

# Banks, Dollar Liquidity, and Exchange Rates\*

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September 30, 2020

PRELIMINARY AND INCOMPLETE

## Abstract

This paper presents a theory of exchange rate determination based on an endogenous liquidity premium. The theory builds on the premise that dollar reserves are the dominant currency in settling international transactions among banks. Financial flows are unpredictable and generate a precautionary demand for dollar reserves when interbank market operate with frictions. We show how the predictions of the model are consistent with the observed empirical relationship between exchange rates, liquidity premia, and dollar-reserve positions.

Keywords: Exchange rates, liquidity premia, monetary policy.

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\*We would like to thank Andy Atkeson, Roberto Chang, Pierre-Olivier Gourinchas, Arvind Krishnamurthy, Martin Eichenbaum, Oleg Itskhoki, Matteo Maggiori, Dmitry Mukhin, Sergio Rebelo, and Helen Rey for useful comments and other conference participants the Bank of Peru-Northwestern Conference on ‘Exchange Rates’ and the 2019 Stanford SITE Conference on ‘International Finance’, the NBER “Frontier and Emerging Markets Conference” for valuable comments. We are especially thankful to people that discussed this paper: Dmitry Mukhin.

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# 1 Introduction

The well-known “disconnect” puzzle in international finance holds that foreign exchange rates show little empirical relationship to the supposed economic drivers of currency values: monetary policy instruments, output, etc.<sup>1</sup> The literature has turned toward deviations from uncovered interest parity (UIP) as the potential “missing link” in the exchange-rate puzzle.<sup>2</sup> Moreover, when measured in historical data on major currency pairs, a consistent pattern emerges: most currencies earn a significant and persistent premium over the US dollar. That is, saving in dollars has typically earned a low return.<sup>3</sup> But the source or sources of these UIP deviations remains a mystery that has not been fully resolved.

In this paper, we develop a theory of these UIP deviations as arising from the demand by financial institutions for liquid dollar assets. We build on the observation that banks that participate in the international payments system (global banks) incur dollar-denominated liabilities. In the international banking system, U.S. dollars are the dominant foreign-currency source of funding. According to the BIS locational banking statistics, in September 2019, the global banking and non-bank financial sector had cross-border dollar liabilities of over \$10 trillion.

Banks domiciled in the U.S. and foreign banks (and other financial institutions, which we will call “banks”) face uncertainty about funding. This uncertainty arises in part from the possibility of withdrawals by depositors, but also from the vicissitudes of very short-term lenders to these banks. While banks might typically turn to other financial institutions as a source of short-term liquidity, in times of uncertainty, the banks must rely on their own stocks of liquid assets to meet the demands of depositors and short-term lenders. U.S. banks might hold reserves at the Federal Reserve to maintain liquidity, but they also generally hold U.S. Treasury and agency obligations that are highly liquid. International banks have no central bank dollar reserves but hold other liquid dollar assets. We build a model of “global” banks, which could be domiciled either in the U.S. or abroad, but which have dollar and non-dollar (euro) liabilities. The global demand of the financial system for liquid dollar assets plays a pivotal role in our story.

Of course, the risk for an individual bank is that if it experiences a net outflow of liabilities, it will end up short of liquid assets to settle those flows. In the case of a shortfall, these banks might seek funding in the interbank market, but that market operates with frictions. In periods where transactions move in one direction, it may be costly to find a counterparty, and there may be times when banks may lose confidence in one another.

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<sup>1</sup>The term “exchange-rate disconnect” was coined by [Obstfeld and Rogoff \(2000\)](#). [Devereux and Engel \(2002\)](#), [Duarte and Stockman \(2005\)](#), and [Itskhoki and Mukhin \(2019\)](#) provide models of disconnect.

<sup>2</sup>[McCallum and Nelson \(1999\)](#), [Kollmann \(2002\)](#), [Bergin \(2006\)](#) and [Itskhoki and Mukhin \(2019\)](#) are examples of DSGE models that include a random deviation from UIP in order to help explain exchange-rate behavior.

<sup>3</sup>See for example, [Lustig and Verdelhan \(2007\)](#) and [Hassan \(2013\)](#) emphasize the relatively low rate of return on nominally risk-free dollar assets, while [Gourinchas and Rey \(2007\)](#) refer more broadly to the “exorbitant privilege” the U.S. enjoys by paying a lower return on its external liabilities than it earns on its foreign assets.

We model how frictions in the settlement of international deposit transactions emerge as a liquidity premium earned by the dollar. In our framework, deviations from UIP arise as a function of monetary policy variables such as the quantity of outside money and policy rates in two currencies, as well as technology parameters such as the a matching efficiency among banks (which captures interbank confidence), the volatility of payments, and relative settlement demand in different currencies. Through the channel of UIP deviations, we link the determination of nominal dollar exchange rates (and a dollar return premium) to the reserve position of banks in different currencies, and to settlement risk and payments technology.

Many theories of the forces behind UIP deviations have focused primordially on a risk-premium or an external financing premium earned by the US dollar. Risk premium models chiefly explain the UIP deviation for dollar bonds as stemming from a greater exposure of currencies other than the dollar to global pricing factors.<sup>4</sup> Theories of the external financing premia can explain the dollar premium as a funding advantage in dollar liabilities. The interpretation in this paper is on an alternative explanation, a liquidity premium. Superficially, the model resembles the early monetary exchange rate model of Lucas (1982).<sup>5</sup> In that model, two currencies earn a liquidity premium over bonds because certain goods must be bought with corresponding currencies. A money demand equation determines prices in both currencies, and relative prices determine the exchange rate. Our model shares the segmentation of transactions and the exchange-rate determination of Lucas. However, in our model, the demand for reserves in either currency stems from the settlement demand by banks. This distinction is important, because our model leads to predictions about the direction of exchange rates as functions of the ratio of reserves to deposits in different currencies, the size and volatility of flows in different currencies and the dispersion of interbank rates in different currencies.

## Literature Review

The deviation from uncovered interest parity, or the expected excess return on foreign interest earning assets, is important not just for understanding international pricing of interest-bearing assets, or the expected depreciation (or appreciation) of the currency, but also for the level of the exchange rate. This point is brought out clearly by Obstfeld and Rogoff (2003), which shows how the expected present value of current and future foreign exchange risk premiums affect the current exchange rate in a simple DSGE model. They refer to this present value as the “level risk premium.”<sup>6</sup>

Potentially, a better understanding of the role of the UIP deviation can help account for the empirical failure of exchange-rate models (Meese and Rogoff, 1983; Obstfeld and Rogoff, 2000),

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<sup>4</sup>See, for example, Lustig et al. (2011).

<sup>5</sup>See also Svensson (1985) and Engel (1992a,b).

<sup>6</sup>This present value plays a key role in the analysis of Engel and West (2005), Froot and Ramadorai (2005), and Engel (2016). See Engel (2014)’s survey of exchange rates for an overview of the effect of the risk premium on exchange rates.

and the excess volatility of exchange rates (Frankel and Meese, 1987; Backus and Smith, 1993; Rogoff, 1996). Much of the literature has been directed toward explaining the expected excess return as arising from foreign exchange risk. Another branch of the literature has explored limits to capital mobility and frictions in asset markets. A third branch has looked at deviations from rational expectations. A line of research closely related to this paper has been the role of the “convenience yield” in driving exchange rates.

***Foreign exchange risk premium.*** The modeling of failures of uncovered interest parity as arising from foreign exchange risk has a long history. Early contributions include Solnik (1974), Roll and Solnik (1977), Kouri (1976), Stulz (1981), and Dumas and Solnik (1995). Much theoretical work has been devoted toward building models of the risk premium that are consistent with the Fama (1984) puzzle, which finds a positive correlation between the expected excess return and the interest rate differential.<sup>7</sup> Bansal and Shaliastovich (2013), Colacito (2009), Colacito and Croce (2011, 2013), Colacito et al. (2018b,a), and Lustig and Verdelhan (2007) examine models with recursive preferences. Verdelhan (2010) presents a model with habit formation to account for the Fama puzzle. Ilut (2012) proposes ambiguity aversion as a solution to the puzzle. Some recent studies, such as Burnside et al. (2011), Farhi and Gabaix (2016), and Farhi et al. (2015), model the risk premium as arising from risks associated with rare events.

***Limited Capital Mobility.*** Other models attribute these uncovered interest parity differentials to financial premia earned by foreign currency because of limited market participation as in the segmented markets models of (Alvarez et al., 2002, 2009; Itskhoki and Mukhin, 2019) or limited capital flows (Gabaix and Maggiori, 2015; Amador et al., 2019). Models in which order flow matters for exchange rate determination also require some frictions in the foreign exchange market. See, for example, Evans and Lyons (2002, 2008). Relatedly, Bacchetta and Van Wincoop (2010) posit that slow adjustment of portfolios can account for the expected excess returns on foreign bonds.

***Deviations from Rational Expectations.*** A simple alternative story for the UIP deviations is that agents expectations are not fully rational. Empirical studies, such as Frankel and Froot (1987), Froot and Frankel (1989), and Chinn and Frankel (2019) have used survey measures of expectations to uncover possible deviations from rational expectations. Models that incorporate systematically skewed expectations include Gourinchas and Tornell (2004) and Bacchetta and Van Wincoop (2006).

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<sup>7</sup>See Tryon (1979) and Bilson (1981) for earlier empirical studies that find this relationship. Engel (1996, 2014) surveys empirical and theoretical models.

*Convenience Yield.* Our model is closely connected to the recent examination of the “convenience yield” - the low return on riskless government liabilities - and exchange rates. We posit that our model provides one possible channel for the emergence of the convenience yield on U.S. government bonds. See [Engel \(2016\)](#), [Valchev \(2020\)](#), [Jiang et al. \(2018\)](#), [Engel and Wu \(2018\)](#).

## 2 Motivating Facts

We begin with a look at the data relating the banking sector’s balance sheet data to the U.S. dollar price of the euro. Our thesis, at its simplest, is that the financial sector increases its demand for dollar liquid assets – U.S. government obligations, including reserves held at the Federal Reserve for banks in the Federal Reserve system – when funding becomes more uncertain. The global banking system relies heavily on U.S. dollars for funding, much of which is raised through money market funding for banks located outside of the U.S.

Here we look at two sources of data for the U.S. banking system. Detailed data on short-term dollar funding, and on liquid dollar assets is not readily available, so we use the U.S. data as a proxy for the dollar-denominated elements of the global banking balance sheets. That is, we presume that foreign banks’ demand for liquid dollar assets responds in a similar way to U.S. banks when faced with uncertainty about dollar funding. This approach is also followed by [Adrian et al. \(2010\)](#), a study that focuses on how the price of risk is related to banks’ balance sheets and the expected change in the exchange rate (rather than the level of the exchange rate, which is our focus here), and presents a simple partial-equilibrium model of the banking sector. More precisely, [Adrian et al. \(2010\)](#) focus on the state of the balance sheet at time  $t$  in forecasting  $e_{t+1} - e_t$ , as they are concerned with understanding the expected excess return on foreign bonds between  $t$  and  $t + 1$ . Our concern is more centered on understanding how changes in the balance sheet between  $t - 1$  and  $t$  contribute to changes in the exchange rate between  $t - 1$  and  $t$ , that is  $e_t - e_{t-1}$ .

We consider two measures of funding to financial intermediaries. The first is used by [Adrian, Etula, and Shin \(2010\)](#), U.S. dollar financial commercial paper (series DTBSPCKFM from FRED, the Federal Reserve Economic Data website maintained by the Federal Reserve Bank of St. Louis.) Of course, another major source of short-term funding to U.S. banks is demand deposits, measured by DEMDEPSL from FRED. We construct a variable that measures the level of funding and the response of financial intermediaries to uncertainty about that funding. We look at the ratio of the sum of reserves held at Federal Reserve banks and government securities held by commercial banks (the sum of RESBALNS and USGSEC from FRED) to short term funding (DTBSPCKFM + DEMDEPSL from FRED). This variable is endogenous in our model, but its movements are a key indicator of how the demand for dollars is affected by the financial sector’s demand for liquid assets when uncertainty increases. As dollar funding becomes more volatile for banks, they will increase their ratio of safe dollar assets to liabilities. That in turn will lead to a global increase in

dollar demand, leading to a dollar appreciation.

Figure 1 plots this ratio of liquid government assets to short-term funding of the financial sector, as well as the ratio of the funding to just demand deposits. During this period, bank reserve balances rose from around 10 billion dollars in August, 2008 to nearly 800 billion dollars one year later, and then continued to climb to a peak of around 2.3 trillion dollars by late 2017 before gradually declining to 1.4 trillion dollars by the end of 2019. However, these liquidity ratios do not show movement anywhere near that magnitude. It is true that they rose during the onset of the global financial crisis, but this movement is not largely driven by the increase in reserves, because demand deposits rose almost proportionately. A large part of the rise in the overall liquidity ratio is driven by a fall in financial commercial paper funding, which works to lower the denominator of the ratio.

In Table 1, we present the parameter estimates of the regression:

$$\Delta e_t = \alpha + \beta_1 \Delta(LiqDepRat_t) + \beta_2 \Delta(i_t - i_t^*) + \beta_3 (\pi_t - \pi_t^*) + \beta_4 LiqDepRat_{t-1} + \epsilon_t \quad (1)$$

In this regression,  $\Delta$  means the “change from  $t - 1$  to  $t$ ” in the variable;  $e_t$  is the log of the euro price of a dollar;  $LiqRat_t$  is the variable described above;  $i_t - i_t^*$  is the difference between the 1-month euro and 1-month dollar LIBOR rates; and,  $\pi_t - \pi_t^*$  is the difference between the European and U.S. year-on-year inflation rates. All data is monthly.

The latter two variables are the traditional monetary determinants of exchange rate movements. As  $i_t - i_t^*$  increases, we should find an appreciation of the euro, and hence  $\beta_2$  should be negative. That is, a relative tightening of monetary policy in Europe should lead to a stronger euro. Also, we should find  $\beta_3$  negative. As much of the empirical literature has found, there is a negative relationship between the change in a country’s inflation rate and its exchange rate. When inflation is rising in a country, markets anticipate future monetary tightening, and that leads to a currency appreciation. In fact, the interest-rate differential itself is not highly significant, so we also report results dropping that variable from the regression (here and in all subsequent regressions.)

If uncertainty is driving the  $LiqRat_t$ , then we should also expect a positive relationship between this variable and  $e_t$ , i.e.,  $\beta_1$  positive. During times of high uncertainty, banks hold greater amounts of liquid dollar assets (reserves and Treasury securities) relative to demand deposits, so  $LiqRat_t$  is higher. That increased demand for safe dollar assets leads to a stronger dollar (an increase in  $e_t$ .)

We also include the lagged level of  $LiqRat_t$ . This is included because the depreciation of the dollar might depend on lagged as well as current levels of this variable. The regressions we report would have the identical fit if we included current and lagged levels of this variable, instead of the change in the variable and the lagged level. We specify the regression as above for two reasons. First, specifying the regression so that the change in the liquidity variable influences the change in the exchange rate leads to a more natural interpretation. Second, while the current and lagged

levels of the variable are highly correlated, which leads to multicollinearity and imprecise coefficient estimates, the change and the lagged level are much less highly correlated.

These variables all are statistically significant and of the correct sign in the regressions reported in Table 1. The table reports regressions for the entire sample, 2001:2-2018:1 and also reports for shorter samples starting in 2005:1. (The beginning and end of our sample is limited by data availability. The series for DTBSPCKFM begins only in 2001:1, and the measure of convenience yield used below, from [Engel and Wu \(2018\)](#) ends in 2018:1.) While the relationship between each variable and the exchange rate is as expected, the fit is tighter for the shorter samples. In the latter samples, all variables are significant at the 1 percent level and the overall fit of the model (as measured by adjusted  $R^2$ ) is quite good.

An important point to make about these regressions is that the dependent variables are not simply market prices. That is, these regressions “explain” exchange rate movements but are not relying on other market prices to do the job. It is the balance sheet variables that play the pivotal role.

We argue that the uncertainty about funding drives the balance sheet variables, but what if we were to include a direct measure of uncertainty in the regression? Many asset-pricing studies have used  $VIX$  to capture market uncertainty, and it seems as though  $VIX$  has power in explaining the movements of many asset prices. However,  $VIX$  does not directly measure uncertainty about dollar funding for banks. Indeed,  $VIX$  might capture some dimensions of that uncertainty, but it also might be measuring global risk, and global risk might be driving the dollar, as in the model of [Farhi and Gabaix \(2016\)](#). In Table 2, we have included the change in  $VIX$  along with the other variables.

Interestingly, while  $VIX$  has the expected positive coefficient (a higher  $VIX$  is associated with a stronger dollar), the other variables remain almost as statistically significant as before, and the magnitude of the coefficients is little changed. That is,  $VIX$  introduces either another dimension of uncertainty about funding that the balance sheet variables do not perfectly measure, or else it works to influence the exchange rate through other channels, because it seems to provide independent information about the currency value. This is especially true in the post-2005 samples, and is reflected in the surprisingly tight fit of the model, with an adjusted  $R^2$  value of 0.31 in the sample that begins in 2005:1.

How are these findings related to the recent studies that find the value of the dollar is related to the “convenience yield” on U.S. Treasury bonds, measured as the difference between the return on LIBOR and U.S. short-term bonds?<sup>8</sup> Let  $\eta_t$  be the measure of this convenience yield (in the U.S. relative to the convenience yield on in other countries.) An increase in  $\eta_t$  has been found to be associated with a stronger U.S. dollar. That is, as U.S. Treasury interest rates fall relative to dollar LIBOR rates, increasing the convenience yield and increasing  $\eta_t$ , the dollar appreciates ( $e_t$

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<sup>8</sup>Jiang et al. (2018), [Engel and Wu \(2018\)](#), [Valchev \(2020\)](#)

risers.) These studies control for other traditional determinants of exchange rates, such as the level of interest rates (either government rates or LIBOR rates), and possibly other monetary factors.

We use as our measure of  $\eta_t$  the baseline measure in Engel and Wu (2018):

$$\eta_t \equiv e_t - f_{t,t+1} + i_t^G - i_t^{G*}$$

where  $f_{t,t+1}$  is the log of forward rate and  $e_t$  is the log of the spot exchange rate, both expressed in euros per dollar. The interest rates are on government bonds,  $i_t^{G*}$  for the U.S., and  $i_t^G$  for Germany.

If covered interest parity holds, so that the forward premium is equal to the difference between the dollar and euro LIBOR rates, this is a relative measure of difference between marketable securities and government bond yield in the U.S. and Germany. Or, more directly, the first three terms  $e_t - f_{t,t+1} + i_t^G$  can be understood as the payoff of a synthetic dollar government bond that is constructed by buying the German government bond, and eliminating exchange rate risk by entering a forward contract. Since the U.S. government bond and the synthetic dollar-denominated German government bond pay equivalent pecuniary returns, the difference between the two gives a measure of the relative difference in liquidity services the U.S. and German government bonds provide.

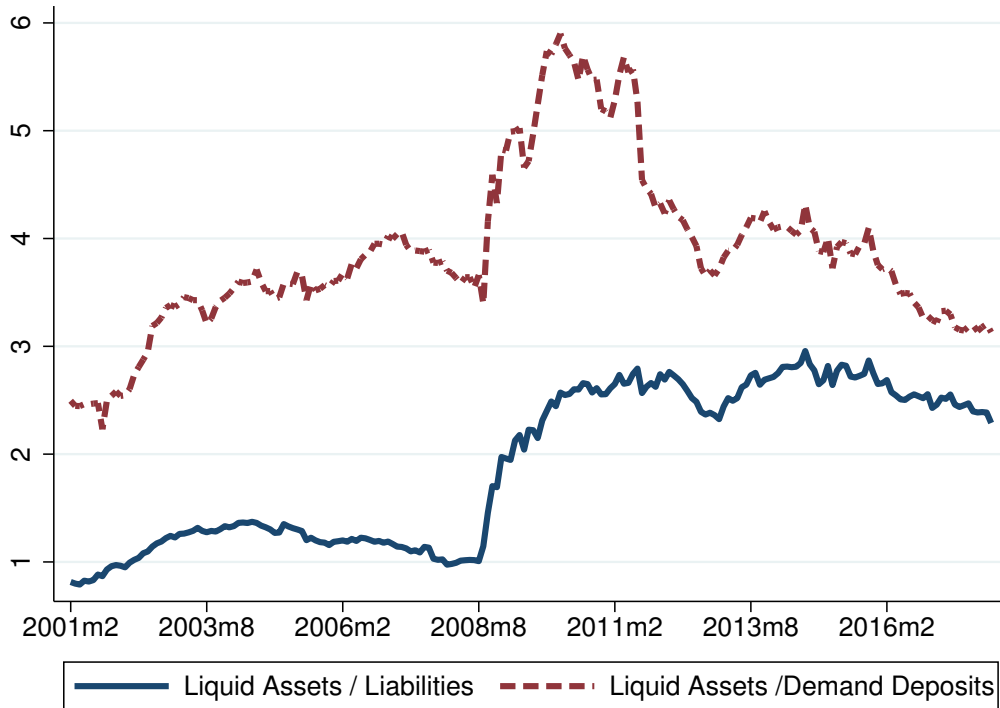


Figure 1: The ratio of liquid assets to short-term liabilities

Table 3 presents the findings from including the change in  $\eta_t$  in the exchange-rate regression.



Some versions of the regression also included the lagged level of  $\eta_t$ . Also, we exclude the interest differential variable,  $\Delta(i_t - i_t^*)$ , from these regressions because it has almost no explanatory power.

Remarkably, we find that not only does the change in  $\eta_t$  have explanatory power for the change in the exchange rate, but when it is included in the regression, the t-statistics increase for the balance sheet variable, the change in  $LiqRat_t$ . Apparently, the convenience yield is correlated with this balance sheet variable but contains information not contained in that variable. We might think of these two variables as imperfectly measuring the forces that lead the banking sector to increase its demand for safe, liquid assets. The orthogonal components each significantly affect the exchange rate.

Not only are all the variables in these regressions individually highly statistically significant, but the fit is remarkably strong. For the entire sample, the adjusted  $R^2$  is as high as 0.21, and for the shorter sample beginning in 2005:1, the adjusted  $R^2$  climbs as high as 0.30. There clearly seems to be a relationship between the measures that capture the need for liquidity by the financial system, and the value of the dollar, especially during times of financial turmoil.

Table 1: Relationship of change of exchange rates and measures of banking liquidity

	01M2-18M1	01M2-18M1	05M1-18M1	05M1-18M1
$\Delta(LiqRat_t)$	0.214*** (3.974)	0.223*** (4.160)	0.234*** (4.198)	0.251*** (4.469)
$\Delta(i_t - i_t^*)$	-1.466 (-1.501)		-2.498** (-2.356)	
$\pi_t - \pi_t^*$	-0.005*** (-3.284)	-0.005*** (-3.227)	-0.005*** (-2.983)	-0.005*** (-2.888)
$(LiqRat_{t-1})$	0.009* (1.843)	0.010** (2.180)	0.009 (1.437)	0.012* (1.783)
constant	-0.011*** (-2.965)	-0.012*** (-3.178)	-0.011* (-1.959)	-0.012** (-2.167)
$N$	204	204	157	157
adj. $R^2$	0.10	0.10	0.15	0.12

$t$  statistics in parentheses.

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 2: Relationship of change of exchange rates and measures of banking liquidity, with VIX

	01M2-18M1	01M2-18M1	05M1-18M1	05M1-18M1
$\Delta(\text{LiqRat}_t)$	0.173*** (3.251)	0.179*** (3.392)	0.177*** (3.336)	0.189*** (3.539)
$\Delta(i_t - i_t^*)$	-1.234 (-1.306)		-2.079** (-2.100)	
$\pi_t - \pi_t^*$	-0.004** (-2.532)	-0.004** (-2.472)	-0.004** (-2.046)	-0.003* (-1.941)
$\Delta\text{VIX}_t$	0.002*** (3.956)	0.002*** (4.038)	0.002*** (4.960)	0.002*** (5.101)
$\text{LiqRat}_{t-1}$	0.009** (1.979)	0.010** (2.284)	0.009 (1.554)	0.011* (1.866)
constant	-0.010*** (-2.808)	-0.011*** (-2.991)	-0.009* (-1.796)	-0.011* (-1.975)
$N$	204	204	157	157
adj. $R^2$	0.17	0.16	0.26	0.25

$t$  statistics in parentheses.

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 3: Relationship of change of exchange rates and measures of banking liquidity, with VIX and convenience yield

	01M2-18M1	01M2-18M1	05M1-18M1	05M1-18M1
$\Delta(\text{LiqRat}_t)$	0.173*** (3.345)	0.140** (2.590)	0.187*** (3.599)	0.153*** (2.739)
$\pi_t - \pi_t^*$	-0.003** (-2.147)	-0.003** (-2.078)	-0.003* (-1.672)	-0.004** (-2.051)
$\Delta\text{VIX}_t$	0.002*** (3.592)	0.002*** (3.619)	0.002*** (4.446)	0.002*** (4.459)
$\Delta\eta_t$	4.909*** (3.235)	6.162*** (3.777)	4.882*** (3.182)	6.076*** (3.568)
$\text{LiqRat}_{t-1}$	0.010** (2.267)	0.011** (2.566)	0.011* (1.876)	0.016** (2.416)
$\eta_{t-1}$		2.297** (1.997)		2.352 (1.583)
constant	-0.010*** (-2.876)	-0.016*** (-3.494)	-0.010* (-1.916)	-0.020** (-2.438)
$N$	204	204	157	157
adj. $R^2$	0.20	0.21	0.29	0.30

$t$  statistics in parentheses.

\*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

### 3 A Model of Banking Liquidity and Exchange Rates

We present a dynamic equilibrium model of global banks that intermediate international financial flows and are subject to idiosyncratic liquidity shocks. The model has two countries, the EU and the US, and two currencies. To fix ideas, we think about the euro as the domestic currency and the dollar as the foreign currency. In each country, there is a continuum of households and a central bank that sets monetary policy. Production of the single tradable consumption good is carried out globally by multinationals. We assume that the law of one price holds.

#### 3.1 Banks

**Timing.** Time is discrete and there is an infinite horizon. Every period is divided in two sub-stages: a lending stage and a balancing stage. In the lending stage, banks make their equity payout,  $Div_t$ , and portfolio decisions. In the balancing stage, banks face liquidity shocks and re-balance their portfolio.

**Notation.** We use “asterisk” to denote the foreign currency (i.e., the “dollar”) variable and “tilde” to denote a real variable. The vector of aggregate shocks is indexed by  $X$ . The exchange rate is defined as the amount of euros necessary to purchase one dollar—hence, a higher  $e$  indicates an appreciation of the dollar.

**Preferences and budget constraint.** Payouts are distributed to households that own bank shares and have linear utility. Banks maximize the net present value of dividends:

$$\sum_{t=0}^{\infty} \beta^t Div_t$$

where  $\beta$  is the discount factor of the household.

Banks enter the lending stage with a portfolio of assets/liabilities and collect/make associated interest payments. The portfolio of initial assets is given by corporate loans,  $l_t$ , and liquid assets  $m_t$ , both in euros and dollars, and corporate loans,  $b$ , denominated in consumption goods. Note that we refer to liquid assets as reserves, for simplicity, but they should be understood as capturing also government bonds—the important property, as we will see, is that these are assets that can be used as settlement instruments. On the liability side, banks issue demand deposits,  $d_t$ , discount window loans,  $w_t$ , and net interbank loans,  $f_t$  (if the bank has borrowed funds,  $f_t$  is positive, and vice versa), again in both currencies. Deposits and interbank market loans have market returns given by  $i^d$  and  $\bar{i}^f$  while central banks set the corridor rates for reserves and discount window, respectively  $i^m$  and  $i^w$ . Meanwhile,  $R^b$  is the real return on loans.

Their budget constraint is given by

$$P_t^* Div_t + \frac{m_{t+1} - d_{t+1}}{e_t} + \tilde{b}_{t+1} P_t^* + m_{t+1}^* - d_{t+1}^* \leq P_t^* \tilde{b}_t R_t^b + m_t^*(1 + i_t^{m,*}) - d_t^*(1 + i_t^{d,*}) + f_t^*(1 + i_t^{f,*}) + w_t^*(1 + i_t^{w,*}) - \frac{m_t(1 + i_t^m) - d_t(1 + i_t^d) + f_t(1 + i_t^f) + w_{t-1}(1 + i_t^w)}{e_t} \quad (2)$$

**Withdrawal shocks.** In the balancing stage, banks are subject to random withdrawal of deposits in either currencies. As in Bianchi and Bigio (2020), withdrawals shows have zero mean—hence deposits are reshuffled but preserved within the banking system. In addition, we assume that the distribution of these shocks is time-varying. As a way to capture the prevalence of the dollar for international settlements, we will be focus on an environment where the volatility of dollar deposits is larger than the euro.

The inflow/outflow of deposits across banks generates, in effect, a transfer of liabilities. We assume that these liabilities are settled using reserves in the same currency of the deposit. Importantly, reserves must remain positive at the end of the period. We denote by  $s^j$  the euro surplus of a bank facing a withdrawal shock  $\omega_t^j$ . This surplus is given by the amount of euro reserves a bank brings from the lending stage minus the withdrawals of deposits:

$$s_t^j = m_{t+1} + \omega_t^j d_{t+1}, \quad (3)$$

Notice that we omit the subscript of bank choices in deposits and reserves, because it is without loss of generality that all banks make the same choices in the lending stage. If a bank faces a negative withdrawal shock, lower than  $\tilde{\omega} \equiv -m/d$ , the bank has a deficit of reserves. Conversely, if the withdrawal shock is larger than  $\tilde{\omega}$ , the bank has a surplus. Notice that if  $m = 0$ , the sign of the surplus has the same sign as the withdrawal shock. A higher liquidity ratio makes more likely that the bank will be in surplus.

Similarly, we have the following surplus in dollars

$$s_t^{j,*} = m_{t+1}^* + \omega_t^{j,*} d_{t+1}^* \quad (4)$$

**Interbank market.** After the withdrawal shocks are realized, we have a distribution of banks in surplus and deficits in both currencies. We assume that there is an interbank for each currency in which banks that have a deficit in one currency borrow from banks that have a surplus in the same currency. These two interbank market behave symmetrically, so it suffices to show how one of the market works.<sup>9</sup>

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<sup>9</sup>We are assuming in the background an extreme form of segmented interbank markets: penalties in dollars and euros are independent because dollar surpluses cannot be used to patch Euro deficits and vice versa. This assumption can be relaxed to some extent but some form of segmentation of asset markets is necessary to obtain liquidity premia. See the discussion below.

We model the interbank market an over-the-counter (OTC) market, which is in line with institutional features of this market (see [Ashcraft and Duffie, 2007](#); [Afonso and Lagos, 2015](#)). Modeling the interbank market using search and matching is also natural considering that the interbank market is a credit market in which banks on different sides of the market (surplus and deficit) must find a counterpart they trust.

As a result of the search frictions, only a fraction of the surplus (deficit) will be lent (borrowed) in the interbank market. We assume, in particular, that each bank gives an order to a continuum of traders to either lend or borrow, as in [Atkeson, Eisfeldt, and Weill \(2015\)](#). A bank with surplus  $s$  is able to lend a fraction  $\Psi^+$  to other banks. The remainder fraction is kept in reserves. Conversely, a bank that has a deficit is able to borrow a fraction  $\Psi^-$  from other banks, and the remainder deficit is borrowed at a penalty rate  $i^w$ . The penalty rate can be thought as the discount window rate or as an overdraft-rate charged by correspondent banks that have access to the Fed's discount window.

The fractions  $\Psi^+$  and  $\Psi^-$  depend on the abundance of reserve deficits relative to surplus. Assuming a constant returns to scale matching function, the probabilities depend entirely on market tightness, defined as

$$\theta_t \equiv S_t^- / S_t^+ \quad (5)$$

where  $S_t^+ \equiv \int_0^1 \max\{s_t^j, 0\} dj$  and  $S_t^- \equiv -\int_0^1 \min\{s_t^j, 0\} dj$  denote the aggregate surplus and deficit, respectively. Notice that because  $m \geq 0$  and  $\mathbb{E}(\omega) = 0$ , we have that in equilibrium  $\theta \leq 1$ . That is, there is a relatively larger mass of banks in surplus than deficit.

The interbank market rate is the outcome of a bargaining problem between banks in deficit and surplus. As in [Bianchi and Bigio \(2020\)](#), the interbank market rate, is the outcome of a bargaining problem between banks in deficit and surplus. There are  $N$  trading rounds, in which banks trade with each other. If banks are not able to match by the  $N$  trading rounds, they deposit the surplus of reserves at the central bank or borrow from the discount window. Throughout the trading, the terms of trade at which banks borrow/lend, i.e., the interbank market rate, depend on the probabilities of finding a match in a future period. We denote by  $\bar{i}^f$  the average interbank market rate at which banks trade on average. Ultimately, we can define a real penalty function  $\chi$  that captures the benefit of having a real surplus or deficit  $\tilde{s}$  upon facing the withdrawal shock as follows:

$$\chi(\theta, \tilde{s}) = \begin{cases} \chi^+(\theta)\tilde{s} & \text{if } \tilde{s} \geq 0, \\ \chi^-(\theta)\tilde{s} & \text{if } \tilde{s} < 0 \end{cases} \quad (6)$$

where  $\chi^+$  and  $\chi^-$  are given by

$$\chi^-(\theta) = \Psi^-(\theta)(R^f - R^m) + (1 - \Psi^-(\theta))(R^w - R^m) \quad (7)$$

$$\chi^+(\theta) = \Psi^+(\theta)(R^f - R^m) \quad (8)$$

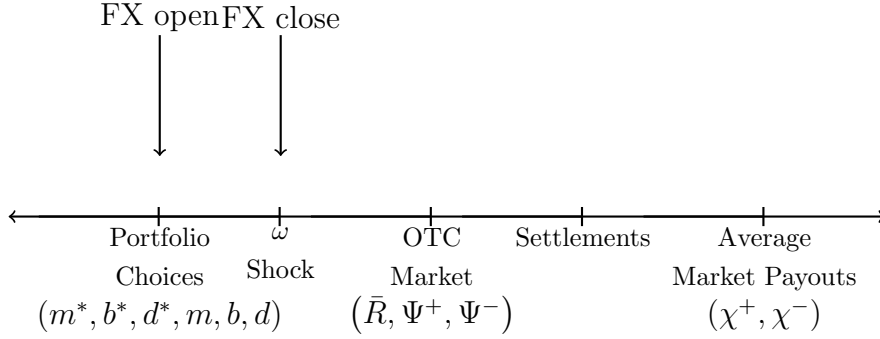


Figure 2: Timeline

In these expressions,  $R^y(X) \equiv \mathbb{E}_X \frac{1+i^y(X)}{1+\pi(X')}$ , denote the gross expected real rate of an asset/liability  $y$  and  $\pi_{t+1}$  denotes the inflation rate between  $t$  and  $t+1$  when the initial state is  $X$ . (When it does not lead to confusion, we streamline the argument  $X$  in these expressions.) Notice that a bank that borrows from the the interbank market or from the discount window, obtains the interest on reserves—hence the cost of being in deficit is given by  $R^f - R^m$  in the former and  $R^w - R^m$  in the latter. By the same token, the benefit from lending in the interbank market in case of surplus is  $R^f - R^m$  as reflected in (8).

Figure (3.1) present a schematic version of the complete timeline within each period. We next turn to describe the bank optimization problem.

**Banks' Problem.** We express the bank's optimization problem in terms of real portfolio holdings  $\{\tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$  to maximize expected real profits subject to the budget constraint. When choosing the portfolio, banks anticipate how the withdrawal shock may lead to a surplus or deficit and the associated costs and benefits. We can show that the problem can be expressed as choosing follows:

**Problem 1.** The recursive problem of a bank is

$$v(n, X) = \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta \mathbb{E}[v(n', X') | X] \quad (9)$$

subject to

the budget constraint:

$$Div + \tilde{b} + \tilde{m}^* + \tilde{m} = n + \tilde{d} + \tilde{d}^* \quad (10)$$

and the evolution of bank networkh:

$$n' = \underbrace{R^b(X)\tilde{b} + R(X)\tilde{m} + R^{m,*}(X)\tilde{m}^* - R^d(X)\tilde{d} - R^{*,d}(X)\tilde{d}^*}_{\text{Portfolio Returns}} + \underbrace{\mathbb{E}_{\omega^*}\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*) + \mathbb{E}_{\omega}\chi(\theta(X), \tilde{m} + \omega\tilde{d})}_{\text{Settlement Costs}} \quad (11)$$

From here, we derive the following auxiliary Lemma that allow us to characterize equilibrium portfolio conditions.

**Lemma 1.** *The solution to (9) is  $v(n, X) = n$  and the law of motion of bank networkh satisfies:*

$$n' = \frac{1}{\beta} (n - Div) + \Pi^*(X)$$

where  $\Pi^*(X)$  are the expected intermediation profits: given expected real returns and market tightness  $\{\theta, \theta^*\}$ ,  $\Pi^*(X)$  solves

$$\begin{aligned} \Pi^*(X) = & \max_{\{\tilde{m}, \tilde{d}^*, \tilde{d}, \tilde{m}^*\}} (R^b(X) - R^{*,d}(X))\tilde{d}^* - (R^b(X) - R^{*,m}(X))\tilde{m}^* \\ & + (R^b(X) - R^d(X))\tilde{d} - (R^b(X) - R^m(X))\tilde{m} + \mathbb{E}_{\omega^*}\chi^*(\theta^*(X), \tilde{m}^* + \omega^*\tilde{d}^*) + \mathbb{E}_{\omega}\chi(\theta(X), \tilde{m} + \omega\tilde{d}). \end{aligned} \quad (12)$$

In equilibrium  $\Pi^*(X) = 0$  and dividends are indeterminate at the individual bank level. Furthermore,  $R^b(X) = 1/\beta$ .

*Proof.* In the appendix □

Central to this optimization problem are the liquidity costs, as captured by  $\chi$  and  $\chi^*$ . Deposits in either currency, have direct interest costs given by the real returns on deposits, but also affects indirectly the banks' settlement needs. Reserves in each currency yield direct real returns, correspondingly, but have the additional indirect benefit of leading to higher average positions in the interbank market.

It is important to note that the bank problem is homogeneous of degree one: As a result, the scale of dollar and euro deposits of each bank is indeterminate for an individual bank. The liquidity ratio and the leverage ratio, however, are not. In effect, the kink in the liquidity cost function creates concavity in the bank objective, generating strictly interior solutions for the ratios.<sup>10</sup> Thus, liquidity risk generates an endogenous bank risk-averse behavior, which will be critical for the determination of the exchange rate, as will become clear below.

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<sup>10</sup>This behavior is analogue to the behavior of productive firms with Cobb-Douglas technologies: firms earn zero-profits, their production scale is indeterminate, but the ratio of production inputs is determined in equilibrium as a function of relative factor prices.

## 3.2 Non-Financial Sector

This section presents the description of the non-financial block: This block is composed of a representative household, one in each country. Households supply labor and save in deposits in both currencies. Firms are multinationals which labor for production and are subject to working capital constraints, giving rise to a demand for loans. This block delivers an endogenous demand schedule for loans, and deposits demands in both currencies.

To keep the model as simple as possible, we purposefully make assumptions so that the decisions for loan demand and deposit supplies are static, in the sense that they do not depend explicitly on future variables. In particular, we will be able to treat loan demand and deposit supply as exogenous schedules with only two parameters: an intercept that controls the scale, and an elasticity that controls how much they respond to changes in interest rates. As we show in the appendix, we obtain the following schedules

$$R_{t+1}^b = \Theta^b (B_t)^\epsilon, \quad \epsilon > 0, \quad \Theta^b > 0, \quad (13)$$

$$R_{t+1}^{*,d} = \Theta^{*,d} (D_t^{*,s})^{-\varsigma^*}, \quad \varsigma > 0, \quad \Theta^{*,d} > 0, \quad (14)$$

$$R_{t+1}^d = \Theta^d (D_t^s)^{-\varsigma}, \quad \varsigma^* > 0, \quad \Theta^d > 0. \quad (15)$$

where  $\epsilon$  is the semi-elasticity of credit demand and  $\varsigma, \varsigma^*$  are the semi-elasticity of the deposit supply with respect to the real return in either currency. These parameters are linked to the production structure and preference parameters in the micro-foundations developed in the appendix.

## 3.3 Government/Central Bank

Both central banks choose the rates for reserves  $i_t^m$  and discount window  $i_t^w$ . Central banks in each country also set the supply of reserves  $\{M_{t+1}, M_{t+1}^*\}$ , which for now, we assume are constant for simplicity. Absent any aggregate shocks, this would imply that the price level would be constant over time in each country.

To balance the payments on reserves and the revenues from discount window loans, we assume that central banks use lump sum taxes/transfers rebated to households from the same country. Because households have linear utility in the tradable consumption good, these lump sum taxes only affect the level of consumption, but has no other implications. Using  $W_t$  to denote the discount window loans, we have the following budget constraint.

$$M_t + T_t + W_{t+1} = M_{t-1}(1 + i_t^m) + W_t(1 + i_t^w).$$



### 3.4 Competitive equilibrium

We study recursive competitive equilibrium where all variables are indexed by the vector of aggregate shocks,  $X$ . We consider shocks to the nominal interest rates on reserves, deposit supply, matching efficiency and the volatility of withdrawals. Without loss of generality, we restrict to symmetric equilibrium, in which all banks choose the same portfolios.

**Definition 1.** Given central bank policies for both countries  $\{M(X), i^m(X), i^w(X), W(X)\}$ ,  $\{M^*(X), i^{m,*}(X), i^{w,*}(X), W^*(X)\}$  a recursive competitive equilibrium is a set of price levels  $\{P(X), P^*(X)\}$ , exchange rates  $e(X)$ , real returns for loans,  $R^b(X)$ , nominal returns for deposits  $\{i^d(X), i^{d,*}(X)\}$ , an interbank market rate  $\bar{i}^f(X)$ , market tightness  $\theta(X)$ , bank portfolios  $\{d(X), d^*(X), m(X), m^*(X), \tilde{b}(X)\}$ , interbank loans and discount window loans  $\{f(X), f^*(X), w(X), w^*(X)\}$  and aggregate quantities of loans  $\{\tilde{B}(X)\}$  and deposits  $\{D(X), D^*(X)\}$  such that:

- (i) Households are on their deposit supply and firms are on their loan demand. That is, equations (13) (15) (14) are satisfied given real returns and quantities  $\{\tilde{B}(X), D(X), D^*(X)\}$ .
- (ii) Banks choose portfolios  $\{\tilde{d}(X), \tilde{d}^*(X), \tilde{m}(X), \tilde{m}^*(X), \tilde{b}(X)\}$  to maximize expected profits, given by (12)
- (iii) The law of one price holds

$$P(X) = P^*(X)e(X) \quad (16)$$

All market clear. For deposits, we have

$$\tilde{d}(X) = D^{s,}(X) \quad , \quad (17)$$

$$\tilde{d}^*(X) = D^{s,*}(X) \quad . \quad (18)$$

For reserves

$$\tilde{m}(X) P(X) = M(X) \quad (19)$$

$$\tilde{m}^*(X) P^*(X) = M^*(X) \quad (20)$$

For loans

$$\tilde{b}(X) = B(X). \quad (21)$$

For interbank loans

$$\Psi^+(X)S^+ = \Psi^-(X)S^-. \quad (22)$$

- (vi) Market tightness  $\theta(X)$  is consistent with the portfolios and the distribution of withdrawals while the matching probabilities  $\{\Psi^+(X), \Psi^-(X)\}$  and the fed funds rate  $\bar{i}^f(X)$  are consistent with market tightness  $\theta$ .

Combining (19) and (20) and using the law of one price (16), we arrive to a condition for the determination of the nominal exchange rate:

$$e(X) = \frac{P(X)}{P^*(X)} = \frac{\frac{M(X)}{\tilde{m}(X)}}{\frac{M^*(X)}{\tilde{m}^*(X)}} \quad (23)$$

Condition (23) is a Lucas-style exchange rate determination equation. Given a real demand for reserves in euro and dollars that emerge from the bank portfolio problem (12), the dollar will be stronger (i.e., higher  $e$ ) the larger is the nominal supply of euro reserves relative to dollar reserves. Similarly, for given nominal supplies of euro and dollar reserves, the dollar will be stronger the larger is the demand for real dollar reserves. The novelty here relative to the canonical model is that liquidity factors play a role in the real demand for currencies, and hence affect the value of the exchange rate. We turn next to analyze this mechanism.

### 3.5 Liquidity Premia and Exchange Rates

To understand how liquidity affects exchange rates, it is useful to inspect the bank portfolio problem, in particular ((12)). Replacing using (6), we can write the expected profits of a bank with portfolio  $(m, m^*, d, d^*)$  as follows:

$$\begin{aligned} \Pi^*(X) = & (R^b(X) - R^{*,d}(X)) \tilde{d}^* - (R^b(X) - R^{*,m}(X)) \tilde{m}^* + (R^b(X) - R^d(X)) \tilde{d} - (R^b(X) - R^m(X)) \tilde{m} \\ & \chi^{*, -}(\theta) \int_{-1}^{-m^*/d^*} (\tilde{m}^* + \omega^* \tilde{d}^*) d\Phi^*(\omega^*) + \chi^{*, +}(\theta) \int_{-m^*/d^*}^{\infty} (\tilde{m}^* + \omega^* \tilde{d}^*) d\Phi^*(\omega^*) + \end{aligned} \quad (24)$$

$$\chi^-(\theta) \int_{-1}^{-m/d} (\tilde{m} + \omega \tilde{d}) d\Phi(\omega) + \chi^+(\theta) \int_{-m/d}^{\infty} (\tilde{m} + \omega \tilde{d}) d\Phi(\omega). \quad (25)$$

First-order condition with respect to  $m^*$  :

$$R^b - R^m = [(1 - \Phi^*(-m^*/d^*))\chi^{*,*}(\theta^*) + \Phi(-m^*/d^*)\chi^{-,*}(\theta^*)]. \quad (26)$$

At the optimum, banks equate the marginal return of investing in loans, which deliver a constant return  $R^b$ , with the return on reserves. The latter is given by  $R^m$  plus the liquidity value of having reserves. If the bank ends up in surplus, which occurs with probability  $1 - \Phi(-m/d)$ , the marginal value is given by  $\chi^+$  and if the bank ends up in deficit which occurs with probability  $\Phi(-m/d)$ , the marginal value is given by  $\chi^-$ . A useful observation to note that is that for a given  $\chi^+, \chi^-$  a higher ratio of reserves to deposits is associated with a smaller  $R^b - R^m$  premium. We label the difference the excess bond premium (EBP), as  $\mathcal{EBP} \equiv R^b - R^m$ .

We have an analogous condition for  $m$ :

$$R^b = R^m + [(1 - \Phi(-m/d))\chi^+(\theta) + \Phi(-m/d)\chi^-(\theta)]. \quad (27)$$

Combining (26) and (27), we obtain a condition that relates the real return differential to the real liquidity premium differential

$$R^m - R^{m*} = [(1 - \Phi^*(-m^*/d^*))\chi^{+,*}(\theta^*) + \Phi(-m^*/d^*)\chi^{-,*}(\theta^*)] - [(1 - \Phi(-m/d))\chi^+(\theta) + \Phi(-m/d)\chi^-(\theta)]. \quad (28)$$

We label this difference, the dollar liquidity premium (DLP),  $\mathcal{DL}\mathcal{P} \equiv R^m - R^{m*}$ .

Using that  $1 + \pi = (1 + \pi^*)e_2/e_1$  by the law of one price, we can express (28) as:

$$\mathbb{E}_t \frac{1 + i_t^m}{1 + \pi_{t+1}} - \mathbb{E}_t \frac{(1 + i_t^{m,*})(1 + e_{t+1})}{(1 + e_t)(1 + \pi_{t+1})} = \underbrace{\mathbb{E}_{\omega^*} [\chi_{m^*}(s^*; \theta^*)] - \mathbb{E}_{\omega} [\chi_m(s; \theta)]}_{\text{dollar liquidity premium } (\mathcal{DL}\mathcal{P})} = \mathcal{LP}. \quad (29)$$

Condition (29) is a *liquidity premium adjusted interest parity condition*. Absent any liquidity premia, we arrive at a canonical uncovered interest parity (UIP) condition; the same is true under satiation in both currencies. However, whenever the marginal liquidity value of a dollar reserve is larger than the marginal liquidity value of a euro reserves (i.e., when the dollar liquidity premium (DLP) is positive), and the nominal rates are equal, this implies that the dollar must be expected to depreciate over time. In effect, the dollar reserve delivers a lower expected real return compensating for the higher liquidity value.

**Theoretical properties.** We now provide a theoretical characterization of how the exchange rate and the liquidity premia vary with some shocks in the model. A useful object for the characterization is the liquidity ratio, which defined in terms of aggregates is given by  $\mu \equiv \frac{M/P}{D}$ .

We first show that a higher supply of dollar deposits appreciate the dollar. The intuition is simple: a higher amount of real dollar deposits increases the demand for real dollar reserves. Given a fixed nominal supply, we must have an appreciation of the dollar. This result is formalized in the following proposition.

**Proposition 1.** *[Scale Effects] A temporary (i.i.d.) rise in dollar deposits appreciates the dollar in equilibrium. That is,  $\frac{\partial e_t}{\partial d_t^*} > 0$ . In addition, the liquidity ratio in dollars falls and the dollar liquidity premium rises. That is,  $\frac{\partial \mu_t}{\partial d_t^*} < 0$  and  $\frac{\partial \mathcal{DL}\mathcal{P}_t}{\partial d_t^*} > 0$ . Moreover, a permanent rise in dollar deposits appreciates the dollar but leaves unchanged the liquidity ratio and the dollar liquidity premium*

*Proof.* In the appendix. □

Proposition 1 highlights that episodes of portfolio re-balancing toward dollar deposits go hand

in hand with appreciations of the dollar. At the same time, the proposition shows that the liquidity ratio falls. That is, the real amount of dollar reserves increase, but less than the increase in deposits. The reason for this result is that the rise in volatility reduces the direct return on dollar reserves. Finally, the proposition also shows that a permanent rise in dollar deposits has an effect over the level of the exchange rate but have no effects, neither on the liquidity ratio nor on the dollar liquidity premium.

Next, we demonstrate that a shock to the volatility of dollar deposits appreciates the dollar.

**Proposition 2.** *[Effects of Dollar Payment Volatility] Assume that shocks satisfy a binominal distribution: :*

$$\omega^* = \begin{cases} \delta^* & \text{with probability } 1/2 \\ -\delta^* & \text{with probability } 1/2. \end{cases}$$

A temporary shock (i.i.d.) to  $\delta^*$  leads to an appreciation of the exchange rate and an increase in the liquidity ratio. In particular, the semi-elasticity of the exchange rate to  $\delta^*$  and the liquidity ratios are given by:

$$\frac{d \log e_t}{d \delta_t^*} = \frac{d \log \mu_t^*}{d \delta_t^*} = \frac{1}{2} \frac{\overbrace{\frac{d[\chi^{*,+} + \chi^{*, -}]}{d \theta^*} \cdot \frac{d \theta_t^*}{d \delta_t^*}}^{>0}}{\underbrace{\frac{\partial[\chi^{*,+} + \chi^{*, -}]}{\partial \theta^*} \cdot \frac{\partial \theta^*}{\partial \mu^*} \cdot \mu^*}_{<0}} > 0.$$

Furthermore, the liquidity and excess bond premia satisfy:

$$\frac{d \log (\mathcal{DL}\mathcal{P})}{d \delta^*} = \left[ \frac{\mathcal{DL}\mathcal{P}}{R^{*,m}} \right]^{-1} \frac{d \log (e)}{d \delta^*} \text{ and } \frac{d \log (\mathcal{EB}\mathcal{P})}{d \delta^*} = \left[ \frac{\mathcal{EB}\mathcal{P}}{R^{*,m}} \right]^{-1} \frac{d \log (e)}{d \delta^*} > 0.$$

*Proof.* In the appendix. □

Proposition 2 presents a key result in the paper. It highlights that episodes of high dollar volatility go hand in hand with appreciations of the dollar. Moreover, a higher volatility also leads to a rise in the dollar liquidity ratio. As we will see below, these results are key to account for the observed empirical relationships that we documented in Section 2. Next, we demonstrate that a shock to the volatility of dollar deposits appreciates the dollar.

Next we derive the effect of a change in the nominal policy rates:

**Proposition 3.** *[Effects of Changes in Policy Rates] Consider a temporary change in the dollar interest rate on reserves,  $i^{*,m}$ . Holding fixed the policy corridor ( $i^{*,w} - i^{*,m}$ ) the policy change leads to an appreciation of the dollar and decrease in the liquidity ratio. In particular, the elasticity of*

the exchange rate and liquidity ratios satisfy:

$$\frac{d \log(e)}{d \log(1 + i^{*,m})} = \frac{d \log \mu_t^*}{d \log(1 + i^{*,m})} = \frac{R^{*,m}}{\left( R^b - \frac{1}{2} \frac{\partial[\chi^{*,+} + \chi^{*, -}]}{\partial \theta^*} \frac{\partial \theta^*}{\partial \mu^*} \mu^* \right)} \in (0, 1).$$

Furthermore, the liquidity premia falls and satisfy

$$\frac{d \log(\mathcal{EBP})}{d(1 + i^{*,m})} = - \left[ \frac{\mathcal{EBP}}{R^{*,m}} \right]^{-1} \left( 1 - \frac{d \log(e)}{d \log(1 + i^{*,m})} \right) < 0.$$

*Proof.* In the appendix. □

The proposition shows that in response to an increase in the US nominal rate, the exchange rate appreciates, and in addition, the liquidity premium falls. The liquidity premium falls because as the rate on dollar reserves go up, the dollar liquidity ratio increases and this reduces the marginal value of an additional unit of reserves. As a result, the appreciation of the exchange rate is less than one-to-one.

## 4 Model Results

### 4.1 Calibration

In the calibration, the model period is one month. We consider a calibration for the two countries that is symmetric, except that the US has a time-varying volatility of dollar deposits and the policy rate in the US shifts; there is a larger scale of dollar deposits but this aspect of the model is immaterial. The volatility in Europe is fixed and so is the policy rate.

For the distributions of  $\omega$  shocks, we assume that these are distributed as two sided exponentials (formally known as Laplace distributions) with shocks centered at zero. Each distribution is therefore indexed by a single time-varying dispersion parameter  $\sigma$  and  $\sigma_t^*$ . All shocks are assumed to follow a an AR(1) process which is approximated by a Markov process numerically. A first subset of parameters is calibrated externally. These parameters are the nominal rates, which we take directly from the data, and the elasticities, which we take from [Bianchi and Bigio \(2020\)](#)—in addition, the relative supply of reserves and the intercept in loan demand that can be normalized.

A second set of parameters is calibrated to second moments. The matching efficiency is calibrated to match an average excess bond premim on loans relative to dollar reserves of 100bps. Moreover, the volatility and autocorrelation is used to match the same moments for the exchange rate. The average dollar withdrawal risk is chosen to match a dollar liquidity premium of 20bps.

Table 4: PARAMETRIZATION

Parameter	Description	Target
Fixed Parameters		
$i_t^m = 2.14\%$	EU Safe Asset Rate	data
$M^*/M$	Relative Supplies of Reserves	normalized to match average $e$
$\Theta^b = 100$	Global loan demand scale	normalization
$\epsilon = -35$	Loan Elasticity	Bianchi and Bigio (2020)
$\Theta^{d,*} = 40$	US Deposit Demand Scale	Liquidity ratio of 20%
$\varsigma^* = 35$	US Deposit Demand Elasticity	Bianchi and Bigio (2020)
$\Theta^d = 40$	EU Deposit Demand Scale	symmetry
$\varsigma = 35$	US Deposit Demand Elasticity	symmetry
$\sigma = 4\%$	EU withdrawal risk	$R^b - R^d = 2\%$
$\lambda^* = 3.1$	US interbank market matching efficiency	$\mathcal{EBP} = R^b - R^{*,m} = 1\%$
$\lambda = 3.1$	EU interbank market matching efficiency	symmetric value of $\lambda^*$
Process for US withdrawal volatility (AR(1) process)		
$\mathbb{E}(\sigma_t^*) = 4\%$	average US withdrawal risk	empirical average $\mathcal{LP}$
$std(\sigma_t^*) = 0.12\%$	standard deviation	empirical std of $\log(e)$
$\rho(\sigma_t^*) = 0.98$	mean reversion coefficient	empirical autocorrelation of $\log(e)$
Process for US policy rate $i^{m,*}$ (AR(1) process)		
$\mathbb{E}(i_t^{*,m}) = 1.95\%$	average annual US policy rate	data
$std(i_t^{*,m}) = 2.1652\%$	std annual US policy rate	data
$\rho(i_t^{*,m}) = 0.99$	autocorrelation annual US policy rate	data

**Moment Fit.** The model and data moments are reported in Table 5. The match to the targeted moments is as desired.

## 4.2 Volatility, Liquidity Premia and Exchange Rates

In this section, we present results of a version of the model with a Markov process for the volatility of dollar withdrawals. Figure 3 shows all endogenous variables as a function of the volatility of the withdrawal of dollar deposits. In line with the results of Proposition 2, we can see that a higher volatility appreciates the dollar and generates a positive liquidity premium. In addition, we can see a rise in the differential rate on deposits. That is, the rate on euro deposits increases relative to the dollar rate as the rise in volatility makes euro deposits more attractive. Furthermore, there is a rise in the loan rate because higher volatility increases, in effect, the liquidity frictions and reduces the demand for loans. Finally, we also see an increase in the dollar liquidity ratio concomitantly with a reduction in the euro liquidity ratio. The latter occurs because a higher lending rate makes euro reserves relatively less attractive (in the absence of any shocks to the euro market).

Figure 4 shows the simulations of the economy for a given path of volatility shocks. The red line denotes the realization of the volatility shock. The overall message, in line with the previous

Table 5: MODEL AND DATA MOMENTS

Statistic	Description	Data/Target	Model
Targets			
$std(\log e)$	Std. Dev. of log exchange rate	0.1538	0.154
$\rho(\log e)$	Autocorrelation of log exchange rate	0.9819	0.9922
$\mathbb{E}(\mathcal{LP})$	Average bond premium	20bps	19.8bps
$\mathbb{E}(\mathcal{EBP})$	Average bond premium	100bps	100.1bps
Non-Targeted			
$std(\log \mu^*)$	Std. Dev. of dollar liquidity ratio	0.422	0.0656
$\rho(\log \mu)$	Autocorrelation of dollar liquidity ratio	0.9961	0.9924
$std(\pi_{eu} - \pi_{us})$	Std. Dev. of inflation differential	1.29	1.84
$\rho(\pi_{eu} - \pi_{us})$	Autocorrelation of inflation differential	0.925	0.98

figure is that episodes of high volatility lead to appreciation of the dollar. Notice that there is mean reversion in the exchange rate and all other variables. Importantly, the simulations are consistent with the empirical analysis presented in Section 2. Indeed, we see a positive correlation between the strength of the dollar and the dollar liquidity ratio,

### 4.3 Regressions with Simulated Data

In this section, we simulate our model, study the second moments and run the same regressions as we did with simulated data. Table (6) shows that the model simulations are consistent with the date time series. Namely, using a regression with the simulated data, we estimate a positive coefficient on the liquidity ratio, just like we found in the data. This result is in line with the comovement observed in the simulations in Figure 4.

Table 6: REGRESSION COEFFICIENTS WITH SIMMULATED DATA

	$\sigma^*$ -shocks only	$i^{*,m}$ -shocks only	both shocks
$\Delta(\text{LiqRat}_t)$	2.2484*** (0.0015)	1.0763*** (0.0440)	1.9735*** (0.0450)
$(\text{LiqRat}_{t-1})$	-0.0007 (0.0004)	-0.0014 (0.0007)	-0.0037 (0.0015)
$\Delta(i_t^m - i_t^{*,m})$		-42.4640*** (1.5185)	-14.5032*** (1.6027)
constant	-0.0 0.01	-0.015 0.008	-0.039 0.0017
adj. $R^2$	0.999	0.9987	0.9953

$t$  statistics in parentheses.

\*\*\*  $p < 0.01$

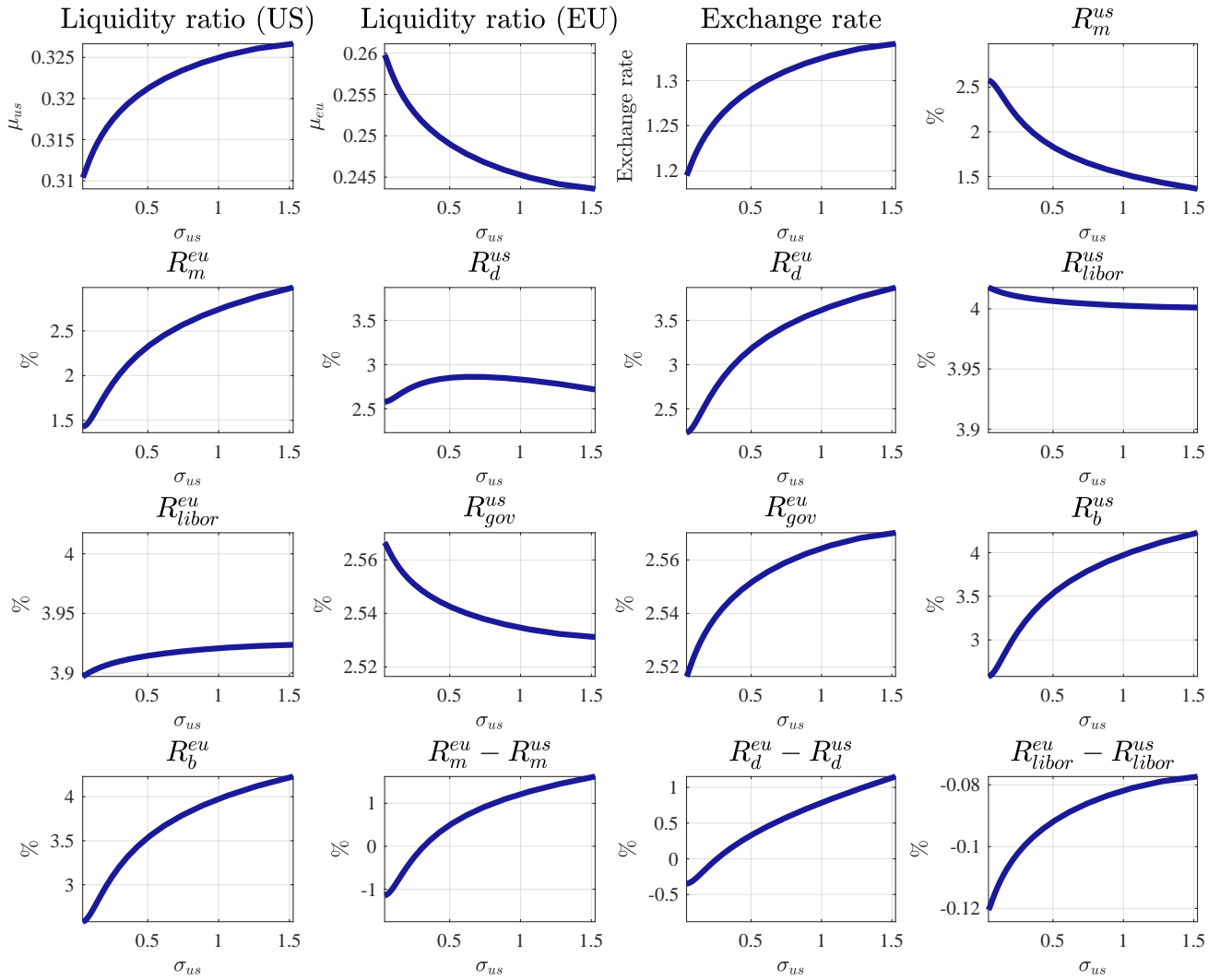


Figure 3: Equilibrium solution for a range of values of volatility



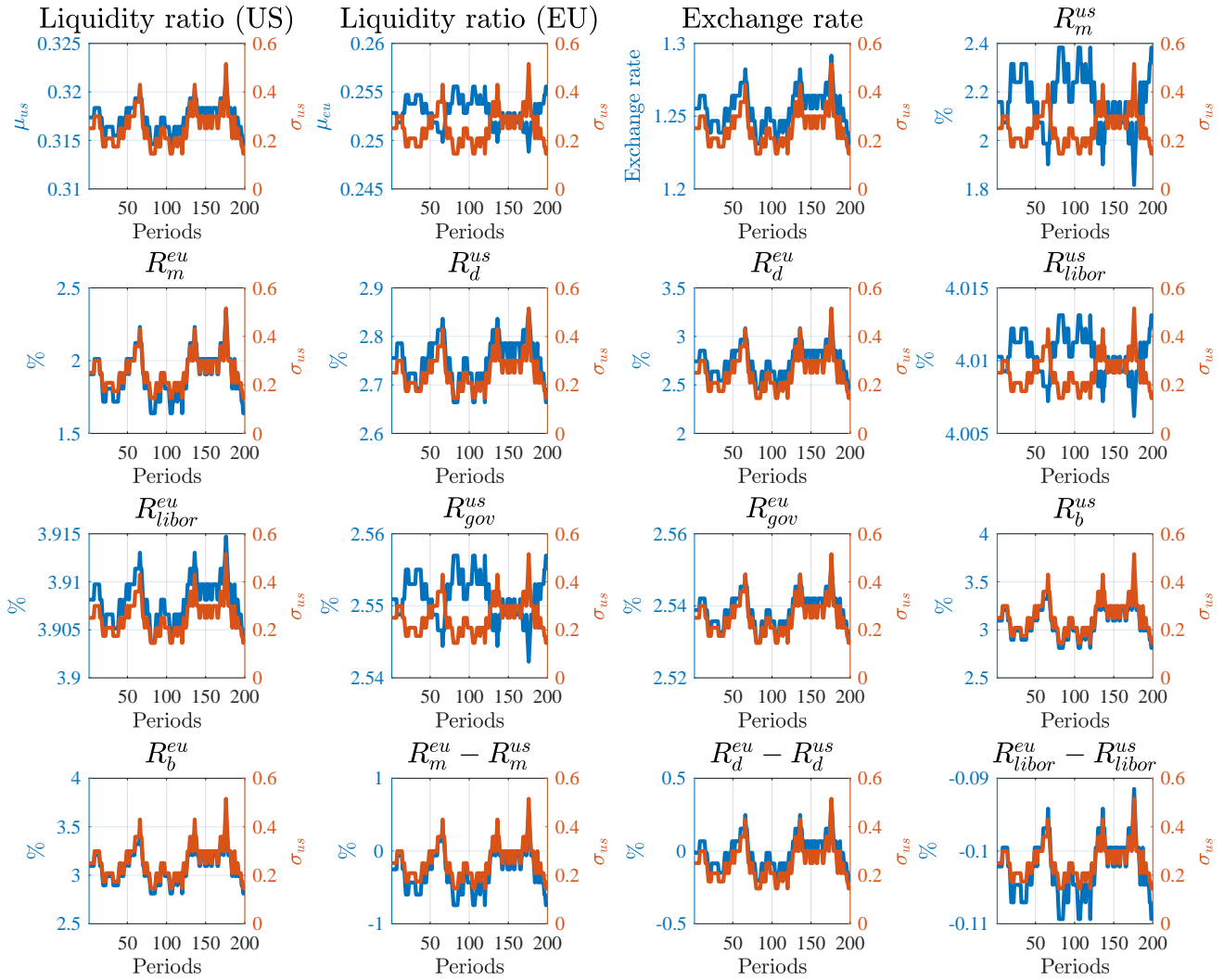


Figure 4: Simulation of the model. Red line is the volatility process

## 5 Conclusion

We developed a theory of exchange rate determination as arising from the demand by financial institutions for liquid dollar assets. Periods of increased funding volatility generate an increase in the dollar liquidity premium and appreciates the dollar. The effect is empirically validated as we document that how a higher liquidity ratio is associated with periods of stronger demand for dollar reserves, and a stronger dollar.

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# A Proofs

## A.1 Preliminary Observations

To produce theoretical results, we derive some observations. First, recall that the market tightness in both currencies is always lower than one:

$$\theta = \frac{\delta - \frac{M/P}{D}}{\delta + \frac{M/P}{D}} < 1.$$

For this reason, we have that tightness is increasing in the size of the dispersion shocks:

$$\frac{\partial \theta}{\partial \delta} = \frac{1}{\mu + \delta} - \frac{\delta - \mu}{(\mu + \delta)^2} = \frac{1}{(\mu + \delta)^2} > 0.$$

Moreover, the penalties are increasing in tightness.

$$\frac{\partial \chi^+}{\partial \delta} = \frac{\partial \chi^+}{\partial \theta} \frac{\partial \theta}{\partial \delta} > 0,$$

and

$$\frac{\partial \chi^-}{\partial \delta} = \frac{\partial \chi^-}{\partial \theta} \frac{\partial \theta}{\partial \delta} > 0.$$

Likewise, we have that the tightness is decreasing in the liquidity ratio:

$$\frac{\partial \theta}{\partial \mu} = -\frac{1 + \theta}{\delta + \frac{M/P}{D}} < 0.$$

We also know that the penalty rates are increasing in tightness, and hence:

$$\frac{\partial \chi^+}{\partial \mu} = \frac{\partial \chi^+}{\partial \theta} \frac{\partial \theta}{\partial \mu} < 0, \text{ and } \frac{\partial \chi^-}{\partial \mu} = \frac{\partial \chi^-}{\partial \theta} \frac{\partial \theta}{\partial \mu} < 0.$$

## A.2 Proof of Lemma 1

Substitute the budget constraint (10) into (11). We obtain:

$$n' = R^b(X)n - R^b(X)Div - (R^b(X) - R^m(X))\tilde{m} - (R^b(X) - R^{m,*}(X))\tilde{m}^* + (R^b(X) - R^d(X))\tilde{d} + (R^b(X) - R^d(X))\tilde{d}^* + (R^b(X) - R^d(X))\tilde{d}^* + (R^b(X) - R^d(X))\tilde{d}^*$$

and thus:

$$n' = R^b(X)n - R^b(X)Div + \Pi^*(X).$$

Conjecture that  $v(n, X) = n$ . Then, substituting  $v(n', X') = n'$ , into (9) we obtain:

$$v(b, X) = \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta \mathbb{E} [R^b(X)n - R^b(X)Div + \Pi(X) | X].$$

Note that if  $\beta R^b(X) \neq 1$ , dividends are either  $\infty$  or  $-\infty$ . Thus, in equilibrium, it must be the case that  $R^b(X) = 1/\beta$ . As a result,

$$\begin{aligned} v(b, X) &= \max_{\{Div, \tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} Div + \beta \mathbb{E} [1/\beta b - 1/\beta Div + \Pi^*(X) | X] \\ &= b + \beta \max_{\{\tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} \mathbb{E} \left[ \Pi \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) \right]. \end{aligned} \quad (30)$$

where

$$\Pi \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) = (R^b(X) - R^d(X)) \tilde{d} + (R^b(X) - R^{*,d}(X)) \tilde{d}^* - (R^b(X) - R^m(X)) \tilde{m} - (R^b(X) - R^{m,*}(X)) \tilde{m}^*$$

Next, consider the first order conditions for  $\{\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$ . We have:

$$d : \Pi_d \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) = R^b(X) - R^d(X) - \mathbb{E}_\omega [\chi_{\tilde{d}}] = 0.$$

$$d^* : \Pi_{d^*} \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) = R^b(X) - R^{*,d}(X) - \mathbb{E}_\omega [\chi_{\tilde{d}^*}^*] = 0.$$

$$m : \Pi_m \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) = R^b(X) - R^m(X) - \mathbb{E}_\omega [\chi_{\tilde{m}}^*] = 0.$$

$$m^* : \Pi_{m^*} \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) = R^b(X) - R^{*,m}(X) - \mathbb{E}_\omega [\chi_{\tilde{m}^*}^*] = 0.$$

Thus, in equilibrium, these conditions must hold. Next, observe that  $\Pi \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right)$  is homogeneous of degree 1 in  $\{\tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}$ . Hence, by Euler's Theorem for Homogeneous Functions:

$$\Pi^*(X) = \max_{\{\tilde{b}, \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}\}} \Pi \left( \tilde{m}^*, \tilde{d}^*, \tilde{d}, \tilde{m}, X \right) = \begin{bmatrix} \Pi_d & \Pi_{d^*} & \Pi_m & \Pi_{m^*} \end{bmatrix} \cdot \begin{bmatrix} d \\ d^* \\ m \\ m^* \end{bmatrix} = 0.$$

Hence, we verify that,  $\Pi^*(X) = 0$  and as a result, replacing this result in (30), we verify the conjecture that:

$$v(e, X) = e.$$



### A.3 Proof of Proposition 1

The proof is by contradiction. Suppose that the dollar depreciates at the time of the shock  $e_t < 1$ . Because we have a temporary shock, we have  $e_{t+1} = 1$  for all  $t > 1$ . Then, using  $i_t^{m,*} = i_t^m$

$$R_t^m = \mathbb{E}_t \frac{1 + i_t^m}{1 + \pi_{t+1}} = \mathbb{E}_t \frac{1 + i_t^{m,*}}{1 + \pi_{t+1}} = \mathbb{E}_t \frac{1 + i_t^{m,*} e_t}{(1 + \pi_t^*) e_{t+1}} < \mathbb{E}_0 \frac{1 + i_t^{m,*}}{1 + \pi_t^*} = R_t^{m,*}$$

The third equality uses law of one price, and the inequality follows from  $e_t < e_{t+1}$ . Hence, we must have the that the expected dollar return is higher. Hence, using the arbitrage condition between dollar and euro reserves (26), we have

$$0 > R^m - R^{m,*} = \left[ \Phi \left( -\frac{m^*}{d^*} \right) \chi^- \left( \frac{m^*}{d^*} \right) + \left( 1 - \Phi \left( -\frac{m^*}{d^*} \right) \right) \chi^{+,*} \right] - \left[ \Phi \left( -\frac{m}{d} \right) \chi^- \left( \frac{m}{d} \right) + \left( 1 - \Phi \left( -\frac{m}{d} \right) \right) \chi^+, \left( \frac{m}{d} \right) \right] \quad (31)$$

which implies that  $\frac{m_t^*}{d_t^*} > \frac{m_t}{d_t}$ . But then since  $d_t^* > d$ , we have  $m_t^* > m$ . Since  $M$  and  $M^*$  remain constant and  $e_t = \frac{\frac{M}{M^*}}{\frac{m_t}{m_t^*}} > \frac{\frac{M}{M^*}}{\frac{m_{t+1}}{m_{t+1}^*}} = e_{t+1} = 1$ . We reached a contradiction.

### A.4 Proof of Proposition 2

We use the implicit function theorem. When the loans supply is perfectly elastic,  $R^b$  is a constant. In this case, we have that the liquidity premium is given by:

$$R^b = (1 + i^m) \frac{P}{\mathbb{E}[p(X')]} + \frac{1}{2} \left[ \chi^+ \left( \frac{\delta - \frac{M/P}{D}}{\frac{M/P}{D} + \delta} \right) + \chi^- \left( \frac{\delta - \frac{M/P}{D}}{\frac{M/P}{D} + \delta} \right) \right].$$

The price level appears in the real rate on reserves, on the value of real balances, and the real penalties  $\{\chi^+, \chi^-\}$ .

**Application of the Implicit Function Theorem.** From the liquidity premium, we have that:

$$(1 + i^m) \frac{P}{\mathbb{E}[p(X')]} \frac{dP}{P} \frac{1}{d\delta} + \frac{1}{2} [\chi^+ + \chi^-] \frac{dP}{P} \frac{1}{d\delta} + \frac{1}{2} \frac{d[\chi^+ + \chi^-]}{d\theta} \frac{d\theta}{d\delta} + \frac{1}{2} \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \frac{d\mu}{dP} \frac{dP}{d\delta} = 0. \quad (32)$$

Holding nominal reserve balances and real deposits fixed, we obtain that

$$\frac{d\mu}{dP} = \frac{d\frac{M}{P} \frac{1}{D}}{dp} = -\mu \frac{dP}{P}$$

Thus, re-arranging (32) we obtain:

$$\frac{d \log(P)}{d\delta} = \frac{dP}{P} \frac{1}{d\delta} = - \frac{\frac{1}{2} \frac{d[\chi^+ + \chi^-]}{d\theta} \frac{d\theta}{d\delta}}{R^m + \frac{1}{2} (\chi^+ + \chi^-)] - \frac{1}{2} \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu}.$$

Since we have that,  $R^b = R^m + \frac{1}{2} (\chi^+ + \chi^-)$ , we write the solution as:

$$\frac{d \log(P)}{d\delta} = - \frac{1}{2} \frac{\frac{d[\chi^+ + \chi^-]}{d\theta} \frac{d\theta}{d\delta}}{R^b - \frac{1}{2} \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu} < 0,$$

where the sign follows from  $\frac{d\theta}{d\delta} > 0$  and  $\frac{\partial\theta}{\partial\mu} < 0$ . The same formula holds for dollars and euros.

Recall that the exchange rate is given by:

$$e \equiv \frac{P}{P^*}.$$

Thus:

$$\frac{de}{d\delta} = \frac{de}{dP^*} \cdot \frac{dP^*}{d\delta} = -e \frac{dP^*}{P^*} \frac{1}{d\delta}$$

and re-arranging we can express as:

$$\frac{d \log(e)}{d\delta^*} = - \frac{dP^*}{P^*} \frac{1}{d\delta} = \frac{\frac{1}{2} \frac{d[\chi^{*,+} + \chi^{*, -}]}{d\theta^*} \frac{d\theta^*}{d\delta^*}}{R^b - \frac{1}{2} \frac{\partial[\chi^{*,+} + \chi^{*, -}]}{\partial\theta^*} \frac{\partial d\theta^*}{\partial\mu^*} \mu^*}.$$

Likewise,

$$\frac{d \log(\mu^*)}{d\delta^*} = \frac{\frac{1}{2} \frac{d[\chi^{*,+} + \chi^{*, -}]}{d\theta^*} \frac{d\theta^*}{d\delta^*}}{R^b - \frac{1}{2} \frac{\partial[\chi^{*,+} + \chi^{*, -}]}{\partial\theta^*} \frac{\partial d\theta^*}{\partial\mu^*} \mu^*}.$$

Consider the liquidity premium

$$\mathcal{LP} = R^m - R^{*,m}.$$

Then,

$$\frac{d \log(\mathcal{LP})}{d\delta^*} = - \frac{1}{\mathcal{LP}} \frac{(1 + i^{*,m})}{\mathbb{E}[p^*(X')]} P^* \frac{dP^*}{P^*} \frac{1}{d\delta^*} = - \left[ \frac{\mathcal{LP}}{R^{*,m}} \right]^{-1} \frac{d \log(P^*)}{d\delta^*} = \left[ \frac{\mathcal{LP}}{R^{*,m}} \right]^{-1} \frac{d \log(e)}{d\delta^*} > 0.$$

Similarly, the excess bond premium satisfies:

$$\frac{d \log(\mathcal{EBP})}{d\delta^*} = \left[ \frac{\mathcal{EBP}}{R^{*,m}} \right]^{-1} \frac{d \log(P^*)}{d\delta^*} > 0.$$

## A.5 Proof of Proposition 2 (Pass-Through)

**Application of the Implicit Function Theorem.** From the liquidity premium, we have that:

$$\frac{\mathbb{E}[p(X')]}{P} + (1+i^m) \frac{\mathbb{E}[p(X')]}{P} \frac{dP}{P} \frac{1}{di^m} + \frac{1}{2}[\chi^+ + \chi^-] \frac{dP}{P} \frac{1}{di^m} + \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \frac{d\mu}{dP} \frac{dP}{di^m} = 0. \quad (33)$$

Using (xxx) and (xxx) we obtain:

$$\frac{\mathbb{E}[p(X')]}{P} + \left( (1+i^m) \frac{\mathbb{E}[p(X')]}{P} + \frac{1}{2}[\chi^+ + \chi^-] - \frac{1}{2} \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu \right) \frac{dP}{P} \frac{1}{di^m} = 0.$$

Therefore, re-arranging terms we have:

$$\frac{d \log(P)}{di^m} = - \frac{\mathbb{E}[p(X')]/P}{\left( R^b - \frac{1}{2} \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu \right)} < 0.$$

We can express this equation in a more elegant way in terms of an elasticity:

$$\frac{d \log(P)}{d \log(1+i^m)} = - \frac{R^m}{\left( R^b - \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu \right)} < 0.$$

We know that:

$$R^m \leq R^b \text{ and } \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu \leq 0$$

with equality only under satiation. Hence,

$$\frac{R^m}{\left( R^b - \frac{\partial[\chi^+ + \chi^-]}{\partial\theta} \frac{\partial\theta}{\partial\mu} \mu \right)} \geq -1$$

with equality only under satiation. Next, using:

$$\frac{d\mu}{dP} = \frac{d \frac{M}{P} \frac{1}{D}}{dp} = -\mu \frac{dP}{P},$$

we obtain:

$$\frac{d \log(\mu^*)}{d \log(1+i^{*,m})} = - \frac{d \log(P)}{d \log(1+i^m)} > 0.$$

We can express this equation in a more elegant way in terms of an elasticity:

$$\frac{d \log(e)}{d \log(1+i^{*,m})} = - \frac{d \log(P^*)}{d \log(1+i^{*,m})} = \frac{R^{*,m}}{\left( R^b - \frac{1}{2} \frac{\partial[\chi^{*,+} + \chi^{*, -}]}{\partial\theta^*} \frac{\partial\theta^*}{\partial\mu^*} \mu^* \right)} > 0.$$

Since away from satiation  $R^{*,m} < R^b$  and  $\frac{\partial[\chi^{*,+} + \chi^{*, -}]}{\partial\theta^*} > 0$ , we have that, away from satiation, the

Reduced	$\Theta^x$	$\epsilon^x$	$\Theta^b$	$\epsilon^b$
Structural	$\bar{X}_t \beta^{1/\gamma^x}$	$\frac{1}{\gamma^x} - 1$	$(\alpha A_{t+1})^{-\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)}$	$\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)$

Table 7: Structural to Reduced form Parameters

exchange rate pass-through is less than 1.

Now consider the liquidity premium

$$\mathcal{DL}\mathcal{P} = R^m - R^{*,m}.$$

Then, we have that:

$$\frac{d\mathcal{DL}\mathcal{P}}{d \log(1 + i^{*,m})} = -R^{*,m} - R^{*,m} \frac{d \log(P)}{\log(1 + i^{*,m})} = -R^{*,m} \left(1 + \frac{d \log(P)}{\log(1 + i^{*,m})}\right).$$

Hence, we have that:

$$\frac{d\mathcal{DL}\mathcal{P}}{d \log(1 + i^{*,m})} = - \left[ \frac{\mathcal{DL}\mathcal{P}}{R^{*,m}} \right]^{-1} \left(1 + \frac{d \log(P)}{\log(1 + i^{*,m})}\right) = - \left[ \frac{\mathcal{DL}\mathcal{P}}{R^{*,m}} \right]^{-1} \left(1 - \frac{d \log(e)}{\log(1 + i^{*,m})}\right) \leq 0,$$

because  $\frac{d \log(P)}{\log(1 + i^{*,m})} \geq -1$  with strict inequality away from satiation. Thus, we also have that:

$$\frac{d\mathcal{EB}\mathcal{P}}{d \log(1 + i^{*,m})} = - \left[ \frac{\mathcal{EB}\mathcal{P}}{R^{*,m}} \right]^{-1} \left(1 - \frac{d \log(e)}{\log(1 + i^{*,m})}\right) \leq 0$$

## B Microfoundations for Deposit Supplies and Loan Demands

The equivalence table from the structural parameters to the reduced form parameters is:

**Household Problem.** Define the household net worth  $e^h = (1 + i_t^d) D + (1 + i_t^G) G + (q_t + r_t^h) \Sigma - T_t^h$ , as the right-hand side of its budget constraint, excluding labor income. Then, substitute  $c^h$  from the budget constraint and employ the definition  $e^h$ . We obtain the following value function:

$$V_t^h(G^h, D, \Sigma) = \max_{\{c^d, c^g, h, G', D', \Sigma'\}} U^d(c^d) + U^g(c^g) - \frac{h^{1+\nu}}{1+\nu} + e^h + \frac{z_t h - (P_t c^g + P_t c^d + D' + G' + q_t \Sigma')}{P_t} + \beta V_{t+1}^h(G', D', \Sigma')$$

subject to  $c^d \leq (1 + i_t^d) \frac{D}{P_t}$  and  $c^g \leq \frac{D}{G_t}$ .

*Step 1 - deposit and bond-goods demand.* The step is to take the first-order conditions for  $\{c^d, c^g\}$ . Since  $\{G, D\}$  enter symmetrically into the problem, we express the formulas in terms of  $x \in \{d, g\}$ , an index that corresponds to each asset. From the first-order conditions with respect to  $\frac{D}{P_t}$  and  $\frac{G}{P_t}$ , we obtain that:

$$c^x(X, t) = \min \left\{ (U_{c^x}^x)^{-1}(1), R_t^x \cdot \frac{X}{P_{t-1}} \right\} \text{ for } x \in \{d, g\}. \quad (34)$$

The expression shows that the deposit- and bond-in-advance constraints bind if the marginal utility associated with their consumption is less than one. Note that

$$U_{c^x}^x(\bar{X}) = (\bar{X})^{\gamma^x} x^{-\gamma^x} \text{ for } x \in \{d, g\}, \quad (35)$$

marginal utility is above 1, for  $X/P_t < \bar{X}$ . Then, the marginal consumption as a function of real balances is:

$$c_{X/P_t}^x(X, t) = \begin{cases} R_t^x & X/P_t < \bar{X} \\ 0 & \text{otherwise} \end{cases} \text{ for } x \in \{d, g\}$$

We return to this conditions below to derive the demand for deposits and bonds by the non-financial sector.

*Step 2 - labor supply.* The first-order condition with respect to labor supply yields a labor supply that only depends on the real wage:

$$h_t^v = z_t/P_t. \quad (36)$$

*Step 3 - deposit and bond demand.* Next, we the derive deposit demand and T-Bill demand. By taking first-order conditions with respect to  $D'/P_t$  and  $G'/P_t$ , the real balances of deposits and bonds.

$$1 = \beta \frac{\partial V_{t+1}^h}{\partial (X'/P_t)} = \beta \left[ \frac{\partial U^x}{\partial c^x} \cdot \frac{\partial c^x}{\partial (X'/P_t)} + \frac{\partial U^h}{\partial c^h} \cdot \frac{\partial c^h}{\partial (X'/P_t)} \right] \text{ for } x \in \{d, g\}.$$

The first equality follows directly from the first-order condition and the second uses the envelope Theorem and the solution for the optimal consumption rule. If we shift the period in (34), by one period, the first-order condition then becomes:

$$\frac{1}{\beta} = \begin{cases} \frac{\partial U^x}{\partial c^x} R_t^x & X/P_t < \bar{X} \\ R_t^x & \text{otherwise} \end{cases} \text{ for } x \in \{d, g\}.$$

Finally, once we employ the definition of marginal utility, we obtain:

$$\frac{1}{\beta} = \begin{cases} (\bar{X})^{\gamma^x} (R_t^x X/P_t)^{-\gamma^x} R_t^x & X/P_t < \bar{X} \\ R_t^x & \text{otherwise} \end{cases} \text{ for } x \in \{d, g\}.$$

Inverting the condition yields:

$$X/P_t = \begin{cases} \bar{X} \beta^{1/\gamma^x} (R_t^x)^{\frac{1}{\gamma^x}-1} & R_t^x < 1/\beta \\ [\bar{X}, \infty) & R_t^x = 1/\beta \end{cases} \text{ for } x \in \{d, g\}.$$

Thus, we have that

$$\Theta_t^x = \bar{X}_t \beta^{1/\gamma^x} \text{ and } \epsilon^x = \frac{1}{\gamma^x} - 1 \text{ for } x \in \{d, g\}.$$

Next, we move to the firm's problem to obtain the demand for loans.

**Firm Problem.** In the appendix, we allow the firm to save in deposits whatever it doesn't spend in wages. From firm's problem, if we substitute the production function into the objective we obtain:

$$P_{t+1} r_{t+1}^h = \max_{B_{t+1}^d \geq 0, x_{t+1}, h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - (1 + i_{t+1}^b) B_{t+1}^d + (1 + i_{t+1}^d) (B_{t+1}^d - z_t h_t)$$

subject to  $z_t h_t \leq B_{t+1}^d$ . Observe that

$$\begin{aligned} P_{t+1} A_{t+1} h_t^\alpha - (1 + i_{t+1}^b) B_{t+1}^d + (1 + i_{t+1}^d) (B_{t+1}^d - z_t h_t) \\ = P_{t+1} A_{t+1} h_t^\alpha - z_t h_t - (i_{t+1}^b - i_{t+1}^d) (B_{t+1}^d + z_t h_t). \end{aligned}$$

*Step 4 - loans demand.* Since  $i_{t+1}^b \geq i_{t+1}^d$ , then it is without loss of generality, that the working capital constraint is binding,  $z_t h_t = B_{t+1}^d$ . Thus, the objective is

$$P_{t+1} A_{t+1} h_t^\alpha - (1 + i_{t+1}^b) z_t h_t.$$

The first-order condition in  $h_t$  yields

$$P_{t+1} \alpha A_{t+1} h_t^{\alpha-1} = (1 + i_{t+1}^b) z_t h_t.$$

Dividing both sides by  $P_t$ , we obtain

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^{\alpha-1} = (1 + i_{t+1}^b) \frac{z_t}{P_t} h_t.$$

Next, we use the labor supply function (36), to obtain the labor demand as a function of the loans rate:

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^\alpha = (1 + i_{t+1}^b) h_t^{\nu+1} \rightarrow R_t^b = \frac{\alpha A_{t+1} h_t^\alpha}{h_t^{\nu+1}}. \quad (37)$$

Once we have the wage bill, and the fact that the working capital constraint is binding,

$$\frac{B_{t+1}^d}{P_t} = h_t \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left( \frac{B_{t+1}^d}{P_t} \right)^{\frac{1}{\nu+1}}. \quad (38)$$

Thus, we can combine (37) and (38) to obtain the demand for loans:

$$R_t^b = \alpha A_{t+1} \left( \frac{B_{t+1}^d}{P_t} \right)^{-1} \left( \frac{B_{t+1}^d}{P_t} \right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B_{t+1}^d}{P_t} = \Theta_t (R_{t+1}^b)^{\epsilon^b} \quad (39)$$

Thus, the coefficients of the loans demand are

$$\Theta_t^b = (\alpha A_{t+1})^{-\epsilon^b} \text{ and } \epsilon^b = \left( \frac{\nu + 1}{\alpha - (\nu + 1)} \right).$$

*Step 5 - deposit and bond demand.* We replace the loans demand (39) into (38), to obtain the labor market equilibrium:

$$h_t = \left( \frac{1}{\alpha A_{t+1}} \right)^{\frac{1}{\alpha - (\nu+1)}} (R_{t+1}^b)^{\frac{1}{\alpha - (\nu+1)}}.$$

We replace (38) into the production function to obtain:

$$y_{t+1} = A_{t+1} \left( \frac{1}{\alpha A_{t+1}} \right)^{\frac{\alpha}{\alpha - (\nu+1)}} (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu+1)}} \rightarrow y_{t+1} = \left( \frac{1}{\alpha} \right)^{\frac{\alpha}{\alpha - (\nu+1)}} A_{t+1}^{\frac{\nu+1-\alpha}{\alpha - (\nu+1)}} (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu+1)}}.$$

The profit of the firm is given by:

$$r_{t+1}^h = y_{t+1} - R_{t+1}^b B_{t+1} \rightarrow r_{t+1}^h = A_{t+1}^{\frac{\nu+1-\alpha}{\alpha - (\nu+1)}} \left( \alpha^{-\frac{\alpha}{\alpha - (\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha - (\nu+1)}} \right) \cdot (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu+1)}}.$$

The asset price  $q_t$  then is determine as:

$$q_t = \sum_{s \geq 1} \beta^s r_s^h.$$

With this, we conclude that output, hours and the firm price are decreasing in current (and future) loans rate.

Note that throughout the proof we use the labor market clearing condition. Then, clearing in the loans and deposit markets, by Walras's law, implies clearing in the goods market. Once

we compute equilibria taking the schedules as exogenous in the bank's problem, it is possible to obtain output and household consumption from the equilibrium rate.

## C Computational Algorithms

### C.1 Algorithm to solve for transitions

We can consider the previous section as a steady-state version of the model, if prices are fixed in both currencies, then policy rates are actual real rates. In this section we consider what happens one period before the steady state, we call that period  $t = 0$ . Assume that the nominal policy rates are given:

$$(1 + i^{*,a}) \text{ for } a \in \{m, w\},$$

for the US and for the EU:

$$(1 + i^a) \text{ for } a \in \{m, w\}.$$

The real rates now satisfy:

$$R^{*,a} = \frac{(1 + i^{*,a})}{(1 + \pi^*)} \text{ for } a \in \{m, w\},$$

for the US and for the EU:

$$R^a = \frac{(1 + i^a)}{(1 + \pi)} \cdot (1 + \Omega) \text{ for } a \in \{m, w\}.$$

Where now we have that:

$$(1 + \pi^*) = \frac{p_{ss}}{p_0}$$

and

$$(1 + \Omega) = \frac{e_{ss}}{e_0}.$$

The values  $\{p_{ss}, e_{ss}\}$  are solved from the steady state solution.

**Algorithm for T-1 of Economy with Deposit Segmentation Step 1.** Conjecture  $\{R_0^m, R_0^{*,m}, R_0^w, R_0^{*,w}\}$ . Solve for the liquidity ratios in Dollars and Euro  $\{\mu, \mu^*, R^d, R^{*,d}\}$  using:

$$R^d + \frac{1}{2}\omega (\chi^+(\mu) - \chi^-(\mu)) = R^{*,d} + \frac{1}{2}\omega^* (\chi^{*,+}(\mu^*) - \chi^{*,-}(\mu^*))$$

$$R^m + \frac{1}{2}(\chi^+(\mu) + \chi^-(\mu)) = R^{*,m} + \frac{1}{2}(\chi^{*,+}(\mu^*) + \chi^{*,-}(\mu^*))$$



$$\Theta^b ((v(1-\mu) + (1-\mu^*))d^*)^\epsilon = R^{*,d} + \frac{1}{2}\omega^* (\chi^+(\mu) - \chi^-(\mu))$$

$$R^m = R^{*,d} + \frac{1}{2}\omega (\chi^+(\mu) - \chi^-(\mu)) - \frac{1}{2} (\chi^+(\mu) + \chi^-(\mu)).$$

**Step 2.** Given the solutions to  $\{R^d, R^{*,d}\}$ , solve  $\{d^*, v\}$  using:

$$d = \left[ \frac{R^d}{\Theta^d} \right]^{1/\zeta}$$

$$v = \left[ \frac{R^d}{\Theta^d} \right]^{1/\zeta} \left[ \frac{R^{*,d}}{\Theta^{*,d}} \right]^{-1/\zeta^*}.$$

**Step 3.** Solve for prices and the exchange rate using the solutions:

$$\mu v d^* = \frac{M}{p_{ss}}$$

$$\mu^* d^* = \frac{e}{p_{ss}} M^*$$

$$p_{ss}^* = e^{-1} p_{ss}.$$

**Step 4.** Update values for real policy rates:

$$R^{*,a} = \frac{(1 + i^{*,a})}{(1 + \pi^*)} \text{ for } a \in \{m, w\},$$

for the US and for the EU:

$$R^a = \frac{(1 + i^a)}{(1 + \pi)} \cdot (1 + \Omega) \text{ for } a \in \{m, w\}.$$

Where now we have that:

$$(1 + \pi^*) = \frac{p_{ss}}{p_0}$$

and

$$(1 + \Omega) = \frac{e_{ss}}{e_0}.$$

## C.2 Algorithm to obtain a Global Solution

The algorithm to obtain the global solution to the model follows the algorithm to produce transitions. First, we define  $s \in \mathcal{S} = \{1, 2, 3, \dots, N^s\}$  to be a finite set of states. We let  $s$  follow a Markov process with transition matrix  $Q$ . Thus,  $s' \sim Q(s)$ . The state now affects the parameters of the model. That is, at each period,  $\{\delta, \lambda, i^{*,m}, i^m, i^{*,w}, i^w, M, M^*, \Theta^d, \Theta^{*,d}, \Theta^{*,m}\}$  are all, potentially, functions of the state  $s$ .

The algorithm proceeds as follows. We define a “greed” parameter  $\Delta^{greed}$  and a tolerance parameters  $\varepsilon^{tol}$ , and construct a grid for  $\mathcal{S}$ . We conjecture a price-level functions  $p_{(0)}(s), p_{(0)}^*(s)$  which produces a price levels in both currencies as a function of the state. As an initial guess, we propose to use  $p_{(0)}(s) = p_{ss}^*$ , and  $p_{(0)}^*(s) = p_{ss}^*$  setting the exchange rate to its steady state level in all periods. We proceed by iterations, setting a tolerance count  $tol$  to  $tol > 2 \cdot \varepsilon^{tol}$ .

**Outerloop 1: Iteration of price functions.** We iterate price functions until they converge.

Let  $n$  be the  $n - th$  step of a given iteration. Given a  $p_{(n)}(s), p_{(n)}^*(s)$ , we produce a new price level functions  $p_{(n+1)}(s), p_{(n+1)}^*(s)$  if  $tol > \varepsilon^{tol}$ .

**Innerloop 1: Solve for real policy rates.** For each  $s$  in the grid for  $\mathcal{S}$ , we solve for

$$\{R^m(s), R^{*,m}(s), R^w(s), R^{*,w}(s)\}.$$

Let  $j$  be the  $j - th$  step of a given iteration. Conjecture values

$$\{R_{(0)}^m(s), R_{(0)}^{*,m}(s), R_{(0)}^w(s), R_{(0)}^{*,w}(s)\}$$

—we propose  $\{R_{ss}^m, R_{ss}^{*,m}, R_{ss}^w, R_{ss}^{*,w}\}$  as an initial guess. We then update

$$\{R_{(j)}^m(s), R_{(j)}^{*,m}(s), R_{(j)}^w(s), R_{(j)}^{*,w}(s)\}$$

until we obtain convergence:

**2.a** Given this guess, we solve for the liquidity ratios in Dollars and Euro  $\{\mu, \mu^*, R^d, R^{*,d}\}$  as a function of the state using:

$$R^d + \frac{1}{2}\omega(\chi^+(\mu) - \chi^-(\mu)) = R^{*,d} + \frac{1}{2}\omega^*(\chi^{*,+}(\mu^*) - \chi^{*,-}(\mu^*))$$

$$R^m + \frac{1}{2}(\chi^+(\mu) + \chi^-(\mu)) = R^{*,m} + \frac{1}{2}(\chi^{*,+}(\mu^*) + \chi^{*,-}(\mu^*))$$

$$\Theta^b((v(1-\mu) + (1-\mu^*))d^*)^\epsilon = R^{*,d} + \frac{1}{2}\omega^*(\chi^+(\mu) - \chi^-(\mu))$$

$$R^m = R^{*,d} + \frac{1}{2}\omega (\chi^+ (\mu) - \chi^- (\mu)) - \frac{1}{2} (\chi^+ (\mu) + \chi^- (\mu)).$$

**2.b** Given the solutions to  $\{R^d (s), R^{*,d} (s)\}$ , solve  $\{d^*, v\}$  using:

$$d = \left[ \frac{R^d}{\Theta^d} \right]^{1/\varsigma}$$

$$v = \left[ \frac{R^d}{\Theta^d} \right]^{1/\varsigma} \left[ \frac{R^{*,d}}{\Theta^{*,d}} \right]^{-1/\varsigma^*}.$$

**2.c** Given  $\{d^* (s), v (s)\}$  we solve for prices  $\{p, p^*, e\}$  using:

$$\mu v d^* = \frac{M}{p} \mu^* d^* = \frac{e}{p} M^* \mathbf{p}^* = e^{-1} p.$$

**2.d** Finally, we update the real policy rates. For that we construct the expected inflation in each currency:

$$\mathbb{E} [\pi^*] = \frac{\sum_{s' \in S} Q (s' | s) p_{(n)}^* (s)}{p^* (s)}$$

and

$$\mathbb{E} [\pi] = \frac{\sum_{s' \in S} Q (s' | s) p_{(n)} (s)}{p (s)}.$$

We then update the policy rates by:

$$R_{(j+1)}^{*,a} = \frac{(1 + i^{*,a})}{(1 + \pi^*)} \text{ for } a \in \{m, w\}$$

and

$$R_{(j+1)}^a = \frac{(1 + i^a)}{(1 + \pi)} \text{ for } a \in \{m, w\}.$$

**2.e** Repeat steps 2.a-2.d, unless

$$\left\{ R_{(j)}^m (s), R_{(j)}^{*,m} (s), R_{(j)}^w (s), R_{(j)}^{*,w} (s) \right\}$$

is close to

$$\left\{ R_{(j+1)}^m (s), R_{(j+1)}^{*,m} (s), R_{(j+1)}^w (s), R_{(j+1)}^{*,w} (s) \right\}.$$

If the real policy rates have converged, update prices according to

$$p_{(n+1)}^* (s) = \Delta^{greed} p^* + (1 - \Delta^{greed}) p_{(n)}^* (s)$$

and

$$p_{(n+1)}(s) = \Delta^{greed} p + (1 - \Delta^{greed}) p_{(n)}^*(s)$$

and proceed back to the outerloop.