Reconnecting Exchange Rate and the General Equilibrium Puzzle*

Yu-chin Chen† — *Univ. of Washington/ABFER*
Ippei Fujiwara‡ — *Keio/ANU/ABFER*
Yasuo Hirose§ — *Keio University*

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Abstract

This paper uses nonlinear general equilibrium (GE) estimations to assess the empirical relevance of macro-volatility shocks in explaining nominal exchange rate dynamics. Embedding stochastic volatilities and limits to international arbitrage in a two-country New Keynesian model with recursive preferences, we use third-order model approximation and full-information Bayesian methods to estimate the GE model with the US and the Euro area data from 1987Q1 to 2008Q4. In contrast to the well-known "exchange rate disconnect,"we find that macroeconomic shocks together with their uncertainties can account for a sizable portion—over 40%—of the observed exchange variations. The remainder, due to a direct international-risk sharing shock, likely reflect various informational or financial frictions that put a barrier on arbitrage. Our results also point to a new challenge in empirical exchange rate modeling which we term the "general equilibrium puzzle." We find that while conditionally, nominal volatility shocks, for example, can deliver coefficient estimates consistent with the uncovered interest rate parity (UIP) puzzle, their contributions unconditionally in the GE settings, are quantitatively insufficient to resolve the puzzle. The presence of multiple shocks, the potential interactions amongst them, and the need for the estimates to fit the full GE dynamics all underscore the importance of evaluating the empirical relevance of any structural mechanism in general equilibrium.

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†Department of Economics, University of Washington. Email: yuchin@uw.edu
‡Faculty of Economics, Keio University; Crawford School of Public Policy, Australian National University; Asian Bureau of Finance and Economic Research. Email: ippei.fujiwara@keio.jp
§Faculty of Economics, Keio University. Email: yhirose@econ.keio.ac.jp

1 Introduction

The nominal exchange rate is an important driver of aggregate fluctuations as well as a key link in the global goods and asset markets. However, endogenizing realistic exchange rate dynamics as observed in the data is a task that has eluded international macroeconomists for decades. While various structural frameworks aim to describe how policy actions and macroeconomic shocks can spill across country borders via the exchange rate, estimation efforts of such general equilibrium models typically find fluctuations in nominal exchange rates to be unrelated to macroeconomic forces.[1] Consequently, empirical evidence for the proposed transmission mechanisms of international policies and shocks through the exchange rate channel remains thin to non-existent, a pattern commonly referred to in the literature as the "exchange rate disconnect."

Exchange rate disconnect has spun a wide range of research agendas aiming to reconnect the exchange rate to the rest of the macroeconomy. In this paper, we evaluate two recent approaches by estimating a full-fledged New Keynesian DSGE model that encompasses two sources of fluctuations: 1) a direct shock to the exchange rate or the international risking-sharing condition; and 2) macroeconomic volatility shocks that induce an endogenous time-varying currency risk premium. We note that since the two approaches emphasize first-moment vs. 2nd-moment shocks, proper comparisons thus require estimating the model up to a third-order approximation and evaluating them along the dimensions of both the means and the variances. Using full-information Bayesian likelihood approach and central-difference Kalman filtering, we estimate directly the nonlinear DSGE system to evaluate how the two sources of shocks contribute to explaining the uncovered interest rate parity (henceforth, UIP) puzzle and excess exchange rate volatility (relative to macro-fundamentals) observed in the data.

Our motivation and contributions are as follows. First, instead of using simulations or partial equilibrium methods, we use general equilibrium (GE) system estimations to let the data distinguish directly the relative contributions of various transmission mechanisms. Second, by moving beyond linearization assumptions, our model remains flexible to nonlinear or cross interactions amongst the macro variables and the exchange rate. As the macro-finance literature has long demonstrated the limitations of linearized systems in modeling asset returns, we incorporate recursive preferences and stochastic volatilities in our GE model and approximate it to the third-order, to help capture the asset price nature of nominal exchange rate.[2] Third, by juxtaposing macro volatility shocks with a direct shock to the exchange rate that reflects limits to international arbitrage, we assess empirically the extent to which exchange rate may be connected nonlinearly to the macroeconomy, such as through a risk premium arisen endogenously to macro uncertainties. By relying on GE Bayesian estimations instead of simulations, we are also able

[1]See, for example, Lubik and Schorfheide (2006). Notable exception is Adolfson, Laseen, Linde, and Villani (2007) for the small open economy but they incorporate rather ad-hoc adjustment costs to capture risks in exchange rates.

[2]For example, Rudebusch and Swanson (2012) introduce Epstein-Zin preferences into a canonical DSGE model to solve the bond term premium puzzle.

to obtain direct estimates of key structural parameters, and evaluate if their relative magnitudes are consistent with theory. Last but not least, while our nonlinear GE estimations show that the exchange rate is not disconnected after all from the rest of the macroeconomy, they also reveal additional challenges in exchange rate modeling that we term the "general equilibrium puzzle." We emphasize two aspects of this GE puzzle, concerning first the presence of multiple shocks, and second the need to fit multiple targets in GE model evaluations. Accordingly, findings based on partial equilibrium or conditional analyses, while capable of delivering important insights, may not extend to the unconditional GE settings in terms of their quantitative impact. While these issues that surface in our GE analyses are by no means new conceptually (nor are they specific to exchange rate modeling), they nevertheless highlight the need for broader GE evaluations in assessing the relevance of various structural mechanisms in explaining empirical exchange rate behavior. We elaborate on these points below.

Ignoring risk considerations, the UIP states that countries with high relative interest rates should expect subsequent currency depreciations to ensure zero expected excess returns, or no arbitrage, from cross-border financial investments. As is well-known since Fama (1984), data consistently show significant and robust positive returns from "carry-trade" strategies that invest in high interest rate currencies with funding from low-interest rate currencies, an empirical regularity known as the forward-premium puzzle or the UIP puzzle. There have been numerous attempts to solve the forward discount puzzle, though as pointed out in Itskhoki and Mukhin (2017), any proposed solutions must also account for the high volatilities present in the exchange rates, but absent in other macroeconomic variables.

This paper focuses on evaluating two alternative mechanisms for explaining exchange rate dynamics: an international risk-sharing shock that encapsulates various frictions to arbitrage, and uncertainty shocks about the macroeconomy which lead to an endogenous time-varying currency risk premium. We incorporate both channels into a standard two-country New Open Economy Macro Economics (NOEM) model, adopting recursive preferences *a la* Epstein and Zin (1989) and Weil (1989) and stochastic volatilities.[3] While the vast majority of the literature using a DSGE setting relies on simulation results, we estimate the model using pre-financial crisis US and the Euro area data, allowing a direct evaluation for the relative contributions of these two channels in explaining the dollar-euro fluctuations.

The two mechanisms we emphasize capture arguments put forth in recent studies. As frictions in financial transactions hinder international arbitrage through the exchange rates, they work as direct shocks to exchange rates themselves.[4] We follow Itskhoki and Mukhin (2017) and model the wedge in the international arbitrage condition as an

[3] Bloom (2009) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015), for example, emphasize the importance of volatility shocks on aggregate fluctuations.

[4] See, for example, Adolfson, Laseen, Linde, and Villani (2007), Alvarez, Atkeson, and Kehoe (2009), Bacchetta and van Wincoop (2010), Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017), who point out the importance of financial frictions in accounting for aggregate fluctuations in open economies.

exogenous shock, without imposing a specific micro foundation.[5] The second mechanism views the exchange rate disconnect as result of linear or first-order approximations. As endogenous risk premiums may arise from covariance between the stochastic discount factor and returns to international financial investments, their omission may result in unexplained exchange rate volatility, or in the Fama regressions, biased coefficient estimates.[6] Moving beyond a linearized framework, one can endogenously generate time-varying currency risks through various channels. As previous attempts to generate endogenous currency risk premiums through first-moment shocks have led to little success, our paper considers shocks to the volatilities of macro variables. If exchange rate fluctuations systematically reflect such endogenized risks, one would then infer that the exchange rate is *not* disconnected from macroeconomic fundamentals. From the literature that endogenizes exchange rate risks, our approach follows most closely Benigno, Benigno, and Nistico (2011) (hereafter BBN). Through simulations, they show that a rise in the volatility of nominal shocks at home enhances the hedging properties of its currency, thereby inducing endogenously a risk premium for foreign currency-holding. Through this channel, uncertainty shocks can be a key driver behind empirical exchange dynamics.[7]

Our paper moves the evaluations of these mechanisms to a GE system estimation framework instead of relying only on single-equation estimations or simulations with calibrated parameters. To identify various shocks to macroeconomic fundamentals in the GE setting, we estimate the two-country NOEM model with a full information Bayesian maximum likelihood approach. The model is solved using perturbation methods up to the third-order approximation in order to allow stochastic volatilities in the fundamental shocks.[8] Because the model is non-linear, the standard Kalman filter is not applicable

[5]The choice to adopt this form of international risk-sharing shock is to facilitate our assessment for the degree of exchange rate disconnect from the rest of the macro dynamics. As we augment the standard linearized GE system with higher order approximations and then volatility shocks, we test how these additional elements help reconnect the exchange rate by lowering the explanatory power of the direct exchange rate shock.

[6]For example, a structural or macroeconomic fundamental shock—especially to volatilities—can simultaneously raises interest rates and appreciates the nominal exchange rates. Backus, Foresi, and Telmer (2001), Duarte and Stockman (2005), Verdelhan (2010), Colacito and Croce (2011), Bansal and Shaliastovich (2012), Benigno, Benigno, and Nistico (2011), Backus, Gavazzoni, Telmer, and Zin (2010), Gourio, Siemer, and Verdelhan (2013) and Engel (2016) are examples in recent literature that aim to solve the UIP puzzle through risk corrections.

[7] A rise in home nominal volatility tends to reduce domestic output and increase inflation, while the domestic nominal interest rate declines relative to the foreign one. Contrary to findings in the previous literature, e.g. Engel and West (2004) and Bacchetta and Wincoop (2006), that only a fraction of the exchange rate volatilities can be accounted for by observed economic fundamentals, BBN show the potential of the uncertainty shocks as a key driver behind empirical exchange dynamics. They show that in line with the findings in McCallum (1994) and Backus, Gavazzoni, Telmer, and Zin (2010), when the monetary policy inertia is high and price stickiness is low, a negative correlation emerges between the expected exchange rate depreciation and interest rate differentials in response to volatility shocks, potentially over-turning the forward premium puzzle.

[8]To gauge the impact of stochastic volatilities, BBN employ the efficient solution method with second-order approximation proposed by Benigno, Benigno, and Nistico (2013), which can account for any distinct and direct effects of volatility shocks, provided that shocks are conditionally linear.

for evaluating the likelihood function. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) use a particle filter to estimate the closed-economy real business cycle models with stochastic volatilities approximated up to the third order. However, it is practically infeasible to use the time-consuming particle filter to estimate the two-country model considered in our paper, given the rich dynamic structures we emphasize. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013). Their filter is much faster than the standard particle filter and its quasi maximum likelihood estimators can be consistent and asymptotically normal for models solved up to the third order.

Our estimation results are broadly consistent with the empirical regularities shown in BBN, in that: (1) an increase in the volatility of the productivity shock depreciates the exchange rate; (2) an increase in the volatility of the monetary policy shock appreciates the exchange rate; and (3) an increase in the volatility of the monetary policy shock produces excess foreign currency returns and deviations from the UIP. Moreover, using our estimated parameters, we show that *conditionally*, several volatility shocks (to e.g. monetary policy and aggregate demand) can generate the negative correlation observed in the Fama (1984) regression between expected nominal exchange rate depreciation and interest rate differentials. We also demonstrate that by approximating the model to 3rd-order, the macro shocks begin to play a larger role in our variance decompositions; together with shocks to their volatilities, they explain 43% of the variance of nominal exchange rate changes. We take these as evidence that the exchange rate is not disconnected from the rest of the macroeconomy, once we move beyond linearization assumptions. Our results also show that the direct financial shock, reflecting limits-to-arbitrage, remain the key driver behind most (57%) of the variations in the nominal exchange rate. Conditionally, the direct shock to risk-sharing can also replicate the negative UIP correlations.

Despite the positive results supporting the empirical relevance of our two proposed transmission mechanisms, our GE estimations also illustrate the limitations of partial or conditional analyses in providing full resolutions to these empirical puzzles. In general equilibrium, the presence of multiple shocks and their potential interactions can mitigate the quantitative relevance of one particular shock. In our analyses, for example, we find that even though several proposed shocks can individually generate the observed Fama coefficient (close to or below zero), simulation data using our GE estimations and all shocks together do not replicate the observed pattern in the data, rather they imply a positive Fama coefficient closer to 1. Putting this aspect of our "general equilibrium" puzzle in a more positive light, we view the conditional results to be insightful in illustrating transmission mechanisms and their qualitative relevance; indeed, our conditional analyses based on full GE estimates provide support for both of the channels we explore. Their ultimate quantitative relevance in resolving the unconditional empirical puzzles observed in data, however, ought to be assessed in the GE framework. On that front, our full model incorporating both of these mechanisms fail to replicate the observe empirical pattern. A second aspect of the GE puzzle is a reminder that in GE estimations, there are multiple dynamics to fit, not just the exchange rate. As we incorporate additional elements into the model to explain one targeted empirical pattern, general equilibrium estimations help ensure they do not come at a cost of deteriorating fit in other parts of the GE system.

The remainder of this paper is organized as follows. Section 2 summarizes the previous studies on the exchange rate dynamics. We also show why the exchange rate disconnect can be mitigated in our model with the recursive preference and uncertainty shocks in line with the reasoning offered by the previous literature. Section 3 presents the model with recursive preferences and stochastic volatilities in open economies. Section 4 shows how we estimate the model in a nonlinear setting by a full-information Bayesian approach. Section 5 presents our main results and discussions. Finally, Section 6 concludes.

2 Related Literature

The literature on the exchange rate disconnect and the forward premium puzzle is vast and dates back many decades. While it has branched out and manifested itself as various well-known empirical puzzles, a simple description of the exchange rate disconnect is that macroeconomic fundamentals – the theoretical determinants of the exchange rate – have essentially no explanatory power for actual exchange rate behavior.[9] The failure of the uncovered interest rate parity condition is at the heart of this problem.

In this section, we briefly discuss the disconnect problem and puts a particular emphasis on the solutions to the UIP puzzle. Then, we explain why our model with recursive preferences *a la* Epstein and Zin (1989) and Weil (1989) can generate an endogenous time-varying currency risk premium, potentially replicating the empirical deviations from the UIP condition. We also discuss a key mechanism for generating UIP deviations of the empirically relevant size, as emphasized in Backus, Gavazzoni, Telmer, and Zin (2010), that hinges on the relative magnitudes between two parameter values: the persistence of monetary policy and that of nominal volatility shocks. Our GE estimations can evaluate this condition directly from the data, as we will show in Section 5.

2.1 Exchange rate disconnect

The absence of arbitrage in the international asset markets implies the UIP condition, which can be expressed in a log-linear form as:

$$\mathbf{E}_t e_{t+1} - e_t = R_t - R_t^* \tag{1}$$

where R and R_t^* denote domestic and foreign interest rates, respectively and e_t is the logged nominal exchange rate. The UIP implies that countries with higher relative interest rates should see their currencies depreciate subsequently on average. Under the

[9] See, for example, Meese and Rogoff (1995), Engel and Rogers (1996), Obstfeld and Rogoff (2001) for a summary of the earlier literature. Evans (2011), Engel (2014), and Burnside (2019) provide additional survey of recent developments.

assumption of rational expectations, this implication is commonly tested in what is referred to as the Fama (1984) or UIP regression:[10]

$$e_t - e_{t-1} = \alpha_0 + \alpha_1 \left(R_{t-1} - R^*_{t-1}\right) + u_t, \tag{2}$$

with $H_0 : a_0 = 0$ and $\alpha_1 = 1$. In a wide range of international data, the literature has consistently found the estimated slope coefficient $\alpha_1 = 1$, to be significantly below one, and often negative, contrary to the theoretical prediction. Structural VARs, such as in Eichenbaum and Evans (1995), also confirm the empirical observation that a country's currency tends to *appreciate* after a positive monetary policy shock.[11] In terms of financial trading, Lustig and Verdelhan (2007) and Burnside, Eichenbaum, and Rebelo (2008), for example, find sizable gains from the carry trade strategy of investing in high interest rate currencies with funding from low interest rate currencies, confirming the robustness of the UIP puzzle in the data.

Besides producing the wrong signs, empirical tests of the UIP generally result in extremely poor fits, with the estimated R-squares near zero. This is indicative of the broader exchange rate disconnect phenomenon, as interest rates and the UIP condition are a key channel through which other macro fundamentals interact with the exchange rate. Engel and West (2004) and Bacchetta and Wincoop (2006), for example, show that the exchange rate volatility is hardly explained by their macroeconomic fundamentals. In a general equilibrium (GE) context, Lubik and Schorfheide (2006) similarly demonstrate the exchange rate disconnect using Bayesian maximum likelihood estimations of a two-country model. Approximating the GE system up to the first order, they find most of the fluctuations in the nominal exchange rates to come from a direct shock to the exchange rate itself.

The above results lead to two natural approaches for connecting the exchange rate to the rest of the macroeconomy.[12] The first is to augment the UIP condition with a time-varying risk premium through which macro fundamentals can influence the exchange rate. If this risk premium is correlated negatively with the interest rate differentials (or monetary policy shocks) and of sufficient magnitudes, one could attribute the UIP puzzle – the negative slope coefficient in the Fama regression eq.(2) – to omitted variable biases. The second approach introduces financial wedge to the UIP condition, representing limits to arbitrage that can arise from financial or informational frictions.

The literature has emphasized various promising channels to motivate a time-varying

[10]The Fama regression can also be expressed in terms of the forward premium,$f_t - e_t$, where f_t is the log forward rate. Under covered interest parity $f_t - e_t = R_t - R^*_t$, a condition that has strong empirical support until recently.

[11]The paper, as well as Scholl and Uhlig (2008) and Bjørnland (2009) further explore the pattern of "delayed overshooting" in exchange rate dynamics.

[12]The two approaches are by no means exhaustive. For example, an important branch of the literature focuses on solving the UIP puzzle with deviations from the rational expectations, such as over-confidence, learning dynamics, or ambiguity aversion. These include Gourinchas and Tornell (2004), Burnside, Han, Hirshleifer, and Wang (2011), Ilut (2012), and Chakraborty and Evans (2008).

risk premium, such as through disaster risks, liquidity premium, or uncertainty shocks.[13] The underlying mechanism relies on the covariance of the stochastic discount factor and the expected payoff to generate an endogenous risk premium that moves in the opposite direction with the relative interest rate, so as to induce the overall exchange rate response consistent with the empirical negative Fama coefficient. Note that to have the negative slope coefficient, movements in the risk premium must be large enough to overcome the changes in relative interest rates. As is well known in the macro-asset pricing literature, consumption dynamics are very smooth and therefore, the stochastic discount factor as well. In order to generate the requisite variations in the stochastic discount factor, non-time-separable utility functions are often adopted; for example, Verdelhan (2010) considers external habit formation while Backus, Foresi, and Telmer (2001), Colacito and Croce (2011), and Bansal and Shaliastovich (2012) employ the recursive preference *a la* Epstein and Zin (1989) and Weil (1989). They show that the negative correlations between exchange rate depreciations and interest rate differentials can be replicated under certain parameter conditions. As for the source of the risk premium, we follow Menkhoff, Sarno, Schmeling, and Schrimpf (2012) and BBN to focus on the role of stochastic volatilities in explaining exchange rate behavior. While Menkhoff, Sarno, Schmeling, and Schrimpf (2012) shows that global foreign exchange volatility risk can explain excess returns from carry trades, BBN considers both real and nominal macro volatility shocks. By simulating a two-country NOEM model with recursive preferences, they find that a rise in the volatility of nominal shocks at home enhance the hedging properties of its currency, thereby inducing endogenously a risk premium for foreign currency-holding. To re-evaluate the exchange rate disconnect, we focus on uncertainty shocks regarding the economic fundamentals and assess their relevance, using GE estimations, in explaining the UIP puzzle and the excess exchange rate volatility.

Limits of arbitrage a la Shleifer and Vishny (1997) naturally leads to the failure of the UIP, as they put a direct wedge in the UIP condition. The possible mechanisms again vary, and portfolio adjustment costs, limited participations, regulatory constraints have all been proposed as the possible "micro-foundation" behind the wedge or friction. For instance, Adolfson, Laseen, Linde, and Villani (2007) assume portfolio adjustment costs based on the lagged real net foreign asset positions, and show that the lagged nominal exchange rate can appear in the UIP and provide better model fits. Alvarez, Atkeson, and Kehoe (2009) incorporate limited participation to affect the risk premium. As inflation stemming from monetary easing lowers the cost of entry and increases the fraction of the agents in the asset market, monetary policy affects the marginal utility of market participants and reduces the currency risk premium. Bacchetta and van Wincoop (2010) resolves the UIP puzzle by employing the infrequent portfolio decisions, which leads to the delayed overshooting and therefore gradually appreciates of the high interest rate currency. Gabaix and Maggiori (2015) extends the model to incorporate large players (financiers) with limited risk-bearing capacities and financial market imperfection. An

[13] Engel (2016) and Valchev (2017) emphasize the importance of liquidity premium stemming from the imperfect substitutability between money and bonds for the liquidity service. Rogoff (1977)'s *peso problem* and disaster risk are another important branch of literature, more recently explored in Gourio, Siemer, and Verdelhan (2013) and Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011).

adverse shock to the financial system can then lead to positive ex ante returns from the carry trade, since financiers cannot fully engage in international arbitrage. Our paper follows Itskhoki and Mukhin (2017) and assumes a direct exogenous shock which hinders the perfect international financial transactions.

We adopt this direct shock approach instead of specifying any explicit forms of financial friction in order to gauge how the standard settings in modern macro finance, namely Epstein and Zin (1989) preferences and uncertainty shocks, can help "reconnect" the exchange rate. Any remainder, e.g. variations attributed to the direct shock, we view as capturing the empirical relevance of the above-mentioned mechanisms.

2.2 Monetary policy and exchange rate dynamics

Backus, Gavazzoni, Telmer, and Zin (2010) and BBN, two papers our modeling approach follows closely, further emphasize the monetary policy reaction functions in replicating the negative Fama slope coefficient. Specifically,the persistence in the monetary policy uncertainty shock process, and the policy inertia, must meet certain parameter restrictions in order to generate the exchange rate dynamics observed in the data. Since one of the advantages of our GE estimations is that we can obtain direct estimates for these relevant parameters, we will briefly present the conditions described in these papers.

For the stochastic volatilities in the model to generate a currency risk premium of the requisite sign and size, Backus, Gavazzoni, Telmer, and Zin (2010) and BBN show that the degree of interest rate smoothing in the Taylor type instrument rule, relative to the degree of persistence in the monetary policy shock, play a key role. Specifically, assuming away any real dynamics for simplicity, Backus, Gavazzoni, Telmer, and Zin (2010) consider the following standard Taylor–type feedback rule for monetary policy:

$$R_t = \gamma_0 + \gamma_1 \pi_t + \gamma_2 R_{t-1} + \varepsilon_t$$

where π_t is inflation, and ε_t is monetary policy shock, assumed to be not only persistent but also heteroskedastic:

$$\begin{aligned} \varepsilon_{t+1} &= \delta_\varepsilon \varepsilon_t + \sqrt{v_t} u^\varepsilon_{t+1}, \\ v_{t+1} &= (1-\delta_v)\,\bar{v} + \delta_v v_t + \sigma^v u^v_{t+1}. \end{aligned}$$

By assuming symmetry between the two countries, they derive the following result for the slope coefficient in equation (2):

$$\alpha_1 = \frac{\delta_v - \gamma_2}{\gamma_1}, \tag{3}$$

This condition shows that in order for monetary policy volatility shocks to produce the empirically observed negative α_1, the policy inertia parameter γ_2 must be larger than the persistence of the volatility shock, δ_v.[14] Since our GE system estimations with 3rd-order approximation can incorporate both monetary volatility shocks as well as its persistence, we can empirically test whether the above condition is generally satisfied. Moreover, we note that since equation (3) is derived under the stringent assumption of no real dynamics, the presence of addition shocks, including real ones, may affect this condition and the sign of the slope coefficient. Direct estimations of the full system therefore can help reveal the overall balance of these considerations, and reveal 1) whether the exchange rate is connected or not from macroeconomic fundamentals through stochastic volatilities, and 2) if the negative slope coefficient in UIP regression can be explained by the endogenous risk premium, as shown in Backus, Gavazzoni, Telmer, and Zin (2010).

3 The Model

The model estimated in this paper is a two-country extension of the standard New Keynesian model but incorporates recursive preferences *a la* Epstein and Zin (1989) and Weil (1989) together with stochastic volatilities in various structural shocks. The world economy consists of the US (the domestic or home country) and the Euro area (the foreign country), which are assumed to be of the same size.[15] In each country, the representative household gains utility from aggregate consumption composed of home and foreign goods, and trades state contingent assets in both domestic and international asset markets. Monopolistically competitive firms produce differentiated goods, and are subject to Calvo (1983)-type staggered price-setting. Monetary authorities adjust the nominal interest rates in response to inflation and output growth. While we assume symmetric households preferences, the two regions differ in price-setting, monetary policy and fundamental shocks. The assumptions with regard to preferences, technology and complete financial markets give us a highly tractable framework for the open economy.

3.1 Household

A representative household in the domestic country maximizes the recursive utility:

$$V_t = \left[u\left(C_t, N_t\right)^{1-\sigma} + \beta \left(\mathbf{E}_t V_{t+1}^{1-\varepsilon}\right)^{\frac{1-\sigma}{1-\varepsilon}} \right]^{\frac{1}{1-\sigma}},$$

where σ measures the inverse of the intertemporal elasticity of substitution, and ε is the coefficient of relative risk aversion. N_t denotes labor supply. Aggregate consumption C_t is a composite of home- and foreign-produced goods, $C_{H,t}$ and $C_{F,t}$, given by

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

[14] Backus, Gavazzoni, Telmer and Zin (2010) show that without the stochastic volatility, the slope coefficent in the Fama regression for the real variables cannot be negative as observed from the data.

[15] This assumption follows from Lubik and Schorfheide (2006). Indeed, the two regions are roughly the same size and have similar per capita income.

with

$$C_{H,t} = \left[\int_0^1 C_{H,t}(j)^{1-\frac{1}{\mu}} \, \mathrm{d}j \right]^{\frac{\mu}{\mu-1}},$$

$$C_{F,t} = \left[\int_0^1 C_{F,t}(j^*)^{1-\frac{1}{\mu}} \, \mathrm{d}j^* \right]^{\frac{\mu}{\mu-1}},$$

where $C_{H,t}(j)$ and $C_{F,t}(j^*)$ are differentiated consumption goods produced by domestic and foreign firms, each of which are indexed by j and j^* respectively. The parameters α, η, and μ are the steady state share of the domestically produced goods consumption in the aggregate consumption, the elasticity of substitution between domestically produced and imported goods, the elasticity of substitution among differentiated products in each country. Following BBN, we specify the instantaneous utility as

$$u(C_t, N_t) := C_t^{\psi} (1 - N_t)^{1-\psi}.$$

The household's utility maximization is subject to the budget constraint:

$$P_t C_t + B_t + \mathbf{E}_t[\frac{m_{t,t+1}}{\pi_{t+1}} D_{t+1}] = R_{t-1} B_{t-1} + D_t + W_t N_t + T_t,$$

where P_t is the consumer price index, B_t is the holding of the domestic bond, $m_{t,t+1}$ is the real stochastic discount factor, $\pi_t := P_t / P_{t-1}$ is CPI inflation, D_t is the state-contingent payoff, R_t is the nominal interest rate, W_t is nominal wage, and T_t is the net transfer from firms and the government.

The optimality conditions for the home household lead to

$$C_{H,t} = (1-\alpha) p_{H,t}^{-\eta} C_t,$$

$$C_{F,t} = \alpha (p_{F,t})^{-\eta} C_t,$$

$$C_t = \frac{\psi}{1-\psi} (1 - N_t) w_t,$$

$$1 = \mathbf{E}_t m_{t,t+1} \frac{R_t}{\pi_{t+1}},$$

$$m_{t,t+1} = \beta \left(\mathbf{E}_t V_{t+1}^{1-\varepsilon} \right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}} V_{t+1}^{\sigma-\varepsilon} \frac{C_{t+1}^{\psi(1-\sigma)-1} (1 - N_{t+1})^{(1-\psi)(1-\sigma)}}{C_t^{\psi(1-\sigma)-1} (1 - N_t)^{(1-\psi)(1-\sigma)}},$$

where $w_t := W_t / P_t$.

A representative household in the foreign country faces a symmetric utility maximization problem to the one in the home country.

3.2 Firms

In the home country, each firm, indexed by j, produces one kind of differentiated goods $Y_t(j)$ by choosing a cost-minimizing labor input $N_t(j)$, given the real wage w_t, subject to the production function:

$$Y_t(j) = A_{W_t} A_t N_t(j),$$

where A_t is a stationary and country-specific technology shock, and $A_{W,t}$ is a non-stationary worldwide technology component that grows at a constant rate γ, i.e.,

$$\frac{A_{W,t}}{A_{W,t-1}} = \gamma.$$

Firms set prices of their products on a staggered basis à la Calvo (1983). In each period, a fraction $1-\theta \in (0,1)$ of firms reoptimizes prices, while the remaining fraction θ indexes prices to a weighted average of the past inflation rate for the domestically produced goods $\pi_{H,t-1} := P_{H,t-1}/P_{H,t-2}$ and the steady-state inflation rate $\bar{\pi}$. Then, firms that reoptimize prices in the current period maximize their expected profit

Each firm sets its price in a monopolistically competitive market to maximize the present discounted value of their profits:

$$\mathbf{E}_t \sum_{n=0}^{\infty} \theta^n m_{t,t+n} \left[\frac{P_{H,t}(j)}{P_{t+n}} \prod_{i=1}^{n} \left(\bar{\pi}^{1-\iota} \pi_{H,t+i-1}^{\iota} \right) - \frac{w_{t+n}}{A_{W_{t+n}} A_{t+n}} \right] Y_{t+n}(j),$$

subject to the firm-level resource constraint

$$Y_t(j) = C_{H,t}(j) + G_{H,t}(j) + C_{H,t}^*(j),$$

and the downward sloping demand curves, which are obtained from the households' optimization problems,

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\mu} (C_{H,t} + G_t),$$

$$C_{H,t}^*(j) = \left[\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right]^{-\mu} C_{H,t}^*,$$

where $\iota \in [0,1]$ denotes the weight of price indexation to past inflation relative to steady-state inflation, $C_{H,t}^*$ is export of the domestically produced goods, $P_{H,t}$ is the producer price index, $P_{H,t}^*$ is the export price of the domestically produced goods in the foreign currency, G_t ($G_{H,t}(j)$) is an external demand component other than consumption.[16] The last equality holds because we assume the law of one price:

$$P_{H,t}(j) = e_t P_{H,t}^*(j),$$

[16] We assume that only domestically produced goods are used for external demand.

where e_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency).

Let $p_{H,t} = P_{H,t}/P_t$. Then, $\pi_{H,t} := P_{H,t}/P_{H,t-1}$ can be expressed as

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}}.$$

Moreover, with the auxiliary variables F_t and K_t, the optimal pricing decision can be written in the recursive form:

$$F_t = \frac{1}{2}p_{H,t}\left(C_{H,t} + G_t + C^*_{H,t}\right) + \theta \mathbf{E}_t m_{t,t+1}\left(\frac{\bar{\pi}^{1-\iota}\pi^{\iota}_{H,t}}{\pi_{H,t+1}}\right)^{1-\mu} F_{t+1},$$

$$K_t = \frac{1}{2}\frac{\mu}{\mu-1}\frac{w_t}{A_t}\left(C_{H,t} + G_t + C^*_{H,t}\right) + \theta \mathbf{E}_t m_{t,t+1}\left(\frac{\bar{\pi}^{1-\iota}\pi^{\iota}_{H,t}}{\pi_{H,t+1}}\right)^{-\mu} K_{t+1}.$$

Under the present price-setting rule, the inflation rate for the domestically produced goods $\pi_{H,t}$ can be related to these auxiliary variables by

$$\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi^{\iota}_{H,t-1}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{1}{1-\mu}} F_t = K_t.$$

By aggregating the firm-level resource constraint, we have

$$Y_t = \Delta_t\left(C_{H,t} + G_t + C^*_{H,t}\right),$$

where $\Delta_t := \int_0^1 \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\mu} dj$ represents price dispersion across firms. The price dispersion term evolves according to

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi^{\iota}_{H,t-1}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{\mu}{\mu-1}} + \theta\left(\frac{\pi_{H,t}}{\bar{\pi}^{1-\iota}\pi^{\iota}_{H,t-1}}\right)^{\mu}\Delta_{t-1}.$$

To specify measurement equations in the subsequent section, we define the output growth rate YGR_t:

$$YGR_t := \frac{Y_t}{Y_{t-1}}.$$

Foreign firms' profit maximization problems are symmetric to those presented above.

3.3 Monetary policy

The monetary authority in the home country adjusts the nominal interest Monetary policy in response to deviations of inflation and output growth from their steady state values.

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\phi_r)\left[\phi_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y \log\left(\frac{Y_t}{\gamma Y_{t-1}}\right)\right] + \log(\varepsilon_{R,t}).$$

where $\phi_r \in [0,1)$ is the degree of interest rate smoothing, and $\phi_\pi, \phi_y \geq 0$ are the degrees of monetary policy responses to inflation and output growth. $\varepsilon_{R,t}$ is an exogenous shock interpreted as an unsystematic component of monetary policy.

The monetary authority in the foreign country also controls the nominal interest rate following the same type of monetary policy rule.

3.4 Exchange rate and international linkage

Recall that the law of one price holds for prices of domestically produced goods:

$$P_{H,t} = e_t P^*_{H,t},$$

where e_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). Note that * indicates variables in the foreign currency. We define the real exchange rate s_t as

$$s_t = \frac{e_t P^*_t}{P_t},$$

where P^*_t is the foreign aggregate price in the foreign currency. Let $p_{H,t} = P_{H,t}/P_t$ and $p_{H,t} = s_t p^*_{H,t}$. Then, we have

$$p_{H,t} = s_t p^*_{H,t}.$$

Similarly, we can obtain

$$p_{F,t} = s_t p^*_{F,t}.$$

From the definition of the real exchange rate, we have a expression for the nominal exchange rate depreciation d_t:

$$d_t := \frac{e_t}{e_{t-1}} = \frac{s_t \pi_t}{s_{t-1}\pi^*_t}.$$

Regarding the international asset market, the value of the asset in the foreign currency is given by

$$\mathbf{E}_t[\frac{m_{t,t+1}}{\pi_{t+1}} D^*_{t+1} e_{t+1}]/e_t.$$

Thus, under the perfect risk sharing, the stochastic discount factor in the foreign currency $m^*_{t,t+1}$ must satisfy

$$\frac{m^*_{t,t+1}}{\pi^*_{t+1}} = \frac{m_{t,t+1}}{\pi_{t+1}}\frac{e_{t+1}}{e_t}.$$

Substituting the optimality conditions for the home and foreign households to this equation, we have

$$\left(\frac{\left(V_{t+1}^{*}\right)^{1-\varepsilon}\mathbf{E}_{t}V_{t+1}^{1-\varepsilon}}{V_{t+1}^{1-\varepsilon}\mathbf{E}_{t}\left(V_{t+1}^{*}\right)^{1-\varepsilon}}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\left[\frac{C_{t+1}^{\psi}\left(1-N_{t+1}\right)^{(1-\psi)}}{\left(C_{t+1}^{*}\right)^{\psi}\left(1-N_{t+1}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t+1}^{*}}{C_{t+1}}s_{t+1}$$
$$=\left[\frac{C_{t}^{\psi}\left(1-N_{t}\right)^{(1-\psi)}}{\left(C_{t}^{*}\right)^{\psi}\left(1-N_{t}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t}^{*}}{C_{t}}s_{t}. \tag{4}$$

Let us denote

$$Q_{t}=\left[\frac{C_{t}^{\psi}\left(1-N_{t}\right)^{(1-\psi)}}{\left(C_{t}^{*}\right)^{\psi}\left(1-N_{t}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t}^{*}}{C_{t}}s_{t}. \tag{5}$$

Then, equation (4) can be written as

$$Q_{t+1}=Q_{t}\left(\frac{\left(V_{t+1}^{*}\right)^{1-\varepsilon}\mathbf{E}_{t}\left(V_{t+1}^{1-\varepsilon}\right)}{V_{t+1}^{1-\varepsilon}\mathbf{E}_{t}\left[\left(V_{t+1}^{*}\right)^{1-\varepsilon}\right]}\right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}}, \tag{6}$$

where we assume that $Q_0 = 1$, implying that the initial state-contingent wealth equalizes the marginal utilities across countries. If the preferences were non-recursive, i.e., $\sigma = \varepsilon$, then $Q_t = 1$ for all t, and hence the risk-sharing condition would be reduced to the one characterized by equation (5) with $Q_t = 1$. Thus, we regard equation (5) as the international risk-sharing condition and introduce a shock Ω_t to this condition as follows:

$$\Omega_{t}Q_{t}=\left[\frac{C_{t}^{\psi}\left(1-N_{t}\right)^{(1-\psi)}}{\left(C_{t}^{*}\right)^{\psi}\left(1-N_{t}^{*}\right)^{(1-\psi)}}\right]^{1-\sigma}\frac{C_{t}^{*}}{C_{t}}s_{t}, \tag{7}$$

Here, Ω_t works as the time-varying financial frictions considered in Itskhoki and Mukhin (2017).

3.5 Exogenous shocks

The following variables are exogenous in the model: country-specific technology A_t, external demand g_t, monetary policy shock $\varepsilon_{R,t}$ in the home country, the corresponding foreign variables A_t^*, g_t^*, $\varepsilon_{R,t}^*$, and the risk-sharing shock Ω_t. The stochastic processes for these variables are given by

$$
\begin{aligned}
\log\left(A_t\right) &= \rho_A \log\left(A_{t-1}\right) + \sigma_{A,t} u_{A,t}, \\
\log\left(g_t\right) &= \left(1-\rho_g\right)\log \bar{g} + \rho_g \log\left(g_{t-1}\right) + \sigma_{g,t} u_{g,t}, \\
\log(\varepsilon_{R,t}) &= \sigma_{\varepsilon_R,t} u_{\varepsilon_R,t}, \\
\log\left(A_t^*\right) &= \rho_A^* \log\left(A_{t-1}^*\right) + \sigma_{A,t}^* u_{A,t}^*, \\
\log\left(g_t^*\right) &= \left(1-\rho_g^*\right)\log \bar{g} + \rho_g^* \log\left(g_{t-1}^*\right) + \sigma_{g,t}^* u_{g,t}^*, \\
\log(\varepsilon_{R,t}^*) &= \sigma_{\varepsilon_R,t}^* u_{\varepsilon_R,t}^*, \\
\log\left(\Omega_t\right) &= \rho_\Omega \log\left(\Omega_{t-1}\right) + \sigma_{\Omega,t} u_{\Omega,t},
\end{aligned}
$$

where $\rho_A, \rho_g, \rho_A^*, \rho_g^*, \rho_\Omega \in [0,1)$ are the autoregressive parameters and $u_{A,t}, u_{g,t}, u_{\varepsilon_R,t}, u_{A,t}^*, u_{g,t}^*, u_{\varepsilon_R,t}^*, u_{\Omega,t} \sim$ i.i.d. $N(0,1)$ are disturbances to the exogenous processes.

The stochastic processes for the volatilities of the shocks are given by

$$
\begin{aligned}
\log\left(\sigma_{A,t}\right) &= \left(1-\rho_{\sigma_A}\right)\log\left(\sigma_A\right) + \rho_{\sigma_A}\log\left(\sigma_{A,t-1}\right) + \tau_A z_{\sigma_A,t}, \\
\log\left(\sigma_{g,t}\right) &= \left(1-\rho_{\sigma_g}\right)\log\left(\sigma_g\right) + \rho_{\sigma_g}\log\left(\sigma_{g,t-1}\right) + \tau_g z_{\sigma_g,t}, \\
\log\left(\sigma_{\varepsilon_R,t}\right) &= \left(1-\rho_{\sigma_{\varepsilon_R}}\right)\log\left(\sigma_{\varepsilon_R}\right) + \rho_{\sigma_{\varepsilon_R}}\log\left(\sigma_{\varepsilon_R,t-1}\right) + \tau_{\varepsilon_R} z_{\sigma_{\varepsilon_R},t}, \\
\log\left(\sigma_{A,t}^*\right) &= \left(1-\rho_{\sigma_A}^*\right)\log\left(\sigma_A^*\right) + \rho_{\sigma_A}^*\log\left(\sigma_{A,t-1}^*\right) + \tau_A^* z_{\tau_A,t}^*, \\
\log\left(\sigma_{g,t}^*\right) &= \left(1-\rho_{\sigma_g}^*\right)\log\left(\sigma_g^*\right) + \rho_{\sigma_g}^*\log\left(\sigma_{g,t-1}^*\right) + \tau_g^* z_{\sigma_g,t}^*, \\
\log\left(\sigma_{\varepsilon_R,t}^*\right) &= \left(1-\rho_{\sigma_{\varepsilon_R}}^*\right)\log\left(\sigma_{\varepsilon_R}^*\right) + \rho_{\sigma_{\varepsilon_R}}^*\log\left(\sigma_{\varepsilon_R,t-1}^*\right) + \tau_{\varepsilon_R}^* z_{\sigma_{\varepsilon_R},t}^*, \\
\log\left(\sigma_{\Omega,t}\right) &= \left(1-\rho_{\sigma_\Omega}\right)\log\left(\sigma_\Omega\right) + \rho_{\sigma_\Omega}\log\left(\sigma_{\Omega,t-1}\right) + \tau_\Omega z_{\sigma_\Omega,t}.
\end{aligned}
$$

where $\rho_{\sigma_A}, \rho_{\sigma_g}, \rho_{\sigma_{\varepsilon_R}}, \rho_{\sigma_A}^*, \rho_{\sigma_g}^*, \rho_{\sigma_{\varepsilon_R}}^*, \rho_{\sigma_\Omega} \in [0,1)$ are the autoregressive parameters, $z_{\sigma_A}, z_{\sigma_g}, z_{\sigma_{\varepsilon_R}}, z_{\sigma_A}^*, z_{\sigma_g}^*, z_{\sigma_{\varepsilon_R}}^*, z_{\sigma_\Omega} \sim$ i.i.d. $N(0,1)$ are the innovation to the stochastic volatilities, and $\tau_A, \tau_g, \tau_{\epsilon_R}, \tau_A^*, \tau_g^*, \tau_{\epsilon_R}^*, \tau_\Omega$ are their respective standard deviations.

3.6 Detrending

To make the model stationary and obtain the steady state, real variables in the home country are detrended by non-stationary worldwide technology component $A_{W,t}$ so that $v_t := V_t/A_{W,t}^{\psi}$, $y_t := Y_t/A_{W,t}$, $c_{H,t} := C_{H,t}/A_{W,t}$, $c_{F,t} := C_{F,t}/A_{W,t}$, $\tilde{w}_t := w_t/A_{W,t}$, and $g_t := G_t/A_{W,t}$. Foreign variables are also detrended in the same manner.

The Steady-state conditions in terms of detrended variables are presented in Appendix A, whereas the detrended system of equations are shown in Appendix B.

4 Solution and Estimation Methods

The model is solved using perturbation methods up to the third-order approximation in order to take account of the stochastic volatilities in the fundamental shocks. To ensure stability, we employ the pruning method developed by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018).

We estimate the model using a full-information Bayesian approach. Because the model is no longer linear, the standard Kalman filter is not applicable to evaluate the likelihood function. Instead, we approximate the likelihood function using the central difference Kalman filter proposed by Andreasen (2013).[17]

To approximate the posterior distribution, this paper exploits the generic Sequential Monte Carlo (SMC) algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015).[18] In the algorithm, a sequence of tempered posteriors are defined as

$$\varpi_n(\vartheta) = \frac{[p(X^T|\vartheta)]^{\tau_n} p(\vartheta)}{\int [p(X^T|\vartheta)]^{\tau_n} p(\vartheta) d\vartheta}, \quad n = 0, ..., N_\tau.$$

The tempering schedule $\{\tau_n\}_{n=0}^{N_\tau}$ is determined by $\tau_n = (n/N_\tau)^\chi$, where χ is a parameter that controls the shape of the tempering schedule. The SMC algorithm generates parameter draws and associated importance weights—which are called particles—from the sequence of posteriors $\{\varpi_n\}_{n=1}^{N_\tau}$; that is, at each stage, $\varpi_n(\vartheta)$ is represented by a swarm of particles $\{\vartheta_n^{(i)}, w_n^{(i)}\}_{i=1}^N$, where N denotes the number of particles. For $n = 0, ..., N_\tau$, the algorithm sequentially updates the swarm of particles $\{\vartheta_n^{(i)}, w_n^{(i)}\}_{i=1}^N$ through importance sampling.[19] Posterior inferences about parameters to be estimated are made based on the particles $\{\vartheta_{N_\tau}^{(i)}, w_{N_\tau}^{(i)}\}_{i=1}^N$ from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(X^T) = \prod_{n=1}^{N_\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_n^{(i)} w_{n-1}^{(i)} \right),$$

where $\tilde{w}_n^{(i)}$ is the incremental weight defined as $\tilde{w}_n^{(i)} = [p(X^T|\vartheta_{n-1}^{(i)})]^{\tau_n - \tau_{n-1}}$. In the subsequent empirical analysis, the SMC algorithm uses $N = 2,000$ particles and $N_\tau = 50$ stages. The parameter that controls the tempering schedule is set at $\chi = 2$ following Herbst and Schorfheide (2014, 2015).

Seven quarterly time series ranging from 1987Q1 to 2008Q4 are used for estimation: the per-capita real GDP growth rate ($100\Delta \log GDP_t, 100\Delta \log GDP_t^*$), the inflation rate of the GDP implicit price deflator ($100\Delta \log PGDP_t, 100\Delta \log PGDP_t^*$), and the three-month nominal interest rate(INT_t, INT_t^*), in the US and the Euro Area, and the nominal exchange rate depreciation of the US dollar against the Euro ($100\Delta \log EXR_t$). The construction of the data basically follows from Lubik and Schorfheide (2006): The US data are extracted from the FRED database maintained by the Federal Reserve Bank of St. Louis, whereas the Euro Area data and the exchange rate series are taken from the Area-Wide

[17] Andreasen (2013) argue that quasi maximum likelihood estimators based on the central difference Kalman filter can be consistent and asymptotically normal for DSGE models solved up to the third order.

[18] Creal (2007) is the first that applied the SMC methods to the estimation of DSGE models.

[19] This process includes one step of a single-block RWMH algorithm.

Model (AWM) database of the European Central Bank.[20] The observation equations that relate the data to model variables are given by

$$\begin{bmatrix} 100\Delta \log GDP_t \\ 100\Delta \log PGDP_t \\ INT_t \\ 100\Delta \log GDP_t^* \\ 100\Delta \log PGDP_t^* \\ INT_t^* \\ 100\Delta \log EXR_t \end{bmatrix} = \begin{bmatrix} \bar{\gamma} \\ \bar{\pi} \\ \bar{r} \\ \bar{\gamma} \\ \bar{\pi} \\ \bar{r} \\ 0 \end{bmatrix} + \begin{bmatrix} 100Y\hat{G}R_t \\ 100\hat{\pi}_t \\ 100\hat{r}_t \\ 100Y\hat{G}R_t \\ 100\hat{\pi}_t \\ 100\hat{r}_t \\ 100\hat{d}_t \end{bmatrix},$$

where $\bar{\gamma} = 100(\gamma - 1)$, $\bar{\pi} = 100(\pi - 1)$, $\bar{r} = 100(R - 1)$, and the hatted variables on the right hand side denote the log deviations from their steady-state values.

Before estimation, parameters regarding the share of foreign goods, the elasticity of substitution between home and foreign goods, the elasticity of substitution across the goods within each country, the share of external demand, the steady-state growth, inflation, and interest rates, and relative risk aversion are fixed at $\bar{\gamma} = 0.346$, $\bar{\pi} = 0.639$, $\bar{r} = 1.274$, $\bar{g}/\bar{y} = 0.18$, $\alpha = 0.13$, $\eta = 1.5$, $\mu = 6$, $\psi = 0.333$, $\epsilon = \epsilon^* = 5$, respectively, to avoid an identification issue. The values for $\bar{\gamma}$, $\bar{\pi}$, $\bar{r}$, and $\bar{g}/\bar{y}$ are set at the sample means of the corresponding data across the two countries so that the ergodic means of the model-implied observables tend to be close the sample means. The other parameter values are chosen based on the calibration in BBN. All the other parameters are estimated; their prior distributions are shown in Table 1. The priors are set according to those used in Smets and Wouters (2007) and the calibrated values in BBN. For the standard deviations of the stochastic volatilities ($\tau_A, \tau_g, \tau_{\epsilon_R}, \tau_A^*, \tau_g^*, \tau_{\epsilon_R}^*, \tau_\Omega$), the prior mean is set in line with the upper bound of the estimated standard deviation of the stochastic volatility regarding the technology shock reported in Fernández-Villaverde and Rubio-Ramírez (2007).

5 Results

We first report the estimation results and show that they are generally robust across model specifications and broadly consistent with estimates in the previous literature. We then discuss how our model can account for the aggregate fluctuations observed in the US-Euro area data, specifically the exchange rate dynamics. We focus on the sources of exchange rate variations, and also present the "general equilibrium puzzle" in the context of the UIP slope coefficient.

5.1 Parameter estimates

Table 2 and 3 report the posterior estimates of parameters. To better understand the effect of higher order approximations and the introduction of stochastic volatilities, we

[20]For the nominal exchange rate series for the period prior to the introduction of the Euro in 1999, the USD-ECU (Euroepan Currency Unit) exchange rate is used.

Table 1: Prior distributions of parameters

Parameter	Distribution	Mean	S.D.
ε	Gamma	5.000	0.500
σ	Gamma	2.000	0.250
θ	Beta	0.667	0.100
ι	Beta	0.500	0.150
θ^*	Beta	0.667	0.100
ι^*	Beta	0.500	0.150
ϕ_r	Beta	0.750	0.100
ϕ_π	Gamma	1.500	0.200
ϕ_y	Gamma	0.125	0.050
ϕ_r^*	Beta	0.750	0.100
ϕ_π^*	Gamma	1.500	0.200
ϕ_y^*	Gamma	0.125	0.050
ρ_A	Beta	0.500	0.150
ρ_g	Beta	0.500	0.150
ρ_A^*	Beta	0.500	0.150
ρ_g^*	Beta	0.500	0.150
ρ_Ω	Beta	0.500	0.150
ρ_{σ_A}	Beta	0.500	0.150
ρ_{σ_g}	Beta	0.500	0.150
$\rho_{\sigma_{\epsilon_R}}$	Beta	0.500	0.150
$\rho_{\sigma_A}^*$	Beta	0.500	0.150
$\rho_{\sigma_g}^*$	Beta	0.500	0.150
$\rho_{\sigma_{\epsilon_R}}^*$	Beta	0.500	0.150
ρ_{σ_Ω}	Beta	0.500	0.150
$100\sigma_A$	Inverse Gamma	5.000	2.590
$100\sigma_g$	Inverse Gamma	5.000	2.590
$100\sigma_{\epsilon_R}$	Inverse Gamma	0.500	0.260
$100\sigma_A^*$	Inverse Gamma	5.000	2.590
$100\sigma_g^*$	Inverse Gamma	5.000	2.590
$100\sigma_{\epsilon_R}^*$	Inverse Gamma	0.500	0.260
$100\sigma_\Omega$	Inverse Gamma	5.000	2.590
τ_A	Inverse Gamma	1.000	0.517
τ_g	Inverse Gamma	1.000	0.517
τ_{ϵ_R}	Inverse Gamma	1.000	0.517
τ_A^*	Inverse Gamma	1.000	0.517
τ_g^*	Inverse Gamma	1.000	0.517
$\tau_{\epsilon_R}^*$	Inverse Gamma	1.000	0.517
τ_Ω	Inverse Gamma	1.000	0.517

also estimate a linearized version of the model and a model *without* stochastic volatilities approximated up to the second and third order for comparison. For each model, the posterior mean and 90 percent highest posterior density intervals for the estimated parameters are presented as well as the SMC-based approximation of log marginal data density $\log p(\mathcal{Y}^T)$.[21]

The marginal data densities $\log p(\mathcal{Y}^T)$ indicate that the higher-order approximation of the model does not contribute to improving the overall fit of the model to the data, and the baseline model with stochastic volatilities deteriorates the fit even further. This is because the higher-order approximation with stochastic volatilities imposes such tighter cross-equation restrictions that do not necessarily improve the empirical performance of the model.

We note that while the estimates for the structural parameters do not differ much across the four specifications, remarkable differences arise in the parameters related to the shocks. First, as the degree of approximation becomes higher, the $AR(1)$ coefficients for structural shocks tend to decrease. In particular, the third-order approximation with stochastic volatilities results in substantially smaller persistence estimates (with the exception of the coefficient on the technology shock in the Euro area ρ_A^*.) In addition, even in the baseline model (the third-order approximation with stochastic volatilities), we see the persistence coefficient on the risk-sharing shock ρ_Ω to be very large and close to unity. The standard deviations of the structural shocks are not much different across the four specifications, except in the case of the risk-sharing shock, where the standard deviation becomes substantially lower in the baseline model. These findings suggest that the risk-sharing shock is absorbing some key empirical properties of the exchange rate (its persistence or random walk-like behavior), and the introduction of the stochastic volatilities do affect its role somewhat.

Since our structural model shares many similarities with the one presented in BBN, we compare our parameter estimates with the values they use for their model calibration. Table 4 show that our posterior mean estimates are mostly very close to their calibrated values, except for the monetary policy output response parameters. As such, we have conducted additional robustness checks to ensure the difference in these parameters do not lead to any qualitative differences in our main results presented below.

5.2 Impulse responses

This subsection demonstrates that our estimated model can broadly replicate the key observations about volatility shocks described in BBN: (1) an increase in the volatility of the productivity shock induces an exchange rate depreciation; (2) an increase in the volatility of the monetary policy shock induces an exchange rate appreciation; and (3) an increase in the volatility of the monetary policy shock also causes deviations from the UIP via increasing the excess return on the foreign currency.

[21] The risk aversion parameter ε in the recursive preferences does not appear in the linearized version of the model.

Table 2: Posterior distributions of parameters

Parameter	Linear		2nd order	
	Mean	90% interval	Mean	90% interval
ε	5.127	[4.308, 5.999]	5.007	[4.389, 5.705]
σ	2.184	[1.875, 2.501]	2.180	[1.962, 2.419]
θ	0.594	[0.495, 0.707]	0.710	[0.665, 0.761]
ι	0.193	[0.048, 0.313]	0.143	[0.048, 0.236]
θ^*	0.672	[0.603, 0.748]	0.633	[0.581, 0.680]
ι^*	0.119	[0.030, 0.199]	0.140	[0.047, 0.234]
ϕ_r	0.790	[0.754, 0.831]	0.817	[0.785, 0.850]
ϕ_π	1.946	[1.715, 2.190]	1.947	[1.703, 2.160]
ϕ_y	0.274	[0.164, 0.383]	0.207	[0.139, 0.275]
ϕ_r^*	0.768	[0.717, 0.815]	0.771	[0.732, 0.816]
ϕ_π^*	2.017	[1.812, 2.244]	2.113	[1.911, 2.307]
ϕ_y^*	0.249	[0.147, 0.347]	0.207	[0.130, 0.288]
ρ_A	0.667	[0.494, 0.813]	0.652	[0.560, 0.732]
ρ_g	0.943	[0.910, 0.977]	0.839	[0.786, 0.884]
ρ_A^*	0.618	[0.530, 0.722]	0.551	[0.453, 0.643]
ρ_g^*	0.954	[0.927, 0.979]	0.968	[0.947, 0.989]
ρ_Ω	0.997	[0.995, 0.999]	0.997	[0.996, 0.999]
$100\sigma_A$	2.138	[1.337, 2.969]	3.003	[2.126, 3.868]
$100\sigma_g$	8.339	[6.913, 9.566]	8.864	[7.495, 10.060]
$100\sigma_{\epsilon_R}$	0.159	[0.135, 0.185]	0.154	[0.133, 0.176]
$100\sigma_A^*$	2.980	[1.916, 4.115]	2.781	[2.055, 3.417]
$100\sigma_g^*$	7.781	[6.613, 8.969]	4.706	[4.108, 5.333]
$100\sigma_{\epsilon_R}^*$	0.160	[0.137, 0.185]	0.161	[0.140, 0.183]
$100\sigma_\Omega$	6.885	[6.059, 7.711]	8.591	[7.538, 9.648]
$\log p(\mathcal{Y}^T)$	-673.902		-683.774	

Notes: Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathcal{Y}^T)$ represents the SMC-based approximation of log marginal data density.

Table 3: Posterior distributions of parameters (cont.)

	3rd order		3rd order with S.V.		No risk-sharing shock	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval
ε	4.388	[4.129, 4.625]	4.331	[3.993, 4.669]	4.139	[3.775, 4.439]
σ	2.615	[2.502, 2.774]	1.879	[1.682, 2.118]	2.427	[2.200, 2.628]
θ	0.708	[0.675, 0.742]	0.525	[0.473, 0.575]	0.521	[0.469, 0.565]
ι	0.140	[0.053, 0.256]	0.340	[0.212, 0.442]	0.587	[0.482, 0.673]
θ^*	0.495	[0.439, 0.539]	0.766	[0.713, 0.827]	0.840	[0.824, 0.858]
ι^*	0.330	[0.260, 0.416]	0.389	[0.248, 0.548]	0.616	[0.471, 0.792]
ϕ_r	0.749	[0.715, 0.793]	0.772	[0.703, 0.836]	0.685	[0.632, 0.725]
ϕ_π	2.208	[2.041, 2.360]	2.103	[1.893, 2.348]	1.803	[1.655, 1.946]
ϕ_y	0.123	[0.096, 0.152]	0.196	[0.164, 0.232]	0.103	[0.069, 0.139]
ϕ_r^*	0.745	[0.714, 0.772]	0.794	[0.733, 0.866]	0.699	[0.655, 0.739]
ϕ_π^*	1.428	[1.329, 1.489]	1.651	[1.462, 1.819]	1.380	[1.245, 1.499]
ϕ_y^*	0.085	[0.054, 0.116]	0.151	[0.099, 0.204]	0.089	[0.056, 0.122]
ρ_A	0.542	[0.456, 0.620]	0.481	[0.363, 0.590]	0.332	[0.126, 0.473]
ρ_g	0.983	[0.965, 1.000]	0.862	[0.757, 0.972]	0.553	[0.356, 0.701]
ρ_A^*	0.562	[0.486, 0.644]	0.822	[0.733, 0.928]	0.930	[0.903, 0.953]
ρ_g^*	0.947	[0.920, 0.988]	0.390	[0.245, 0.507]	0.581	[0.502, 0.649]
ρ_Ω	0.997	[0.995, 0.999]	0.955	[0.927, 0.990]	-	-
ρ_{σ_A}	-	-	0.683	[0.588, 0.780]	0.251	[0.090, 0.373]
ρ_{σ_g}	-	-	0.513	[0.373, 0.692]	0.386	[0.268, 0.512]
$\rho_{\sigma_{\epsilon_R}}$	-	-	0.739	[0.612, 0.882]	0.378	[0.304, 0.462]
$\rho_{\sigma_A}^*$	-	-	0.567	[0.454, 0.710]	0.105	[0.061, 0.146]
$\rho_{\sigma_g}^*$	-	-	0.337	[0.193, 0.461]	0.241	[0.156, 0.335]
$\rho_{\sigma_{\epsilon_R}}^*$	-	-	0.362	[0.189, 0.528]	0.356	[0.196, 0.501]
ρ_{σ_Ω}	-	-	0.389	[0.262, 0.498]	-	-
$100\sigma_A$	2.948	[2.218, 3.630]	2.048	[1.452, 2.528]	1.396	[1.014, 1.728]
$100\sigma_g$	8.108	[6.955, 9.136]	9.235	[8.100, 10.928]	4.616	[3.417, 5.520]
$100\sigma_{\epsilon_R}$	0.217	[0.172, 0.268]	0.144	[0.106, 0.186]	0.200	[0.143, 0.253]
$100\sigma_A^*$	1.749	[1.370, 2.117]	5.293	[4.034, 6.461]	11.140	[9.235, 13.468]
$100\sigma_g^*$	4.038	[3.405, 4.580]	7.734	[6.522, 8.799]	8.034	[6.393, 9.945]
$100\sigma_{\epsilon_R}^*$	0.285	[0.148, 0.430]	0.168	[0.107, 0.223]	0.179	[0.133, 0.227]
$100\sigma_\Omega$	6.589	[5.940, 7.360]	4.652	[3.833, 5.407]	-	[-, -]
τ_A	-	-	0.538	[0.408, 0.674]	1.087	[0.782, 1.427]
τ_g	-	-	0.862	[0.545, 1.115]	1.227	[0.851, 1.573]
τ_{ϵ_R}	-	-	1.339	[1.016, 1.686]	0.736	[0.570, 0.888]
τ_A^*	-	-	0.720	[0.582, 0.877]	0.987	[0.894, 1.121]
τ_g^*	-	-	1.162	[0.972, 1.338]	1.430	[1.142, 1.725]
$\tau_{\epsilon_R}^*$	-	-	1.287	[1.032, 1.553]	1.245	[0.930, 1.591]
τ_Ω	-	-	0.635	[0.486, 0.774]	-	-
$\log p(\mathcal{Y}^T)$	-775.060		-807.321		-919.449	

Notes: Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 2,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(\mathcal{Y}^T)$ represents the SMC-based approximation of log marginal data density.

Table 4: Comparison to BBN

Parameter	Our estimate	BBN's calibration
ε	4.33	5.00
σ	1.88	2.00
θ	0.53	0.66
θ^*	0.77	0.75
ϕ_r	0.77	0.76
ϕ_π	2.10	1.41
ϕ_y	0.20	0.66
ϕ_{r^*}	0.79	0.84
ϕ_{π^*}	1.65	1.37
ϕ_{y^*}	0.15	1.27

We present the impulse responses for the set of main observed macro variables (YGR_t, π_t, R_t, YGR_t^*, π_t^*, R_t^*, d_t), nominal interest rate differential ($R_t - R_t^*$), and the excess return on the foreign currency ($d_{t+1} + R_t^* - R_t$). Figure 1 considers a volatility shock to home technology and Figure 2 a volatility shock to home monetary policy, both at the ergordic mean of the state variables based on the (posterior mean) parameter estimates obtained in our baseline estimation. We see that a positive uncertainty shock to home technology causes a currency appreciation upon impact and subsequent depreciation. In response to a home monetary volatility shock, on the other hand, the exchange rate appreciates upon impact, and we see positive deviation from the UIP as demonstrated in BBN.[22] We note also that the quantitative impacts these volatility shocks created are small.

In addition to the macro volatility shocks, Figure 3 presents the impulse responses to the direct (level) risk-sharing shock that we also incorporated into our model. Not surprisingly, the direct shock depreciates the dollar upon impact. We then see a persistent negative deviation from the UIP as foreign currency excess return declines. This direct shock also affects other macroeconomic variables to a substantial degree, especially compared to the marginal quantitative impacts of the volatility shocks.

The impulse responses for the all other shocks are presented in the Appendix.

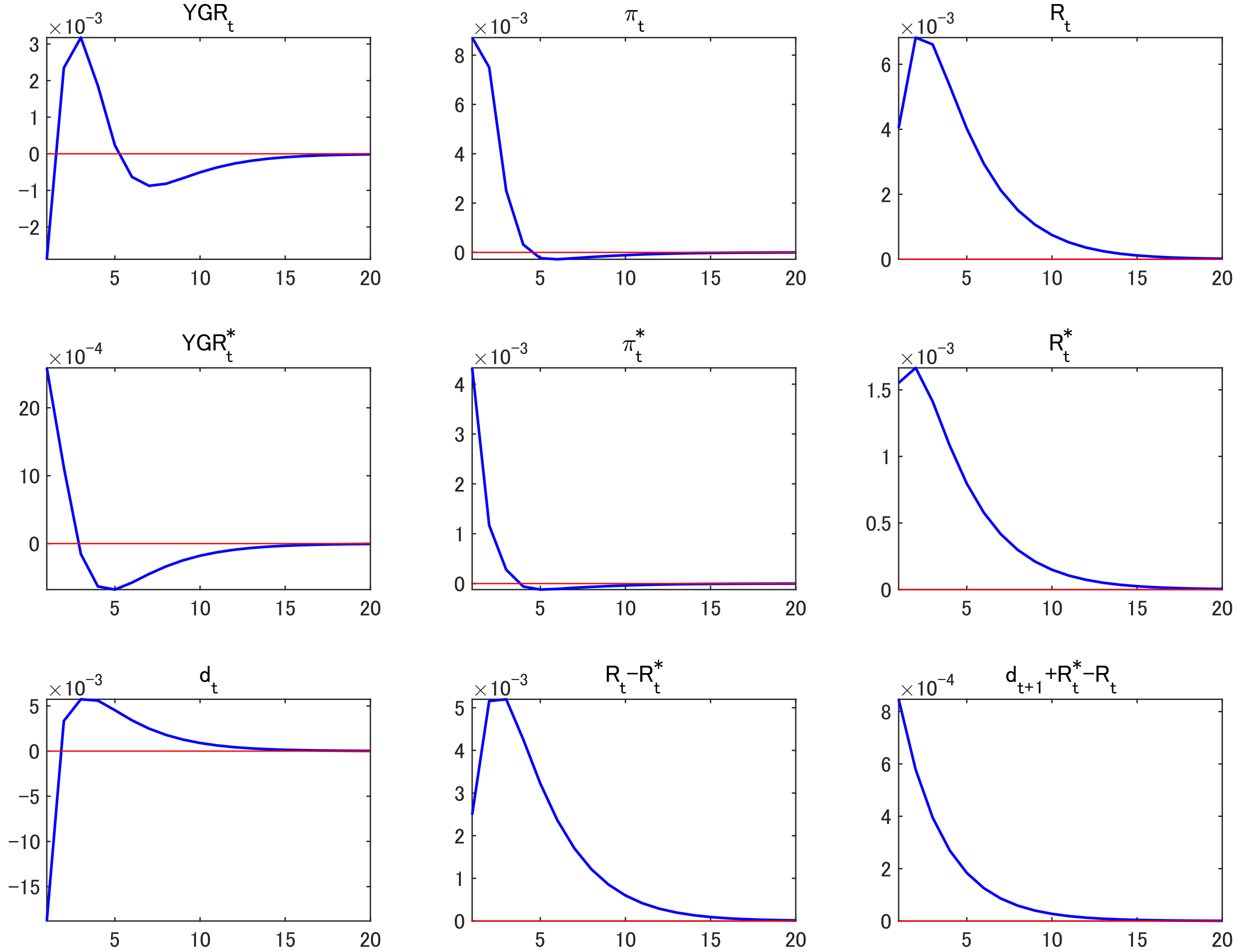
5.3 Accounting for exchange rate volatility

Under log-linearization, the equilibrium conditions presented in section 3 imply the following modified UIP condition, which directly incorporates the risk-sharing shock Ω_t:

$$\hat{R}_t - \hat{R}_t^* = \mathbf{E}_t \hat{d}_{t+1} + \hat{\Omega}_t - \mathbf{E}_t \hat{\Omega}_{t+1}, \quad (8)$$

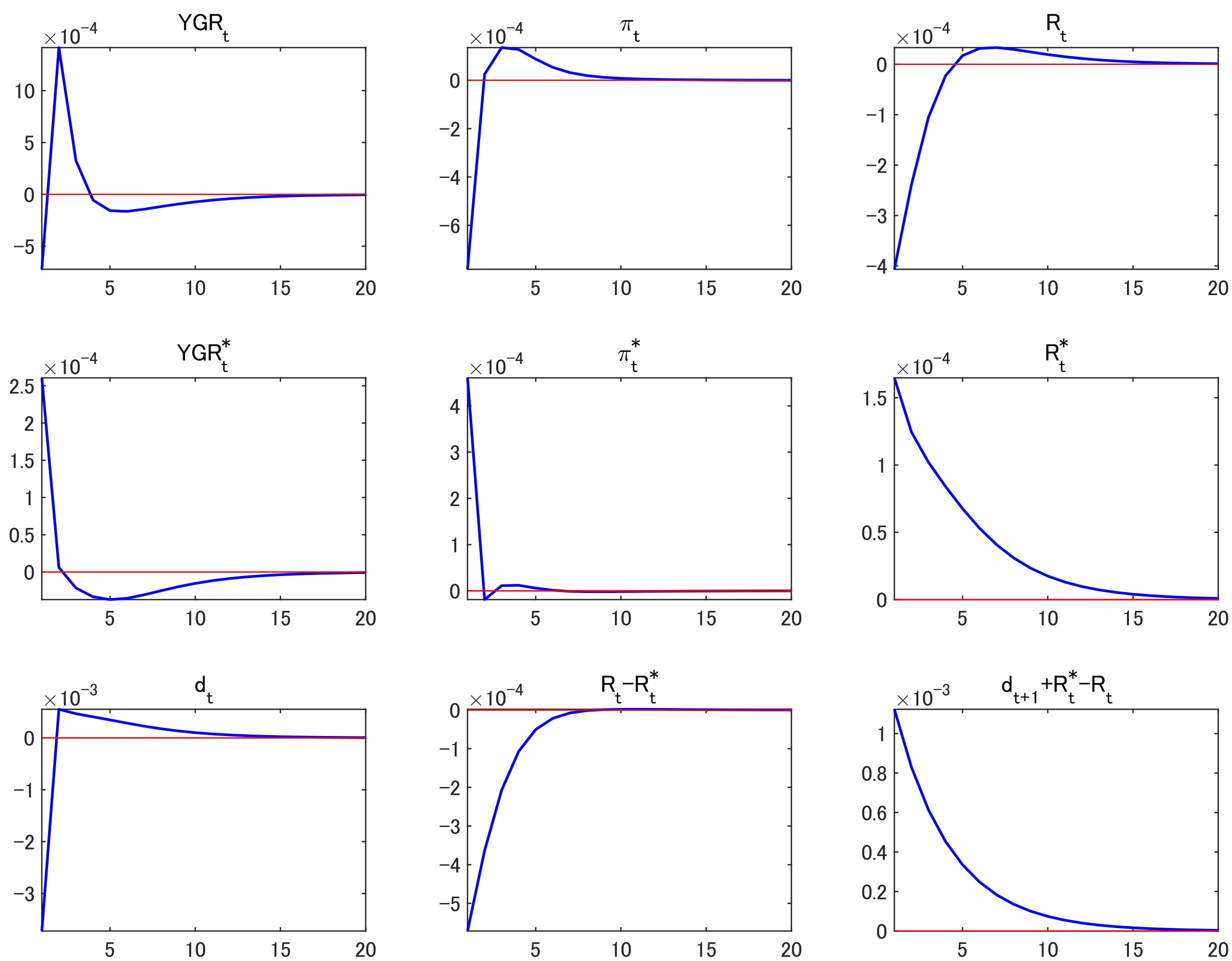
[22] We refer interested readers to BBN for more detailed discussions on the explanations and economic intuitions behind these results.

Figure 1: Responses to volatility shock to home technology



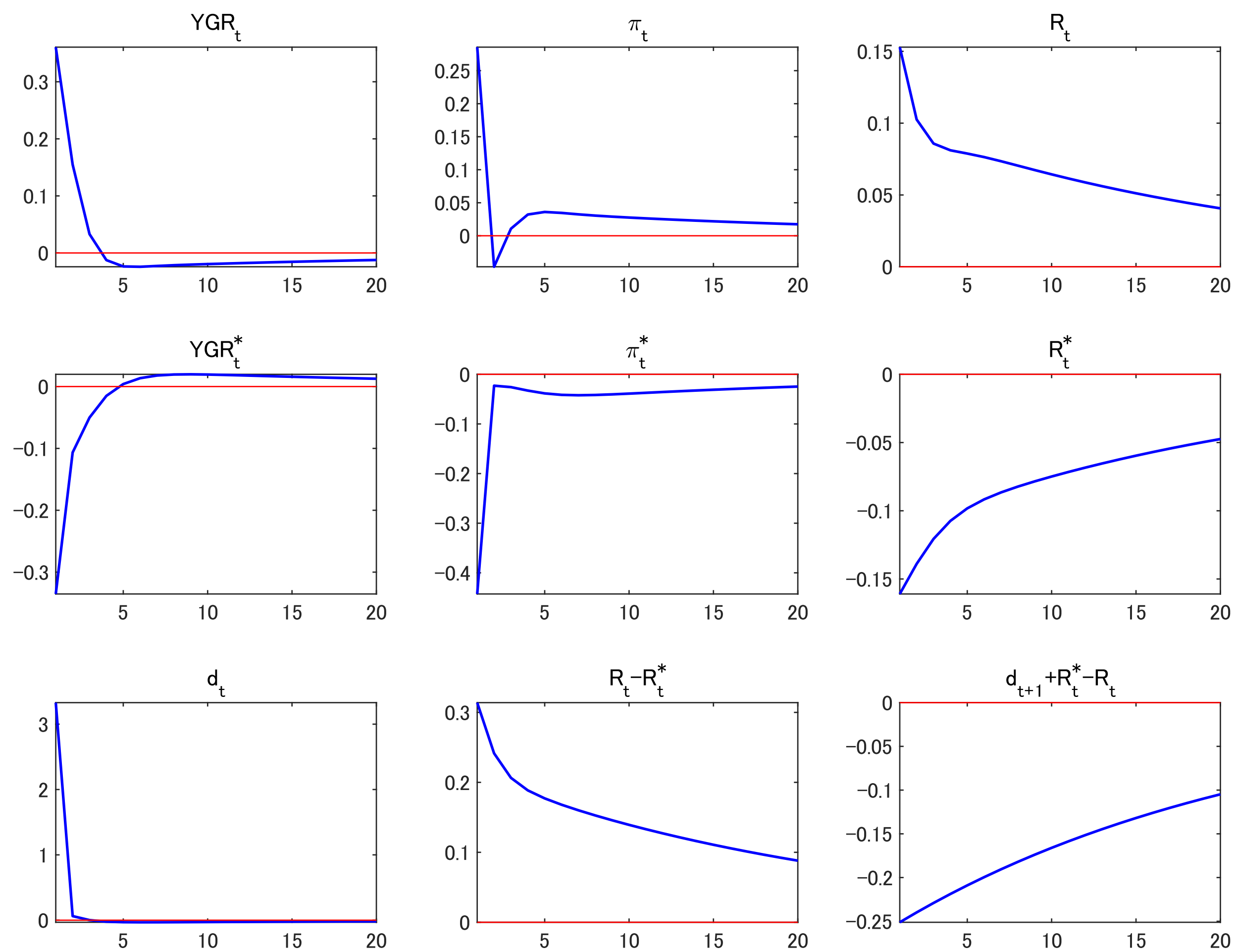
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home technology, given the posterior mean estimates of parameters in the baseline model.

Figure 2: Responses to volatility shock to home monetary policy



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 3: Responses to risk sharing shock



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.

where the circumflex denotes log deviation from the steady state value. We note that since deviations from the UIP are now directly captured by this risk-sharing shock, all deviations from the UIP in the data will be attributable to this risk-sharing shock, implying its contributions to exchange rate dynamics. Moving beyond linearization, on the other hand, we note that the international risk-sharing condition, given by equation (7) earlier, is as follows:

$$\Omega_t Q_t = \left[\frac{C_t^{\psi} \left(1 - N_t\right)^{(1-\psi)}}{\left(C_t^*\right)^{\psi} \left(1 - N_t^*\right)^{(1-\psi)}} \right]^{1-\sigma} \frac{C_t^*}{C_t} s_t,$$

where the time-varying Q_t evolves according to (6). Higher order approximations would thus imply additional higher order terms or endogenous risk premiums in the resulting UIP equation. Compared to in the linear case, the contribution of the risk-sharing shock to overall exchange rate fluctuations is therefore expected to decrease.

To assess the relative contributions of the two channels, we again consider four specifications: 1) a linearized version of the model, 2) the model without stochastic volatilities and approximated to the second order; 3) the model without stochastic volatilities and approximated to the third order, and 4) the full (baseline) model with stochastic volatilities and solved by third-order approximations. We use the posterior mean estimates from each specification to compute the variances of the observed variables the model can explain with *one shock excluded at a time*. We adopt this method because standard variance decompositions would underestimate the contributions of each shock, as they cannot capture cross-terms or interactions among shocks that arise in nonlinear settings. To evaluate the contribution of a shock more appropriately, we measure how its *exclusion* affect the data variations the model can explain. To compute these variances, each model is simulated for 10,100 periods with the first 100 observations discarded.

The numbers reported in each row of Table 5 indicate the fraction of fluctuations in the observed column variable the model can explain without the particular structural shocks shown in the left column. We consider the same set of level shocks in all specifications: u_A, u_g, u_{ϵ_R}, u_A^*, u_g^*, $u_{\epsilon_R}^*$, and u_Ω, denoting shocks to home technology, home external demand, home monetary policy, their foreign counterparts indicated with * superscripts, and also the international risk-sharing condition. In the last panel, we add in the volatility shocks to the respective level shocks: $z_{\sigma A}$, $z_{\sigma g}$, $z_{\sigma \epsilon_R}$, $z_{\sigma A}^*$, $z_{\sigma g}^*$, $z_{\sigma \epsilon_R}^*$, and $z_{\sigma \Omega}$. The last column shows the results for nominal exchange rate changes, d_t. We see that excluding the international risk-sharing shock, the linear model with its remaining macroeconomic shocks can explain only 14% of the exchange rate volatility, implying that 86% of the exchange rate fluctuations are driven by the direct risk-sharing shock. From the 2nd panel, we see that approximating the model to the second-order actually deteriorates the collective contribution from macro shocks slightly, to 11%. Nevertheless, accounting for nonlinearities up to the third order shows substantial improvements in the fraction of exchange rate variance explained;in the final panel, we see that the macro shocks, with their stochastic volatilities incorporated, can explain 43% of the observed exchange rate fluctuations. This result is consistent BBN's findings based on simulations, that macroeconomic uncertainties can induce a time-varying exchange rate risk premium that acts as

Table 5: Relative variances explained by each shock

		$\Delta \log Y_t$	$\log \pi_t$	$\log R_t$	$\Delta \log Y_t^*$	$\log \pi_t^*$	$\log R_t^*$	d_t
Linear								
w/o:	u_A	0.690	0.280	0.382	0.994	0.954	0.940	0.977
	u_g	0.423	0.962	0.793	1.000	0.992	0.997	0.970
	$u_{\epsilon R}$	0.959	0.920	0.984	1.000	0.996	1.000	0.983
	u_A^*	0.985	0.933	0.919	0.667	0.242	0.307	0.963
	u_g^*	0.999	0.995	0.997	0.456	0.966	0.840	0.968
	$u_{\epsilon R}^*$	1.000	0.996	0.999	0.955	0.952	0.985	0.989
	u_Ω	0.925	0.920	0.940	0.929	0.896	0.921	0.141
2nd order								
w/o:	u_A	0.837	0.331	0.377	0.986	0.952	0.913	0.979
	u_g	0.351	0.943	0.807	1.000	0.992	0.989	0.989
	$u_{\epsilon R}$	0.952	0.934	0.971	1.000	0.992	0.998	0.979
	u_A^*	0.979	0.936	0.951	0.590	0.262	0.270	0.965
	u_g^*	0.999	0.999	1.003	0.717	0.992	0.945	0.988
	$u_{\epsilon R}^*$	1.000	0.997	0.999	0.949	0.947	0.984	0.990
	u_Ω	0.886	0.815	0.880	0.730	0.903	0.923	0.105
3rd order								
w/o:	u_A	0.830	0.315	0.290	0.936	0.935	0.910	0.936
	u_g	0.342	0.946	0.868	0.975	0.985	0.967	0.898
	$u_{\epsilon R}$	0.931	0.925	0.959	0.999	0.995	0.998	0.970
	u_A^*	0.986	0.934	0.942	0.621	0.355	0.394	0.989
	u_g^*	0.999	0.998	1.000	0.765	0.976	0.885	0.991
	$u_{\epsilon R}^*$	0.999	0.988	0.997	0.821	0.790	0.964	0.932
	u_Ω	0.952	0.855	0.917	0.825	0.957	0.865	0.279
3rd order with SV								
w/o:	$z_{\sigma A}$	0.905	0.734	0.815	0.998	0.991	0.997	0.984
	$z_{\sigma g}$	0.480	0.952	0.808	1.000	0.996	0.998	0.967
	$z_{\sigma \epsilon_R}$	0.913	0.692	0.957	1.000	0.996	1.000	0.920
	$z_{\sigma A}^*$	0.996	0.963	0.871	0.823	0.457	0.427	0.877
	$z_{\sigma g}^*$	0.999	0.999	0.998	0.475	0.994	0.993	0.998
	$z_{\sigma \epsilon_R}^*$	1.000	0.987	0.992	0.946	0.938	0.976	0.933
	$z_{\sigma\Omega}$	0.986	0.971	0.940	0.997	0.964	0.959	0.711
	u_A, $z_{\sigma A}$	0.830	0.523	0.664	0.998	0.984	0.993	0.975
	u_g, $z_{\sigma g}$	0.295	0.932	0.722	1.000	0.994	0.994	0.956
	$u_{\epsilon R}$, $z_{\sigma \epsilon_R}$	0.907	0.673	0.956	1.000	0.996	1.000	0.915
	u_A^*, $z_{\sigma A}^*$	1.003	0.936	0.777	0.717	0.169	0.108	0.816
	u_g^*, $z_{\sigma g}^*$	0.999	0.998	0.998	0.338	0.992	0.988	0.997
	$u_{\epsilon R}^*$, $z_{\sigma \epsilon_R}^*$	1.000	0.985	0.992	0.940	0.927	0.975	0.926
	u_Ω, $z_{\sigma\Omega}$	0.976	0.932	0.859	0.995	0.934	0.941	0.425

Notes: The table shows the variances of the output growth rate, the inflation rate, the nominal interest rate in the home and foreign countries, and the nominal exchange rate depreciation excluding each shock, relative to those with all the shocks, given the posterior mean estimates of parameters.

a key source behind exchange rate fluctuations.[23]

Last but not least, our results show that despite significant contributions from the macro side, the direct risk-sharing shock still accounts for more than half (57%) of the exchange rate fluctuations. These findings support the arguments put forth in e.g. Itskhoki and Mukhin (2017) on the importance of financial frictions in accounting for the exchange rate dynamics and aggregate fluctuations in the open economy. To confirm this point, we re-estimate the baseline model *excluding* the direct risk-sharing shock, and report the estimates in the last two columns of Table 3. We see that the price indexation parameters and several $AR(1)$ coefficients all become larger, to account for the persistence in the observed variables that was captured by the risk-sharing shock in the baseline specification. The log marginal data density $\log p(\mathcal{Y}^T)$ is also substantially lower (-919.4) than that in the baseline estimation (-807.3), indicating a much worse model fit.

We note that the high (estimated) persistence of the risk-sharing shock is likely behind why it plays such a significant role in explaining exchange rate fluctuations. As reported in Tables 2 and 3, the mean estimates for the $AR(1)$ coefficient ρ_Ω are all around 0.99 under the three specifications without stochastic volatilities. Even with stochastic volatilities incorporated, the coefficient estaimte remains high at 0.96. This dynamics appears consistent with the (near) random-walk behavior of nominal exchange rates in the data.

Overall, our results show that by allowing higher order approximations and stochastic volatilities, the exchange rate behavior is no longer disconnected from the macroeconomy. Yet, our findings also point to the indispensable role a direct risk-sharing shock plays in explaining most of variations in the Euro-dollar exchange rate.

5.4 The UIP puzzle

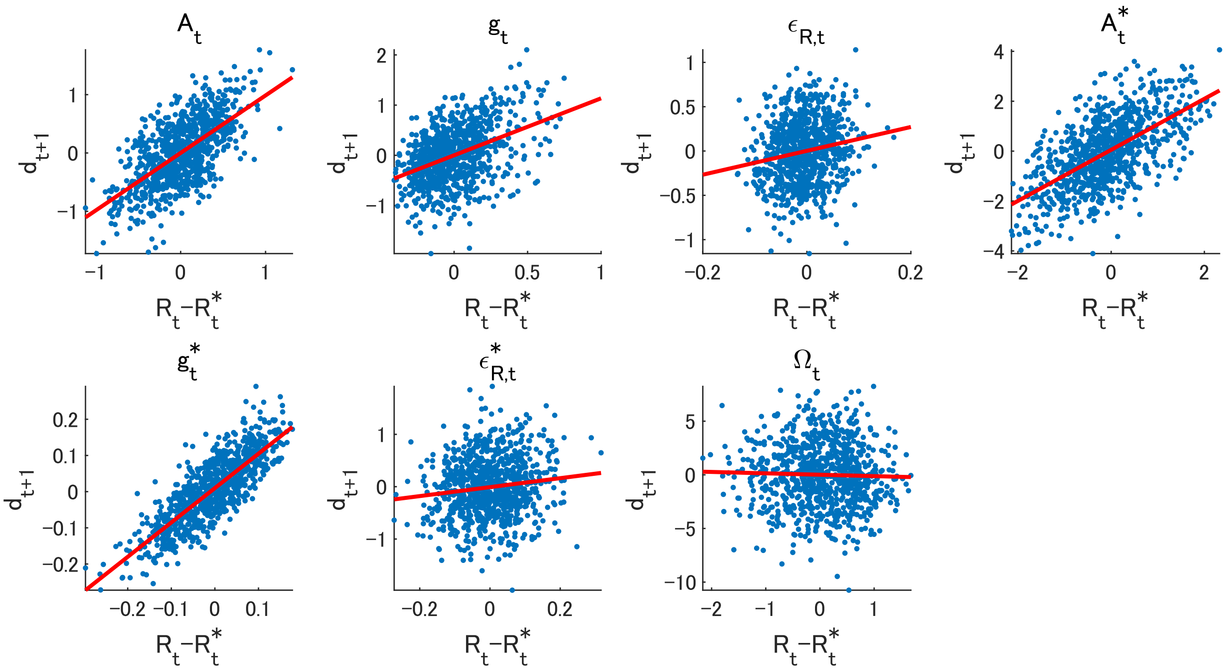
We now turn to examine how the different sources of shocks may be behind the UIP puzzle, namely the negative Fama coefficient observed in the data. We construct a set of artificial time-series data by shutting down all but a single shock each time, and examine which individual shock can replicate the empirical negative correlation between interest rate differentials and subsequent nominal exchange rate changes (i.e. high relative interest rate currency appreciates subsequently.)

Considering the set of level shocks first, Figure 4 presents plots of the UIP regressions using simulated series of the exchange rate depreciation d_{t+1} and the nominal interest rate differentials $R_t - R_t^*$ driven by a single shock, indicated at the top of the graph.[24] We note that *all* of the macro level shocks - to home and foreign technology (A_t and A_t^*), demand (g_t and g_t^*), and monetary policy ($\varepsilon_{R,t}$ and $\varepsilon_{R,t}^*$) - generate a *positive* slope coefficient, in

[23] These finding also support the view in Bloom (2009) and Fernandez-Villaverde, Guerron-Quintana, Kuester, and Rubio-Ramirez (2015) that uncertainties can play a major role in explaining aggregate economic fluctuations observed in the data.

[24] Given the posterior mean estimates of parameters in the baseline estimation, the model is simulated for 1,100 periods, and the first 100 observations are discarded.

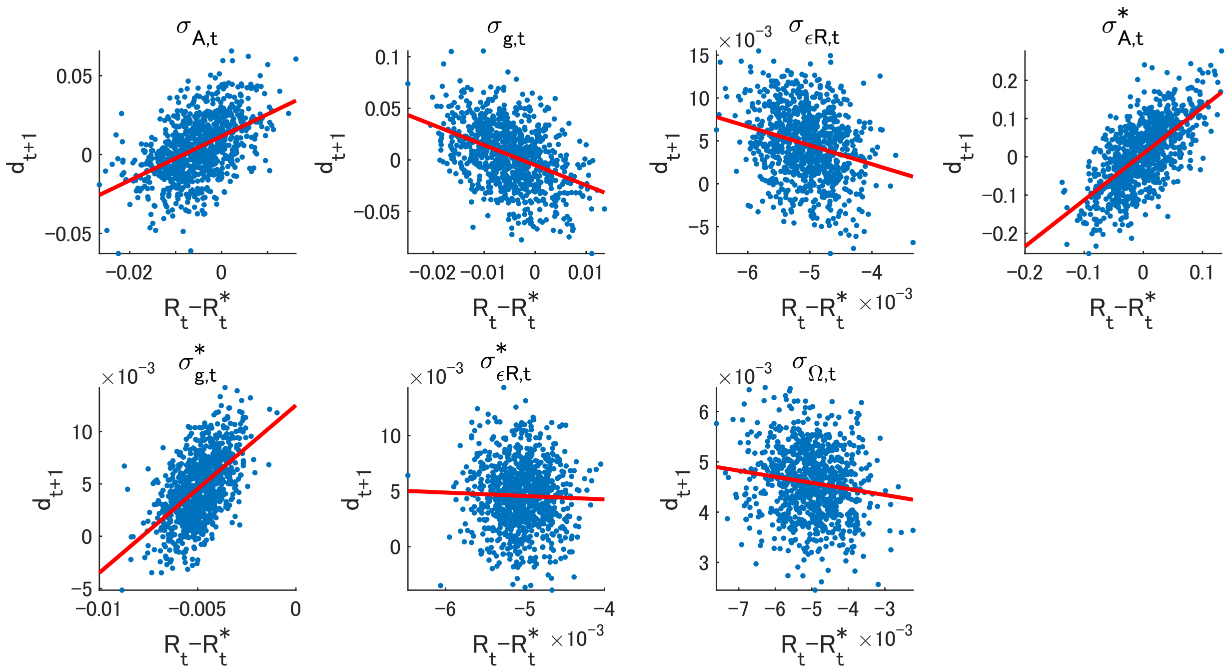
Figure 4: UIP regressions based on simulated series driven by each level shock



line with the theoretical no-arbitrage implication of the UIP, and thus unable to account for the contrary empirical regularities This outcome is not surprising as it is consistent with the robustness of the UIP puzzle found in previous literature. The notable figure is that last one, where the risk-sharing shock, Ω_t, generates a Fama coefficient in line with the empirics: close to and slightly below zero. This reaffirms our observation based on the variance decomposition analyses above: the direct risk-sharing shock may play a key role in explaining exchange rate dynamics.

We next look at how the volatility shocks perform in replicating the empirical negative- to no-correlation between interest rate differentials and exchange rate changes. Figure 5 depicts UIP regression results using simulated series driven by one individual volatility shock at a time (together with the corresponding level shock). Here we see more promising results: while the volatility shocks to technology ($\sigma_{A,t}$ and $\sigma^*_{A,t}$) and foreign demand ($\sigma^*_{g,t}$) generate positive Fama coefficients, volatility shocks to monetary policy both at home and abroad ($\sigma_{\varepsilon R,t}$ and $\sigma^*_{\varepsilon R,t}$), to home demand, and to the risk-sharing wedge ($\sigma_{\Omega,t}$) all replicate the negative UIP slope coefficients observed in the literature. [DOUBLE-CHECK or delete: The results for the monetary policy uncertainty shocks also confirm the mechanism proposed in BBN. Higher uncertainty in nominal shocks makes the home currency a good hedge (less risky), while at the same time can raise domestic nominal interest rates under certain monetary policy rules. This would a imply positive excess return for the carry-trade strategy, since risk premium compensates investors for holding the riskier foreign currency. In addition, as explained in Section 2.2, the relative sizes of the persistence parameters between monetary policy and stochastic volatilities are cru-

Figure 5: UIP regressions based on simulated series driven by each volatility shock



cial for generating the target exchange rate dynamics. Backus, Gavazzoni, Telmer, and Zin (2010) and BBN emphasize that the slope of the UIP regression is more likely to be negative when interest rate smoothing is more active and the persistence of the nominal volatility shock is low.[25]

5.5 The general equilibrium puzzle

The above exercises have identified several shocks that can replicate the empirical regularity of a mildly negative Fama coefficient: 1) level and the volatility shocks to the risk-sharing condition, 2) volatility shocks to home external demand, and 3) volatility shocks to both home and foreign monetary policy. This set of shocks generally echo findings and mechanisms proposed in previous papers that rely on simulations with calibrated parameters. However, we note that these findings are all based on partial equilibrium or conditional analyses, where all shocks but the proposed one are assumed to be absent. The actual empirical UIP puzzle, on the other hand, is a pattern that manifests unconditionally in general equilibrium. As such, the actual empirical relevance of these proposed mechanisms, in the general equilibrium context, needs to be further evaluated. Put it dif-

[25]There are more stringent restrictions in deriving an exact condition for generating a negative slope coefficient, such as low price stickiness, symmetric policy rules, and little interference from real shocks. We do not test the conditions directly but conducted simulation exercises with a model with low price stickiness, symmetric monetary policy rules, and no risk-sharing shock. We confirm that the higher the monetary policy intertia, the lower the Fama coefficient; our simple exercise did not find changes in the persistence of nominal stochastic volatility shocks to affect the coefficient estimate by much.

Table 6: Fama coefficients in actual vs. simulated data

	Fama Coeff. $\widehat{a}_1$	$95\%CI$	R^2
data	0.0477	$[-1.4919, \quad 1.5873]$	0.00
simulation with all shocks	0.7049	$[0.5302, \quad 0.8796]$	0.06
simulation without Ω_t	1.0839	$[0.9945, \quad 1.1732]$	0.36

ferently for our particular context, besides the set of shocks that produced the desired negative Fama coefficient, there is the complement set of shocks that support the theoretical UIP condition instead. Their relative contributions in general equilibrium need to be assessed in order to determine whether the proposed models and mechanisms are empirically relevant.

Our paper has the unique advantage that our full model is estimated directly in general equilibrium, so all parameter values are obtained to fit not just one target variable (e.g. the exchange rate) but the full set of relevant open-economy macro dynamics.[26] Given these estimates, we can further examine the aggregate impact of all the proposed shocks, to determine whether the successes in conditional analyses extend quantitatively to general equilibrium settings. To this end, we first discuss the stochastic volatilities identified as potential solutions to the UIP puzzle. We see from the impulse responses presented above that the effects of volatility shocks on the exchange rate (and also on other macro variables) is miniscule in magnitude, especially compared to the impact the international risk-sharing (level) shock Ω_t creates. We thus conduct the general equilibrium analysis focusing on the risk-sharing shock Ω_t instead, as the quantitative impact of volatility shocks on macro variables will be dominated. To do so, we use the parameter estimates from the model to generate simulation data, and see if the with all the shocks together, their aggregate impact still implies a Fama coefficient close to the one from the actual data.

Table 6 presents the estimated Fama regression coefficient $\widehat{a}_1$ based on three sets of data: 1) the actual data; 2) simulated data from the baseline model with all shocks and 3rd-order model approximation; and 3) simulated data from the model excluding the risk-sharing shock Ω_t. We see that while incorporating the risk-sharing shock does slower the Fama coefficient (from 1.08 to 0.70), the baseline model with the full set of shocks all together still generate a Fama slope coefficient that is significantly positive.

The finding from the GE analysis leaves us with ”the general equilibrium puzzle” of exchange rate dynamics. While our results show that the exchange rate is not disconnected from macro fundamentals, and that the risk-sharing shock can explain a large

[26]The importance of this ”multiple target” aspect of the general equilibrium estimation is well-understood in the broader macroeconomic literature. Models and mechanisms that fit one aspect of the economy, e.g. dynamics of the real variables, can fail miserably at explaining another, e.g. asset returns. As discussed earlier, we choose to incorporate stochastic volatilities and recursive preferences as they have been shown in the close-economy settings to provide reasonable fit of both real variables and asset returns.

fraction of the overall exchange rate volatility, their collective impact on actual exchange rate, unconditionally, is not quantitatively large enough to resolve the UIP puzzle. In other words, results derived from partial equilibrium or conditional analyses can offer important qualitative insights about potential transmission mechanism, but their overall quantitative relevance needs to be evaluated in general equilibrium, due to what the presence of multiple shocks as well as multiple time series dynamics to explain simultaneously.

6 Conclusion

In this paper, we have estimated the two country New Keynesian model with the recursive preferences and stochastic volatilities using higher order approximation and the central difference Kalman filter. According to the estimation results, the direct shock to the international risk-sharing condition, which represents the time-varying financial frictions that hinder the international arbitrage, is a major driver for the observed exchange rate dynamics and aggregate fluctuations. We also find that macroeconomic shocks, together with shocks to their volatility, can explain a significant portion of dollar-euro dynamics as well. By allowing for higher-order terms and volatility shocks, we thus reconnect the exchange rate with the macroeconomy, while also providing empirical support, via direct general equilibrium estimations, that some sources of arbitrage friction play a major role in driving exchange rate dynamics. The exact micro-foundation behind this direct shock warrants additional investigation in general equilibrium, so are other mechanisms that can generate endogenous risk premiums. Additional mechanisms, such as the stochastic volatilities of *news shocks*, following ideas developed in Fujiwara, Hirose, and Shintani (2011) and Schmitt-Grohe and Uribe (2012), may be incorporated into the nonlinear of GE estimations using the framework presented in this paper. From a structural perspective, one could consider a setup with local currency pricing as in Betts and Devereux (2000) and Devereux and Engel (2002), which tend to generate more disconnected exchange rate behavior.

Most importantly, our explorations in hope of resolving the UIP puzzle instead point to an important observation that we call the general equilibrium puzzle. We show that while certain shocks or mechanisms may work well in partial equilibrium analyses and offer important insights into the possible mechanisms behind certain empirical observations, they ultimate relevance in explaining the actual data would still need to be evaluated in general equilibrium, where multiple shocks are present, and multiple observable series are to be explained simultaneously To this end, our model that combined the key insights from two recent strands of literature is not successful in overturning the UIP puzzle.

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A Steady state

To avoid nonstationarity, we need to assume

$$\bar{\pi} = \bar{\pi}^*$$

at the steady state.

We parameterize g/y instead of g. Thus, Then, g that appears in the subsequent steady-state conditions are given by

$$g = \frac{\frac{\psi(\mu-1)}{\mu-\psi}}{\left(\frac{\bar{g}}{\bar{y}}\right)^{-1} + \frac{\psi(\mu-1)}{\mu-\psi} - 1}.$$

A.1 Domestic

$$\pi = \bar{\pi},$$

$$p_H = 1,$$

$$\pi_H = \bar{\pi},$$

$$R = \frac{\bar{\pi}}{\beta\gamma^{\psi(1-\sigma)-1}},$$

$$m = \beta\gamma^{\psi(1-\sigma)-1},$$

$$\tilde{w} = \frac{\mu-1}{\mu},$$

$$c = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right),$$

$$N = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right) + g,$$

$$y = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g\right) + g,$$

$$c_H = \left(1-\alpha\right)c,$$

$$c_F = \alpha c,$$

$$v = \left\{\frac{\left[c^{\psi}\left(1-N\right)^{1-\psi}\right]^{1-\sigma}}{1-\beta\gamma^{\psi}}\right\}^{\frac{1}{1-\sigma}}$$

$$f = \frac{y}{2\left(1-\theta\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$k = \frac{y}{2\left(1-\theta\beta\gamma^{\psi(1-\sigma)}\right)},$$

$$\Delta = 1,$$
$$YGR = \gamma.$$

A.2 Foreign

$$\pi^* = \bar{\pi}^*,$$
$$p_F^* = 1,$$
$$\pi_F^* = \bar{\pi}^*,$$
$$R^* = \frac{\bar{\pi}^*}{\beta\gamma^{\psi(1-\sigma)-1}},$$
$$m^* = \beta\gamma^{\psi(1-\sigma)-1},$$
$$\tilde{w}^* = \frac{\mu-1}{\mu},$$
$$c^* = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g^*\right),$$
$$N^* = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g^*\right)+g^*,$$
$$y^* = \frac{\psi(\mu-1)}{\mu-\psi}\left(1-g^*\right)+g^*,$$
$$c_H^* = \alpha c^*,$$
$$c_F^* = \left(1-\alpha\right)c^*,$$
$$v^* = \left\{\frac{\left[c^{*\psi}\left(1-N^*\right)^{1-\psi}\right]^{1-\sigma}}{1-\beta\gamma^{\psi}}\right\}^{\frac{1}{1-\sigma}},$$
$$f^* = \frac{y^*}{2\left(1-\theta^*\beta\gamma^{\psi(1-\sigma)}\right)},$$
$$k^* = \frac{y^*}{2\left(1-\theta^*\beta\gamma^{\psi(1-\sigma)}\right)},$$
$$\Delta^* = 1,$$
$$YGR^* = \gamma.$$

A.3 International

$$s = 1,$$
$$Q = 1,$$

and

$$d = 1.$$

B Detrended system of equations

The detrended system of equations consists of 35 equations as shown below.

B.1 Domestic

$$c_t := \left[(1-\alpha)^{\frac{1}{\eta}} c_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} c_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

$$v_t^{1-\sigma} = \left[c_t^{\psi} (1-N_t)^{1-\psi} \right]^{1-\sigma} + \beta\gamma^{\psi} \left(\mathbf{E}_t \left[v_{t+1}^{1-\varepsilon} \right] \right)^{\frac{1-\sigma}{1-\varepsilon}},$$

$$\log\left(\frac{R_t}{R}\right) = \phi_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\phi_r)\left[\phi_\pi \log\left(\frac{\pi_t}{\bar{\pi}}\right) + \phi_y \log\left(\frac{y_t}{y_{t-1}}\right) \right] + \log(\varepsilon_{R,t}),$$

$$c_{H,t} = (1-\alpha)\, p_{H,t}^{-\eta} c_t,$$

$$c_{F,t} = \alpha \left(s_t p_{F,t}^{*} \right)^{-\eta} c_t,$$

$$c_t = \frac{\psi}{1-\psi} (1-N_t)\, \tilde{w}_t,$$

$$1 = \mathbf{E}_t m_{t,t+1} \frac{R_t}{\pi_{t+1}},$$

$$m_{t,t+1} = \beta \left[\mathbf{E}_t (v_{t+1})^{1-\varepsilon} \right]^{\frac{\varepsilon-\sigma}{1-\varepsilon}} (v_{t+1})^{\sigma-\varepsilon} \gamma^{\psi(1-\sigma)-1} \frac{c_{t+1}^{\psi(1-\sigma)-1} (1-N_{t+1})^{(1-\psi)(1-\sigma)}}{c_t^{\psi(1-\sigma)-1} (1-N_t)^{(1-\psi)(1-\sigma)}},$$

$$\pi_{H,t} = \frac{p_{H,t}\pi_t}{p_{H,t-1}},$$

$$f_t = \frac{1}{2} p_{H,t} \left(c_{H,t} + g_t + c_{H,t}^{*} \right) + \gamma\theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota} \pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{1-\mu} f_{t+1},$$

$$k_t = \frac{1}{2} \frac{\mu}{\mu-1} \frac{\tilde{w}_t}{A_t} \left(c_{H,t} + g_t + c_{H,t}^{*} \right) + \gamma\theta \mathbf{E}_t m_{t,t+1} \left(\frac{\bar{\pi}^{1-\iota} \pi_{H,t}^{\iota}}{\pi_{H,t+1}} \right)^{-\mu} k_{t+1},$$

$$y_t = A_t N_t,$$

$$\left[\frac{1 - \theta \left(\frac{\bar{\pi}^{1-\iota} \pi_{H,t-1}^{\iota}}{\pi_{H,t}} \right)^{1-\mu}}{1-\theta} \right]^{\frac{1}{1-\mu}} f_t = k_t,$$

$$y_t = \Delta_t \left(c_{H,t} + g_t + c_{H,t}^{*} \right),$$

$$\Delta_t = (1-\theta)\left[\frac{1-\theta\left(\frac{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}{\pi_{H,t}}\right)^{1-\mu}}{1-\theta}\right]^{\frac{\mu}{\mu-1}} + \theta\left(\frac{\pi_{H,t}}{\bar{\pi}^{1-\iota}\pi_{H,t-1}^{\iota}}\right)^{\mu}\Delta_{t-1},$$

$$YGR_t := \gamma\frac{y_t}{y_{t-1}}.$$

B.2 Foreign

$$c_t^* := \left[(\alpha)^{\frac{1}{\eta}}\left(c_{H,t}^*\right)^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}}\left(c_{F,t}^*\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$(v_t^*)^{1-\sigma} = \left[(c_t^*)^{\psi}\left((1-N_t)^*\right)^{1-\psi}\right]^{1-\sigma} + \beta\gamma^{\psi}\left(\mathbf{E}_t\left[\left(v_{t+1}^*\right)^{1-\varepsilon}\right]\right)^{\frac{1-\sigma}{1-\varepsilon}},$$

$$\log\left(\frac{R_t^*}{R^*}\right) = \phi_r^*\log\left(\frac{R_{t-1}^*}{R^*}\right) + (1-\phi_r^*)\left[\phi_\pi^*\log\left(\frac{\pi_t^*}{\bar{\pi}^*}\right) + \phi_y^*\log\left(\frac{y_t^*}{y_{t-1}^*}\right)\right] + \log(\varepsilon_{R,t}^*),$$

$$c_{H,t}^* = \alpha\left(\frac{p_{H,t}}{s_t}\right)^{-\eta}c_t^*,$$

$$c_{F,t}^* = (1-\alpha)\left(p_{F,t}^*\right)^{-\eta}c_t^*,$$

$$c_t^* = \frac{\psi}{1-\psi}\left(1-N_t^*\right)\tilde{w}_t^*,$$

$$1 = \mathbf{E}_t m_{t,t+1}^*\frac{R_t^*}{\pi_{t+1}^*},$$

$$m_{t,t+1}^* = \beta\left(\mathbf{E}_t\left(v_{t+1}^*\right)^{1-\varepsilon}\right)^{\frac{\varepsilon-\sigma}{1-\varepsilon}}\left(v_{t+1}^*\right)^{\sigma-\varepsilon}\gamma^{\psi(1-\sigma)-1}\frac{\left(c_{t+1}^*\right)^{\psi(1-\sigma)-1}\left(1-N_{t+1}^*\right)^{(1-\psi)(1-\sigma)}}{(c_t^*)^{\psi(1-\sigma)-1}\left(1-N_t^*\right)^{(1-\psi)(1-\sigma)}},$$

$$\pi_{F,t}^* = \frac{p_{F,t}^*\pi_t^*}{p_{F,t-1}^*},$$

$$f_t^* = \frac{1}{2}p_{F,t}^*\left(c_{F,t} + c_{F,t}^* + g_t^*\right) + \gamma\theta^*\mathbf{E}_t m_{t,t+1}^*\left[\frac{\left(\bar{\pi}^*\right)^{1-\iota}\left(\pi_{F,t}^*\right)^{\iota}}{\pi_{F,t+1}^*}\right]^{1-\mu} f_{t+1}^*,$$

$$k_t^* = \frac{1}{2}\frac{\mu}{\mu-1}\frac{\tilde{w}_t^*}{A_t^*}\left(c_{F,t} + c_{F,t}^* + g_t^*\right) + \gamma\theta^*\mathbf{E}_t m_{t,t+1}^*\left[\frac{\left(\bar{\pi}^*\right)^{1-\iota}\left(\pi_{F,t}^*\right)^{\iota}}{\pi_{F,t+1}^*}\right]^{-\mu} k_{t+1}^*,$$

$$y_t^* = A_t^* N_t^*,$$

$$\left[\frac{1-\theta^*\left(\frac{\left(\bar{\pi}^*\right)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}^*}\right)^{1-\mu}}{1-\theta^*}\right]^{\frac{1}{1-\mu}} f_t^* = k_t^*,$$

$$y_t^* = \Delta_t^*\left(c_{F,t} + c_{F,t}^* + g_t^*\right),$$

$$\Delta_t^* = (1-\theta^*)\left[\frac{1-\theta^*\left[\frac{(\bar{\pi}^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}{\pi_{F,t}}\right]^{1-\mu}}{1-\theta^*}\right]^{\frac{\mu}{\mu-1}} + \theta^*\left[\frac{\pi_{F,t}^*}{(\bar{\pi}^*)^{1-\iota}\left(\pi_{F,t-1}^*\right)^{\iota}}\right]^{\mu}\Delta_{t-1}^*,$$

$$YGR_t^* := \gamma\frac{y_t^*}{y_{t-1}^*}.$$

B.3 International

$$c_t^{\psi(1-\sigma)-1}(1-N_t)^{(1-\psi)(1-\sigma)}s_t = \Omega_t Q_t (c_t^*)^{\psi(1-\sigma)-1}(1-N_t^*)^{(1-\psi)(1-\sigma)},$$

$$Q_{t+1} = Q_t\left(\frac{\left(v_{t+1}^*\right)^{1-\varepsilon}\mathbf{E}_t\left(v_{t+1}\right)^{1-\varepsilon}}{\left(v_{t+1}\right)^{1-\varepsilon}\mathbf{E}_t\left(v_{t+1}^*\right)^{1-\varepsilon}}\right)^{\frac{\sigma-\varepsilon}{1-\varepsilon}},$$

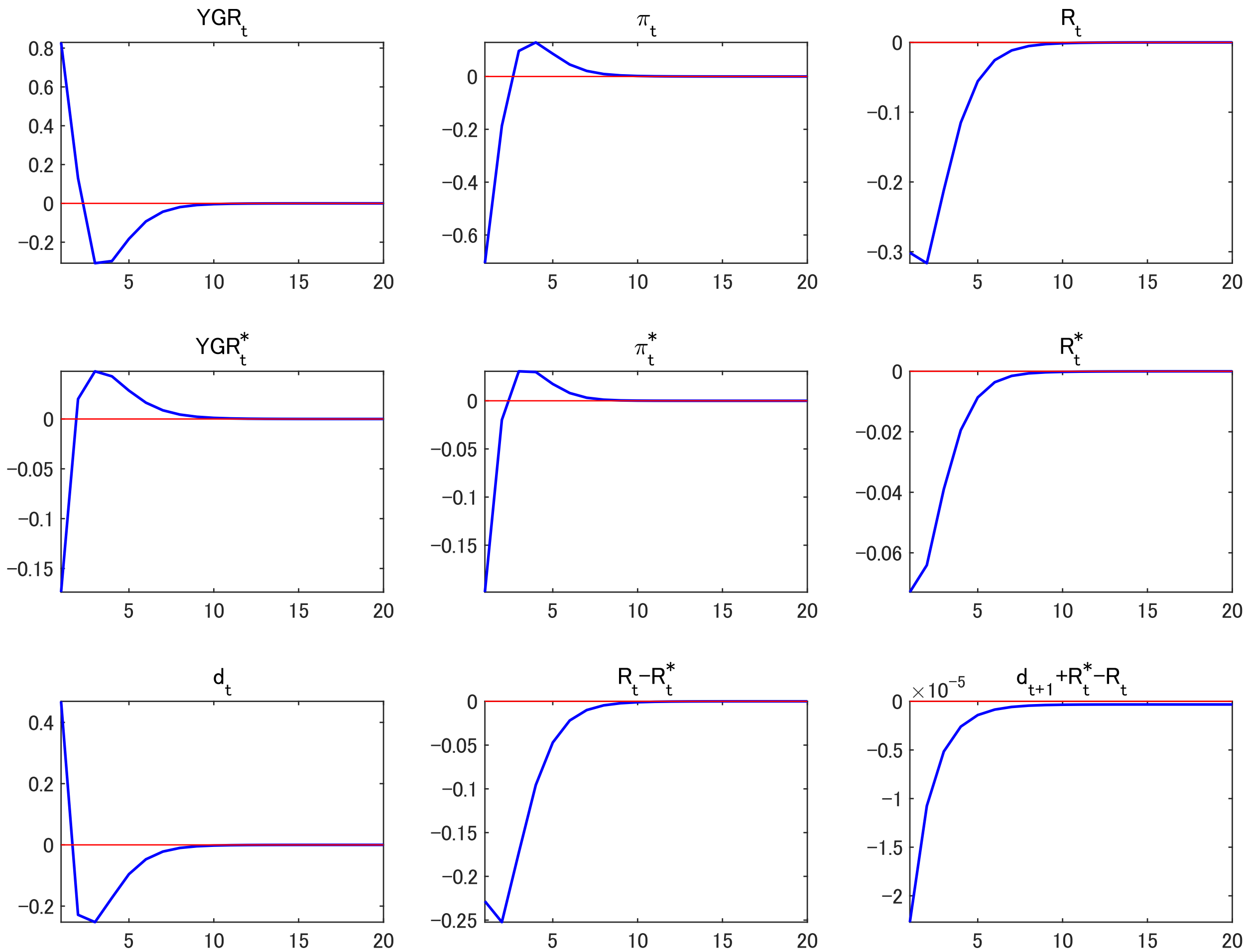
and

$$d_t = \frac{s_t\pi_t}{s_{t-1}\pi_t^*}.$$

C Impulse responses to the other shocks

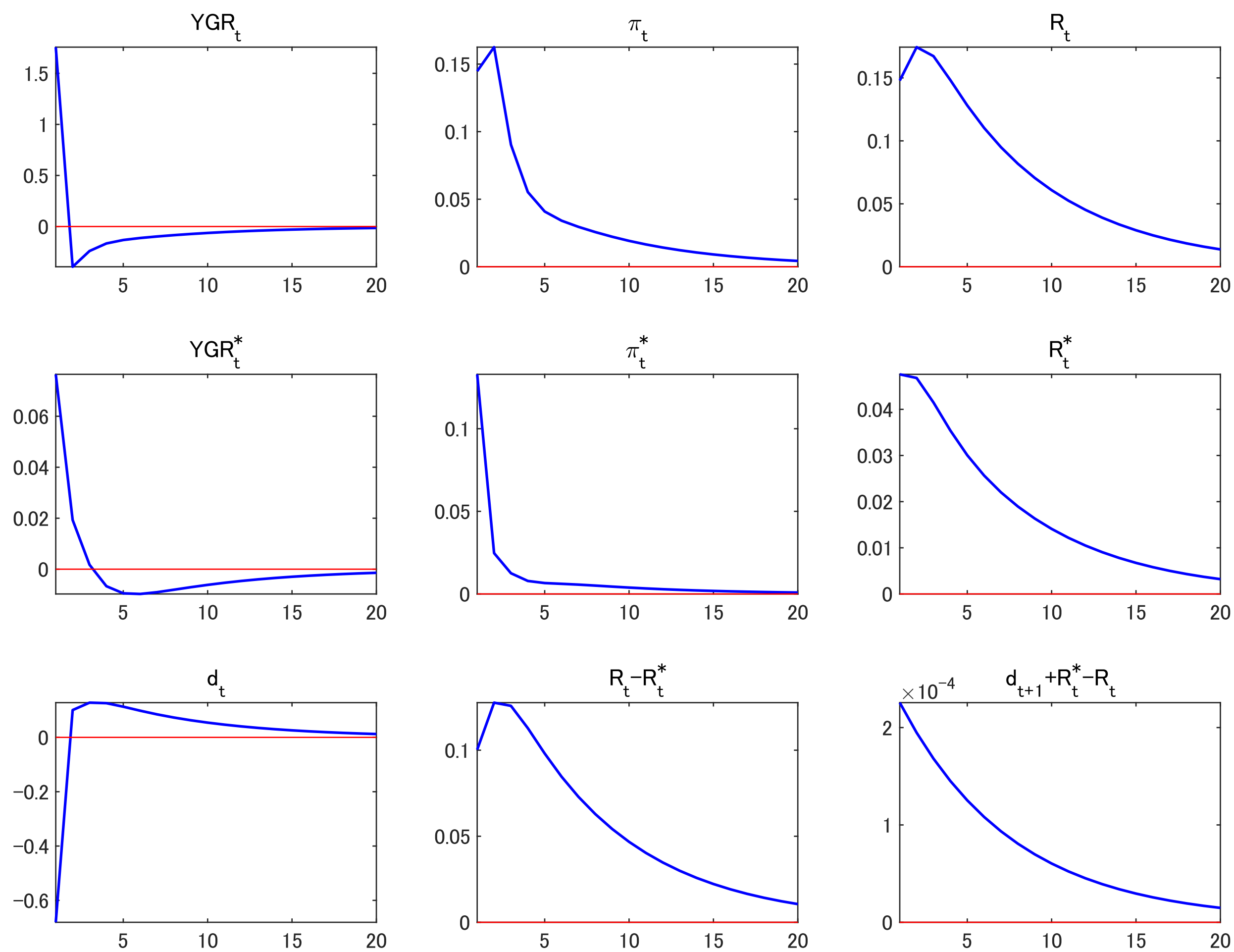
In what follows, the figures 6–16 show the impulse responses of the observed variables (YGR_t, π_t, R_t, YGR_t^*, π_t^*, R_t^*, d_t), nominal interest rate differential ($R_t - R_t^*$), and the excess return on the foreign currency ($d_{t+1} + R_t^* - R_t$) to the other shocks that are not reported in Section 5.2, given the posterior mean estimates of parameters in the baseline estimation.

Figure 6: Responses to home technology shock



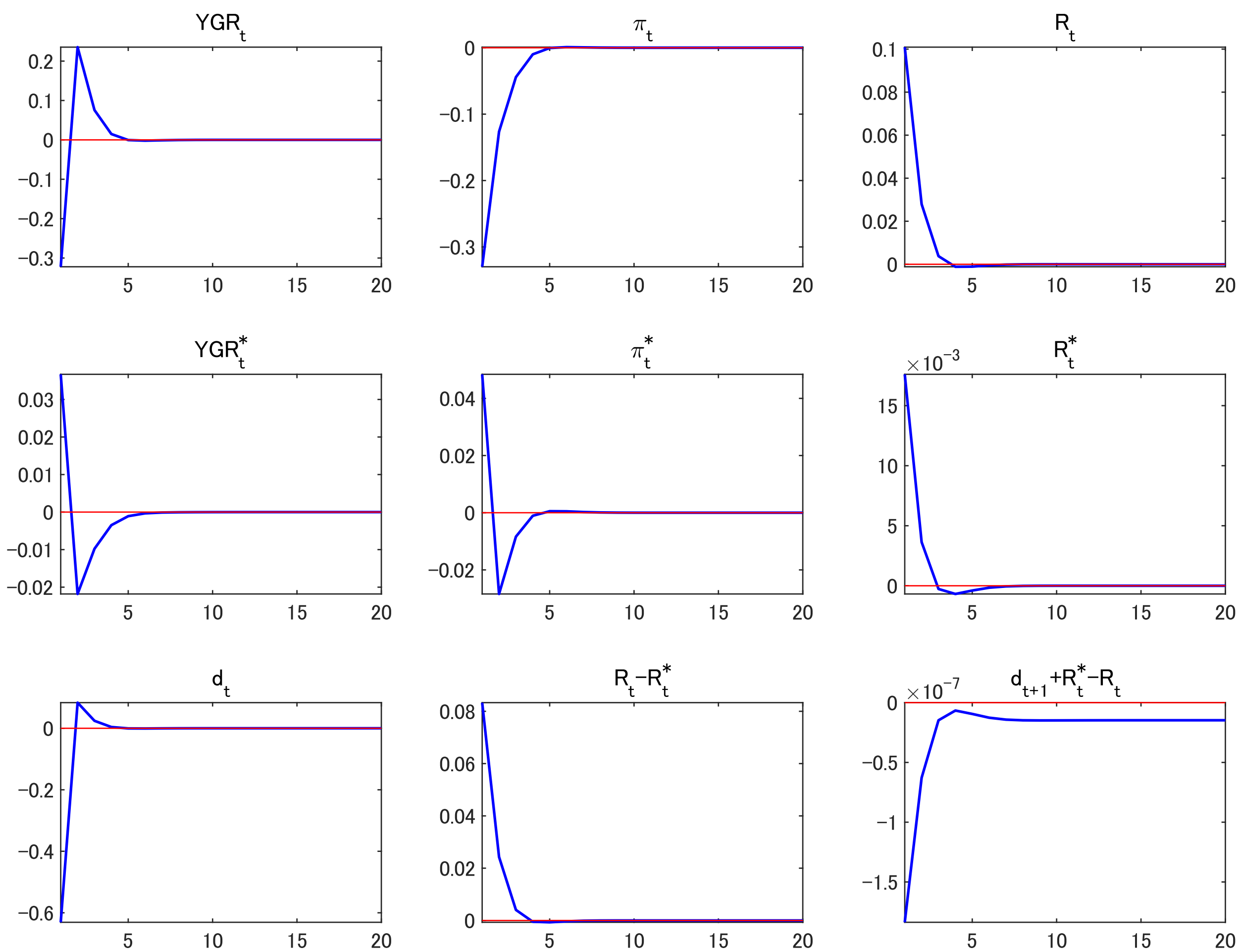
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home technology, given the posterior mean estimates of parameters in the baseline model.

Figure 7: Responses to home external demand shock



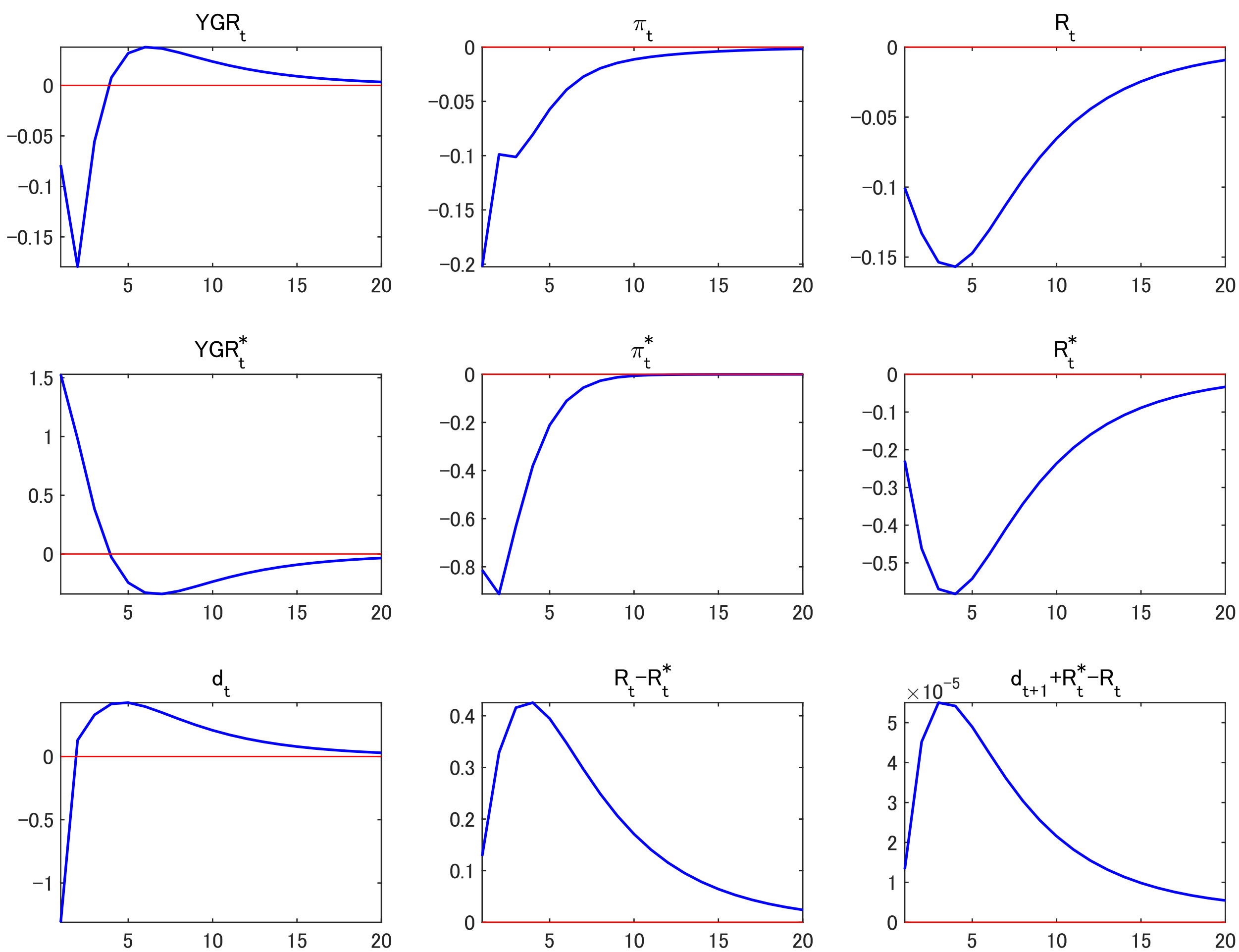
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home external demand, given the posterior mean estimates of parameters in the baseline model.

Figure 8: Responses to home monetary policy shock



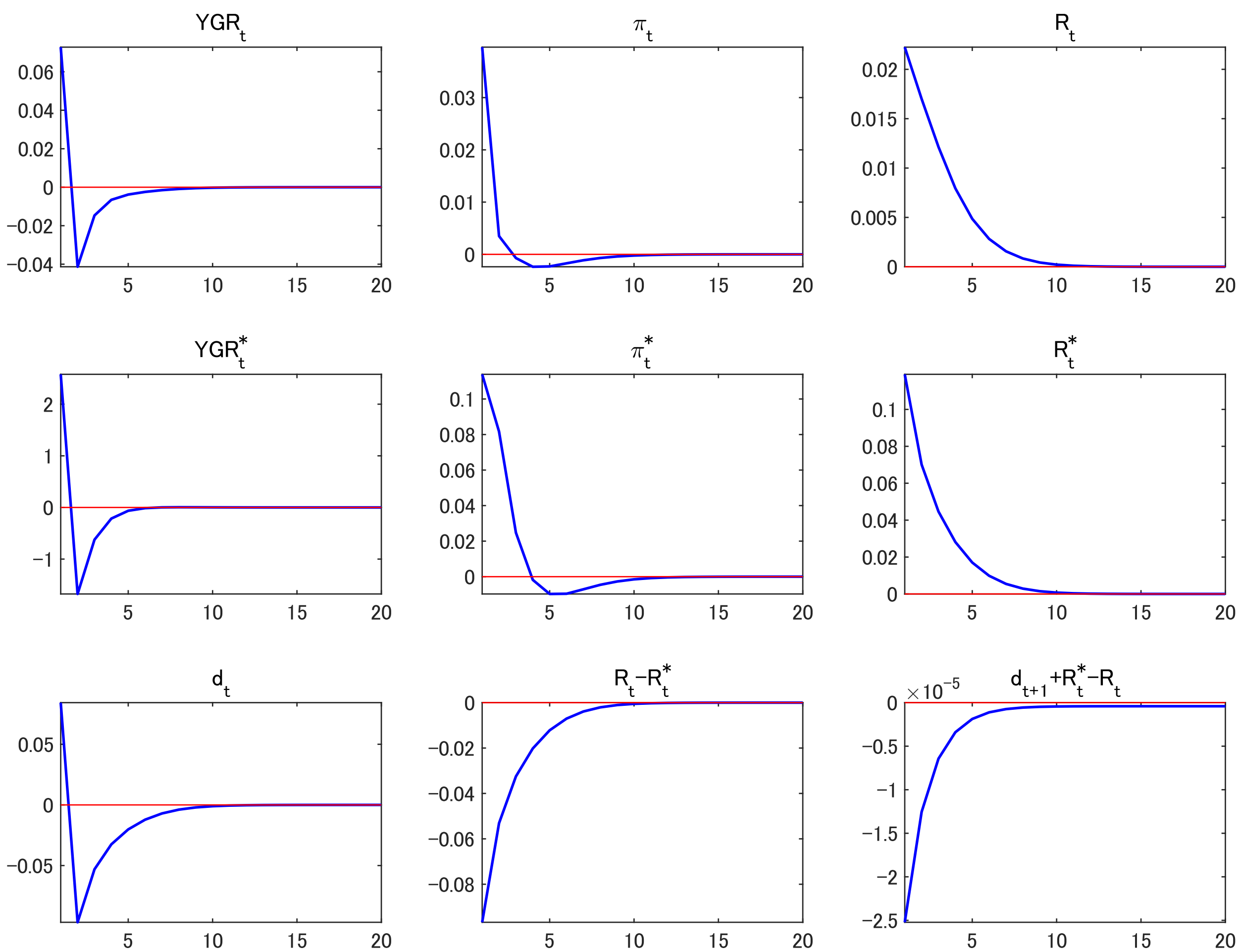
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to home monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 9: Responses to foreign technology shock



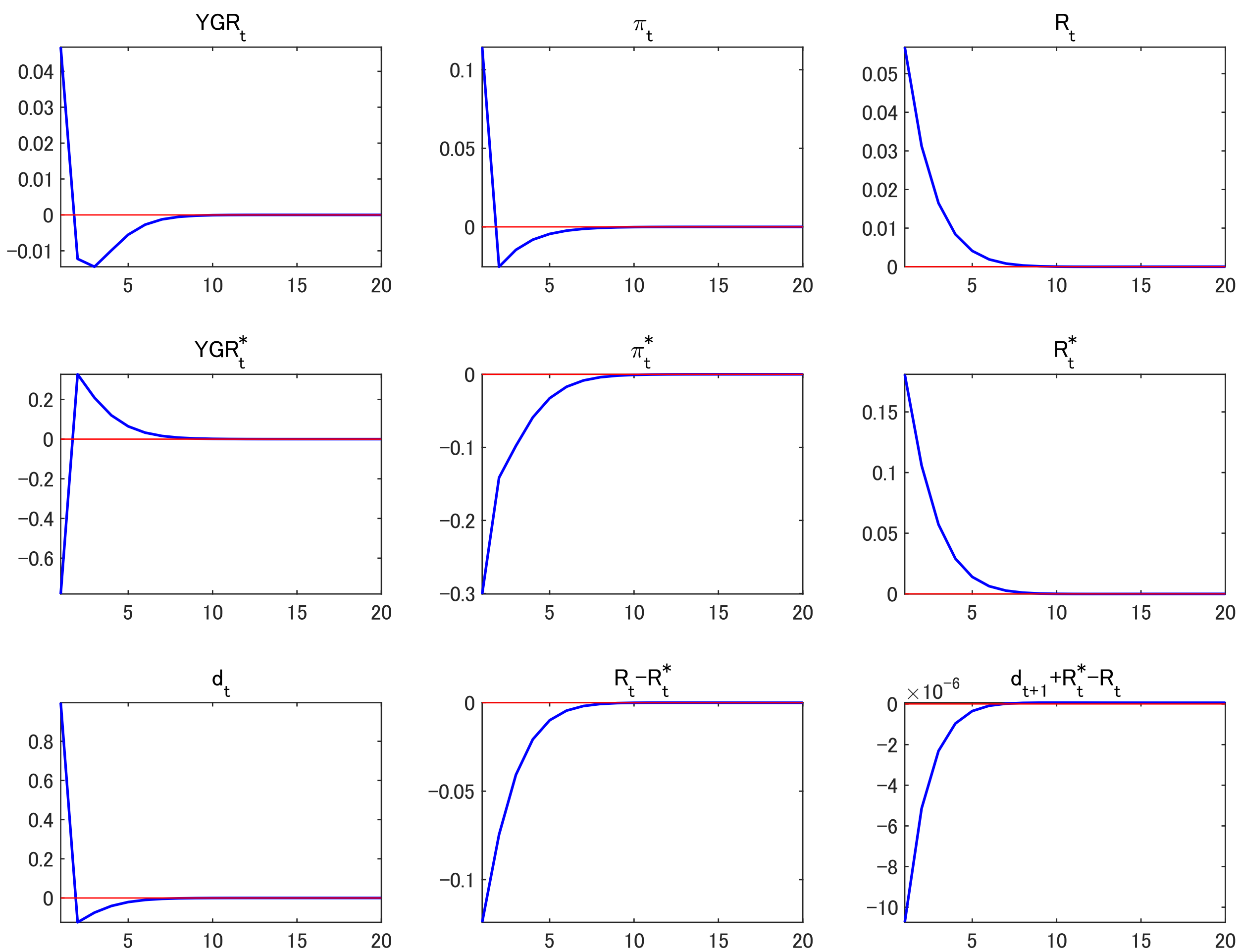
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

Figure 10: Responses to foreign external demand shock



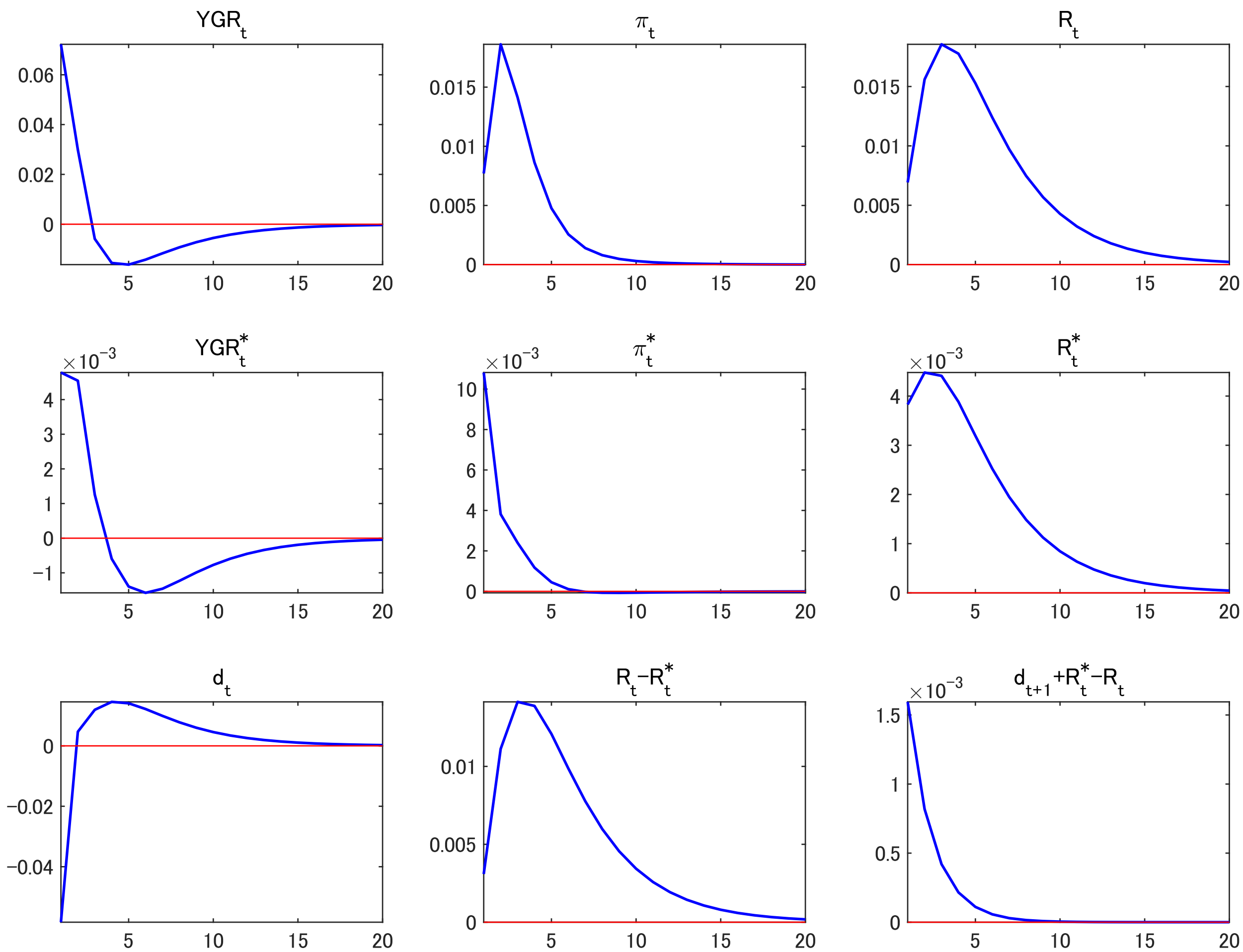
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign external demand, given the posterior mean estimates of parameters in the baseline model.

Figure 11: Responses to foreign monetary policy shock



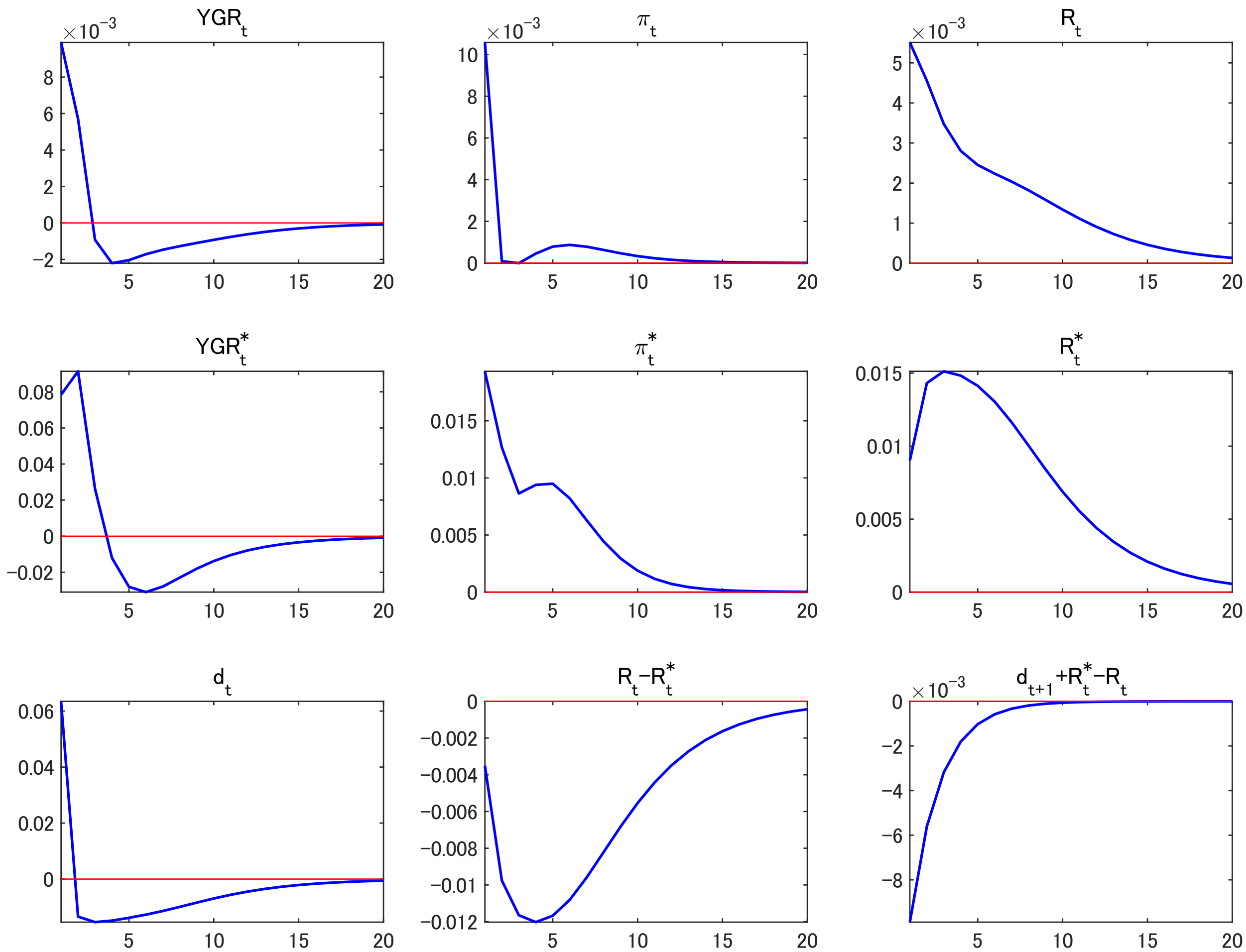
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 12: Responses to volatility shock to home external demand



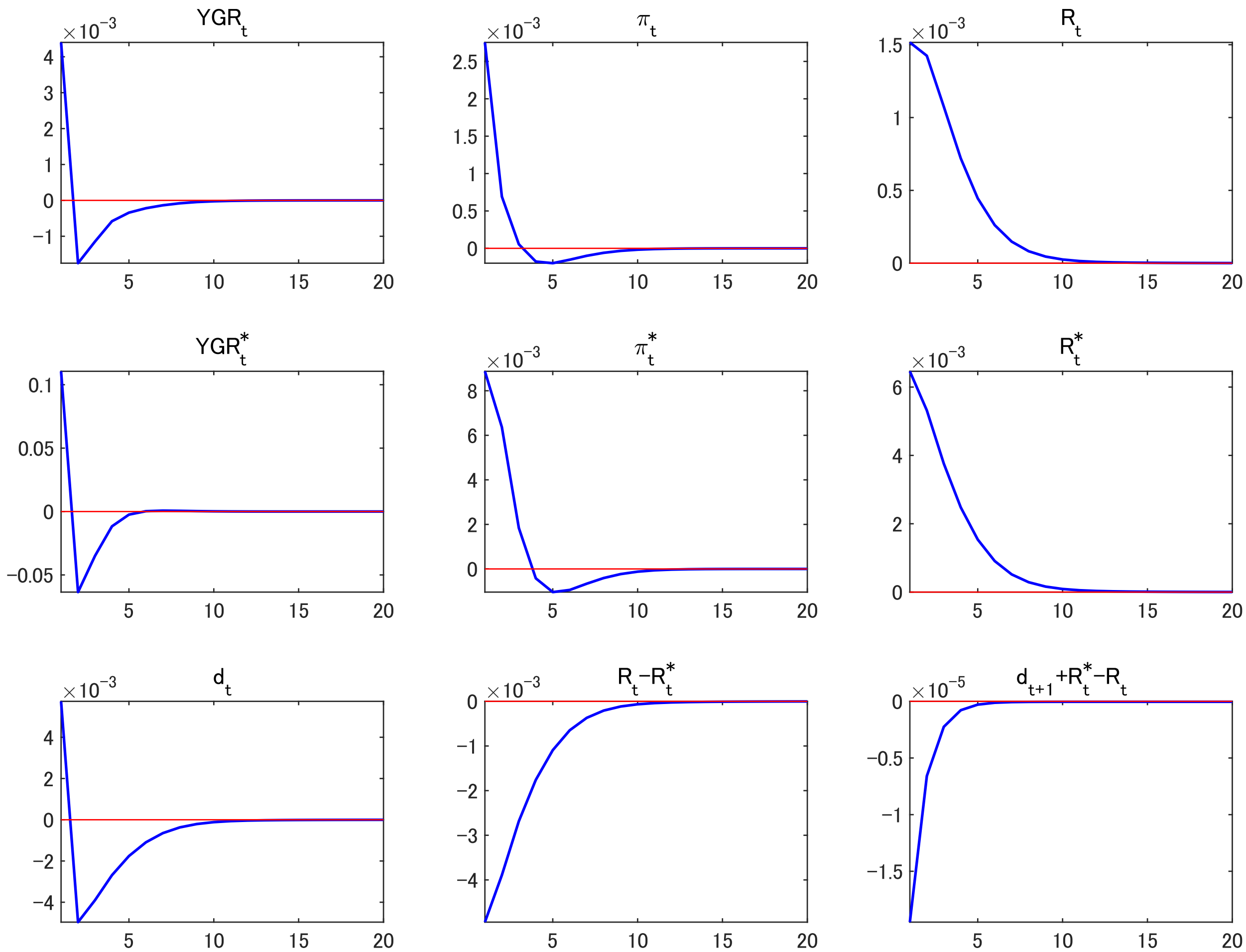
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to home external demnad, given the posterior mean estimates of parameters in the baseline model.

Figure 13: Responses to volatility shock to foreign technology



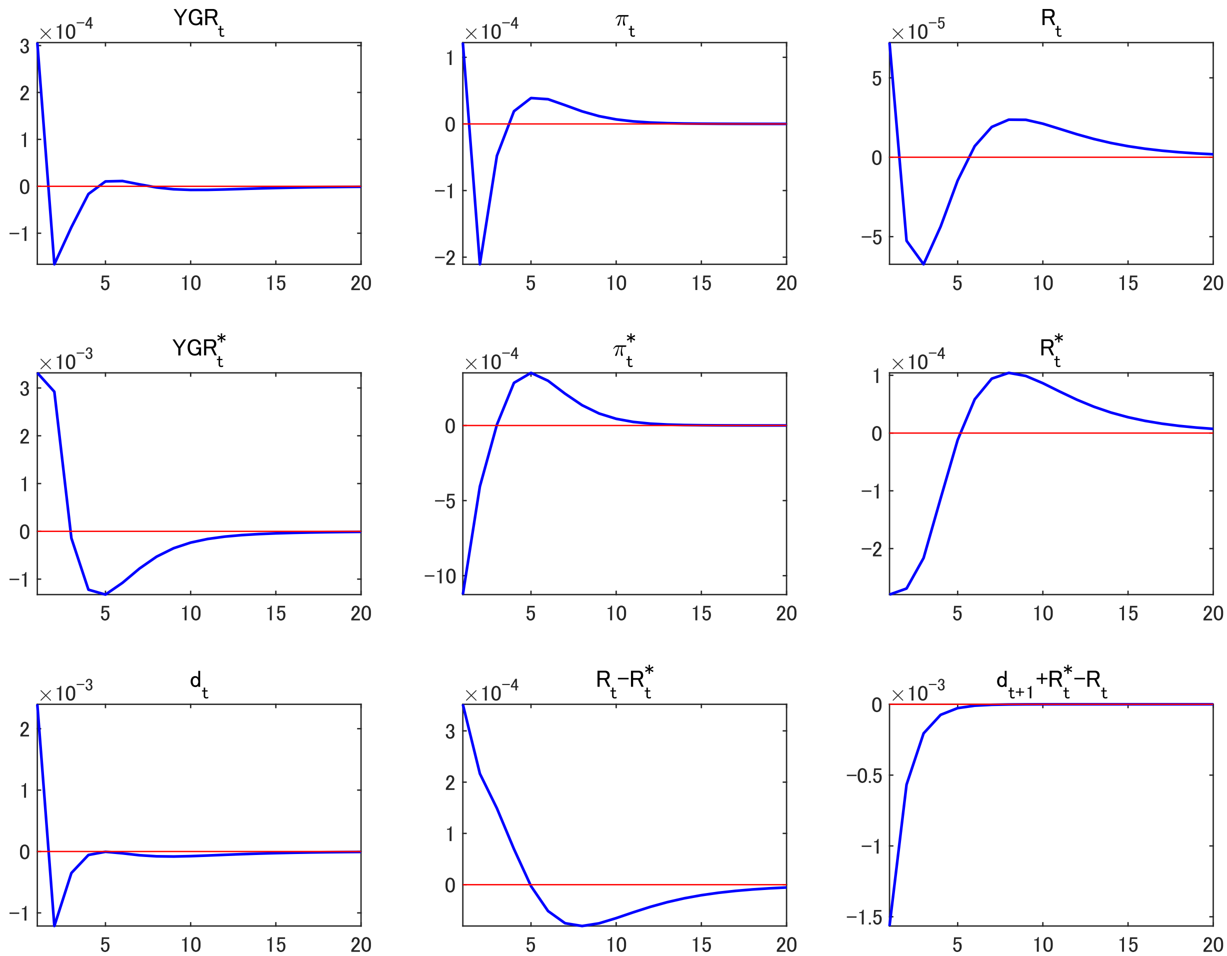
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign technology, given the posterior mean estimates of parameters in the baseline model.

Figure 14: Responses to volatility shock to foreign external demand



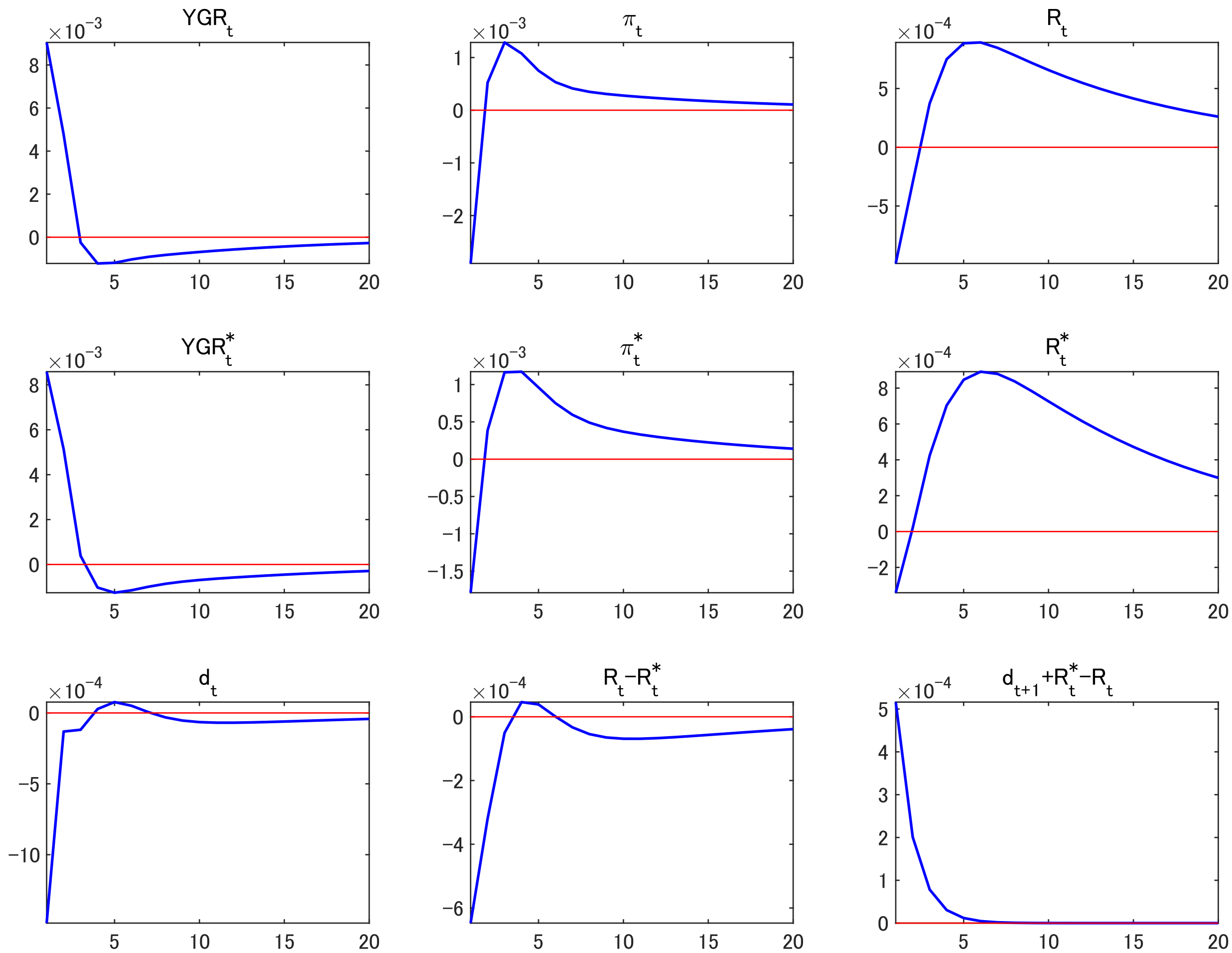
Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign external demand, given the posterior mean estimates of parameters in the baseline model.

Figure 15: Responses to volatility shock to foreign monetary policy



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to foreign monetary policy, given the posterior mean estimates of parameters in the baseline model.

Figure 16: Responses to volatility shock to risk sharing



Note: This figure shows the impulse responses of output growth, inflation, the nominal interest rate in both countries, depreciation of the nominal exchange rate, nominal interest rate differential, and deviation from the UIP, in terms of deviations from steady-state values, to a one-standard-deviation volatility shock to the risk-sharing condition, given the posterior mean estimates of parameters in the baseline model.