the Family Disutility of Labor

In the calculations that follow, we require the derivatives of $z$ and $f$, evaluated in steady state. In the case of the standard model, these calculations are trivial. We compute the derivatives for our model with involuntary unemployment here. We drop the $j$ subscript for convenience, as well as the stochastic shock. From the definition of the inverse function,

$$m_t = f \left( Q \left( m_t \right) \right).$$

We find the derivatives of $f$ by differentiating this expression twice with respect to $m_t$:

$$\begin{aligned} 1 &= f' \left( Q \left( m_t \right) \right) Q' \left( m_t \right) \\ 0 &= f'' \left( Q \left( m_t \right) \right) \left[ Q' \left( m_t \right) \right]^2 + f' \left( Q \left( m_t \right) \right) Q'' \left( m_t \right) \end{aligned}$$

From the first expression,

$$f' \left( Q \left( m_t \right) \right) = \frac{1}{Q' \left( m_t \right)}.$$

Substituting this into the second expression and solving:

$$0 = f'' \left( Q \left( m_t \right) \right) \left[ Q' \left( m_t \right) \right]^2 + \frac{Q'' \left( m_t \right)}{Q' \left( m_t \right)},$$

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so that
$$f''\left(Q\left(m_t\right)\right) = -\frac{Q''\left(m_t\right)}{\left[Q'\left(m_t\right)\right]^3},$$

From (E.3),
$$Q'\left(m_t\right) = \eta + \left(\sigma_L + 1\right)a^2\varsigma\sigma_L m_t^{\sigma_L}$$
$$Q''\left(m_t\right) = \left(\sigma_L + 1\right)a^2\varsigma\sigma_L^2 m_t^{\sigma_L - 1},$$

so that, in steady state,
$$f_h = \frac{1}{\eta + \left(\sigma_L + 1\right)a^2\varsigma\sigma_L m^{\sigma_L}} \qquad \text{(E.4)}$$
$$f_{hh} = -\frac{\left(\sigma_L + 1\right)a^2\varsigma\sigma_L^2 m^{\sigma_L - 1}}{\left[\eta + \left(\sigma_L + 1\right)a^2\varsigma\sigma_L m^{\sigma_L}\right]^3},$$

where $m$ denotes the steady state value of $m$, computed below.

Let,
$$Z\left(m_t\right) = \log\left[Q\left(m_t\right)\left(e^{F + \varsigma\left(1 + \sigma_L\right)m_t^{\sigma_L}} - 1\right) + 1\right] - \frac{a^2\varsigma^2\left(1 + \sigma_L\right)\sigma_L^2}{2\sigma_L + 1}m_t^{2\sigma_L + 1} - \eta\varsigma\sigma_L m_t^{\sigma_L + 1}$$
$$h_t = Q\left(m_t\right) \equiv m_t\eta + a^2\varsigma\sigma_L m_t^{\sigma_L + 1},$$

so that
$$z\left(h_t\right) \equiv Z\left(Q^{-1}\left(h_t\right)\right) = Z\left(f\left(h_t\right)\right),$$

and
$$z_h\left(h_t\right) = Z'\left(f\left(h_t\right)\right)f_h\left(h_t\right)$$
$$z_{hh}\left(h_t\right) = Z''\left(f\left(h_t\right)\right)\left[f_h\left(h_t\right)\right]^2 + Z'\left(f\left(h_t\right)\right)f_{hh}\left(h_t\right).$$

Evaluating this in steady state,
$$z_h = Z'f_h, \quad z_{hh} = Z''f_h^2 + Z'f_{hh}.$$

In this case,
$$\sigma_z \equiv \frac{z_{hh}h}{z_h} = \frac{\left[Z''f_h^2 + Z'f_{hh}\right]h}{Z'f_h} = \frac{Z''f_h h}{Z'} + \frac{f_{hh}h}{f_h}$$

From (E.4),
$$f_h = \frac{1}{Q'}, \quad f_{hh} = -\frac{Q''}{\left[Q'\right]^3},$$

so that
$$\sigma_z = \frac{Z''f_h h}{Z'} + \frac{f_{hh}h}{f_h}$$
$$= \frac{Z''h}{Z'Q'} - \frac{Q''h}{\left[Q'\right]^2}$$
$$= \frac{Q}{Q'}\left[\frac{Z''}{Z'} - \frac{Q''}{Q'}\right]$$

## E.2.2. First Order Condition Associated with Family Wage Setting

Consider the monopoly wage setter, $j$, that has an opportunity to reoptimize the wage rate. The objective function with $h_{t+i,j}$ substituted out using labor demand, (4.5), and ignoring terms beyond the control of the monopolist, is as follows:

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[ -z \left( \zeta_{t+i}, \left( \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right) \right.$$

$$\left. + \upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left( \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right],$$

where

$$\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}$$

is the nominal wage rate of the monopolist which sets wage $\tilde{W}_t$ in period $t$ and cannot reoptimize again afterward. Also, $z$ is the function described in the case of our model with unemployment, and it is (4.20) in the case of the standard model. We adopt the following scaling convention:

$$w_t = \frac{\tilde{W}_t}{W_t}, \quad \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \quad \psi_{z^+,t} = \upsilon_t P_t z_t^+.$$

With this notation, the objective can be written,

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[ -z \left( \zeta_{t+i}, \left( \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right) + \psi_{z^+,t+i} w_t^{\frac{1}{1-\lambda_w}} \bar{w}_t X_{t,i} \left( \frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right],$$

where:

$$X_{t,i} = \frac{\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+i} \pi_{t+i-1} \cdots \pi_{t+1} \mu_{z^+,t+i} \cdots \mu_{z^+,t+1}}.$$

Differentiating with respect to $w_t$,

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left[ -z_{h,t+i}^t \frac{\lambda_w}{1-\lambda_w} w_t^{\frac{\lambda_w}{1-\lambda_w}-1} \left( \frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right.$$

$$\left. + \frac{1}{1-\lambda_w} \psi_{z^+,t+i} w_t^{\frac{1}{1-\lambda_w}-1} \bar{w}_t X_{t,i} \left( \frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right],$$

where

$$z_{h,t+i}^t \equiv z_h \left( \zeta_{t+i}, \left( \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right).$$

Here, $h_{2,t+i}^t$ denotes the marginal utility of labor in period $t+i$, for a monopolist who last reoptimized the wage rate in period $t$. Dividing and rearranging,

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left( \frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} [\psi_{z^+,t+i} w_t \bar{w}_t X_{t,i} - \lambda_w z_{h,t+i}^t] = 0. \qquad \text{(E.5)}$$

The first object in square brackets is the marginal utility real wage in period $t + i$ and the second is a markup, $\lambda_w$, over the marginal utility cost of working. According to (E.5) the monopolist attempts to set a weighted average of the term in square brackets to zero. The structure of $z_{z,t+i}^t$ makes it difficult to express (E.5) in recursive form. This is because we have not found a way to express $z_{h,t+1}^t = Z_t z_{h,t+1}^{t+1}$, for some variable, $Z_t$. The expression, (E.5), is recursive after linearizing it about steady state. Thus,

$$
\hat{z}_{h,t+i}^t \equiv \frac{dz_h \left( \zeta_{t+i}, \left( \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)}{z_h \left( \zeta, w^{\frac{\lambda_w}{1-\lambda_w}} H \right)},
$$

where a variable without a time subscript denotes non-stochastic steady state. Expanding this expression:

$$
\hat{z}_{h,t+i}^t = \sigma_\varsigma \hat{\zeta}_{t+i} + \alpha_{h,1} \left( \hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+i} + \hat{X}_{t,i} \right) + \sigma_z \hat{H}_{t+i},
$$

where,

$$
\sigma_\varsigma \equiv \frac{z_{h\zeta} \zeta}{z_h}, \quad \sigma_z \equiv \frac{z_{hh} H}{z_h}, \quad \alpha_{h,1} \equiv \frac{\lambda_w}{1 - \lambda_w} \sigma_z.
$$

Also,

$$
\hat{X}_{t,i} = \widehat{\bar{\pi}}_{w,t+i} + \cdots + \widehat{\bar{\pi}}_{w,t+1} - \hat{\pi}_{t+i} - \hat{\pi}_{t+i-1} - \cdots - \hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1}.
$$

However, note:

$$
\widehat{\bar{\pi}}_{w,t+1} = \kappa_w \hat{\pi}_t.
$$

Then,

$$
\hat{X}_{t,i} = -\Delta_{\kappa_w} \hat{\pi}_{t+i} - \Delta_{\kappa_w} \hat{\pi}_{t+i-1} - \cdots - \Delta_{\kappa_w} \hat{\pi}_{t+1} - \hat{\mu}_{z^+,t+i} - \cdots - \hat{\mu}_{z^+,t+1},
$$

where

$$
\Delta_{\kappa_w} \equiv 1 - \kappa_w L,
$$

where $L$ denotes the lag operator.

Write out (E.5) in detail:

$$
H_t [\psi_{z^+,t} w_t \bar{w}_t - \lambda_w z_{h,t}^t]
$$
$$
+ \beta \xi_w \left( \frac{\bar{w}_t}{\bar{w}_{t+1}} X_{t,1} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} [\psi_{z^+,t+1} w_t \bar{w}_t X_{t,1} - \lambda_w z_{h,t+1}^t]
$$
$$
+ (\beta \xi_w)^2 \left( \frac{\bar{w}_t}{\bar{w}_{t+2}} X_{t,2} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} [\psi_{z^+,t+2} w_t \bar{w}_t X_{t,2} - \lambda_w z_{h,t+2}^t] + \dots = 0
$$

In expanding this expression, we can simply set the terms outside the square brackets to their steady state values. The reason is that the term inside the brackets are equal to zero in steady state. Thus, the expansion of the previous expression about steady state:

$$
\begin{aligned}
&H[d\left(\psi_{z^+,t} w_t \bar{w}_t\right) - \lambda_w d\left(z_{h,t}^t\right)] \\
&+\beta\xi_w H[d\left(\psi_{z^+,t+1} w_t \bar{w}_t X_{t,1}\right) - \lambda_w d\left(z_{h,t+1}^t\right)] \\
&+ \left(\beta\xi_w\right)^2 H[d\left(\psi_{z^+,t+2} w_t \bar{w}_t X_{t,2}\right) - \lambda_w d\left(z_{h,t+2}^t\right)] + ... = 0
\end{aligned}
$$

or,

$$
\begin{aligned}
&H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t\right) - \lambda_w z_h \hat{z}_{h,t}^t] \\
&+\beta\xi_w H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,1}\right) - \lambda_w z_h \hat{z}_{h,t+1}^t] \\
&+ \left(\beta\xi_w\right)^2 H[\psi_{z^+}\bar{w}\left(\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,2}\right) - \lambda_w z_h \hat{z}_{h,t+2}^t] + ... = 0
\end{aligned}
$$

Note that in steady state, $\psi_{z^+}\bar{w} = \lambda_w z_h$, so that, after multiplying by $1/\left(H\psi_{z^+}\bar{w}\right)$, we obtain:

$$
\begin{aligned}
&\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t - \hat{z}_{h,t}^t \\
&+\beta\xi_w[\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,1} - \hat{z}_{h,t+1}^t] \\
&+ \left(\beta\xi_w\right)^2 [\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t + \hat{X}_{t,2} - \hat{z}_{h,t+2}^t] + ... = 0
\end{aligned}
$$

Substitute out for $\hat{z}_{h,t+i}^t$ and $\hat{X}_{t,i}$ :

$$
\begin{aligned}
0 = \; &\hat{\psi}_{z^+,t} + \hat{w}_t + \widehat{\bar{w}}_t - \left[\sigma_\varsigma \hat{\zeta}_t + \alpha_{h,1}\hat{w}_t + \sigma_z \hat{H}_t\right] \\
\\
&+\beta\xi_w[\hat{\psi}_{z^+,t+1} + \hat{w}_t + \widehat{\bar{w}}_t - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \\
&- \left(\sigma_\varsigma \hat{\zeta}_{t+1} + \alpha_{h,1}\left(\hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+1} - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right)\right) + \sigma_z \hat{H}_{t+1}\right)] \\
\\
&+ \left(\beta\xi_w\right)^2 [\hat{\psi}_{z^+,t+2} + \hat{w}_t + \widehat{\bar{w}}_t - \left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \\
&- \left(\sigma_\varsigma \hat{\zeta}_{t+2} + \alpha_{h,1}\left(\begin{array}{c} \hat{w}_t + \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t+2} \\ -\left(\Delta_{\kappa_w}\hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2}\right) - \left(\Delta_{\kappa_w}\hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1}\right) \end{array}\right) + \sigma_z \hat{H}_{t+2}\right)] + ...
\end{aligned}
$$

Collecting terms:

$$
\begin{aligned}
0 &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[ \hat{\psi}_{z^+,t+j} - \left( \sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j} \right) \right] + \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \hat{w}_t \\
&\quad + \frac{1 - \alpha_{h,1}\beta\xi_w}{1 - \beta\xi_w} \widehat{\tilde{w}}_t + \alpha_{h,1} \sum_{j=1}^{\infty} (\beta\xi_w)^j \, \widehat{\tilde{w}}_{t+j} \\
&\quad - (1 - \alpha_{h,1})\, \beta\xi_w \left[ \left( \Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right) \right] \\
&\quad - (1 - \alpha_{h,1}) (\beta\xi_w)^2 \left[ \left( \Delta_{\kappa_w} \hat{\pi}_{t+2} + \hat{\mu}_{z^+,t+2} \right) + \left( \Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right) \right] \\
&\quad - ...
\end{aligned}
$$

or,

$$
\begin{aligned}
0 &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[ \hat{\psi}_{z^+,t+j} - \left( \sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j} \right) \right] + \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \hat{w}_t \\
&\quad + \frac{1 - \alpha_{h,1}\beta\xi_w}{1 - \beta\xi_w} \widehat{\tilde{w}}_t + \sum_{j=1}^{\infty} (\beta\xi_w)^j \left[ \alpha_{h,1} \widehat{\tilde{w}}_{t+j} - \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \left( \Delta_{\kappa_w} \hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j} \right) \right].
\end{aligned}
$$

Note

$$
\begin{aligned}
S_t &= X_t + \beta\xi_w X_{t+1} + (\beta\xi_w)^2 X_{t+2} + ... \\
&= X_t + \beta\xi_w \overbrace{[X_{t+1} + \beta\xi_w X_{t+2} + ...]}^{S_{t+1}},
\end{aligned}
$$

so that the log-linearized first order condition can be written:

$$
0 = F_{,t} + \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \hat{w}_t + \frac{1 - \alpha_{h,1}\beta\xi_w}{1 - \beta\xi_w} \widehat{\tilde{w}} + G_t, \tag{E.6}
$$

where

$$
\begin{aligned}
F_t &= \sum_{j=0}^{\infty} (\beta\xi_w)^j \left[ \hat{\psi}_{z^+,t+j} - \left( \sigma_\varsigma \hat{\zeta}_{t+j} + \sigma_z \hat{H}_{t+j} \right) \right] \\
&= \hat{\psi}_{z^+,t} - \left( \sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \beta\xi_w F_{t+1} \\
G_t &= \sum_{j=1}^{\infty} (\beta\xi_w)^j \left[ \alpha_{h,1} \widehat{\tilde{w}}_{t+j} - \frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \left( \Delta_{\kappa_w} \hat{\pi}_{t+j} + \hat{\mu}_{z^+,t+j} \right) \right] \\
&= \beta\xi_w \alpha_{h,1} \widehat{\tilde{w}}_{t+1} - \frac{(1 - \alpha_{h,1})\beta\xi_w}{1 - \beta\xi_w} \left( \Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right) + \beta\xi_w G_{t+1}
\end{aligned}
$$

Note:

$$
\left( 1 - \beta\xi_w L^{-1} \right) F_t \equiv F_t - \beta\xi_w F_{t+1} = \hat{\psi}_{z^+,t} - \left( \sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) \tag{E.7}
$$

$$
\left( 1 - \beta\xi_w L^{-1} \right) G_t \equiv G_t - \beta\xi_w G_{t+1} = \beta\xi_w \alpha_{h,1} \widehat{\tilde{w}}_{t+1} - \frac{(1 - \alpha_{h,1})\beta\xi_w}{1 - \beta\xi_w} \left( \Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right)
$$

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We now obtain a second restriction on $\hat{w}_t$ using the relation between the aggregate wage rate and the wage rates of individual households:

$$W_t = \left[ (1 - \xi_w) \left( \tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left( \tilde{\pi}_{w,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w}.$$

Dividing both sides by $W_t$:

$$1 = (1 - \xi_w)(w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left( \frac{\tilde{\pi}_{w,t} W_{t-1}}{W_t} \right)^{\frac{1}{1-\lambda_w}}.$$

Note,

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\bar{w}_t z_t^+ P_t}{\bar{w}_{t-1} z_{t-1}^+ P_{t-1}} = \frac{\bar{w}_t \mu_{z^+,t} \pi_t}{\bar{w}_{t-1}},$$

so that

$$1 = (1 - \xi_w)(w_t)^{\frac{1}{1-\lambda_w}} + \xi_w \left( \frac{\bar{w}_{t-1} \tilde{\pi}_{w,t}}{\bar{w}_t \mu_{z^+,t} \pi_t} \right)^{\frac{1}{1-\lambda_w}}.$$

Differentiate and make use of $w = 1$, $\tilde{\pi}_w = \mu_{z^+} \pi$:

$$0 = (1 - \xi_w) \frac{1}{1 - \lambda_w} \hat{w}_t + \xi_w \frac{1}{1 - \lambda_w} \left[ \widehat{\bar{w}}_{t-1} + \widehat{\tilde{\pi}}_{w,t} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \hat{\pi}_t \right],$$

or,

$$\hat{w}_t = -\frac{\xi_w}{1 - \xi_w} \left[ \widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right].$$

Use this expression to substitute out for $\hat{w}_t$ in (E.6):

$$\frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \frac{\xi_w}{1 - \xi_w} \left[ \widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right] = F_t + \frac{1 - \beta\xi_w \alpha_{h,1}}{1 - \beta\xi_w} \widehat{\bar{w}}_t + G_t.$$

Multiply by $(1 - \beta\xi_w L^{-1})$ and use (E.7):

$$\frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \frac{\xi_w}{1 - \xi_w} \left( 1 - \beta\xi_w L^{-1} \right) \left[ \widehat{\bar{w}}_{t-1} - \widehat{\bar{w}}_t - \hat{\mu}_{z^+,t} - \Delta_{\kappa_w} \hat{\pi}_t \right]$$

$$= \hat{\psi}_{z^+,t} - \left( \sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \left( 1 - \beta\xi_w L^{-1} \right) \frac{1 - \beta\xi_w \alpha_{h,1}}{1 - \beta\xi_w} \widehat{\bar{w}}_t$$

$$+ \beta\xi_w \alpha_{h,1} \widehat{\bar{w}}_{t+1} - \frac{(1 - \alpha_{h,1}) \beta\xi_w}{1 - \beta\xi_w} \left( \Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right),$$

or,

$$\frac{1 - \alpha_{h,1}}{1 - \beta\xi_w} \frac{\xi_w}{1 - \xi_w} \left[ \begin{array}{c} \widehat{\bar{w}}_{t-1} - \beta\xi_w \widehat{\bar{w}}_t - \widehat{\bar{w}}_t + \beta\xi_w \widehat{\bar{w}}_{t+1} - \hat{\mu}_{z^+,t} \\ + \beta\xi_w \hat{\mu}_{z^+,t+1} - \Delta_{\kappa_w} \hat{\pi}_t + \beta\xi_w \Delta_{\kappa_w} \hat{\pi}_{t+1} \end{array} \right]$$

$$= \hat{\psi}_{z^+,t} - \left( \sigma_\varsigma \hat{\zeta}_t + \sigma_z \hat{H}_t \right) + \frac{1 - \beta\xi_w \alpha_{h,1}}{1 - \beta\xi_w} \left[ \widehat{\bar{w}}_t - \beta\xi_w \widehat{\bar{w}}_{t+1} \right]$$

$$+ \beta\xi_w \alpha_{h,1} \widehat{\bar{w}}_{t+1} - \frac{(1 - \alpha_{h,1}) \beta\xi_w}{1 - \beta\xi_w} \left( \Delta_{\kappa_w} \hat{\pi}_{t+1} + \hat{\mu}_{z^+,t+1} \right).$$

Note that the wage does not simply enter via nominal wage inflation. To see this, note

$$\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} = \hat{\pi}_{w,t} - \hat{\mu}_{z^+,t} - \hat{\pi}_t,$$

where $\hat{\pi}_{w,t}$ denotes nominal wage inflation. But, it is not simply $\widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1}$ that enters in this expression. That is, if we tried to express the above expression in terms of nominal wage inflation, we would simply add another variable to it, $\hat{\pi}_{w,t}$, without subtracting any, such as the real wage, $\widehat{\bar{w}}_t$. Collecting terms:

$$
\begin{aligned}
0 = & \ E_t[\eta_0\widehat{\bar{w}}_{t-1} + \eta_1\widehat{\bar{w}}_t + \eta_2\widehat{\bar{w}}_{t+1} + \eta_3\hat{\pi}_{t-1} + \eta_4\hat{\pi}_t + \eta_5\hat{\pi}_{t+1} + \eta_6\hat{\mu}_{z^+,t} + \eta_7\hat{\mu}_{z^+,t+1} \quad \text{(E.8)} \\
& + \eta_8\hat{\psi}_{z^+,t} + \eta_9\hat{H}_t + \eta_{10}\hat{\zeta}_t],
\end{aligned}
$$

where

$$
\begin{aligned}
\eta_0 &= \frac{1-\alpha_{h,1}}{1-\beta\xi_w}\frac{\xi_w}{1-\xi_w}, \quad \eta_1 = -\eta_0\left(1+\beta\xi_w\right) - \frac{(1-\beta\xi_w\alpha_{h,1})}{1-\beta\xi_w}, \\
\eta_2 &= \beta\xi_w\left(\eta_0 + \frac{(1-\beta\xi_w\alpha_{h,1})}{1-\beta\xi_w} - \alpha_{h,1}\right), \quad \eta_3 = \eta_0\kappa_w, \\
\eta_4 &= -\eta_0\left(1+\kappa_w\beta\xi_w\right) - \frac{(1-\alpha_{h,1})\beta\xi_w}{1-\beta\xi_w}\kappa_w, \\
\eta_5 &= \eta_0\beta\xi_w + \frac{(1-\alpha_{h,1})\beta\xi_w}{1-\beta\xi_w}, \\
\eta_6 &= -\eta_0, \ \eta_7 = \eta_5, \ \eta_8 = -1, \ \eta_9 = \sigma_z, \ \eta_{10} = \sigma_\varsigma.
\end{aligned}
$$

Note that (E.8) is the same for the standard model and for our model with involuntary unemployment. The difference between the two models has only to do with the construction of $\sigma_\varsigma$ and $\sigma_z$.

The wage equation can be thought of, for computational purposes, as a nonlinear equation, if we treat

$$\widehat{\bar{w}}_t = \frac{\bar{w}_t - \bar{w}}{\bar{w}},$$

and the other hatted variables in the same way.

### E.3. Other Equilibrium Conditions

### E.3.1. Firms

We let $s_t$ denote the firm's marginal cost, divided by the price of the homogeneous good. The standard formula, expressing this as a function of the factor inputs, is as follows:

$$s_t = \frac{\left(\frac{r_t^k P_t}{\alpha}\right)^\alpha \left(\frac{W_t R_t^f}{1-\alpha}\right)^{1-\alpha}}{P_t z_t^{1-\alpha}}.$$

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When expressed in terms of scaled variables, this reduces to:

$$s_t = \left(\frac{\bar{r}_t^k}{\alpha}\right)^\alpha \left(\frac{\bar{w}_t R_t^f}{1-\alpha}\right)^{1-\alpha}. \tag{E.9}$$

Productive efficiency dictates that $s_t$ is also equal to the ratio of the real cost of labor to the marginal product of labor:

$$s_t = \frac{\left(\mu_{\Psi,t}\right)^\alpha \bar{w}_t R_t^f}{(1-\alpha)\left(\frac{k_{i,t}}{\mu_{z^+,t}}/H_{i,t}\right)^\alpha}. \tag{E.10}$$

The only real decision taken by intermediate good firms is to optimize price when it is selected to do so under the Calvo frictions. According The first order necessary conditions associated with price optimization are, after scaling:[58]

$$E_t\left[\psi_{z^+,t}y_t + \left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}}\beta\xi_p F_{t+1}^f - F_t^f\right] = 0 \tag{E.11}$$

$$E_t\left[\lambda_f\psi_{z^+,t}y_t s_t + \beta\xi_p\left(\frac{\tilde{\pi}_{f,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}}K_{t+1}^f - K_t^f\right] = 0, \tag{E.12}$$

$$\mathring{p}_t = \left[(1-\xi_p)\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{\lambda_f} + \xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_f}{1-\lambda_f}}\right]^{\frac{1-\lambda_f}{\lambda_f}}, \tag{E.13}$$

$$\left[\frac{1-\xi_p\left(\frac{\tilde{\pi}_{f,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right]^{(1-\lambda_f)} = \frac{K_t^f}{F_t^f}, \tag{E.14}$$

$$\tilde{\pi}_{f,t} \equiv (\pi_{t-1})^{\kappa_d}(\pi)^{1-\kappa_d}. \tag{E.15}$$

In terms of scaled variables, the law of motion for the capital stock is as follows:

$$\bar{k}_{t+1} = \frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t + \Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{E.16}$$

The aggregate production relation is:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_f}{\lambda_f-1}}\left[\epsilon_t\left(\frac{1}{\mu_{\Psi,t}}\frac{1}{\mu_{z^+,t}}\bar{k}_t u_t\right)^\alpha H_t^{1-\alpha} - \phi\right].$$

---

[58]When we linearize about the steady state and set $\varkappa_d = 0$, we obtain,

$$\hat{\pi}_t - \hat{\bar{\pi}}_t = \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1} - \hat{\bar{\pi}}_{t+1}\right) + \frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1} - \hat{\bar{\pi}}_t\right) - \frac{\kappa_d\beta\left(1-\rho_\pi\right)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t + \frac{1}{1+\kappa_d\beta}\frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d}\widehat{mc}_t,$$

where a hat indicates log-deviation from steady state.

Finally, the resource constraint is:

$$y_t = g_t + c_t + i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}.$$

### E.3.2. Family

We now derive the equilibrium conditions associated with the household, apart from the wage condition, which was derived in a previous subsection. The Lagrangian representation of the household's problem is:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \{[\ln\left(C_t - bC_{t-1}\right) - z\left(h_{t,j}, \zeta_t\right)]$$

$$\upsilon_t \left[\begin{array}{c} W_{t,j}h_{t,j} + X_t^k \bar{K}_t + \left(R_{t-1} - \tau^R\left(R_{t-1} - 1\right)\right)B_t \\ +a_{t,j} - P_t\left(C_t + \frac{1}{\Psi_t}I_t\right) - B_{t+1} - P_t P_{k',t}\Delta_t \end{array}\right]$$

$$+\omega_t \left[\Delta_t + (1-\delta)\bar{K}_t + \left(1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)I_t - \bar{K}_{t+1}\right]\}$$

The first order condition with respect to $C_t$ is:

$$\frac{1}{C_t - bC_{t-1}} - E_t\frac{b\beta}{C_{t+1} - bC_t} = \upsilon_t P_t,$$

or, after expressing this in scaled terms and multiplying by $z_t^+$:

$$\psi_{z^+,t} = \frac{1}{c_t - b\frac{c_{t-1}}{\mu_{z^+,t}}} - \beta b E_t\frac{1}{c_{t+1}\mu_{z^+,t+1} - bc_t}. \tag{E.17}$$

The first order condition with respect to $\Delta_t$ is, after rearranging:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}. \tag{E.18}$$

The first order condition with respect to $I_t$ is:

$$\omega_t \left[1 - \tilde{S}\left(\frac{I_t}{I_{t-1}}\right) - \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + E_t\beta\omega_{t+1}\tilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = \frac{P_t\upsilon_t}{\Psi_t}.$$

Making use of (E.18), multiplying by $\Psi_t z_t^+$, rearranging and using the scaled variables,

$$\psi_{z^+,t}p_{k',t}\left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] \tag{E.19}$$

$$+\beta\psi_{z^+,t+1}p_{k',t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{z^+,t+1}\mu_{\Psi,t+1} = \psi_{z^+,t},$$

Optimality of the choice of $\bar{K}_{t+1}$ implies the following first order condition:

$$\omega_t = \beta E_t\upsilon_{t+1}X_{t+1}^k + \beta E_t\omega_{t+1}\left(1-\delta\right) = \beta E_t\upsilon_{t+1}\left[X_{t+1}^k + P_{t+1}P_{k',t+1}\left(1-\delta\right)\right],$$

using (E.18). Using (E.18) again,

$$\upsilon_t = E_t\beta\upsilon_{t+1}\left[\frac{X_{t+1}^k + P_{t+1}P_{k',t+1}\left(1-\delta\right)}{P_tP_{k',t}}\right] = E_t\beta\upsilon_{t+1}R_{t+1}^k, \tag{E.20}$$

where $R_{t+1}^k$ denotes the rate of return on capital:

$$R_{t+1}^k \equiv \frac{X_{t+1}^k + P_{t+1}P_{k',t+1}\left(1-\delta\right)}{P_tP_{k',t}}$$

Multiply (E.20) by $P_t z_t^+$ and express the results in scaled terms:

$$\psi_{z^+,t} = \beta E_t\psi_{z^+,t+1}\frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{E.21}$$

Expressing the rate of return on capital, (4.13), in terms of scaled variables:

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}}\frac{u_{t+1}\bar{r}_{t+1}^k - a(u_{t+1}) + (1-\delta)p_{k',t+1}}{p_{k',t}}. \tag{E.22}$$

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = a'\left(u_t\right),$$

or, in scaled terms,

$$\bar{r}_t^k = a'\left(u_t\right). \tag{E.23}$$

The first order condition with respect to $B_{t+1}$ is:

$$\upsilon_t = \beta\upsilon_{t+1}R_t.$$

Multiply by $z_t^+ P_t$ :

$$\psi_{z^+,t} = \beta E_t\frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}\pi_{t+1}}R_t. \tag{E.24}$$

### E.4. Steady State

We describe two strategies for computing the steady state. In each case, the strategy is applied to our model with involuntary unemployment, and we indicate what changes are required for the standard model. The first steady state strategy takes all the model parameters as given and computes the endogenous variables. The second imposes values for several endogenous variables and solves for an equal number of parameters.

### E.4.1. First Algorithm

Consider the equilibrium conditions associated with price setting. In steady state, these reduce to (we have used that $\tilde{\pi}^f = \pi$ using (??)):

$$
\begin{aligned}
F^f &= \frac{\psi_{z^+} y}{1 - \beta \xi_p} \\
K^f &= \frac{\lambda_f \psi_{z^+} ys}{1 - \beta \xi_p} \\
\mathring{p} &= 1 \\
\frac{K^f_t}{F^f_t} &= 1
\end{aligned}
$$

These equations imply

$$
(1) s = \frac{1}{\lambda_f}.
$$

Thus,

$$
(2) s = \left(\frac{\bar{r}^k}{\alpha}\right)^\alpha \left(\frac{\bar{w} R^f}{1 - \alpha}\right)^{1-\alpha} = \frac{1}{\lambda_f} \rightarrow \bar{w} = \frac{(1 - \alpha)}{R^f} \left[\frac{1}{\lambda_f} \left(\frac{\alpha}{\bar{r}^k}\right)^\alpha\right]^{\frac{1}{1-\alpha}},
$$

$$
(3) s = \frac{(\mu_\Psi)^\alpha \bar{w} R^f}{\epsilon (1 - \alpha) \left(\frac{\bar{k}}{\mu_{z^+}}/H\right)^\alpha} = \frac{1}{\lambda_f}, \rightarrow \frac{\bar{k}}{H} = \mu_{z^+} \left[\frac{\lambda_f (\mu_\Psi)^\alpha \bar{w} R^f}{\epsilon (1 - \alpha)}\right]^{\frac{1}{\alpha}}
$$

where

$$
(4) R^f = \nu^f R + 1 - \nu^f
$$

The equilibrium conditions associated with wage setting is:

$$
(5) \bar{w} = \lambda_w \frac{z_h}{\psi_{z^+}},
$$

which is the usual wage markup equation. Also,

$$
(6) h = H.
$$

The consumption-compensated elasticity of labor supply is

$$
(7) \sigma_z = \frac{z_{hh} h}{z_h}.
$$

This reduces to $\sigma_L$ in the case of the standard model, and is derived above in the case of the model with involuntary unemployment.

Also,

$$
(8) \left[1 - \frac{1 - \delta}{\mu_{z^+} \mu_\Psi}\right] \bar{k} = \Upsilon i,
$$

and

$$(9)\psi_{z+} = \frac{1}{c}\frac{\mu_{z+} - \beta b}{\mu_{z+} - b},$$

$$p_{k'} = 1$$

Consider the utilization adjustment cost function. We specify that as follows, with $\sigma_b = \bar{r}^k$ :

$$a(u) = 0.5\sigma_b\sigma_a u^2 + \sigma_b(1 - \sigma_a)u + \sigma_b((\sigma_a/2) - 1)$$

$$a(1) = \sigma_b\sigma_a + \sigma_b(1 - \sigma_a) - \sigma_b$$

Then, $a'(u) = \bar{r}^k$ implies

$$u = 1, \ a(1) = 0.$$

We use the latter two results in what follows. The household intertemporal Euler equation for capital implies:

$$(10)1 = \beta\frac{R^k}{\pi\mu_{z+}},$$

with

$$(11)R^k = \frac{\pi}{\mu_\Psi}\left[\bar{r}^k + 1 - \delta\right].$$

The intertemporal Euler equation for nominal bonds:

$$(12)1 = \frac{\beta R}{\mu_{z+}\pi}.$$

The production function and resource constraint:

$$(13)y = \epsilon\left(\frac{1}{\mu_\Psi}\frac{1}{\mu_{z+}}\bar{k}\right)^\alpha H^{1-\alpha} - \phi,$$

$$(14)y = g + c + i$$

The 17 variables whose steady state values are to be determined are:

$$y, c, i, R, h, H, \bar{k}, \bar{r}^k, R^k, \bar{w}, \psi_{z+}, s, R^f, u, \bar{p}, m, \sigma_z.$$

Here is a strategy for solving these equations. Equation (1) produces $s$; $R$ from (12); $R^f$ from (4); $R^k$ from (10); $\bar{r}^k$ from (11); $\bar{w}$ from (2); $\bar{k}/H$ from (3).

The remaining variables can be found using a one dimensional search. Fix a value for $h$. By (6), we have $H$; from $\bar{k}/H$ we have $\bar{k}$; from (13) we have $y$; from (8) we have $i$; from (14) and

$$\eta_g = \frac{g}{y},$$

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we have $c$; from (9) we have $\psi_{z+}$; in the case of the model with involuntary unemployment, compute $m$ from the steady state version of (2.10):

$$(15) h = m\eta + a^2 \varsigma \sigma_L m^{\sigma_L + 1};$$

compute $\sigma_z$ using (7) or $\sigma_z = \sigma_L$ in the case of the standard model; we are now in a position to evaluate (5):

$$f(h) = \bar{w} - \lambda_w \frac{z_h}{\psi_{z+}}.$$

Adjust $h$ until $f(h) = 0$.

In the case of our model with involuntary unemployment, we proceed as follows. Compute $u$ from

$$(16) u = \frac{m - h}{m}.$$

Let $\bar{p}$ denote the steady state probability that the marginal household with $l = 0$ finds a job. According to (??):

$$(17) \bar{p} = \eta + \varsigma a^2 (1 + \sigma_L) m^{\sigma_L}.$$

This algorithm solves for 17 endogenous variables using 17 equations.

In the case of the standard model, $m$ solves the steady state version of (4.22):

$$\psi_{z+} \bar{w} = \varsigma (m)^{\sigma_L},$$

and then the unemployment rate is computed using $m$ and $h$.

### E.4.2. A Second Algorithm

We find it convenient to shift three endogenous labor market variables to the list of exogenous variables:

$$u, \ m, \ \bar{p}, \ \sigma_z.$$

Corresponding to this, we shift four parameters to the list of endogenous variables:

$$F, \varsigma, a, \eta.$$

Thus, we must solve for the following 17 variables:

$$y, c, i, R, h, H, \bar{k}, \bar{r}^k, R^k, \bar{w}, \psi_{z+}, s, R^f, F, \varsigma, a, \eta.$$

We compute the steady state as follows. As before, equation (1) produces $s$; $R$ from (12); $R^f$ from (4); $R^k$ from (10); $\bar{r}^k$ from (11); $\bar{w}$ from (2); $\bar{k}/H$ from (3). Compute $h$ from (16); $H$ is found from (6); $\bar{k}$ from $\bar{k}/H$; $y$ from (13); $i$ from (8); $c$ from (14) and

$$\eta_g = \frac{g}{y};$$

$\psi_{z+}$ from (9).

Equations (5), (7), (15), and (17) remain to be solved for $F, \varsigma, a, \eta$. Fix values of $F$ and $a$. We compute $\eta$ as follows. Rewrite (15):

$$m^{\sigma_L+1} = \frac{h - m\eta}{a^2 \varsigma \sigma_L}.$$

Multiply both sides of (17) by $m$ and substitute out for $m^{\sigma_L+1}$ from the previous expression:

$$\frac{\sigma_L}{1 + \sigma_L} \left( \bar{p} + \frac{\eta}{\sigma_L} \right) m = h$$

Substitute this into (16):

$$u = \frac{m - h}{m} = 1 - \frac{\sigma_L}{1 + \sigma_L} \left( \bar{p} + \frac{\eta}{\sigma_L} \right) = \frac{1 + \sigma_L (1 - \bar{p}) - \eta}{1 + \sigma_L},$$

or,

$$\eta = 1 + \sigma_L (1 - \bar{p}) - (1 + \sigma_L) u,$$

so that we now have $\eta$ (this must be non-negative). Rewriting (15), we have $\varsigma$ :

$$(15') \quad \varsigma = \frac{h - m\eta}{a^2 m^{\sigma_L+1} \sigma_L}.$$

Next, adjust $F$ and $a$ so that (5) and (7) are satisfied.

### E.4.3. Steady State Replacement Ratio

Here, we compute the steady state consumption of employed and non-employed households. By the analog of (2.8), the incentive constraint for the $j^{th}$ family is:

$$c_{t,j}^w = bC_{t-1} + \left( c_{t,j}^{nw} - bC_{t-1} \right) e^{F + \zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}}.$$

The $j^{th}$ family's resource constraint is:

$$h_{t,j} c_{t,j}^w + (1 - h_{t,j}) c_{t,j}^{nw} = C_t.$$

Substituting out for $c_{t,j}^w$ from the incentive constraint, we obtain:

$$bC_{t-1} + \left( c_{t,j}^{nw} - bC_{t-1} \right) e^{F + \zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} + \frac{(1 - h_{t,j})}{h_{t,j}} c_{t,j}^{nw} = \frac{C_t}{h_{t,j}}.$$

Rearrange,

$$c_{t,j}^{nw} = \frac{C_t + \left( e^{F + \zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1 \right) h_{t,j} bC_{t-1}}{h_{t,j} \left( e^{F + \zeta_t(1+\sigma_L)m_{t,j}^{\sigma_L}} - 1 \right) + 1}.$$

Scaling,

$$\tilde{c}^{nw}_{t,j} = \frac{c_t + \left(e^{F+\zeta_t(1+\sigma_L)m^{\sigma_L}_{t,j}} - 1\right) h_{t,j}\frac{b}{\mu_{z+}}c_{t-1}}{h_{t,j}\left(e^{F+\zeta_t(1+\sigma_L)m^{\sigma_L}_{t,j}} - 1\right) + 1}.$$

In steady state:

$$\tilde{c}^{nw} = c\frac{1 + \left(e^{F+\zeta(1+\sigma_L)m^{\sigma_L}} - 1\right) h\frac{b}{\mu_{z+}}}{h\left(e^{F+\zeta(1+\sigma_L)m^{\sigma_L}} - 1\right) + 1},$$

and

$$\tilde{c}^{w} = \frac{b}{\mu_{z+}}c + \left(c^{nw} - \frac{b}{\mu_{z+}}c\right)e^{F+\zeta(1+\sigma_L)m^{\sigma_L}}$$

## F. Aggregate Hours Worked

Given our linear approximation, and the assumptions that imply that steady state is undistorted by wage frictions, we have

$$\hat{h}_t = \hat{H}_t.$$

Although this is a well known result (see, e.g., Yun (1996)), we derive it here for completeness. Recall,

$$h_t \equiv \int_0^1 h_{j,t}dj.$$

Invert the demand for labor, (4.5), to obtain an expression in terms of $h_{j,t}$. Substitute this into the expression for $h_t$ to obtain:

$$h_t = H_t \int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj, \tag{F.1}$$

where

$$\mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Here, $W_t$ denotes the aggregate wage rate, which one obtains by substituting (4.4) into (4.5):

$$W_t = \left[\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} dj\right]^{1-\lambda_w}.$$

Because all families are identical in steady state (see the discussion after (4.9)), $\mathring{w}_j = 1$ for all $j$. Totally differentiating (F.1),

$$\hat{h}_t = \hat{H}_t + \int_0^1 \widehat{\mathring{w}}_{j,t}dj.$$

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Thus, to determine the percent deviation of aggregate employment from steady state, we require the integral of the percent deviations of type $j$ wages from the aggregate wage, over all $j$. We now show that this integral is, to first order, equal to zero.

Express the integral in (F.1) as follows:

$$h_t = \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t,$$

say, where

$$\mathring{w}_t \equiv \left[ \int_0^1 \mathring{w}_{j,t}^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}}. \tag{F.2}$$

Pursuing logic that is standard in the Calvo price/wage setting literature we obtain:

$$W_t = \left[ (1-\xi_w) \left( \tilde{W}_t \right)^{\frac{1}{1-\lambda_w}} + \xi_w \left( \tilde{\pi}_{w,t} W_{t-1} \right)^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w} \tag{F.3}$$

$$\mathring{w}_t = \left[ (1-\xi_w) w_t^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mathring{w}_{t-1} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right]^{\frac{1-\lambda_w}{\lambda_w}}, \tag{F.4}$$

where:

$$w_t \equiv \frac{\tilde{W}_t}{W_t}, \quad \pi_{w,t} \equiv \frac{W_t}{W_{t-1}},$$

and $\tilde{W}_t$ denotes the wage set by the $1 - \xi_w$ families that have the opportunity to reoptimize in the current period. Because all families are identical in steady state

$$w = \mathring{w} = \frac{\tilde{\pi}_w}{\pi_w} = 1, \tag{F.5}$$

where $\tilde{\pi}_{w,t}$ is defined in (4.9) and $\pi_{w,t}$ denotes wage inflation:

$$\pi_{w,t} \equiv \frac{W_t}{W_{t-1}}.$$

Dividing (F.3) by $W_t$ and solving,

$$w_t = \left[ \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w}. \tag{F.6}$$

Differentiating (F.4) and (F.6) in steady state:

$$\widehat{\mathring{w}}_t = (1-\xi_w) \hat{w}_t + \xi_w \left( \widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} + \widehat{\mathring{w}}_{t-1} \right) \tag{F.7}$$

$$\hat{w}_t = -\frac{\xi_w}{1-\xi_w} \left( \widehat{\tilde{\pi}}_{w,t} - \hat{\pi}_{w,t} \right)$$

Using the latter to substitute out for $\hat{w}_t$ in (F.7):

$$\widehat{\hat{w}}_t = \xi_w \widehat{\hat{w}}_{t-1}.$$

Thus, to first order the wage distortions evolve according to a stable first order difference equation, unperturbed by shocks. For this reason, we set

$$\widehat{\hat{w}}_t = 0, \tag{F.8}$$

for all $t$.

Totally differentiating (F.2) and using (F.5), (F.8):

$$\int_0^1 \widehat{\hat{w}}_{j,t} dj = 0.$$

That is, to first order, the integral of the percent deviations of individual wages from the aggregate is zero.

## G. Technical Appendix: Tables

Table A1: Structural Parameters of Small Model Held Fixed

| Parameter | Value | Description |
|---|---|---|
| $\beta$ | $1.03^{-.25}$ | Discount factor |
| $g_A$ | 1.0047 | Technology growth |
| $\xi_p$ | 0.75 | Price stickiness |
| $\lambda_f$ | 1.2 | Price markup |
| $\rho_R$ | 0.8 | Taylor rule: interest smoothing |
| $r_\pi$ | 1.5 | Taylor rule: inflation |
| $r_y$ | 0.2 | Taylor rule: output gap |
| $\gamma$ | 0.001 | Diffusion speed of technology into gov. consumption |
| $\eta_g$ | 0.2 | Government consumption share on GDP |

Table A2: The Impact of Imperfect Information in the Small Model

| Variable | Involuntary Unemp. Model (Imperfect Information) | Full Information Model | | Standard Model | Description |
|---|---|---|---|---|---|
| | | Fixed Structural Params[b] | Obser-vational Equivalent[c] | Obser-vational Equivalent[d] | |
| *Steady State Properties* | | | | | |
| $m$ | 0.67 | 0.69 | 0.67 | 0.63 | Labor force |
| $h$ | 0.63 | 0.68 | 0.63 | 0.63 | Employment |
| $u$ | 0.056 | 0.015 | 0.056 | n.a. | Unemployment rate |
| $\bar{p}$ | 0.95 | 0.99 | 0.95 | n.a. | Max $p(e)$ |
| $1/\sigma_z$ | 2.0 | 0.80 | 2.0 | 2.0 | Labor supply elast. |
| $1/\kappa^{Okun}$ | 2.0 | 1.64 | 2.0 | n.a. | Okun's law coefficient |
| $c^{nw}/c^w$ | 0.18 | 1.0 | 1.0 | 1.0 | Replacement ratio |
| | 0.189 | | n.a. | n.a. | Price (% of $C$) of info.[a] |
| *Structural Parameters[e]* | | | | | |
| $a$ | 0.53 | 0.53 | 0.74 | n.a. | Slope, $p(e)$ |
| $\eta$ | 0.86 | 0.86 | 0.86 | n.a. | Intercept, $p(e)$ |
| $\varsigma$ | 4.64 | 4.64 | 2.45 | 1.67 | Slope, labor disutility |
| $F$ | 1.39 | 1.39 | 1.83 | n.a. | Intercept, labor dis.util. |
| $\sigma_L$ | 13.31 | 13.31 | 13.31 | 0.5 | Power, labor disutility |

[a] Percent increase in consumption in steady state of involuntary unemployment model that makes

steady state utility in that model

equal to steady state utility of model with full information.

[b] Full information model with same structural parameters as involuntary unemployment model.

[c] Full information model with parameter values in Table A1, plus parameter values in bottom panel

of this table, chosen so that full information model steady state properties

in the top panel (except $c^{nw}/c^w$) coincide with those in involuntary unemployment model.

[d] Standard model with parameter values in Table A1, plus parameter values for $\varsigma$ and

$\sigma_L$ so that $h$ and $1/\sigma_z$ coincide with those of the involuntary unemployment model.

[e] Model structural parameter values are those listed in Table A1 plus the ones indicated

in the bottom panel of this table.