The Procyclicality of Expected Credit Loss Provisions*

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Abstract

The Great Recession has pushed accounting standards for banks’ loan loss provisioning to shift from an incurred loss approach to an expected credit loss approach. IFRS 9 and the incoming update of US GAAP imply a more timely recognition of credit losses but also greater responsiveness to changes in aggregate conditions, which raises procyclicality concerns. This paper develops and calibrates a recursive ratings-migration model to assess the impact of different provisioning approaches on the cyclicality of banks’ profits and regulatory capital. The model is used to analyze the effectiveness of potential policy responses to the procyclicality problem.

Keywords: credit loss allowances, expected credit losses, incurred losses, rating migrations, procyclicality.

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1 Introduction

Banks’ delayed recognition of credit losses under the incurred loss (IL) approach to loan loss provisioning was argued to contribute to the severity of the Global Financial Crisis (Financial Stability Forum, 2009). By provisioning “too little, too late,” it might have prevented banks from being more cautious in good times and reduced the pressure for prompt corrective action in bad times. Based on this diagnosis, the G-20 called for a more forward-looking approach. As a result, the International Accounting Standards Board and the US Financial Accounting Standards Board developed reforms, namely IFRS 9 (entered into force in 2018) and an update of US GAAP (scheduled for 2021), which, with some differences, coincide in adopting an expected credit loss (ECL) approach to provisioning.\(^1\)

Under the new approach, loan loss provisions are intended to represent best unbiased estimates of the discounted credit losses expected to emerge over some specified horizons. In the case of IFRS 9, the horizon depends on the credit quality of the exposures, varying from one year for those without deteriorated credit quality (stage 1) to the residual lifetime of the credit instrument for exposures with deteriorated credit quality (stage 2) or already impaired (stage 3). In contrast, the so-called current expected credit loss (CECL) model envisaged by US GAAP opts for using the residual lifetime horizon for all exposures. The general perception is that the ECL approach will increase the reliability of bank capital as a measure of solvency and facilitate prompt corrective action (Cohen and Edwards, 2017).

There are concerns, however, that, absent the capacity of banks to anticipate adverse shifts in aggregate conditions sufficiently in advance, the point-in-time (PIT) nature of the estimates of ECLs might imply a more abrupt deterioration of profits and capital when the economy enters recession or a crisis starts, as it will be then and not before when the bulk of the implied future credit losses will be recognized (European Systemic Risk Board, 2017). The fear is that banks’ or markets’ reaction to such an increase (or to its impact on profits and regulatory capital) cause or amplify asset sales or a credit crunch, and end up producing negative feedback effects on the evolution of the economy, making the contraction more severe.

\(^1\)See International Accounting Standards Board (2014) and Financial Accounting Standards Board (2016) for details.
This paper develops a recursive model with which to compare the impact on profits and capital of IFRS 9 and CECL relative to their less forward-looking alternatives (namely, IL and the one-year expected loss approach behind the internal-ratings based approach to capital regulation). We assess the importance of the procyclical effects and the effectiveness of several policy options for their containment, including the reinforcement or active use of the loss absorbing capital buffers introduced in Basel III.

We address the modeling of ECLs in the context of a ratings-migration model with a compact description of credit risk categories, the economic cycle represented as a Markov chain, and loan maturity modeled as random. Each of these modeling strategies has a well-established tradition in economics and finance and their combination prevents us from having to keep track of loan vintages or a more complex state space, producing a model which is overall highly tractable.\(^2\) The model is calibrated to capture the evolution of credit risk in a typical portfolio of European corporate loans over the business cycle and to compare the cyclical behavior of impairment allowances, profit or loss (P/L), and common equity tier 1 (CET1) across the various provisioning approaches.\(^3\) For the calibration, we use evidence on the sensitivity of rating migration matrices and credit loss parameters to business cycles, as in Bangia et al. (2002). We find that the more forward-looking methods of IFRS 9 and CECL imply significantly larger impairment allowances, sharper on-impact responses to the arrival of an average recession than the old IL and one-year expected loss used by IRB banks, and the quicker recovery of P/L and CET1 after the initial impact.

The arrival of a typical recession implies on-impact increases in IFRS 9 and CECL provisions equivalent to about a third of a bank’s fully loaded capital conservation buffer (CCB) or, equivalently, about twice as large as those implied by the IL approach.\(^4\) This suggests

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\(^2\)Ratings-migration models are extensively used in credit risk modeling (see Trueck and Rachev, 2009, for an overview) and Grünberger (2012) provides an early application of them to the analysis of IFRS 9. Hamilton (1989) showed the possibility of representing the cyclical phases and turning points identified in business cycle dating (e.g. by the NBER) using a binary Markov chain, and Bangia et al. (2002) and Repullo and Suarez (2013), among others, have used such representation in applications regarding fluctuations in credit risk. The modeling of debt maturity as random started with Leland and Toft (1996) and has been recently applied in a banking context by He and Xiong (2012), He and Milbradt (2014), and Segura and Suarez (2017), among others.

\(^3\)The case of a bank fully specialized in European corporate loans must be interpreted as a “laboratory case” with which to run “controlled experiments” about the performance of the different provisioning methods.

\(^4\)Under Basel III, banks’ reporting earnings must retain them until reaching a CCB (or buffer of capital
that the differential impact under the new approaches is sizeable but still suitably absorbable if banks’ CCBs are sufficiently loaded when the shock hits. As we show in extensions included in an appendix, the results depend on the degree to which turning points imply bigger or smaller surprises relative to what banks have anticipated in advance. We show that the sudden arrival of a contraction that is anticipated to be more severe or longer than average will tend to produce sharper responses, while forecasting a recession one or several years in advance would allow to significantly smooth away its impact on P/L and capital.

According to our analysis, if banks fail to anticipate turning points well in advance or to adopt additional precautions during good times, the more forward-looking provisioning methods may paradoxically mean that banks experience more sudden falls in regulatory capital right at the beginning of contractionary phases of the business cycle. Banks might accommodate these effects by consuming the capital buffers accumulated during good times, by cutting dividends or by issuing new equity. However, when confronted with these choices, banks often undertake at least part of the adjustment by reducing their risk-weighted assets (RWAs), for example by cutting the origination of new loans, selling some assets or rebalancing towards safer ones.\(^5\) This gives rise to the procyclical concerns emphasized in the title of the paper.

As discussed in the section devoted to the policy analysis, a full assessment of the procyclical impact of the new provisions would require taking care of multiple moving parts, including endogenous decisions and general equilibrium effects. In this paper we only look at some first-round, partial equilibrium effects that, we think, may help gauge the direction and intensity of the procyclical effects. We measure such intensity through the unconditional annual probability with which the bank described in the model needs to raise new capital to avoid violating its minimum regulatory capital requirements. We examine potentially mitigating policies such as increasing the target size of the CCB, actively using the countercyclical capital buffer (CCyB) of Basel III, introducing prudential buffers based on stress

\(^5\)See, for example, Mésonnier and Monks (2015), Aiyar, Calomiris, and Wieladek (2016), Behn, Haselmann and Wachtel (2016), Gropp et al. (2016), and the references therein. The evidence in these papers is consistent with average bank responses to the ESRB Questionnaire on Assessing Second Round Effects that accompanied the EBA stress test in 2016. The questionnaire examined the way in which banks would expect to reestablish their desired levels of capitalization after exiting the adverse scenario.
test results, and cyclically smoothing the inputs used in the estimation of ECLs. For reasons explained in the discussion of the results, we find that introducing a CCB add-on (possibly calibrated on the basis of stress test results) is not only the simplest but also the most effective of the compared policy options (that is, the one able to achieve a more significant reduction in the unconditional probability of having to recapitalize the bank).

We are aware of just a few papers trying to assess the cyclical behavior of impairment allowances under the new provisioning approaches, none of which addresses the analysis of potential mitigating policies. Cohen and Edwards (2017) develop such analysis from a top-down perspective and relying on the historical evolution of aggregate bank credit losses in a number of countries. Chae et al. (2018) use a more bottom-up approach based on credit loss data for first-lien mortgages originated in California between years 2002 and 2015. Krüger, Rösch, and Scheule (2018) use historical simulation methods on portfolios constructed using bonds from Moody’s Default and Recovery Database.

The first two papers posit the conclusion that, if banks are capable to perfectly foresee the incoming credit losses several years in advance, the new approaches will show smaller spikes in impairment allowances than the incurred loss approach when the economy situation deteriorates.\(^6\) This is consistent with what we obtain in the extension in which banks can anticipate the turning points in the evolution of the economy several periods in advance, albeit such capacity is not consistent with the difficulties faced by econometricians and professional forecasters in predicting recessions.\(^7\) The results in Krüger, Rösch, and Scheule (2018) are closer to ours and lead the authors to conclude that the new provisioning rules “will further increase the procyclicality of bank capital requirements” (p. 114).

The paper is organized as follows. Section 2 describes the model. Section 3 develops formulas for measuring impairment losses under the various provisioning approaches and for assessing their effects on P/L and CET1. Section 4 calibrates the model and uses it to analyze the effects of the arrival of a typical recession under the various measures. Section 5

\(^6\)Chae et al. (2017) also show that, if instead the inputs of the credit loss model are predicted using a high-order autoregressive (AR) model based on information available at the time of producing the ECL estimate, the implied provisions (ALLL) spike only once the housing crisis starts. In their words, the “AR forecast is not able to forecast the inflection point of home prices which leads to large increases in ALLL in early 2009.”

\(^7\)See Section B.4 of Appendix B for further discussion.
discusses the findings on procyclicality and analyzes the effectiveness of various policies that might be used to mitigate the problem. Section 6 concludes the paper. Appendices A, B, and C contain further details about the model, the calibration, and several complementary results.

2 The model

We consider a bank operating in an infinite-horizon discrete-time economy in which dates (regarded year-end dates for accounting reporting purposes) are denoted by $t$. The bank’s assets consist solely of a portfolio of loans whose individual credit risk, in the absence of aggregate risk, is fully described by a rating category $j$. The dynamics of loan origination, rating migration, default, and maturity of the loans that make up the bank’s loan portfolio are described in the first subsection below. The assumptions on the capital structure of the bank, the regulatory environment, and the pricing of the loans appear in the second subsection. For expositional clarity, we first present the main assumptions and formulas of the model in a version without aggregate risk. Then in the third subsection below we comment on the way in which the complete model incorporates aggregate risk. The notationally more intensive equations for the complete model are presented in Appendix A.

2.1 The bank’s loan portfolio

Each of the loans held by the bank can belong to one of three credit rating categories: standard ($j=1$), substandard ($j=2$) or non-performing ($j=3$). We denote the measure of loans of each category in the portfolio of the bank at date $t$ as $x_{j,t}$. In each date, the bank originates a measure $e_{1,t} > 0$ of standard loans with a principal normalized to one, and an endogenous contractual interest payment per period equal $c$. Each loan’s principal remains constant and equal to one up to maturity, which for performing loans ($j=1,2$) is assumed to occur randomly and independently at the end of each period with a constant probability $\delta_j$.\(^8\) This analytically convenient assumption implies that, conditional on remaining in rating $j$, a loan’s expected life span is $1/\delta_j$ and that, by the law of large numbers, the stream of cash

\(^8\)Allowing for $\delta_1 \neq \delta_2$ may help capture the possibility that longer maturity loans get early redeemed with different probabilities depending on their credit quality.
flows due to maturing loans is very similar to the one that would emerge with a portfolio of perfectly-staggered fixed-maturity loans.

The tree in Figure 1 summarizes the contingencies over the life of a loan.

Figure 1. Possible transitions of a loan rated \( j \). Possible contingencies between two dates and their implications for payoffs and continuation value. Variables on each branch describe marginal conditional probabilities.

Non-performing loans (NPLs, \( j=3 \)) are also assumed to be resolved randomly and independently with probability \( \delta_j \) per period, producing a terminal payoff \( 1-\lambda \), so \( \lambda \) is a loan’s loss rate at resolution. NPLs pay no interest and never return to the performing categories, so they accumulate in category \( j=3 \) up to their resolution.\(^9\)

Performing loans at \( t \), irrespectively of their maturity happening at \( t+1 \) or not, default independently with probability \( PD_j \) at \( t+1 \). Each loan that defaults between \( t \) and \( t+1 \) enters the stock of NPLs with an independent probability \( 1-\delta_3/2 \) and gets resolved within

\(^9\)For calibration purposes, it is possible to account for potential gains from the unmodeled interest accrued while in default or from returning to performing categories by adjusting the loss rate \( \lambda \).
the same period with the complementary probability $\delta_3/2$. Maturing loans that do not default pay back their principal of one plus interest $c$.

Performing loans at $t$ that do not mature at $t+1$ pay interest $c$, migrate to rating $i \neq j$ ($i=1,2$) independently with probability $a_{ij}$, and stay in rating $j$ independently with probability $a_{jj} = 1 - a_{ij} - PD_j$.

### 2.2 The bank’s capital structure and loan pricing

The bank originating and holding the loans is assumed to be perfectly competitive, fully solvent, and with access to funding from risk-neutral investors who face an opportunity cost of funds between any two periods constant and equal to $r$. To keep things simple, we assume that the bank finances its activity with one-period debt $d_t$ and equity (or more technically, CET1) $k_t$, and is subject to capital regulation as per a bank operating under the internal ratings-based (IRB) approach of Basel III.$^{11}$

We model the evolution of the bank’s capital structure by assuming that the bank is a minimizer of the equity funding $k_t$ required to support its loan portfolio under the prevailing capital regulation and provisioning regime.$^{12}$ The latter determines the loan loss allowances $LL_t$ associated with the bank’s loan portfolio at each date $t$.

In Section 3 we provide expressions for $LL_t$ under different provisioning approaches as well as further details on the law of motion of $k_t$, and its implications for the dividend payments and the recapitalization needs of the bank under our assumptions.

### 2.3 Adding aggregate risk

We introduce aggregate risk in the model by considering an aggregate state variable $s_t$ whose evolution affects the key parameters governing loan portfolio dynamics and credit losses in the structure described above. For our baseline results, we assume that $s_t$ follows a Markov

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$^{10}$We divide $\delta_3$ by two to reflect the fact that, if loans default uniformly during the period between $t$ and $t+1$, they will, on average, have just half a period to be resolved. Given that the calibration relies on one-year periods and the resolution rate is large, this refinement is important to realistically describe NPL dynamics. The model could easily accommodate alternative assumptions on same-period resolutions.

$^{11}$The case in which the bank follows the standardized approach (SA) is analyzed in Appendix C.

$^{12}$We could rationalize banks’ minimization of its CET1 as the result of imperfections that make equity financing more costly than debt financing. However, for simplicity, we formally consider the limit case where the excess cost of equity financing goes to zero so that loan pricing (as described below) does not depend on the bank’s capital structure.
chain with two states $s=1,2$ and time-invariant transition probabilities $p_{s's} = \text{Prob}(s_{t+1} = s'|s_t = s)$. We interpret $s=1$ and $s=2$ as identifying expansion and contraction periods, respectively.

3 Analysis

In this section we develop the formulas describing the dynamics of the bank’s loan portfolio, the measurement of impairment losses under the various provisioning approaches, the pricing of the loans, and the evolution of the bank’s profits and regulatory capital, under the model assumptions introduced in the previous section.

3.1 Portfolio dynamics

By the law of large numbers, the evolution of the loan portfolio can be represented by the following difference equation:

$$x_t = M x_{t-1} + e_t$$

where

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix}$$

is the vector that describes the loans in each rating category $j=1,2,3$;

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} (1 - \delta_1)a_{11} & (1 - \delta_2)a_{12} & 0 \\ (1 - \delta_1)a_{21} & (1 - \delta_2)a_{22} & 0 \\ (1 - \delta_3/2)PD_1 & (1 - \delta_3/2)PD_2 & (1 - \delta_3) \end{pmatrix}$$

is the matrix that accounts for the migrations across categories of the non-matured, non-resolved loans, and

$$e_t = \begin{pmatrix} e_{1t} \\ 0 \\ 0 \end{pmatrix}$$

accounts for the new loans originated at each date, which we write reflecting the fact that, as previously specified, all loans have rating $j=1$ at origination.

When computing some of the moments relevant for the calibration of the model, we weight each rating category by its share in the steady-state portfolio $x^*$ that would asymptotically be reached, in the absence of aggregate risk, if the amount of newly originated loans is equal.
at all dates \((e_t = e \text{ for all } t)\). Such steady-state portfolio can be obtained as the vector that solves:

\[
x = Mx + e \Leftrightarrow (I - M)x = e,
\]

that is,

\[
x^* = (I - M)^{-1}e.
\]

### 3.2 Measuring impairment losses

In the following subsections we provide formulas for impairment allowances, \(LL_t\), under six different approaches: incurred losses, \(IL_t\); discounted one-year expected losses, \(EL_t^{1Y}\); prudential expected losses under the IRB approach, \(EL_t^{IRB}\); discounted lifetime expected losses, \(EL_t^{LT}\); current expected credit losses, \(EL_t^{CECL}\); and IFRS 9 impairment allowances, \(EL_t^{IFRS9}\). Defining \(EL_t^{1Y}\) and \(EL_t^{LT}\) is useful to understand the mixed-horizon approach in IFRS 9 and to compare the size of the various measures. Eventually our quantitative analysis will focus on \(IL_t\), \(EL_t^{IRB}\), \(EL_t^{CECL}\), and \(EL_t^{IFRS9}\).

#### 3.2.1 Incurred losses

Under the narrowest interpretation, the incurred loss approach prescribes the provisioning, on an expected loss basis, of the exposures for which there is clear evidence of impairment, which in our model would be the NPLs in \(x_{3t}\). Thus, provisions at year \(t\) under the IL approach would be

\[
IL_t = \lambda x_{3t},
\]

since the loss rate \(\lambda\) is the expected loss given default (LGD) of the bank’s NPLs at date \(t\).\(^{13}\)

#### 3.2.2 Discounted one-year expected losses

As a reference for the criterion used for the measurement of the impairment allowances applied to stage 1 exposures under IFRS 9, we define the one-year discounted expected losses of the bank as

\[
EL_t^{1Y} = \lambda [\beta (PD_1 x_{1t} + PD_2 x_{2t}) + x_{3t}]
\]

\(^{13}\)Under our assumptions, the losses associated with loans defaulted between dates \(t - 1\) and \(t\) which are resolved within such period, \(\lambda(\delta_3/2)(PD_1 x_{1t-1} + PD_2 x_{2t-1})\), are directly recorded in the P/L of year \(t\) and do not enter \(IL_t\).
where $\beta = 1/(1+c)$ is the discount factor based on the contractual interest rate of the loans, $c$. Under this approach, impairment allowances for loans performing at $t$ (rated $j=1, 2$) are computed as the discounted expected losses due to default events expected to occur within the immediately incoming year. This approach is forward-looking, but the forecasting horizon is limited to one year. Instead, for NPLs ($j=3$), the default event has already happened and the allowances equal the (non-discounted) expected LGD of the loans, exactly as in $IL_t$.

In matrix notation, which will be useful when comparing the different impairment allowance measures later on, $EL_t^{1Y}$ can also be expressed as

$$EL_t^{1Y} = \lambda (\beta b x_t + x_{3t}),$$

where

$$b = (PD_1, PD_2, 0).$$

### 3.2.3 Prudential expected losses under the IRB approach

When the Basel Committee on Banking Supervision (BCBS) introduced the IRB approach to capital requirements, the idea was to set capital requirements so as absorb the bank’s unexpected credit losses (over a one year horizon) while assuming that the corresponding expected losses would have been already recognized (and subtracted from available capital) via provisions. This led to the introduction of a prudential notion of expected losses which can be formally expressed as

$$EL_t^{IRB} = \lambda [PD_1 x_{1t} + PD_2 x_{2t} + x_{3t}] = \lambda (b x_t + x_{3t}),$$

where the only difference with respect to (9) is the absence of discounting. As described in detail in Appendix A, in the presence of aggregate risk further differences appear due to the fact that the BCBS specifies that instead of PIT estimates of $PD_j$ and $\lambda$, (11) must be fed with so-called through-the-cycle (TTC) PDs and downturn LGDs.

### 3.2.4 Discounted lifetime expected losses

To capture discounted lifetime expected losses in the way IFRS 9 stipulates for stage 2 exposures, we define

$$EL_t^{LT} = \lambda b \left( \beta x_t + \beta^2 M x_t + \beta^3 M^2 x_t + \beta^4 M^3 x_t + ... \right) + \lambda x_{3t},$$

where

$$b = (PD_1, PD_2, 0).$$
which considers the discounted expected losses due to default events occurring over the whole residual lifetime of the existing loans. The formula above reflects that the losses expected from currently performing loans at any future year \( t + \tau \), with \( \tau = 1, 2, 3 \ldots \) can be found as \( \lambda b M^{\tau - 1} x_t \), where \( b \) contains the relevant one-year-ahead PDs (see (10)) and \( M^{\tau - 1} x_t \) gives the projected composition of the portfolio at each future year \( t + \tau - 1 \). It also reflects that the allowance for the NPLs simply equals the expected LGD of the affected loans, as in the other approaches.

Equation (12) can also be expressed as

\[
EL^\text{LT}_t = \beta \lambda b (I + \beta M + \beta^2 M^2 + \beta^3 M^3 + \ldots) x_t + \lambda x_{3t},
\]

where the parenthesis is the infinite sum of a geometric series of matrices. Thus, we can write \( EL^\text{LT}_t \) as

\[
EL^\text{LT}_t = \lambda (\beta b B x_t + x_{3t}),
\]

where

\[
B = (I - \beta M)^{-1}.
\]

### 3.2.5 Current expected credit losses (CECL)

Under the CECL approach all performing loans \((j=1, 2)\) are provisioned on a discounted lifetime basis but, differently from IFRS 9, the discount rate is not the contractual interest rate of each loan but a reference risk free-rate. This gives:

\[
EL^\text{CECL}_t = \lambda (\beta_r b B x_t + x_{3t}),
\]

where the only difference with respect to (14) is at the use of \( \beta_r = 1/(1 + r) \) rather than \( \beta = 1/(1 + c) \).

### 3.2.6 Impairment allowances under IFRS 9

IFRS 9 adopts, for performing loans, a mixed-horizon approach that combines the discounted one-year and lifetime expected loss approaches described above. Specifically, the allowances for loans that have not suffered a significant increase in credit risk since origination (“stage 1” loans or, in our model, the loans in \( x_{1t} \)) are computed as in \( EL^\text{1Y}_t \) while the allowances for
performing loans with deteriorated credit quality ("stage 2" loans or, in our model, the loans in $x_{2t}$) are computed as in $EL^L_{t}$. Finally, for NPLs ("stage 3" loans or, in our model, the loans in $x_{3t}$), the allowance equals the (non-discounted) expected LGD, as under all the other approaches. Combining the formulas obtained in (9) and (14), the impairment allowances under IFRS 9 can be described as

$$EL^{IFRS9}_{t} = \lambda \beta b \begin{pmatrix} x_{1t} \\ 0 \\ 0 \end{pmatrix} \lambda \beta B \begin{pmatrix} 0 \\ x_{2t} \\ 0 \end{pmatrix} + \lambda x_{3t}. \quad (17)$$

### 3.2.7 Comparing the various impairment measures

The definitions provided above clearly imply that

$$IL_t \leq EL^Y_{t} \leq EL^{IFRS9}_{t} \leq EL^{LT}_{t} \leq EL^{CECL}_{t}, \quad (18)$$

where the last inequality follows from the fact that $c \geq r \geq 0$, which means $\beta_r \geq \beta$. Regarding $EL^{IRB}_{t}$, it is obviously the case that $EL^{IRB}_{t} \geq IL_t$ and that the absence of discounting would, without aggregate risk, also imply $EL^{IRB}_{t} \geq EL^Y_{t}$, while the comparison with other measures would be generally ambiguous. However, as further described in Appendix A, once aggregate risk is introduced, the usage of TTC PDs and downturn LGDs also produces ambiguity with respect to the comparison with $EL^Y_{t}$ and the remaining expected-loss-based measures.

### 3.3 Implications for profits and the dynamics of regulatory capital

Under our assumptions, the bank’s only assets are its loans and its only liabilities are one-period debt, $d_t$, loan loss allowances, $LL_t$, and CET1, $k_t$. So the bank’s balance sheet at the end of any period $t$ can be described as

$$\begin{array}{c|c}
  x_{1t} & d_t \\
  x_{2t} & LL_t \\
  x_{3t} & k_t \\
\end{array} \quad (19)$$

with the law of motion of $x_t$ described by (1) and the law of motion of $k_t$ given by

$$k_t = k_{t-1} + PL_t - \text{div}_t + \text{recap}_t, \quad (20)$$
where $PL_t$ is the result of the P/L account at the end of period $t$, $\text{div}_t \geq 0$ are cash dividends paid at the end of period $t$, and $\text{recap}_t \geq 0$ are injections of CET1 at the end of period $t$. Under these assumptions, the dynamics of $d_t$ can be recovered residually from the balance sheet identity, $d_t = \Sigma_{j=1,2,3} x_{jt} - LL_t - k_t$.

The result of the P/L account can in turn be written as

$$PL_t = \left\{ \sum_{j=1,2} \left[ c(1-PD_j) - \frac{\delta_3}{2}PD_jx_{jt} \right] x_{jt-1} - \delta_3 \lambda x_{jt} - \lambda x_{jt-1} - \delta_3 \right\} - \sum_{j=1,2,3} x_{jt-1}L_t - k_{t-1} - LL_t,$$

(21)

where the first term contains the income from performing loans net of realized losses on defaulted loans resolved during period $t$, the second term is the interest paid on $d_{t-1}$, and the third term is the variation in credit loss allowances between periods $t-1$ and $t$. So, other things equal, any increase in $\Delta LL_t$ has a negative contemporaneous impact on $PL_t$ and, through (20), on the capital available for the bank to operate in the subsequent period.

Under the assumption that the bank minimizes the use of CET1 subject to the prescriptions of Basel III, its dividends and equity injections are determined as

$$\text{div}_t = \max[(k_{t-1} + PL_t) - 1.3125k_t, 0],$$

(22)

$$\text{recap}_t = \max[k_t - (k_{t-1} + PL_t), 0].$$

(23)

To explain these expressions, notice that $k_{t-1} + PL_t$ would be the inertial CET1 of the bank at date $t$ without discretionary adjustments ($\text{div}_t = \text{recap}_t = 0$). However, the bank need to operate with CET1 of at least $k_t$ at all times, so it has to recapitalize whenever such minimum is otherwise not reached (equation (23)). On the other hand, dividends cannot be paid until the barrier $k_t = 1.3125k_t$ given by the fully loaded CCB is not reached. So effectively the bank manages its CET1 within the bands $k_t$ and $\bar{k}_t$, thus following a simple $sS$-rule based entirely on existing capital regulations.\footnote{The CCB requires the bank to retain profits, whenever feasible, until reaching a fully loaded buffer equal to 2.5% of its RWAs. Regulatory RWAs equal 12.5 (or $1/0.08$) times the bank’s minimal required capital $k_t$. Thus a fully loaded CCB amounts to a multiple $0.025 \times 12.5 = 0.3125$ of $k_t$.}

\footnote{The working of the $sS$ rule proposed here implicitly assumes the absence of fixed costs associated with the raising of new equity. If such costs were to be introduced, the optimal rule would imply, as in Fischer, Heinkel, and Zechner (1989), discrete recapitalizations to an endogenous level within the bands if the lower band were to be otherwise passed.}
Finally, for corporate loan portfolios operated under the IRB approach, BCBS (2017, paragraph 53) establishes that the minimum capital requirement is

$$k_{IRB}^t = \sum_{j=1}^{2} \gamma_j x_{jt}, \quad (24)$$

with

$$\gamma_j = \lambda \left[ \frac{(1/\delta_j) - 2.5} {1 - 1.5m_j} \right] \Phi \left( \frac{\Phi^{-1}(PD_j) + \text{cor}_j^{0.5}\Phi^{-1}(0.999)} {1 - \text{cor}_j^{0.5}} \right) - PD_j, \quad (25)$$

where $m_j = [0.11852 - 0.05478 \ln(PD_j)]^2$ is a maturity adjustment coefficient, $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution, and $\text{cor}_j$ is a correlation coefficient fixed as $\text{cor}_j = 0.24 - 0.12(1 - \exp(-50PD_j))/(1 - \exp(-50)).$\footnote{In (25) we measure the maturity of performing loans as the expected residual maturity $1/\delta_j$ implied by our formulation.}

So the baseline results below will be based on assuming $k_t = k_t^{IRB}.$\footnote{Our analysis abstracts from the existence of “regulatory filters” addressing possible discrepancies between accounting provisions (the relevant $LL_t$) and prudential expected losses such as $EL_t^{IRB}$. These filters currently establish that if $EL_t^{IRB}$ exceeds $LL_t$, the difference, $EL_t^{IRB} - LL_t$, must be subtracted from CET1. By contrast, if $EL_t^{IRB} - LL_t < 0$, the difference can be added back as tier 2 capital up to a maximum of 0.6% of the bank’s credit RWAs. Therefore, we assess the impact of alternative provisioning methods on bank capital dynamics in the case prudential regulators accept the corresponding method as the one used to define prudential expected losses too.}

### 3.4 Loan rates under competitive pricing

Taking advantage of the recursivity of the model, we can obtain the bank’s ex-coupon value of loans rated $j$ at any given date, $v_j$, by solving the following system of Bellman-type equations:

$$v_j = \mu \left[ (1 - PD_j)c + (1 - PD_j)\delta_j + PD_j(\delta_j/2)(1 - \lambda) + m_1 v_1 + m_2 v_2 + m_3 v_3 \right], \quad (26)$$

for $j=1,2,$ and

$$v_3 = \mu \left[ \delta_3(1 - \lambda) + (1 - \delta_3)v_3 \right], \quad (27)$$

where $\mu = 1/(1+r)$ is the discount factor of the risk neutral bank.\footnote{For calibration purposes, the discount rate $r$ does not need to equal the risk-free rate. One might adjust the value of $r$ to reflect the marginal weighted average costs of funds of the bank or even an extra element capturing (in reduced form) a mark-up applied on that cost if the bank is not perfectly competitive.} Intuitively, the square brackets in (26) and (27) contain the payoffs and continuation value that a loan rated $j=1,2$
or \( j=3 \), respectively, will produce in the contingencies that, in each case, can occur one period ahead (weighted by the corresponding probabilities).

In (26), contingencies producing payoffs are, in order of appearance, the payment of interest on non-defaulted loans, the repayment of principal by the non-defaulted loans that mature, and the recovery of terminal value on defaulted loans resolved within the period. The last three terms contain continuation value under the three rating categories that can be reached one period ahead. Similarly, (27) reflects the terminal value recovered if an NPL is resolved within the period and the continuation value kept otherwise.

Under perfect competition, the value of extending a unit-size loan of standard quality \((j=1)\) must equal the value of its principal, \( v_1 = 1 \), so that the bank obtains zero net present value from its origination. Thus we obtain the endogenous contractual interest rate of the loan, \( c \), as the one that solves this equation.

4 Baseline quantitative results

4.1 Calibration

Table 1 describes the calibration of the baseline model under a parameterization intended to represent a typical portfolio of corporate loans issued by EU banks. Given the absence of detailed publicly available microeconomic information on such a portfolio, the calibration relies on matching aggregate variables taken from recent European Banking Authority (EBA) reports and European Central Bank (ECB) statistics using rating migration and default probabilities consistent with the Global Corporate Default reports produced by Standard & Poor’s (S&P) over the period 1981-2015.\(^{19}\) The probabilities of default (PDs) and yearly probabilities of migration across our standard and substandard categories are extracted from S&P rating migration data using the procedure described in Appendix B. These probabilities are consistent with the alignment of our standard category \((j=1)\) with ratings AAA to BB in the S&P classification and our substandard category \((j=2)\) with ratings B to C.

In a nutshell, to reduce the \(7 \times 7\) rating-migration probabilities and the seven PDs extracted from S&P data to the \(2 \times 2\) migration probabilities and two PDs that appear in

\(^{19}\)We use reports equivalent to S&P (2016) published in years 2003 and 2005-2016, which provide the relevant information for each of the years between 1981 and 2015.
matrix $M$ (equation (3)), we calculate weighted averages that take into account the steady-state composition that the loan portfolio would have under its 7-ratings representation in the absence of aggregate risk. To obtain this composition, we assume that loans have an average duration of 5 years (or $\delta_1=\delta_2=0.2$) as reflected in Table 1, that they have a rating BB at origination, and that they then evolve (through improvements or deteriorations in their credit quality before defaulting or maturing) exactly as in our model, but with the seven non-default rating categories in the original S&P data.

Table 1  
**Calibration of the baseline model**

<table>
<thead>
<tr>
<th>Parameters without variation with the aggregate state</th>
<th>Expansion (s'=1)</th>
<th>Contraction (s'=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banks’ discount rate $r$</td>
<td>1.8%</td>
<td></td>
</tr>
<tr>
<td>Persistence of the expansion state ($s=1$) $p_{11}$</td>
<td>0.852</td>
<td></td>
</tr>
<tr>
<td>Persistence of the contraction state ($s=2$) $p_{22}$</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters that can vary with aggregate arrival state</th>
<th>Expansion (s'=1)</th>
<th>Contraction (s'=2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yearly probability of migration 1 → 2 if not maturing $a_{21}$</td>
<td>6.16%</td>
<td>11.44%</td>
</tr>
<tr>
<td>Yearly probability of migration 2 → 1 if not maturing $a_{12}$</td>
<td>6.82%</td>
<td>4.47%</td>
</tr>
<tr>
<td>Yearly probability of default if rated $j=1$ $PD_1$</td>
<td>0.54%</td>
<td>1.91%</td>
</tr>
<tr>
<td>Yearly probability of default if rated $j=2$ $PD_2$</td>
<td>6.05%</td>
<td>11.50%</td>
</tr>
<tr>
<td>Loss given default $\lambda$</td>
<td>30%</td>
<td>40%</td>
</tr>
<tr>
<td>Average time to maturity if rated $j=1$ $1/\delta_1$</td>
<td>5 years</td>
<td>5 years</td>
</tr>
<tr>
<td>Average time to maturity if rated $j=2$ $1/\delta_2$</td>
<td>5 years</td>
<td>5 years</td>
</tr>
<tr>
<td>Yearly probability of resolution of NPLs $\delta_3$</td>
<td>44.6%</td>
<td>44.6%</td>
</tr>
<tr>
<td>Newly originated loans per period (all rated $j=1$) $e_1$</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Following the model formulation with aggregate risk described in Appendix A and the interpretation of the aggregate state as describing expansion vs. contraction periods, we allow for state-variation in the probabilities of loans migrating across rating categories and into default in a way consistent with the historical correlation between those variables (as observed in S&P rating-migration data) and the US business cycle as dated by the National Bureau of Economic Research (NBER).\(^{20}\) The dynamics of the aggregate state as parameterized in Table 1 imply that the average duration of expansion and contraction periods is 6.75 years and 2 years, respectively, meaning that the system spends about 77% of the time in state $s=1$. Expansions are characterized by significantly smaller PDs among both standard and substandard loans than contractions. During a contraction, the probability of standard loans

being downgraded (or, under IFRS 9, moved into stage 2) is almost double than during an expansion and the probability of substandard loans recovering standard quality (or returning to stage 1) is reduced by about one-third. See Section B.2 of Appendix B for further details.

Under this calibration, the unconditional average yearly PDs for our standard and sub-standard categories are 0.9% and 7.3%, respectively. As shown in Table 2, given the composition of the ergodic portfolio, the unconditional average annual loan default rate equals 1.9%, which is below the average 2.5% PD for non-defaulted corporate exposures that EBA (2013, Figure 12) reports for the period from the first half of 2009 to the second half of 2012 for a sample of EU banks operating under the IRB approach. Conditional on being in an expansion and in a contraction, our calibration implies average annual loan default rates for performing loans of 1.36% and 3.43%, respectively.

We also allow for state variation in the loss rate experienced at resolution. We set $\lambda$ equal to 40% during contractions and 30% during expansions. This is consistent with the cyclical evolution of average realized LGDs on European unsecured loans to large non-financial corporations reported by Brumma and Winckle (2017, Exhibit 3). An LGD of 40% in contractions is also consistent with the (downturn) LGD prescribed by BSBC (2017, paragraph 70) for unsecured corporate loans under the foundation IRB approach. The cyclical variation is also consistent with that documented by Bruche and González-Aguado (2010) for senior unsecured corporate bonds.

To keep the potential sources of cyclical variation under control, we maintain the parameters determining the effective maturity of performing loans, the speed of resolution of NPLs, and the flow of entry of new loans as time invariant.

Banks’ discount rate $r$ is fixed at 1.8% so as to obtain an unconditional average of the contractual loan rate $c$ equal to 2.49%, which is very close to the 2.52% average interest rate of new corporate loans made by Euro Area banks in the period from January 2010 to September 2016.\(^{21}\)

\(^{21}\text{We use the Euro area (changing composition), annualised agreed rate/narrowly defined effective rate on euro-denominated loans other than revolving loans and overdrafts, and convenience and extended credit card debt, made by banks to non-financial corporations (see http://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=124.MIR.M.U2.B.A2.A.A.R.A.2240.EUR.N).}\)
Table 2
Endogenous variables under the baseline calibration
(IRB bank, percentage of mean exposures unless indicated)

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Expansions</th>
<th>Contractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yearly contractual loan rate c (%)</td>
<td>2.47</td>
<td>2.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of standard loans (%)</td>
<td>81.35</td>
<td>3.48</td>
<td>82.68</td>
<td>76.85</td>
</tr>
<tr>
<td>Share of substandard loans (%)</td>
<td>15.46</td>
<td>1.90</td>
<td>14.59</td>
<td>18.42</td>
</tr>
<tr>
<td>Share of NPLs (%)</td>
<td>3.19</td>
<td>1.05</td>
<td>2.73</td>
<td>4.73</td>
</tr>
<tr>
<td>Realized default rate (% of performing loans)</td>
<td>1.89</td>
<td>0.90</td>
<td>1.36</td>
<td>3.43</td>
</tr>
<tr>
<td>Impairment allowances:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incurred losses</td>
<td>1.04</td>
<td>0.37</td>
<td>0.87</td>
<td>1.60</td>
</tr>
<tr>
<td>Prudential expected losses under IRB</td>
<td>2.00</td>
<td>0.47</td>
<td>1.80</td>
<td>2.69</td>
</tr>
<tr>
<td>CECL expected losses</td>
<td>4.36</td>
<td>0.58</td>
<td>4.06</td>
<td>5.36</td>
</tr>
<tr>
<td>IFRS 9 allowances</td>
<td>2.43</td>
<td>0.61</td>
<td>2.14</td>
<td>3.42</td>
</tr>
<tr>
<td>Stage 1 allowances</td>
<td>0.22</td>
<td>0.05</td>
<td>0.20</td>
<td>0.32</td>
</tr>
<tr>
<td>Stage 2 allowances</td>
<td>1.17</td>
<td>0.20</td>
<td>1.07</td>
<td>1.51</td>
</tr>
<tr>
<td>Stage 3 allowances</td>
<td>1.04</td>
<td>0.37</td>
<td>0.87</td>
<td>1.60</td>
</tr>
<tr>
<td>IRB minimum capital requirement</td>
<td>9.05</td>
<td>0.08</td>
<td>9.04</td>
<td>9.10</td>
</tr>
<tr>
<td>IRB minimum capital requirement + CCB</td>
<td>11.88</td>
<td>0.10</td>
<td>11.86</td>
<td>11.94</td>
</tr>
</tbody>
</table>

Regarding the resolution of NPLs, we set $\delta_3$ equal to 44.6% in order to produce an unconditional average fraction of NPLs consistent with the 5% average PD, including defaulted exposures that the EBA (2013, Figure 10) reports for the earliest period in its study, namely the first half of 2008.\textsuperscript{22} This value of $\delta_3$ implies an average time to resolution for NPLs of 2.24 years, which is very close to the 2.42-year average duration of corporate insolvency proceedings across EU countries documented by the EBA (2016, Figure 13).

Finally, the assumed size of the flow of newly originated loans, $e_1=1$, only provides a normalization and solely affects the average size of the bank’s total exposures. Further, we report most variables as a percentage of the bank’s total mean exposures (assets) making the absolute value of those exposures irrelevant in the analysis.

\textsuperscript{22}We take this observation, right before experiencing the full negative impact of the Global Financial Crisis, as the best proxy in the data for the model’s steady state. As shown in Table 2, with this procedure, we obtain an average 3.2% share of defaulted exposures in the ergodic portfolio, right inbetween the 2.5% and 4.4% reported by the EBA (2013, Figure 8) for corporate loans in the second half of 2008 and the first half of 2009, respectively. Conditional on being in an expansion and in a recession, the mean share of defaulted exposures equals 2.73% and 4.73%, respectively.
4.2 Size, volatility and cyclicality of the impairment measures

Table 2 reports unconditional means, standard deviations, and means conditional on each aggregate state for a number of endogenous variables. The variation in the aggregate state causes a significant variation in the composition of the bank’s loan portfolio. Not surprisingly, the shares of substandard and non-performing loans increase during contractions, and the overall realized default rate is more than double during contractions than during expansions.

The mean relative sizes of the various impairment allowances are ranked as predicted above. While for the considered portfolio, impairment allowances under IFRS 9 ($EL_{t}^{IFRS9}$) essentially double those associated with the IL approach ($IL_{t}$), the incoming CECL approach ($EL_{t}^{CECL}$) more than quadruples them. Note that higher level of allowances associated with IFRS 9 comes mostly from stage 2 loans in spite of the fact that these loans only represent a modest 15.5% in the loan portfolio. Interestingly, impairments measured under CECL and IFRS 9 are the ones exhibiting greater volatility and mean variation across states (130 and 128 basis points of mean exposures, respectively), followed by the prudential expected losses used by IRB banks and the IL impairments (91 and 73 basis points, respectively).

The decomposition by stage shown for IFRS 9 reveals that allowances associated with NPLs, followed by those associated with substandard loans, are those that contribute most to cross-state variation in loan loss provisions (73 and 44 basis points, respectively). However, NPLs are treated in the same way by all measures, which implies that the differing mean cross-state variation of the alternative measures must stem from the treatment of standard loans (which is similar in $EL_{t}^{IRB}$ and $EL_{t}^{IFRS9}$, but quite different in $IL_{t}$ and $EL_{t}^{CECL}$) and stage 2 loans (which is similar in $EL_{t}^{CECL}$ and $EL_{t}^{IFRS9}$, but quite different in $IL_{t}$ and $EL_{t}^{IRB}$) or from the cyclical shift of loans across stages 1 and 2 (under $EL_{t}^{IFRS9}$).

Finally, Table 2 also reports the descriptive statistics of the implied overall minimum capital requirement ($k$) and the minimum requirement plus the CCB ($\tilde{k}$) that would apply to the bank under our calibration.

4.3 Impact on the cyclicality of profits and regulatory capital

Table 3 summarizes the impact of the various provisioning approaches on P/L and CET1. The unconditional mean of P/L differs across them, reflecting that different levels of pro-
visions imply *de facto* different levels of debt financing for the same portfolio and, hence, different amounts of interest expense. Confirming what one might expect after observing the volatility ranking of the impairment measures in Table 2, P/L is significantly more volatile and variable across aggregate states under the more forward-looking $EL^{CECL}$ and $EL^{IFRS9}$ than under $EL^{IRB}$ or $IL$.

The more forward-looking impairment measures are the ones that make the bank, on average, more CET1-rich in expansion states and less CET1-rich in contraction states; that is, those that render CET1 more procyclical in this sense. In any case, the reported quantitative differences for this variable are not huge, in part because under our assumptions on the bank’s management of its CET1, the range of variation in CET1 under any of the impairment measures is limited by the regulatory-determined bands of the $sS$-rule described in equations (22) and (23). As explained above, the bank adjusts its CET1 to remain within those bands by paying dividends or raising new equity.

Thus, a suitable way to assess the potential procyclicality associated with each impairment measure is to look at the frequency and size (conditional on them being strictly positive) of dividends and recapitalizations. Quite intuitively, under all measures we obtain that dividend distributions only occur (if at all) during expansions, while recapitalizations only occur (if at all) during contractions.

Dividends are most frequently paid under $EL^{CECL}$, $EL^{IFRS9}$, $EL^{IRB}$, and $IL$, in this order, mostly reflecting the above mentioned differences in leverage and interest expense implied by the levels of provisioning. In terms of recapitalization needs, $EL^{IFRS9}$ involves a significantly higher probability of having to recapitalize the bank in contractions (18.2%) than any other the other measures (for which the probability ranges between 12.72% for $EL^{IRB}$ and 13.42% for $EL^{CECL}$).\(^{23}\)

\(^{23}\)However, these effects become counterbalanced by the fact that, when strictly positive, the average size of the recapitalizations needed under $EL^{IFRS9}$ is slightly lower than that under $EL^{Y}$.\(^{19}\)
Table 3  
Endogenous variables under the baseline calibration  
(IRB bank, percentage of mean exposures unless otherwise indicated)

<table>
<thead>
<tr>
<th></th>
<th>IL</th>
<th>EL²RB</th>
<th>EL²CECL</th>
<th>EL²FRS9</th>
</tr>
</thead>
<tbody>
<tr>
<td>P/L</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional mean</td>
<td>0.18</td>
<td>0.20</td>
<td>0.25</td>
<td>0.21</td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>0.41</td>
<td>0.45</td>
<td>0.56</td>
<td>0.52</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>-0.59</td>
<td>-0.65</td>
<td>-0.81</td>
<td>-0.84</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.42</td>
<td>0.47</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td>CET1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional mean</td>
<td>11.33</td>
<td>11.33</td>
<td>11.37</td>
<td>11.31</td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>11.56</td>
<td>11.59</td>
<td>11.70</td>
<td>11.65</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>10.52</td>
<td>10.43</td>
<td>10.21</td>
<td>10.14</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.85</td>
<td>0.85</td>
<td>0.83</td>
<td>0.86</td>
</tr>
<tr>
<td>Probability of dividends being paid (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>50.46</td>
<td>52.53</td>
<td>58.35</td>
<td>54.27</td>
</tr>
<tr>
<td>Conditional, expansions</td>
<td>65.40</td>
<td>68.07</td>
<td>75.62</td>
<td>70.33</td>
</tr>
<tr>
<td>Conditional, contractions</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.85</td>
<td>0.85</td>
<td>0.83</td>
<td>0.86</td>
</tr>
<tr>
<td>Dividends, if positive</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>0.40</td>
<td>0.42</td>
<td>0.44</td>
<td>0.42</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Probability of having to recapitalize (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>2.92</td>
<td>2.91</td>
<td>3.06</td>
<td>4.16</td>
</tr>
<tr>
<td>Conditional, expansions</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Conditional, contractions</td>
<td>12.77</td>
<td>12.72</td>
<td>13.42</td>
<td>18.20</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.53</td>
<td>0.56</td>
<td>0.46</td>
<td>0.48</td>
</tr>
</tbody>
</table>

4.4 Effects of the arrival of a contraction

Figure 2 shows the effects of the arrival of a contraction at $t=0$ (that is, the realization of $s_0=2$) after having spent a long enough period in the expansion state (that is, having had $s_t=1$ for sufficiently many dates prior to $t=0$). The results shown in the figure are equivalent to those typical of impulse response functions in macroeconomic analysis. From $t=1$ onwards the aggregate state follows the Markov chain calibrated in Table 1, thus making the trajectories followed by the variables depicted in the figure stochastic. The figure depicts the average trajectories resulting from simulating 10,000 paths.
The higher amount of loans becoming substandard immediately after a recession arrives makes the effects of the arrival of a recession persistent over time, despite the relatively short duration of the contraction state under our baseline calibration (2 periods on average). This can be seen in Panel A of Figure 2, which depicts the evolution of NPLs.

The results regarding the evolution of the various impairment measures over the same time span appear in Panel B of Figure 2. The average trajectories of the impairment allowances $IL_t$, $EL^{IRB}_t$, $EL^{CECL}_t$, and $EL^{IFRS9}_t$ are reported as a percentage of the total initial loans. The levels of the series at $t=-1$ reflect the different sizes of the impairment allowances obtained after a long expansion phase under each of the compared measurement methods. When the recession arrives at $t=0$, all the measures move upwards. The most forward looking ones, $EL^{CECL}_t$ and $EL^{IFRS9}_t$, peak on average after one period and then enter a pattern of exponential decay, driven by maturity, defaults, migration of substandard loans back to the standard category, and the continued origination of new standard-quality loans. The measures based on incurred or one-year ahead expected losses, $IL_t$ and $EL^{IRB}_t$ react more slowly, peaking on average after two periods, and then also fall gradually.

Therefore the on-impact responses of $EL^{CECL}_t$ and $EL^{IFRS9}_t$ to the arrival of a recession are larger than those of $IL_t$ and $EL^{IRB}_t$. In the case of $EL^{IFRS9}_t$ part of the reactivity to the arrival of the recession is due to the so-called “cliff effect” associated with the change in the provisioning horizon when exposures shift from stage 1 to stage 2. In both cases the additional reactivity is also related to the effects of updating the PIT forecast of the losses on currently performing loans which are expected beyond the one-year horizon (stage 2 exposures under IFRS 9 and all performing exposures under CECL).

The implications for P/L are described in Panel C of Figure 2. Each measure spreads over time the (same final average) impact of the shock on P/L in a different manner. $EL^{IFRS9}_t$ and $EL^{CECL}_t$ front-load very similarly the impact of the shock: P/L becomes very negative on impact, but then turns positive and returns to normal pretty quickly afterwards. With $IL_t$ and, to a lesser extent, $EL^{IRB}_t$, P/L is affected much less on impact but remains negative for several periods.

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24 Variations of the experiment that simultaneously shut down or reduce origination of new loans for a number periods can be easily performed without losing consistency. Experiencing lower loan origination after $t=0$ delays the process of reversion to the steady state but does not qualitatively affect the results.
Panel A. Non-performing loans
Panel B. Impairment allowances
Panel C. P/L
Panel D. CET1

Figure 2. Effects of the arrival of a contraction

Average responses to the arrival of $s=2$ after a long period in $s=1$
(IRB bank, as a percentage of average exposures).

Panel D of Figure 2 shows the implications for an IRB bank’s CET1. Before the shock hits, at $t=-1$, the bank has a fully loaded CCB. The average positions of the bands $k$ and $\bar{k}$ reflected in the figure exhibit some time variation as a result of the change in RWAs that follows the temporary deterioration in the composition of the loan portfolio, implying a buffer on top of the minimum required capital of more than 2.5% of total assets. The different impacts of the alternative provisioning methods on CET1 essentially mirror the previously commented impact on P/L.

The on-impact effect of the arrival of contraction under CECL and IFRS 9 implies consuming about one percentage point of the roughly three percentage points of initial assets.
represented by the CCB, but CET1 tends to recover on average afterwards. With $EL^{IRB}$ and, especially, $IL$ the on-impact effect is much smaller (about half) than under the new ECL provisions, but CET1 tends to deteriorate further in subsequent periods (though on average not going below the trajectories obtained under $EL^{CECL}$ and $EL^{IFRS9}$).

The fact that the trajectories depicted in Figure 2 are average trajectories is important for interpreting these results. For example, in Panel D, the average trajectory of CET1 lies within the bands determined by the average trajectories of $\bar{k}$ and $\tilde{k}$ but this does not mean that the bank does not need to recapitalize (or does not pay dividends) over all the possible trajectories. On the contrary, many of the trajectories go upward and touch the upper band for paying dividends (e.g. if the contraction ends and does not return), while a few other go downward, touch the lower band, and force the bank to recapitalize (e.g. if the contraction lasts long enough or another contraction follows soon after an initial recovery). This explains the compatibility between the results in this figure and the probabilities of positive dividend payments and recapitalization needs reported in Table 2.

![Figure 3. CET1 after the arrival of a contraction (IRB bank)](image)

500 simulated trajectories of CET1 under $EL^{IRB}$ and $EL^{IFRS9}$ following the arrival of $s=2$ after a long period in $s=1$ (IRB bank, as a percentage of average exposures)

To further illustrate the difference between average and realized trajectories, Figure 3 shows 500 simulated trajectories for CET1 under $EL^{IRB}$ and $EL^{IFRS9}$. Under IFRS 9, it takes four consecutive years in the contraction state ($s=2$) for a bank to deplete its CCB
and require a recapitalization. By contrast, under the one-year expected loss approach, the CCB would be used up only after five years in the contraction state.

5 Implications and policy analysis

The results in prior sections show that the loan loss provisions under CECL and IFRS 9 will imply a more up-front recognition of impairment losses upon the arrival of a recession. This means that the on-impact declines in P/L and CET1 will be larger than under prior provisioning approaches. In our model, a fall in CET1 first reduces a bank’s CCB (if positive), forcing it to cancel its dividends. If the losses are large enough or persist for a number of periods, the CCB may be fully depleted, pushing the bank to raise new equity in order to continue complying with the minimum capital requirement without reducing the size of the loan portfolio. In reality, if the bank owners dislike cancelling dividends, operating without fully loaded buffers or raising new equity capital, the above effects might translate into loan sales or a reduction in the origination of new loans. Specifically, imperfections pressing banks to show up good capital ratios or a stable dividend flow, or making equity issuance costly (as in, e.g., Bolton and Freixas, 2006, Allen and Carletti, 2008, or Plantin, Sapra and Shin, 2008) might lead the bank to reduce its assets.\textsuperscript{25}

Of course the potential disadvantages of a sharper contraction of credit or larger asset sales at the beginning of a recession should be weighed against the advantages of having financial statements that reflect the weakness or strength of the reporting institutions in a more timely and reliable way, as there is evidence suggesting that this helps resolve bank crises in a prompter, safer, and more effective manner.\textsuperscript{26} In this sense, the policy analysis contained below is made without prejudging or attempting to assess the net welfare effects of the new provisions. Instead, we explore the effectiveness of several policy options in reducing

\textsuperscript{25}Concerns on this type of reaction are at the heart of the motivation for macroprudential policies. As put by Hanson, Kashyap and Stein (2011, p. 5), “in the simplest terms, one can characterize the macroprudential approach to financial regulation as an effort to control the social costs associated with excessive balance sheet shrinkage on the part of multiple financial institutions hit with a common shock.”

the impact of the new provisions on the bank’s need to be recapitalized. In other words, we take the bank’s unconditional recapitalization probability as a proxy of the potential procyclical effects.\footnote{In all the baseline results, recapitalization needs only emerge during contractions so using the conditional probability of recapitalization during contractions would provide, up to a scale parameter, an equivalent metric.}

As in discussions on the procyclicality of Basel capital requirements (Kashyap and Stein, 2004, and Repullo and Suarez, 2013), several factors can reduce the procyclical effects associated with the impact of loan loss provisions on recapitalization needs. First, banks may react to the new provisioning methods by increasing their voluntary capital buffers or by undertaking less cyclical investments. Second, even if CECL and IFRS 9 provisions further reduce banks’ lending capacity during recessions, credit demand may also contract during recessions, mitigating the implications. Yet recent evidence (including Mésonnier and Monks, 2015, Aiyar, Calomiris, and Wieladek, 2016, Behn, Haselmann, and Wachtel, 2016, Gropp et al., 2016, Jiménez et al., 2017) suggests that banks tend to accommodate sudden increases in required capital (or falls in available regulatory capital) by reducing bank lending, especially during contractions, producing negative effects on economic activity.

If this were the case, authorities could consider policies such as the ones explored in the rest of this section, namely: (i) increasing the CCB, (ii) using the CCyB by activating it to a level above zero during expansions and releasing it during contractions, (iii) introducing prudential buffers based on stress test results, and (iv) smoothing the credit risk parameters used in the calculation of the new provisions so as to get closer to a TTC approach.

### 5.1 Increasing the capital conservation buffer

A first straightforward way to address the concern about the higher recapitalization probability implied by CECL and IFRS 9 is to increase the CCB. Our baseline results above were obtained under the 2.5\% of RWAs target specified by Basel III. Panel A in Figure 4 compares the unconditional probability of the needing a recapitalization under each of the new provisioning methods if such target is permanently increased with an add-on ranging from 0\% to 2.5\% of the bank’s RWAs.

As shown in the figure, such an add-on, by increasing the bank’s loss absorption capacity
through earnings retention when profits are positive, reduces the probability of having to recapitalize the bank. For a total CCB target of 5%, the recapitalization probability would be below 0.5% per year under both CECL and IFRS 9.

Panel A. CCB add-on

Panel B. CCyB activation

Figure 4. Effects of increasing or activating the regulatory buffers
Unconditional probabilities of recapitalization (%) as a function of the add-on to or level of activation of the corresponding buffer, measured as percentage points of RWAs

5.2 Activation of the countercyclical capital buffer

The CCyB of Basel III works similarly to a discretionary, time-varying add-on to the CCB. Specifically, macroprudential authorities are intended to set such an add-on (called the CCyB rate) in the range from 0% to 2.5% of RWAs on the basis of the evolution of the credit cycle. Under Basel III, rises in the CCyB rate must be announced one year before their effective application, while CCyB reductions (releases) apply immediately.

To formally capture these features, we modify the upper band \( \overline{k}_t \) that drives the dynamics of CET1 in the model by making it now equal to the sum of the time-invariant CCB target of 2.5% of RWAs and, whenever activated, a CCyB rate of \( \alpha \in [0, 0.025] \):

\[
\overline{k}_t^{\text{CCyB}} = \overline{k}_t [1.3125 + (\alpha/0.08)(2 - s_t)(2 - s_{t-1})\ldots(2 - s_{t-T})].
\]  

(28)

This formulation implies that the CCyB is activated whenever the economy is and has been in an expansion for the last \( T \) periods \( (s_{t-\tau} = 1 \text{ for } \tau = 0, \ldots T) \), while the CCyB turns zero
otherwise.\textsuperscript{28}

Panel B in Figure 4 shows the recapitalization probabilities obtained with a two year implementation lag ($T = 2$) and CCyB rates ranging from 0\% to 2.5\% of RWAs. Interestingly, the CCyB is effective in reducing the need to recapitalize the bank but only down to about a probability of 1.5\% and 2\% per year for CECL and IFRS 9, respectively. The lower effectiveness of the CCyB relative to a CCB add-on of the same maximum size is due to the CCyB implementation lag, which delays its build-up, thus increasing the odds that the arrival of a contraction catches the bank with an overall CCB+CCyB below its maximum size.

![Panel A. CCB add-on of 1%](image1)

![Panel B. CCyB activation at 1%](image2)

**Figure 5. Evolution of CET1 under alternative buffer policies**

500 simulated trajectories of CET1 under \( EL^{IFRS9} \) and alternative buffer policies following the arrival of \( s=2 \) after a long period in \( s=1 \).

(IRB bank, as a percentage of average exposures)

Further intuition on the different behavior of a CCB add-on and an actively managed CCyB of similar sizes can be obtained from Figure 5, which shows simulated paths for CET1 under IFRS 9 provisioning and each of the two countercyclical tools. Buffer releases upon the arrival of contractions and implementation lags subsequent to the arrival of expansions explain why the upper band and average trajectory of CET1 in Panel B fluctuate more than their counterparts in Panel A. The following table reflects the quantitative difference in the

\textsuperscript{28}To avoid a counterintuitive pay of dividends out of the release of the CCyB at the start of a recession, we add in this part of the analysis the constraint that the bank is not allowed to pay dividends while in the contraction state.
performance of the two tools for a common rate of 1%:

<table>
<thead>
<tr>
<th></th>
<th>$EL^{CECL}$</th>
<th>$EL^{FRS9}$</th>
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</thead>
<tbody>
<tr>
<td>Baseline model</td>
<td>3.06%</td>
<td>4.16%</td>
</tr>
<tr>
<td>CCB add-on of 1%</td>
<td>1.22%</td>
<td>1.59%</td>
</tr>
<tr>
<td>CCyB activation at 1%</td>
<td>1.83%</td>
<td>2.23%</td>
</tr>
</tbody>
</table>

5.3 Stress test based buffers

Macroprudential stress testing has become part of supervisors’ toolkit in the aftermath of the Global Financial Crisis. What started in some jurisdictions as a one-off exercise intended to bring calm to investors concerned about the solvency of the banks, has consolidated as a systematic recursive way to assess and ensure banks’ resiliency. At the risk of oversimplifying, macroprudential stress testing consists on defining some adverse scenario going forward over a number of years, estimating the capital that banks would need to resist such scenario, and demanding banks (in a harder or softer manner) to have or otherwise raise or accumulate such level of capital within a reasonable period of time. In the two parts of this subsection, we consider two possible manners of formally defining the relevant adverse scenarios and the implicit capital requirements (or capital buffering requirements) associated with macroprudential stress testing. They differ in the frequency with which the size of the required buffer gets recalibrated (recursively or just once at the peak of expansion periods) and in the hardness of the attached capital constraint (which may work as a compulsory capital requirement or, more softly, as a CCB add-on). We consider two variations without the aim to be exhaustive but to illustrate the importance of the details even in a highly stylized setup such as the one provided by our model.

5.3.1 Recursive stress test requirement

In this extension we assume that a macroprudential stress test is performed every year. The stress test performed in year $t$ consists in considering an adverse scenario in which the economy is in recession in year $t + 1$ and up to, at least, year $t + T$, where $T$ is the length of the adverse scenario. The hypothetical dynamics of CET1 over the adverse scenario is computed under the assumption that the bank starts at $t$ with capital $\bar{k}_t = k_{t-1} + PL_t$ and does not make any discretionary capital adjustment up to year $t + T$ so that its capital
follows the difference equation $k_{t+i} = \tilde{k}_{t+i-1} + \tilde{P}L_{t+i}$ for $i=1,...,T$, where $\tilde{P}L_{t+i}$ denotes the P/L generated at year $t+i$ under the projected trajectories of capital and the aggregate state (which can be found by properly feeding the formula for $PL_{t+i}$ used in the baseline model). In this context, the recursive stress test requirement is an add-on $\alpha_i^{ST}$ to the minimum capital requirement of the bank at $t$ defined as

$$\alpha_i^{ST} = \max(\tilde{k}_i + \text{def}_i^{ST} - k_i, 0),$$

where $\text{def}_i^{ST} = k_{t+T} - \tilde{k}_{t+T}$ is the capital deficit detected in the stress test exercise. Intuitively, the add-on $\alpha_i^{ST}$ is designed to guarantee that the bank can go through the adverse scenario without violating the minimum capital requirement at the end of it. The effect of the add-on is to effectively rise the overall minimum capital requirement of the bank at $t$ to $k_t + \alpha_i^{ST}$, forcing it to adapt its dividend and equity raising policy at $t$ accordingly.\textsuperscript{29}

Under this formulation, we simulate the dynamics of the model and, in particular, $PL_t$ and $k_t$ assuming that the bank is subject to the corresponding stress test based add-on $\alpha_i^{ST}$ to the minimum capital requirement in every period $t$. We consider adverse scenarios of different severity as described by $T$. As a summary of the results, Figure 6 depicts the unconditional mean of the add-on $\alpha_i^{ST}$ (Panel A) and the unconditional yearly probability of having to recapitalize the bank (Panel B) for different values of $T$. Clearly, as different provisioning methods imply different trajectories for $PL_t$ and $k_t$, the value of $\alpha_i^{ST}$ can be different for each provisioning method.

These results show that recursively requesting the bank to be able to cope with a sufficiently adverse scenario without needing a future recapitalization has the effect of dramatically increasing the probability of having to recapitalize the bank throughout its lifetime. So, paradoxically, the attempt to avoid recapitalization needs along the hypothesized adverse scenarios ends up producing much more frequent recapitalization needs along the bank’s lifetime than in the absence of the considered stress test requirement.\textsuperscript{30}

\textsuperscript{29}Specifically, dividends and equity injections of the bank in year $t$ would be driven by:

$$\text{div}_t = \max\{\min[(k_{t-1} + PL_t) - 1.3125,k_t, (k_{t-1} + PL_t) - (k_t + \alpha_i^{ST})], 0\},$$

$$\text{recap}_t = \max[(k_t + \alpha_i^{ST}) - (k_{t-1} + PL_t), 0],$$

which are immediate adaptations of (22) and (23) to the new requirement.

\textsuperscript{30}The flat sections in the curves represented in Panel B of Figure 6 evidence that the need to satisfy the
5.3.2 CCB-like stress test requirement

We now consider a much softer stress test requirement: one that, instead of effectively rising the minimal required capital of the bank, takes the form of a CCB add-on such as the one analyzed in subsection 5.1 but calibrated on the basis of the results of some one-off stress test performed at the peak of an expansion period. Specifically, we denote such CCB add-on as $\alpha^{ST}$ and find it by considering the situation reached by the bank at a reference year $t = 0$ after spending a sufficiently large number of periods in the expansion state. In such a situation, the bank would have both the reference CCB of Basel III and the new add-on $\alpha^{ST}$ fully loaded, so its regulatory capital would be $\tilde{k}_0 = k_0 (1.3125 + \alpha^{ST}/0.08)$. We then suppose that at $t=1$, the economy shifts to the contraction state and remains in such state up to, at least, year $t = T$, and the bank does not recapitalize or pay any dividend between $t = 1$ and $t = T$. In this setup, we numerically find the size of the add-on $\alpha^{ST}$ for which the bank would complete the hypothesized trajectory at $T$ with just enough capital to comply with the minimum requirement, that is, with $\tilde{k}_T = k_T$.\footnote{We set $\alpha^{ST} = 0$ whenever this procedure yields $\alpha^{ST} < 0$, which explains the flat sections of the curves depicted in Figure 7.} So, by construction, the CCB add-

---

Figure 6. Recursive stress test requirement

Average add-on $\alpha^{ST}_t$ to required capital (in percent of RWAs) and unconditional probability of recapitalization (%) for different lengths $T$ of a recursive adverse scenario (in years, on the x-axis)
on guarantees that, by the end of this one-off adverse scenario, the bank has just enough CET1 to avoid being recapitalized.

Panel A. Required buffer size $\alpha^{ST}$

Panel B. Recapitalization probability

Figure 7. CCB-like stress test requirement

Required buffer size (in percent of RWAs) and unconditional probability of recapitalization for different lengths $T$ of a one-off adverse scenario (in years, on the x-axis)

Figure 7 depicts the calibrated values for $\alpha^{ST}$ (Panel A) and the resulting unconditional probabilities of recapitalization (Panel B) for different lengths $T$ of the one-off adverse scenario. The figure shows that, for example, the CCB add-on $\alpha^{ST}$ that would leave the bank with just enough capital not to need a recapitalization after a 5-year contraction scenario ($T=5$) is 0.71% under IFRS 9 and 0.44% under CECL. As shown in Panel B of this figure, with this calibration of the CCB add-on, the bank would have a very similar unconditional probability of being recapitalized under the two new provisioning methods.

Formally, a CCB-like stress test requirement as the one just described is entirely equivalent to a CCB add-on of equal size, so they only differ in the narrative leading to their calibration. However, in practice, a CCB add-on explicitly calibrated on the basis of a stress test may have the advantage of adapting better to structural changes and bank heterogeneity than a common CCB add-on fixed once and for all.
5.4 Smoothing the inputs

A key difference between the provisions implied by CECL and IFRS 9 and the prudential expected losses under the IRB approach is that the latter aim to avoid cyclicality by stipulating the use of TTC PDs and downturn LGDs, rather than PIT estimates of both of them. In this subsection we consider how $EL^{CECL}$ and $EL^{IFRS9}$ would perform if they were modified in that direction. Accordingly, both measures would still differ from $EL^{IRB}$ in the horizon over which (some of) the expected losses are projected, but not in the state-variation of the PDs and LGDs. These modifications can be interpreted either as a change in the accounting standards or, less ambitiously, as a policy implemented in the form of prudential adjustments (CET1 reductions or add-backs) based on the differences between the PIT accounting expected losses and the relevant TTC prudential expected losses.\footnote{The second option would not require reforming the new accounting standards but would have the drawback of generating discrepancies between accounting and regulatory capital, which might erode investors’ confidence in the reliability of both numbers. See BCBS (2016) for a description of alternatives for the regulatory treatment of accounting provisions.}

The following table shows the results associated with (i) smoothing only the PDs by using TTC rather than PIT estimates of them, and (ii) smoothing also the LGDs by using downturn LGDs rather than PIT estimates of them:

<table>
<thead>
<tr>
<th></th>
<th>$EL^{CECL}$</th>
<th>$EL^{IFRS9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline model</td>
<td>3.06%</td>
<td>4.16%</td>
</tr>
<tr>
<td>Provisions based on TTC PDs</td>
<td>2.33%</td>
<td>3.17%</td>
</tr>
<tr>
<td>Provisions based on TTC PDs and downturn LGDs</td>
<td>2.31%</td>
<td>4.05%</td>
</tr>
</tbody>
</table>

Interestingly, smoothing the PDs is effective in reducing the procyclicality of both CECL and IFRS 9, while smoothing also the LGDs via the proposed procedure is, in incremental terms, only very marginally countercyclical under CECL and rather procyclical under IFRS 9. To explain this last result, recall that part of the cyclical performance of IFRS 9 is due to the cliff effects associated with the reclassification of exposures between stages 1 and 2. Relying on downturn LGDs rather than PIT LGDs appears to increase the differential provisioning needs behind the cliff effects, possibly because under a PIT approach part of the default losses associated with stage 2 exposures in the contraction state are estimated to actually realize (and produce recoveries) when the economy is back in the expansion state, thus implying LGDs lower than the downturn LGDs.
As one can clearly see, smoothing the inputs relevant for the estimation of credit losses helps reducing the procyclical effects of CECL and IFRS 9 but not as much as the CCB add-ons considered in Figure 4.

6 Conclusions

We have described a simple recursive model for the assessment of the level and cyclical implications of the new ECL approaches to loan loss provisioning under IFRS 9 and the incoming update of US GAAP. We have calibrated the model to represent a bank with a portfolio of EU corporate loans, and compared its performance under alternative provisioning methods: the old incurred loss approach, the prudential one-year expected losses associated with IRB capital requirements, the lifetime CECL provisions of US GAAP, and the mixed-horizon ECL provisions of IFRS 9.

Our results suggest that the loan loss provisions implied by IFRS 9 and the CECL approach will rise more suddenly than their predecessors when the cyclical position of the economy switches from expansion to contraction. This implies that P/L and, without the application of regulatory filters, CET1 will decline more severely at the beginning of downturns. The baseline quantitative results of the paper suggest that the arrival of an average recession might imply on-impact losses of CET1 twice as large as those under the incurred loss approach and equivalent to about one third of the fully loaded CCB of the analyzed bank.

As shown in Appendix C, the timing and the importance of the cyclical effects depend on the anticipated and unanticipated severity (duration) of recessions as well as the extent to which cyclical turning points can be anticipated in advance. Greater severity exacerbates the cyclical effects, while greater capacity to anticipate the arrival of a contraction allows to absorb part of the cyclical losses prior to the start of the contraction.

While the early and decisive recognition of forthcoming losses may have significant advantages (e.g. in terms of transparency, market discipline, inducing prompt supervisory intervention, etc.), it may also imply, via its effects on regulatory capital, a loss of lending capacity for banks at the very beginning of a contraction, potentially contributing, through feedback effects, to its severity. In this paper we have gauged the direction and intensity of
the procyclical effects by looking at some first-round, partial equilibrium effects through the eyes of our simple model.

Specifically, in the discussion of potentially mitigating policies, we have focused the analysis on the unconditional annual probability with which the bank described in the model needs to raise new capital to avoid violating its minimum regulatory capital requirements. After examining policies such as increasing the CCB, actively using the CCyB, introducing prudential buffers based on stress test results or calculating the new provisions using a prudential TTC approach rather than their current PIT approach, we conclude that introducing a CCB add-on (possibly calibrated on the basis of stress test results) would be not only the simplest but also the policy option with the highest effectiveness in terms of reducing the unconditional probability of having to recapitalize the bank.
References


European Banking Authority (2013), Report on the Pro-cyclicality of Capital Requirements under the Internal Ratings Based Approach, December.

European Banking Authority (2016), Report on the Dynamics and Drivers of Nonperforming Exposures in the EU Banking Sector, July.


A Model equations in the presence of aggregate risk

In this appendix we extend the equations presented in the main text to the case in which aggregate risk affects the key parameters governing credit risk and, potentially, loan origination. We capture aggregate risk by introducing an aggregate state variable that can take two values \( s_t \in \{1, 2\} \) at each date \( t \) and follows a Markov chain with time-invariant transition probabilities \( p_{s's} = \text{Prob}(s_{t+1} = s'|s = s) \). The approach can be trivially generalized to deal with a larger number of aggregate states.

In order to measure expected losses corresponding to default events in any future date \( t \), we have to keep track of the aggregate state in which the loans existing at \( t \) were originated, \( z=1, 2 \), the aggregate state at time \( t \), \( s=1, 2 \), and the credit quality or rating of the loan at \( t \), \( j=1, 2, 3 \). Thus, it is convenient to describe (stochastic) loan portfolios held at any date \( t \) as vectors of the form

\[
y_t = \begin{pmatrix}
x_t(1, 1, 1) \\
x_t(1, 1, 2) \\
x_t(1, 1, 3) \\
x_t(1, 2, 1) \\
x_t(1, 2, 2) \\
x_t(1, 2, 3) \\
x_t(2, 1, 1) \\
x_t(2, 1, 2) \\
x_t(2, 1, 3) \\
x_t(2, 2, 1) \\
x_t(2, 2, 2) \\
x_t(2, 2, 3)
\end{pmatrix},
\]

where component \( x_t(z, s, j) \) denotes the measure of loans at \( t \) that were originated in aggregate state \( z \), are in aggregate state \( s \) and have rating \( j \).\(^{33}\)

Our assumptions regarding the evolution and payoffs of the loans between any date \( t \) and \( t+1 \) are as follows. Loans rated \( j=1, 2 \) at \( t \) mature at \( t+1 \) with probability \( \delta_j(s') \), where \( s' \) denotes the aggregate state at \( t+1 \) (unknown at date \( t \)). In the case of NPLs \( (j=3) \), \( \delta_3(s') \) represents the independent probability of a loan being resolved, in which case it pays back a fraction \( 1 - \tilde{\lambda}(s') \) of its unit principal and exits the portfolio. Conditional on \( s' \), each loan rated \( j=1, 2 \) at \( t \) which matures at \( t+1 \) defaults independently with probability \( PD_j(s') \), being resolved within the period with probability \( \delta_3(s')/2 \) or entering the stock of NPLs \( (j=3) \) with probability \( 1 - \delta_3(s')/2 \). Maturing loans that do not default pay back their

\(^{33}\)Along a specific history (or sequence of aggregate states), for any \( z \) and \( j \), the value of \( x_t(z, s, j) \) will equal 0 whenever \( s_t \neq s \).
principal of one plus the contractual interest $c_z$, established at origination.

Conditional on $s'$, each loan rated $j=1,2$ at $t$ which does not mature at $t+1$ goes through one of the following exhaustive possibilities. First, default, which occurs independently with probability $PD_j(s')$, and in which case one of two things can happen: (i) it is resolved within the period with probability $\delta_3(s')/2$; or (ii) it enters the stock of NPLs ($j=3$) with probability $1 - \delta_3(s')/2$. Second, migration to rating $i \neq j$ ($i=1,2$), in which case it pays interest $c_z$ and continues for one more period; this occurs independently with probability $a_{ij}(s')$. Third, continuation in rating $j$, in which case it pays interest $c_z$ and continues for one more period; this occurs independently with probability

$$a_{jj}(s') = 1 - a_{ij}(s') - PD_j(s').$$

### A.1 Portfolio dynamics under aggregate risk

Under aggregate risk, the dynamics of the loan portfolio between any dates $t$ and $t+1$ is no longer deterministic, but driven by the realization of the aggregate state variable at $t+1$, $s_{t+1}$. To describe the dynamics of the system compactly, let the binary variable $s_{t+1} = 1$ if $s_{t+1} = 1$ and $s_{t+1} = 0$ if $s_{t+1} = 2$. The dynamics of the system can be described as

$$y_{t+1} = G(\xi_{t+1})y_t + g(\xi_{t+1}),$$

where

$$G(\xi_{t+1}) = \begin{pmatrix} \xi_{t+1} M(1) & \xi_{t+1} M(1) \\ (1 - \xi_{t+1}) M(2) & (1 - \xi_{t+1}) M(2) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$g(\xi_{t+1})^T = (\xi_{t+1} e_1(1), 0, 0, 0, 0, 0, 0, 0, 0, 0, (1 - \xi_{t+1}) e_1(2), 0, 0),$$

$$\xi_{t+1} = \begin{cases} 1 & \text{if } u_{t+1} \in [0, p_{1s}], \\ 0 & \text{otherwise}, \end{cases}$$

$$s_{t+1} = \xi_{t+1} + 2 (1 - \xi_{t+1}),$$

$u_{t+1}$ is an independently and identically distributed uniform random variable with support $[0,1]$, $e_1(s')$ is the (potentially different across states $s'$) measure of new loans originated at $t+1$, and $0_{6 \times 6}$ denotes a $6 \times 6$ matrix full of zeros.

### A.2 Incurred losses

Incurred losses measured at date $t$ would be those associated with NPLs that are part of the bank’s portfolio at date $t$. Thus, the incurred losses reported at $t$ would be given by

$$IL_t = \sum_{z=1,2} \sum_{s=1,2} \lambda(s) x_t(z, s, 3),$$
where $\lambda(s)$ is the expected LGD on an NPL conditional on being at state $s$ in date $t$. This can be more compactly expressed as

$$IL_t = \hat{b}y_t,$$  \hspace{1cm} (A.2)

where $\hat{b} = (0, 0, \lambda(1), 0, 0, \lambda(2), 0, 0, \lambda(1), 0, 0, \lambda(2))$.

The expected LGD conditional on each current state $s$ can be found as functions of the previously specified primitives of the model (state-transition probabilities, probabilities of resolution of the defaulted loans in subsequent periods, and loss rates $\tilde{\lambda}(s')$ suffered if resolution happens in each of the possible future states $s'$) by solving the following system of recursive equations:

$$\lambda(s) = \sum_{s'=1,2} p_{s's} \left[ \delta_3(s')\tilde{\lambda}(s') + (1 - \delta_3(s'))\lambda(s') \right],$$  \hspace{1cm} (A.3)

for $s=1,2$.

**A.3 Discounted one-year expected losses**

Based on the loan portfolio held by the bank at $t$, provisions computed on the basis of the discounted one-year expected losses add to the incurred losses written above the losses stemming from default events expected to occur within the year immediately following. Since a period in the model is one year, the corresponding allowances are given by

$$EL_t^{1Y} = (b_{\beta} + \hat{b})y_t,$$  \hspace{1cm} (A.4)

where $b_{\beta} = (\beta_1 b, \beta_2 b)$, $b_z = 1/(1 + c_z)$, and $b = (b_{11}, b_{12}, 0, b_{21}, b_{22}, 0)$, with

$$b_{s'j} = \sum_{s'=1,2} p_{s's} PD_j(s') \left\{ [\delta_3(s')/2] \tilde{\lambda}(s') + [1 - \delta_3(s')/2] \lambda(s') \right\},$$  \hspace{1cm} (A.5)

for $j=1,2$. The coefficients defined in (A.5) attribute one-year expected losses to loans rated $j=1,2$ in state $s$ by taking into account their PD and LGD over each of the possible states $s'$ that can be reached at $t + 1$, where the corresponding $s'$ are weighted by their probability of occurring given $s$. The losses associated these one-year ahead defaults are discounted using the contractual interest rate of the loans, $c_z$, as set at their origination. In Section A.10, we derive an expression for the endogenous value of such rate under our assumptions on loan pricing. As for the loans that are already non-performing ($j=3$) at date $t$, the term $\hat{b}y_t$ in (A.4) implies attributing their conditional-on-$s$ LGD to them, exactly as in (A.2).
A.4 Prudential expected losses under the IRB approach

Differences between the BCBS prescriptions on expected losses for IRB loan portfolios and the above definition of $EL^1_t$ include the absence of discounting ($\beta = 1$), the preference for using TTC (rather than PIT) PDs, and the usage of a downturn LGD that reflects the depressed recoveries obtained under adverse circumstances. Thus, the prudential expected losses defined by the BCBS for IRB portfolios can be found as

$$EL^{IRB}_t = \bar{b} y_t,$$

(A.6)

with $\bar{b} = (\bar{b}_1, \bar{b}_2, \bar{b}_3, \bar{b}_1, \bar{b}_2, \bar{b}_3, \bar{b}_1, \bar{b}_2, \bar{b}_3)$ and $\bar{b}_j = \bar{PD}_j \lambda(2)$, where

$$\bar{PD}_j = \sum_{i=1,2} \pi_i PD_j(s_i)$$

(A.7)

for $j=1, 2$, is the TTC PD for loans rated $j$ (with $\pi_i$ denoting the unconditional probability of aggregate state $i$) and $\bar{PD}_3 = 1$.

A.5 Discounted lifetime expected losses

Impairment allowances computed on an lifetime-expected basis imply taking into account not just the default events that may affect the currently performing loans in the next year, but also those occurring in any subsequent period. Building on prior notation and the same approach explained for the model without aggregate risk, these provisions can be computed as

$$EL^{LT}_t = b_\beta y_t + b_\beta M_\beta y_t + b_\beta M_\beta^2 y_t + b_\beta M_\beta^3 y_t + \ldots + \bar{b} y_t$$

$$= b_\beta (I + M_\beta + M_\beta^2 + M_\beta^3 + \ldots) y_t + \bar{b} y_t$$

$$= b_\beta (I - M_\beta)^{-1} y_t + \bar{b} y_t = (b_\beta B_\beta + \bar{b}) y_t,$$

(A.8)

with

$$M_\beta = \begin{pmatrix} \beta_1 M_p & 0_{6 \times 6} \\ 0_{6 \times 6} & \beta_2 M_p \end{pmatrix},$$

$$M_p = \begin{pmatrix} p_{11} M(1) & p_{12} M(1) \\ p_{21} M(2) & p_{22} M(2) \end{pmatrix},$$

$$M(s') = \begin{pmatrix} m_{11}(s') & m_{12}(s') & 0 \\ m_{21}(s') & m_{22}(s') & 0 \\ (1 - \delta_3(s')/2) PD_1(s') & (1 - \delta_3(s')/2) PD_2(s') & (1 - \delta_3(s')) \end{pmatrix},$$

and $m_{ij}(s') = (1 - \delta_j(s')) a_{ij}(s')$. 

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A.6 Current expected credit losses (CECL)

CECL as stipulated by US GAAP relies on a discounted lifetime notion of expected losses like \( EL_{LT} \) above, but uses a discount factor \( \beta_r = 1/(1 + r) \) based on a reference risk free rate rather than \( \beta = 1/(1 + c) \), which is based on the contractual interest rate of the loans. So, similarly to (A.8), we can write:

\[
EL_t^{CECL} = b_{\beta_r} (I - M_{\beta_r})^{-1} y_t + \hat{b} y_t = (b_{\beta_r} B_{\beta_r} + \hat{b}) y_t, \tag{A.9}
\]

where \( b_{\beta_r} = (\beta, b, \beta, b) \) and

\[
M_{\beta_r} = \begin{pmatrix}
\beta_r M_p & 0_{6 \times 6} \\
0_{6 \times 6} & \beta_r M_p
\end{pmatrix}.
\]

A.7 Discounted expected losses under IFRS 9

IFRS 9 adopts a hybrid approach that combines the one-year-ahead and lifetime approaches described in equations in (A.4) and (A.8). Specifically, it applies the one-year-ahead measurement to loans whose credit quality has not increased significantly since origination. For us, these are the loans with \( j=1 \), namely those in the components \( x_t(z, s, 1) \) of \( y_t \). By contrast, it considers the lifetime expected losses for loans whose credit risk has significantly increased since origination. For us, these are the loans with \( j=2 \), namely those in the components \( x_t(z, s, 2) \) of \( y_t \).

As in the case without aggregate risk, it is convenient to split vector \( y_t \) into a new auxiliary vector

\[
\tilde{y}_t = \begin{pmatrix}
x_t(1,1,1) \\
x_t(1,1,1) \\
x_t(1,2,1) \\
x_t(1,2,1) \\
x_t(2,1,1) \\
x_t(2,2,1) \\
x_t(2,2,1)
\end{pmatrix},
\]

which contains the loans with \( j=1 \), and the difference

\[
\tilde{y}_t = y_t - \hat{y}_t,
\]

which contains the rest.
This allows to express loan loss provisions under IFRS 9 as:\textsuperscript{34}

\[ EL^\text{IFRS9}_t = b_\beta \hat{y}_t + b_\beta B_\beta \tilde{y}_t + \hat{b}_t. \] (A.10)

### A.8 Comparison between the various allowance measures

The above definitions clearly imply

\[ EL^\text{IFRS9}_t = EL^\text{LT}_t - b_\beta (B_\beta - I) \tilde{y}_t \leq EL^\text{LT}_t \] (A.11)

and

\[ EL^\text{IFRS9}_t = EL^\text{1Y}_t + b_\beta (B_\beta - I) \tilde{y}_t \geq EL^\text{1Y}_t \geq IL_t. \] (A.12)

Additionally, the definitions of \( \beta \) and \( \beta_r \), together with the fact that \( c \geq r \geq 0 \), imply

\[ EL^\text{CECL}_t \geq EL^\text{LT}_t. \] (A.13)

### A.9 Implications for profits and regulatory capital

By trivially extending the formula derived for the case without aggregate risk, the result of the P/L account with aggregate risk can be written as

\[ PL_t = \sum_{z=1,2} \left\{ \sum_{j=1,2} \left[ c_z (1-PD_j(s_t)) - \frac{\delta_t}{2} PD_j(s_t) \tilde{\lambda}(s_t) x_{t-1}(z, s_t, j) - \delta_t \tilde{\lambda}(s_t) x_{t-1}(z, s_t, 3) \right] \right\} \\
- r \left( \sum_{z=1,2} \sum_{j=1,2,3} x_{t-1}(z, s_t, j) LL_{t-1} - k_{t-1} \right) - \Delta LL_t, \] (A.14)

which differs from (21) in the dependence of a number of the relevant parameters on the aggregate state at the end of period \( t, s_t \).

Dividends and equity injections are determined exactly as in (1) and (10).

Finally, the minimum capital requirement under the IRB approach is now given by

\[ k^\text{IRB}_t = \sum_{j=1,2} \gamma_j(s_t) x_{jt}, \] (A.15)

where

\[ \gamma_j(s_t) = \tilde{\lambda}(s_t) \left[ \frac{1}{\left( \sum_{j'} P_{s's_t, s_j(s_j)} \right) - 2.5} \right] m_j \left[ \Phi \left( \frac{\Phi^{-1} (PD_j) + \text{corr}_j 0.5 \Phi^{-1} (0.999))}{(1 - \text{corr}) 0.5} \right) - PD_j \right]. \] (A.16)

\textsuperscript{34}These definitions clearly imply \( EL^\text{IFRS9}_t = EL^\text{LT}_t - b_\beta (B_\beta - I) \tilde{y}_t \leq EL^\text{LT}_t \) and \( EL^\text{IFRS9}_t = EL^\text{1Y}_t + b_\beta (B_\beta - I) \tilde{y}_t \geq EL^\text{1Y}_t \). Additionally, the definitions of \( \beta \) and \( \beta_r \), together with the fact that \( c \geq r \geq 0 \), imply \( EL^\text{IRB}_t \geq EL^\text{1Y}_t \) and \( EL^\text{CECL}_t \geq EL^\text{LT}_t \).
\[ \mu_j = [0.11852 - 0.05478 \ln(\overline{PD}_j)]^2, \quad \text{and} \quad \overline{w}_j = 0.24 - 0.12(1 - \exp(-50\overline{PD}_j))/(1 - \exp(-50)). \]

Equation (A.7) implies assuming that the bank follows a strict TTC approach to the calculation of capital requirements (which avoids adding cyclicality to the system through this channel).\(^{35}\)

**A.10 Determining the contractual loan rate**

Taking advantage of the recursivity of the model, for given values of the contractual interest rates \(c_z\) of the loans originated in each of the aggregate states \(z=1,2\), one can obtain the ex-coupon value of a loan originated in state \(z\), when the current aggregate state is \(s\) and their current rating is \(j\), \(v_j(z, s)\), by solving the system of Bellman-type equations given by:

\[
v_j(z, s) = \mu \sum_{s' = 1, 2} p_{s's} \left[ (1 - PD_j(s'))c_z + (1 - PD_j(s'))\delta_j(s') + PD_j(s')(\delta_3(s')/2)(1 - \overline{\lambda}(s')) \right. \\
\left. + m_{1j}(s')v_1(z, s') + m_{2j}(s')v_2(z, s') + m_{3j}(s')v_3(z, s') \right],
\]

(A.17)

for \(j \in \{1, 2\}\) and \((z, s) \in \{1, 2\} \times \{1, 2\}\), and

\[
v_j(z, s) = \mu \sum_{s' = 1, 2} p_{s's} [\delta_3(s')(1 - \overline{\lambda}(s')) + (1 - \delta_3(s'))v_3(z, s')]\]

for \(j=3\) and \((z, s) \in \{1, 2\} \times \{1, 2\}\).

Under perfect competition and using the fact that all loans are assumed to be of credit quality \(j=1\) at origination, the interest rates \(c_z\) can be found as those that make \(v_1(z, z) = 1\) for \(z=1,2\), respectively.

\(^{35}\) A PIT approach would imply setting \(\overline{PD}_j(s_t) = \sum_{s't} p_{s's_t} PD_j(s')\) instead of \(\overline{PD}_j\) and \(\overline{\lambda}(s_t)\) instead of \(\overline{\lambda}(s_2)\) in (A.16).
B Calibration details

B.1 Migration and default rates for our two non-default states

We calibrate the migration and default probabilities of our two non-default loan categories using S&P rating migration data based on a finer rating partition. To map the S&P partition into our partition, we start considering the $7 \times 7$ matrix $\tilde{A}$ obtained by averaging the yearly matrices provided by S&P global corporate default studies covering the period from 1981 to 2015. This matrix describes the average yearly migrations across the seven non-default ratings in the main S&P classification, namely AAA, AA, A, BBB, BB, B and CCC/C.\footnote{We have reweighted the original migration rates in S&P matrices to avoid having “non-rated” as an eighth possible non-default category to which to migrate.} Under our convention, each element $\tilde{a}_{ij}$ of this matrix denotes a loan’s probability of migrating to S&P rating $i$ from S&P rating $j$, and the yearly probability of default corresponding to S&P rating $j$ can be found as $\tilde{PD}_j = 1 - \sum_{i=1}^{7} \tilde{a}_{ij}$. With the referred data, we obtain $\tilde{A}$:

$$
\tilde{A} = \begin{pmatrix}
0.8960 & 0.0054 & 0.0005 & 0.0002 & 0.0000 & 0.0000 & 0.0007 \\
0.0967 & 0.9073 & 0.0209 & 0.0022 & 0.0008 & 0.0006 & 0.0000 \\
0.0048 & 0.0798 & 0.9161 & 0.0463 & 0.0034 & 0.0026 & 0.0022 \\
0.0010 & 0.0056 & 0.0557 & 0.8930 & 0.0626 & 0.0034 & 0.0039 \\
0.0005 & 0.0007 & 0.0044 & 0.0465 & 0.8343 & 0.0618 & 0.0112 \\
0.0003 & 0.0009 & 0.0017 & 0.0082 & 0.8090 & 0.8392 & 0.1390 \\
0.0006 & 0.0002 & 0.0002 & 0.0013 & 0.0079 & 0.0432 & 0.5752
\end{pmatrix}, \quad \text{(B.1)}
$$

which implies

$$
\tilde{PD}^T = (0.0000, 0.0002, 0.0005, 0.0023, 0.0100, 0.0493, 0.2678).
$$

To calibrate our model, we want to first collapse the above seven-state Markov process into the two-state one specified in our benchmark model without aggregate risk. We want to obtain its $2 \times 2$ transition probability matrix, which we denote $A$, and the implied probabilities of default in each state, $PD_j = 1 - \sum_{i=1}^{2} a_{ij}$ for $j=1,2$. To collapse the seven-state process into the two-state process, we assume that the S&P states 1 to 5 (AAA, AA, A, BBB, BB) correspond to our state 1 and S&P states 6 to 7 (B, CCC/C) to our state 2. We also assume that all the loans originated by the bank belong to the BB category, so that the vector representing the entry of new loans in steady state under the S&P classification is $e^T = (0, 0, 0, 0, 1, 0, 0)$. Under these assumptions, we produce an average PD for the steady state portfolio of the model without aggregate risk of 1.88%, slightly below the 2.5% average PD on non-defaulted exposures of reported by the EBA (2013, Figure 12) for the period from the first half of 2009 to the second half of 2012 for a sample of EU banks using the IRB approach.
The steady state portfolio under the S&P classification can be found as \( z^* = [I_{7 \times 7} - \tilde{M}]^{-1}c \), where the matrix \( \tilde{M} \) has elements \( \tilde{m}_{ij} = (1 - \delta_j)\mu_{ij} \) and \( \delta_j \) is the independent probability of a loan rated \( j \) maturing at the end of period \( t \). For the calibration we set \( \delta_j = 0.20 \) across all categories, so that loans have an average maturity of five years. The “collapsed” steady state portfolio \( x^* \) associated with \( z^* \) has \( x^*_1 = \sum_{j=1}^{5} z^*_j \) and \( x^*_2 = \sum_{j=6}^{7} z^*_j \).

For the collapsed portfolio, we construct the \( 2 \times 2 \) transition matrix \( M \) (that accounts for loan maturity) as

\[
M = \begin{pmatrix}
\frac{\sum_{j=1}^{5} \sum_{i=1}^{5} \tilde{m}_{ij} z^*_j}{x^*_1} & \frac{\sum_{j=6}^{7} \sum_{i=1}^{5} \tilde{m}_{ij} z^*_j}{x^*_1} \\
\frac{\sum_{j=1}^{5} \sum_{i=6}^{7} \tilde{m}_{ij} z^*_j}{x^*_2} & \frac{\sum_{j=6}^{7} \sum_{i=6}^{7} \tilde{m}_{ij} z^*_j}{x^*_2}
\end{pmatrix}
\]

(\( B.2 \))

where the probabilities of default for the collapsed categories are found as

\[
PD_1 = \frac{\sum_{j=1}^{5} PD_j z^*_j}{x^*_1},
\]

(\( B.3 \))

and

\[
PD_2 = \frac{\sum_{j=6}^{7} PD_j z^*_j}{x^*_2}.
\]

(\( B.4 \))

Putting it in words, we find the moments describing the dynamics of the collapsed portfolio as weighted averages of those of the original distribution, with the weights determined by the steady state composition of the collapsed categories in terms of the initial categories.

### B.2 State contingent migration matrices

Calibrating the full model with aggregate risk on which we base our quantitative analysis requires calibrating state contingent versions of the matrix \( M \) found in (\( B.2 \)), namely the matrices \( M(s) \) for aggregate states \( s = 1, 2 \) that appear in the formulas derived in Appendix A. We find \( M(1) \) and \( M(2) \) following a procedure analogous to that used to obtain \( M \) in (\( B.2 \)) but starting from state-contingent versions, \( \tilde{A}(1) \) and \( \tilde{A}(2) \), of the \( 7 \times 7 \) migration matrix \( \tilde{A} \) in (\( B.1 \)). As described in B.1, we can go from each \( \tilde{A}(s) \) to the maturity adjusted matrix \( \tilde{M}(s) \) with elements \( \tilde{m}_{ij}(s) = (1 - \delta_j)\mu_{ij} \) and then find the elements of \( M(s) \) as weighted averages of the elements of \( \tilde{M}(s) \). To keep things simple, we use the same unconditional weights as in (\( B.2 \)), implying

\[
M(s) = \begin{pmatrix}
\frac{\sum_{j=1}^{5} \sum_{i=1}^{5} \tilde{m}_{ij}(s) z^*_j}{x^*_1} & \frac{\sum_{j=6}^{7} \sum_{i=1}^{5} \tilde{m}_{ij}(s) z^*_j}{x^*_1} \\
\frac{\sum_{j=1}^{5} \sum_{i=6}^{7} \tilde{m}_{ij}(s) z^*_j}{x^*_2} & \frac{\sum_{j=6}^{7} \sum_{i=6}^{7} \tilde{m}_{ij}(s) z^*_j}{x^*_2}
\end{pmatrix}
\]

(\( B.2(s) \))

for aggregate states \( s = 1, 2 \).
where

\[ PD_1(s) = \frac{\sum_{j=1}^{5} \tilde{PD}_j(s)z_j^*}{x_1^*}, \]

\[ PD_2(s) = \frac{\sum_{j=6}^{7} \tilde{PD}_j(s)z_j^*}{x_2^*}, \]

with \( \tilde{PD}_j(s) = 1 - \sum_{i=1}^{7} \tilde{a}_{ij}(s). \)

We calibrate \( \tilde{A}(1) \) and \( \tilde{A}(2) \) exploring the business cycle sensitivity of S&P yearly migration matrices previously averaged to find \( \tilde{A} \). We identify state \( s=1 \) with expansion years and \( s=2 \) with contraction years. We use the years identified by the NBER as the start of the recession to identify the entry in state \( s=2 \) and assume that each of the contractions observed in the period from 1981 to 2015 lasted exactly two years. This is consistent with the NBER dating of US recessions except for the recession started in 2001, to which the NBER attributes a duration of less than one year. However, the behavior of corporate ratings migrations and defaults around such recession does not suggest it was shorter for our purposes than the other three. To illustrate this, Figure B.1 depicts the time series of two of the elements of the yearly default rates \( \tilde{PD}_j \) and migration matrices \( \tilde{A} \) whose cyclical behavior is more evident: (i) the default rate among BB exposures \( \tilde{PD}_5 \) and (ii) the migration rate from a B rating to a CCC/C rating \( \tilde{a}_{7,6} \). Year 2002 emerges clearly as a year of marked deterioration in credit quality among exposures rated BB and B.

\[ \text{Figure B.1. Sensitivity of default and migrations rates to aggregate states} \]

Selected yearly S&P default and downgrading rates. Grey bars identify 2-year periods following the start of NBER recessions.

In light of this, we estimate \( \tilde{A}(2) \) by averaging the yearly counterparts of \( \tilde{A} \) extracted from S&P data for years 1981, 1982, 1990, 1991, 2001, 2002, 2008 and 2009, and \( \tilde{A}(1) \) by
averaging those corresponding to all the remaining years. This leads to

\[
\begin{pmatrix}
0.8923 & 0.0057 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0000 \\
0.1012 & 0.9203 & 0.0209 & 0.0023 & 0.0007 & 0.0003 & 0.0000 \\
0.0039 & 0.0668 & 0.9228 & 0.0500 & 0.0036 & 0.0025 & 0.0027 \\
0.0010 & 0.0058 & 0.0495 & 0.8939 & 0.0668 & 0.0036 & 0.0043 \\
0.0007 & 0.0002 & 0.0040 & 0.0429 & 0.8484 & 0.0679 & 0.0117 \\
0.0000 & 0.0009 & 0.0020 & 0.0084 & 0.0680 & 0.8511 & 0.1548 \\
0.0000 & 0.0002 & 0.0001 & 0.0009 & 0.0059 & 0.0360 & 0.5860 \\
\end{pmatrix},
\]

implying

\[
\begin{pmatrix}
0.9087 & 0.0044 & 0.0003 & 0.0005 & 0.0002 & 0.0000 & 0.0030 \\
0.0786 & 0.8632 & 0.0209 & 0.0014 & 0.0013 & 0.0017 & 0.0000 \\
0.0077 & 0.1237 & 0.8936 & 0.0340 & 0.0026 & 0.0027 & 0.0009 \\
0.0010 & 0.0050 & 0.0767 & 0.8899 & 0.0482 & 0.0028 & 0.0024 \\
0.0000 & 0.0022 & 0.0057 & 0.5878 & 0.7865 & 0.0411 & 0.0095 \\
0.0013 & 0.0007 & 0.0008 & 0.0076 & 0.1245 & 0.7988 & 0.0858 \\
0.0027 & 0.0002 & 0.0006 & 0.0025 & 0.0143 & 0.0676 & 0.5389 \\
\end{pmatrix},
\]

implying

\[
\begin{pmatrix}
0.9000, 0.0001, 0.0002, 0.0014, 0.0063, 0.0386, 0.2405 \\
0.0786, 0.8632, 0.0209, 0.0014, 0.0013, 0.0017, 0.0000 \\
0.0077, 0.1237, 0.8936, 0.0340, 0.0026, 0.0027, 0.0009 \\
0.0010, 0.0050, 0.0767, 0.8899, 0.0482, 0.0028, 0.0024 \\
0.0000, 0.0022, 0.0057, 0.0767, 0.7865, 0.0411, 0.0095 \\
0.0013, 0.0007, 0.0008, 0.0076, 0.1245, 0.7988, 0.0858 \\
0.0027, 0.0002, 0.0006, 0.0025, 0.0143, 0.0676, 0.5389 \\
\end{pmatrix},
\]

Finally, we set \(p_{12} = \text{Prob}(s_{t+1} = 1|s_t = 2)\) equal to 0.5 so that contractions have an expected duration of two years, and \(p_{21} = \text{Prob}(s_{t+1} = 2|s_t = 1)\) equal to 0.148 so that expansion periods have the same average duration as the ones observed in our sample period, \((35-8)/4 = 6.75\) years.

### B.3 Calibrating defaulted loans’ resolution rate

The yearly probability of resolution of NPLs, \(\delta_3\), is calibrated so that the model without aggregate risk fed with unconditional means of the credit risk parameters matches the 5% average probability of default including defaulted exposures (\(P\text{DID}\)) that the EBA (2013, Figure 10) reports for the second half of 2008, right before the stock of NPLs in Europe got inflated by the impact of the Global Financial Crisis. The value of \(P\text{DID}\) for the steady state portfolio obtained in the absence of aggregate risk can be computed as

\[
P\text{DID} = \frac{PD_1 x_1^* + PD_2 x_2^* + x_3^*}{\sum_{j=1}^{3} x_j^*},
\]

where \(PD_1\) and \(PD_2\) are the unconditional mean probabilities of default for standard and substandard loans obtainable from S&P data using (B.3) and (B.4), respectively (and the
procedure explained around those equations). Solving for $x_3^*$ in (B.5) allows us to set a target for $x_3^*$ consistent with the target for $PDID$:

$$x_3^* = \frac{PD_1x_1^* + PD_2x_2^* - (x_1^* + x_2^*)PDID}{PDID - 1}. \quad (B.6)$$

The law of motion of NPLs evaluated at the steady state implies

$$x_3^* = (1 - \delta_3/2)PD_1x_1^* + (1 - \delta_3/2)PD_2x_2^* + (1 - \delta_3)x_3^*, \quad (B.7)$$

where it should be noted that the dynamic system in (1) allows us to compute $x_1^*$ and $x_2^*$ independently from the value of $\delta_3$. But, then, solving for $\delta_3$ in (B.7) yields

$$\delta_3 = \frac{2(PD_1x_1^* + PD_2x_2^*)}{PD_1x_1^* + PD_2x_2^* + 2x_3^*}, \quad (B.8)$$

which allows us to calibrate $\delta_3$ using $x_1^*$, $x_2^*$, and the target for $x_3^*$ found in (B.6).

### B.4 Can professional forecasters predict recessions?

The baseline quantitative results of the model are based on the assumption that changes in the aggregate state $s_t \in \{1, 2\}$ cannot be predicted beyond what the knowledge of the time-invariant state transition probabilities of the Markov chain followed by $s_t$ allows (that is, attributing some probability to the continuation in the prior state and a complementary one to switching to the other state). At the other side of the spectrum, several papers assess the cyclical properties of the new ECL approach to provisions using historical data and the assumption of perfect foresight or that banks can perfectly foresee the losses coming in some specified horizon (e.g. two years in Cohen and Edwards, 2017, or two quarters in Chae et al., 2017). In Section C.3 of the main text, we explore how our own results get modified if banks can foresee the arrival of a recession one year in advance. However, banks’ capacity to anticipate turning points in the business cycle, and especially switches from expansion to recession, can be contended. There is a long research tradition in econometrics trying to predict turning points but the state of the question can be summarized by saying that there are a variety of indicators which allow to “nowcast” recessions (that is, to state that the economy has just entered a recession) but have little or no capacity to “forecast” recessions (see Harding and Pagan, 2010).

The same disappointing conclusion arises from the observation of the so-called Anxious Index published by the Federal Reserve Bank of Philadelphia (at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/anxious-index). Such index reflects professional forecasters’ median estimate of the probability of experiencing negative GDP growth in the quarter following the one in which the forecasters’ views are surveyed. The index, which can be traced back to mid 1968 thanks to the data maintained
by the Federal Reserve Bank of Philadelphia, is reproduced in Figure B.1. As one can see, it does not systematically rise above good-times levels in the proximity of U.S. recessions (marked as grey shaded areas), with the main exception of the second oil crisis in 1980.

![Figure B.2. The Anxious Index](https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/anxious-index)

**Figure B.2. The Anxious Index**

Professional forecasters’ median probability of decline in Real GDP in the following quarter. Grey bars identify NBER recessions.
C Complementary results

C.1 Quantitative results under SA capital requirements

For portfolios operated under the SA approach, the regulatory minimum capital requirement applicable to loans to corporations without an external rating is just 8% of the exposure net of its “specific provisions,” a regulatory concept related to impairment allowances (BCBS, 2017, paragraphs 1, 38 and 90). Assuming that all the loans in $x_t$ correspond to unrated borrowers and that all the loan loss allowances $LL_t$ qualify as specific provisions, this implies that

$$k_t^{SA} = 0.08 \left( \sum_{j=1,2,3} x_{jt} - LL_t \right).$$  \hspace{1cm} (B.9)

Capital requirements for banks following the standardized approach (SA banks) apply to exposures net of specific provisions and, hence, are sensitive to how those provisions are computed. Thus, Table C.1 includes the same variables as Table 3 for IRB banks together with details on the minimum capital requirement implied by each of the impairment measurement methods. Except for the minimum capital requirement and the implied size of a fully loaded CCB, all the other variables in Table 2 are equally valid for IRB and SA banks.

The results in Table C.1 are qualitatively very similar to those described for an IRB bank in Table 3, with some quantitative differences that are worth commenting on. It turns out that, in our calibration, an SA bank holding exactly the same loan portfolio as an IRB bank would be able to support it with lower average levels of CET1 (between 130 basis points and 210 basis points lower, depending on the impairment measurement method). Therefore, in a typical year, our SA bank features de facto higher leverage levels, and hence higher interest expenses than its IRB counterpart. This explains why its P/L is slightly lower than that of an IRB bank. This difference explains most of the level differences which can be seen in the remaining variables in Table C.1.

When comparing impairment measurement methods in the case of an SA bank, the differences are very similar to those observed in Table 3 for IRB banks. The higher state-dependence of the more forward-looking measures explains the higher cross-state differences in CET1, dividends and probabilities of needing capital injections under such measures. As for IRB banks, the differences associated with IFRS 9 relative to either the incurred loss approach or the one-year expected loss approach are significant, but not huge.
### Table C.1

**Endogenous variables under SA capital requirements**
(SA bank, as a percentage of mean exposures unless otherwise indicated)

<table>
<thead>
<tr>
<th></th>
<th>$IL$</th>
<th>$EL^{IRB}$</th>
<th>$EL^{CECL}$</th>
<th>$EL^{IFRS9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P/L</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional mean</td>
<td>0.15</td>
<td>0.17</td>
<td>0.20</td>
<td>0.17</td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>0.38</td>
<td>0.42</td>
<td>0.51</td>
<td>0.48</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>-0.62</td>
<td>-0.69</td>
<td>-0.85</td>
<td>-0.88</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.43</td>
<td>0.47</td>
<td>0.60</td>
<td>0.59</td>
</tr>
<tr>
<td><strong>Minimum capital requirement</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional mean</td>
<td>7.75</td>
<td>7.52</td>
<td>6.95</td>
<td>7.42</td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>7.79</td>
<td>7.57</td>
<td>7.03</td>
<td>7.49</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>7.62</td>
<td>7.35</td>
<td>6.71</td>
<td>7.18</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.14</td>
<td>0.16</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td><strong>CET1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional mean</td>
<td>9.67</td>
<td>9.39</td>
<td>8.72</td>
<td>9.26</td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>9.90</td>
<td>9.65</td>
<td>9.06</td>
<td>9.61</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>8.84</td>
<td>8.46</td>
<td>7.54</td>
<td>8.07</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.88</td>
<td>0.88</td>
<td>0.84</td>
<td>0.90</td>
</tr>
<tr>
<td><strong>Probability of dividends being paid (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>50.92</td>
<td>50.47</td>
<td>57.35</td>
<td>52.32</td>
</tr>
<tr>
<td>Conditional, expansions</td>
<td>65.99</td>
<td>65.41</td>
<td>74.33</td>
<td>67.81</td>
</tr>
<tr>
<td>Conditional, contractions</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Dividends, if positive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>0.34</td>
<td>0.37</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td><strong>Probability of having to recapitalize (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unconditional</td>
<td>3.68</td>
<td>3.92</td>
<td>4.45</td>
<td>4.66</td>
</tr>
<tr>
<td>Conditional, expansions</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Conditional, contractions</td>
<td>16.13</td>
<td>17.18</td>
<td>19.50</td>
<td>20.40</td>
</tr>
<tr>
<td><strong>Recapitalization, if positive</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditional mean, expansions</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Conditional mean, contractions</td>
<td>0.54</td>
<td>0.54</td>
<td>0.50</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Comparing the results in Tables 3 and C.1 leads to the conclusion that the effects on SA banks of IFRS 9 and CECL are quantitatively very similar to those on IRB banks. As for IFRS 9, this is further confirmed by Figure C.1, which shows the counterpart of Figure 3 for a bank operating under the SA approach. It depicts 500 simulated trajectories for CET1 under $IL$ and $EL^{IFRS9}$. As in Figure 3, it takes four consecutive years in the contraction state ($s=2$) for an SA bank under IFRS 9 to use up its CCB and require a recapitalization, while under the incurred loss method, the CCB would be fully depleted only after (roughly)
five years in the contraction state.\footnote{In this case, the dashed lines that delimit the band within which CET1 evolves are averages across simulated trajectories and across provisioning methods, since the sizes of the minimum capital requirement and the minimum capital requirement plus the fully-loaded CCB depend on the size of the corresponding provisions.}

\begin{figure}[h]
\begin{center}
\begin{tabular}{cc}
\textbf{Panel A. CET1 under IL} & \textbf{Panel B. CET1 under } EL^{IFRS9} \\
\includegraphics[width=0.45\textwidth]{panelA} & \includegraphics[width=0.45\textwidth]{panelB}
\end{tabular}
\end{center}
\caption{CET1 after the arrival of a contraction (SA bank)}
\end{figure}

\begin{itemize}
\item 500 simulated trajectories of CET1 under IL and EL^{IFRS9} following the arrival of s=2 after a long period in s=1 (SA bank, as a percentage of average exposures)
\end{itemize}

\subsection{C.2 Especially severe crises}

In this section, we explore whether the severity of crises and the potential anticipation of a particularly severe crisis make a difference in terms of our assessment of the cyclicality of the new more forward-looking provisioning methods (IFRS 9 and CECL) vis-a-vis the prior less forward-looking measures (incurred losses and one-year expected losses). For brevity, we again focus on IRB banks and on the comparison of one of the more forward looking approaches. IFRS 9, with just one of the alternatives, namely the one-year expected loss approach (the one so far prescribed by regulation for IRB banks).

\subsubsection{C.2.1 Unanticipatedly long crises}

We first explore what happens with the dynamic responses analyzed in the benchmark calibration with aggregate risk when we condition them on the realization of the contraction state $s=2$ for four consecutive periods starting from $t=0$. So, as in the analysis shown in Figure 2, we assume that the bank starts at $t=-1$ with the portfolio and impairment allowances resulting from having been in the expansion state ($s=1$) for a long enough period, and that at $t=0$ the aggregate state switches to contraction ($s=2$).
In Figure C.2, we compare the average response trajectories already shown in Figure 2 (where, from $t=1$ onwards, the aggregate state evolves stochastically according to the Markov chain calibrated in Table 1) with trajectories conditional on remaining in state $s=2$ for at least up to date $t=3$ (four years).\footnote{In the conditional trajectories, the aggregate state is again assumed to evolve according to the calibrated Markov chain from $t=4$ onwards.}

When a crisis is longer than expected, the largest differential impact of $EL^{IFRS9}$ relative to $EL^{IRB}$ still happens in the first year of the crisis ($t=0$), since $EL^{IFRS9}$ front-loads the expected beyond-one-year losses of the stage 2 loans. In years two to four of the crisis...
(t=1,2,3) the differential impact of IFRS 9 (compared to one-year) expected losses on P/L lessens before it switches sign (after t=4). In the first years of the crisis, $EL^{IFRS9}$ leaves CET1 closer to the recapitalization band and, in the fourth year ($t=3$), the duration of the crisis forces the bank to recapitalize only under $EL^{IFRS9}$. However, $EL^{IFRS9}$ supports a quicker recovery of profitability and, hence, CET1 after $t=4$.

C.2.2 Anticipatedly long crises

We now turn to the case in which crises can be anticipated to be long from their outset. To study this case, we extend the model to add a third aggregate state that describes “long crises” ($s=3$) as opposed to “short crises” ($s=2$) or “expansions” ($s=1$). To streamline the analysis, we make $s=2$ and $s=3$ have exactly the same impact on credit risk parameters as prior $s=2$ in Table 2, and keep the impact of $s=1$ on credit risk parameters also exactly the same as in Table 2. The only difference between states $s=2$ and $s=3$ is their persistence, which determines the average time it takes for a crisis period to come to an end. Specifically, we consider the following transition probability matrix for the aggregate state:

$$
\begin{pmatrix}
    p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{pmatrix} =
\begin{pmatrix}
    0.8520 & 0.6348 & 0.250 \\
0.1221 & 0.3652 & 0 \\
0.0259 & 0 & 0.750
\end{pmatrix},
$$

(B.10)

which implies an average duration of four years for long crises ($s=3$), 1.6 years for short crises ($s=2$), and the same duration as in our benchmark calibration for periods of expansion ($s=1$). The parameters in (B.10) are calibrated to make $s=3$ to occur with an unconditional frequency of 8% (equivalent to suffering an average of two long crises per century) and to keep the unconditional frequency of $s=1$ at the same 77% as in our benchmark calibration.

In Figure C.3 we compare the average response trajectories that follow the entry in state $s=2$ (thin lines) or state $s=3$ (thick lines) after having spent a sufficiently long period in state $s=1$. Therefore, the figure illustrates the average differences between entering a “normal” short crisis or a “less frequent” long crisis at $t=0$. Opposite to $EL^{IRB}$ (whose credit risk parameters are state-independent), $EL^{IFRS9}$ behave differently across short and long crises from the very first period because it factors in the lower probability of a recovery at $t=1$ under $s=3$ than under $s=2$. $EL^{IFRS9}$ is also more reactive in subsequent periods because, for stage 2 loans, it takes into account the losses further into the future.

These differences also explain the larger initial impact of a severe crisis on P/L and CET1 under IFRS 9 than under the prudential expected losses of the IRB approach. As a result, at the onset of an anticipated long crisis, $EL^{IFRS9}$ pushes CET1 closer to the recapitalization band and the difference with respect to $EL^{IRB}$ increases. Quantitatively, however, the effect on CET1 is still moderate, using up on impact less than half of the fully loaded CCB. Of
course, later on in the long crisis, $EL^{IFRS9}$ results, on average, in a quicker recovery of profitability and CET1 than $EL^{IRB}$.

As a quantitative summary of the implications of an anticipatedly long crisis, the following table reports the unconditional yearly probabilities of the bank needing equity injections, under each of the impairment measures compared, in the baseline model and in the current extension:

<table>
<thead>
<tr>
<th></th>
<th>$IL$</th>
<th>$EL^{IRB}$</th>
<th>$EL^{CECL}$</th>
<th>$EL^{IFRS9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline model</td>
<td>2.92%</td>
<td>2.91%</td>
<td>3.06%</td>
<td>4.16%</td>
</tr>
<tr>
<td>Model with anticipated long crises</td>
<td>3.83%</td>
<td>3.82%</td>
<td>5.18%</td>
<td>5.33%</td>
</tr>
</tbody>
</table>
Note that the anticipated long crises significantly increase the probability of having to recapitalize banks under the CECL approach, making its procyclicality very similar (as measured by this variable) to the one obtained under IFRS 9.

C.3 Better foreseeable crises

We now consider the case in which some crises can be foreseen one year in advance. Similar to the treatment of long crises in the previous subsection, we formalize this by introducing a third aggregate state, $s=3$, which describes normal or expansion states in which a crisis (transition to state $s'=2$) is expected in the next year with a larger than usual probability. So we make $s=3$ identical to $s=1$ in all respects (that is, the way it affects the PDs, rating migration probabilities, and LGDs of the loans, et cetera) except in the probability of switching to aggregate state $s' = 2$ in the next year.

To streamline the analysis, we look at the case in which $s=3$ is followed by $s'=1$ with probability one and assume that half of the crisis are preceded by $s = 3$ (while the other half are preceded, as before, by $s = 1$, which means that they are not seen as coming). Adjusting the transition probabilities to imply the same relative frequencies and expected durations of non-crisis versus crisis periods as the baseline calibration in Table 1, the matrix of state transition probabilities used for this exercise is

\[
\begin{pmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{pmatrix}
= \begin{pmatrix}
0.8391 & 0.5 & 0 \\
0.0740 & 0.5 & 1 \\
0.0869 & 0 & 0
\end{pmatrix}.
\]

The thick lines in Figure C.4 show the average response trajectories to the arrival of the pre-crisis state $s'=3$ at $t=-1$ after having spent a long time in the normal state $s=1$. We compare $EL^{IRB}$ and $EL^{IFRS9}$ and include, using thin lines, the results of the baseline model (regarding the arrival of $s'=2$ at $t=0$ having been in $s=1$ for a long period). The results confirm the notion that being able to better anticipate the arrival of a crisis helps to considerably soften its impact on IFRS 9 provisions and the subsequent effects on P/L and CET1.

\[39\text{See Section B.4 of Appendix B for a discussion of the degree to which professional forecasters are able to predict recessions.}\]
Finally, as in the previous extension, the following table reports the unconditional yearly probabilities of the bank needing equity injections under each of the compared provisioning methods, in the baseline model and in the current extension. Indeed, crises that are better anticipated imply a lower yearly probability that the bank needs an equity injection (and the reduction in such probability relative to the baseline model is more sizeable under IFRS 9). Yet, the ranking of the various provisioning methods in terms of this variable remains the same as in the baseline model, with IFRS 9 performing the worst:

<table>
<thead>
<tr>
<th></th>
<th>IL</th>
<th>EL_{IRB}</th>
<th>EL_{CECL}</th>
<th>EL_{IFRS9}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline model</td>
<td>2.92%</td>
<td>2.91%</td>
<td>3.06%</td>
<td>4.16%</td>
</tr>
<tr>
<td>Model with better foreseeable crises</td>
<td>2.43%</td>
<td>2.45%</td>
<td>2.45%</td>
<td>2.90%</td>
</tr>
</tbody>
</table>

**Figure C.4. Better foreseeable crises**

Average responses to the arrival of pre-crisis state at $t=-1$ after long in $s=1$ (thick lines). Thin lines describe the arrival of $s=2$ at $t=0$ in the baseline model (IRB bank, as a percentage of average exposures)
C.4 Other possible extensions

In this section, we briefly describe additional extensions that the model could accommodate at some cost in terms of notational, computational, and calibration complexity.

Multiple standard and substandard ratings Adding more rating categories within the broader standard and substandard categories would essentially imply expanding the dimensionality of the vectors and matrices described in the baseline model and in the aggregate-risk extension. If loans were assumed to be originated in more than just one category, the need to keep track of the (various) contractual interest rates for discounting purposes means we would need to expand the dimensionality of the model further. Alternatively, an equivalent and potentially less notationally cumbersome possibility would be to consider the same number of portfolios as different-at-origination loans and to aggregate across them the impairment allowances and the implications for P/L and CET1.

Relative criterion for credit quality deterioration This extension would be a natural further development of the previous one and only relevant for the assessment of IFRS 9. Under IFRS 9, the shift to the lifetime approach (“stage 2”) for a given loan is supposed to be applied not when an absolute substandard rating is attained, but when the deterioration in terms of the rating at origination is significant in relative terms, for example because the rating has fallen by more than two or three notches. This distinction is relevant if operating under a ratings scale that is finer than the one we have used in our analysis. As in the case with the above-mentioned multiple standard and substandard ratings, keeping the analysis recursive under the relative criterion for treating loans as “stage 1” or “stage 2” loans in IFRS 9 would require considering as many portfolios as different-at-origination loan ratings and to rewrite the expressions for provisions so as to impute lifetime expected losses to the components of each portfolio whose current rating is significantly lower than the initial rating.