Liquidity, liquidity everywhere, not a drop to use

Why flooding banks with central bank reserves may not expand liquidity[1]

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Abstract

Central bank balance sheet expansion is financed by commercial banks. It involves not just a substitution of liquid central bank reserves for other assets held by commercial banks, but also a counterpart increase in commercial bank liabilities, such as short-term deposits issued to finance reserves. Banks typically also write a variety of other claims on reserve holdings. Normally, central bank balance sheet expansion will enhance the net future availability of liquidity to the system. However, in episodes of stress when a large fraction of claims on liquidity are exercised, the demand for liquidity can be significantly greater than the availability of reserves. Furthermore, at such times some liquid commercial banks may hoard reserves to bolster their own prospects, contributing significantly to liquidity shortages. Therefore, because central bank balance sheet expansion operates through commercial bank balance sheets, it need not eliminate future episodes of liquidity stress, it may even exacerbate them. This may also attenuate any positive effects of central bank balance sheet expansion on economic activity.

Despite a significant expansion in central bank balance sheets, some markets like the US money market have experienced increasing interest rate volatility, including significant spikes in the repo rate, notably in September 2019 (see Copeland, Duffie and Yang (2021), Correa, Du, and Liao (2021), D'Avernas and Vanderweyer (2021), IAWG Treasury Market Surveillance Report (2021) and Yang (2021)). This apparent disruption in money markets that depend intimately on the availability of liquidity seems puzzling when the cash and central bank reserves held by the US private sector at the end of 2019 were around 4 times their holdings before the Global Financial Crisis in 2007. Greater liquid holdings do not seem to have made markets for liquidity immune to liquidity shocks. Indeed, markets were disrupted yet again in March 2020 at the onset of the COVID-19 pandemic, and the banking system was found short in its ability to accommodate the demand for liquidity. [2,3] In response, the Federal Reserve expanded its balance sheet yet more (see, for example, Kovner and Martin (2020)), buying financial assets from the private sector and placing large quantities of liquid reserves with it (or promising to do so). Where had all the prior liquidity gone? Our paper focuses on this question, not so much to explain the micro-underpinnings of interest rate spikes, for which there is an extensive literature now (see, for example Copeland, Duffie, and Yang (2021)), but to analyze more general theoretical underpinnings of the consequences of central bank balance sheet expansion. This will help us gauge how the system might react to a serious liquidity shock in the real sector if one were to hit.

It seems natural that the liquid central bank reserves issued to finance central bank balance sheet expansion should enhance the supply of liquidity, bringing down illiquidity premia in the market, and reducing the cost to firms of financing. Yet this view neglects three key private sector responses. First, central banks effectively issue these reserves to commercial banks (henceforth "banks"), which typically finance them with short-term liabilities such as deposits, an offsetting claim on liquidity. Second, the liquid reserves themselves get further encumbered through regulations and bank activities. Third, and perhaps most novel, in times of liquidity stress banks may hoard spare liquidity, again for reasons stemming from bank activities. Put differently, central bank balance sheet expansion typically has a counterpart effect on commercial bank balance sheets, which could raise the future demand for liquidity, in some cases by enough to exceed the injected supply.

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[2] This was the case especially for corporate debt, but segments of the US Treasuries market also experienced significant illiquidity, see Duffie (2020), Fleming and Ruela (2020), He, Nagel and Song (2020), Liang and Parkinson (2020), Schrimp, Shin and Sushko (2020)), and Vissing-Jorgensen (2020).
[3] Corporates drew down significantly on bank credit lines, see Kashyap (2020) and Acharya, Engle and Steffen (2021); and, dealer banks appear to have faced regulatory constraints in extending their balance-sheets for market-making, see Boyarchenko, Kovner and Shachar (2020), Breckenfelder and Ivashina (2021), Kargar *et al.* (2021), and Vissing-Jorgensen (2020).

Let us elaborate on all this. We assume the central bank wants to expand its balance sheet, buying financial assets from the public markets with newly issued reserves. We take any direct effect of the asset purchases on economic activity as given, so as to focus on what happens to liquidity after that. We assume the reserves eventually find their way back to commercial bank balance sheets (so cash holdings with the public do not go up). Key in the analysis that follows is the mix of how banks finance these reserves. A number of authors (Calomiris and Kahn (1991), Dang, Gorton, and Holmstrom (2010), Flannery (1986), and Gorton and Pennacchi (1990), among others) have argued that banks have a comparative advantage in issuing short-term or demandable debt. Others (see, for example, Diamond and Dybvig (1984) or Stein (2012)) have attributed an implicit liquidity/money premium to demandable bank liabilities that makes them relatively attractive for investors, and Diamond and Rajan (2001) argue that one leads to the other. We are agnostic as to why longer-term financing (that is, capital) is costlier for banks, but assume functional forms that make it so. Naturally then, banks finance a large portion of the reserve expansion with demandable claims. [4]

Indeed, the evidence suggests this is the case (see Exhibit 1). The Federal Reserve bought financial assets between November 2010 and June 2011 ("QE II"), between September 2012 and October 2014 ("QE III"), and between March 2020 to the end of 2020 (the Pandemic Intervention which is still continuing). Exhibit 1 from the Flow of Funds data, suggests that commercial banks increased their assets considerably over the same period – so central bank reserves did not simply substitute for existing bank assets. Furthermore, bank deposit issuance was a multiple of the increase in commercial bank holdings of reserve balances and repos in each case. Of course, banks may also have expanded their holdings of other liquid assets such as vault cash and securities over these periods, but the increase in deposits exceeds even these. Indeed, in both QEII and the Pandemic purchases, the increase in bank deposits exceeds the overall increase in bank assets, while in QE III, it is 80 percent of the increase (the period of QE III was also one when bank loans went up considerably, along with bank liquid assets). In both QE III and the Pandemic Intervention, uninsured deposits account for the majority of the deposit financing. In Exhibit 2, we plot the cumulative increase in outstanding reserves against the rise in uninsured deposits over the last two decades, which confirms this financing pattern cumulates across programs.[5] These data inform our modeling choices.

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[4] Such financing could initially happen near-automatically – the central banks buys financial assets from non-banks, who deposit the proceeds in their banks, giving the commercial banks both reserves and offsetting deposits. Of course, banks and non-banks might eventually want to rebalance, but banks in aggregate have to hold the reserves.
[5] Note that there is a mechanical increase in uninsured deposits – and reduction in insured ones – at the end of 2012, estimated to be around $1.4 trillion, due to the expiration of Federal Deposit Insurance Corporation (FDIC)'s Transaction Account Guarantee (TAG) Program, which after the collapse of Lehman Brothers provided unlimited

We assume that after commercial banks get reserves, make loans, and set their capital structure to accord with these assets, there is a probability that the economy will become *liquidity stressed*, and the demand for liquidity in the real economy will increase significantly. Demand will be concentrated on some banks. Call these the *stressed* banks. We assume that their wholesale depositors, fearful of any loss, withdraw their cash in such states, increasing the stressed banks' need for funds.

This is where the second effect of financing central bank reserve expansion through commercial bank balance sheets appears; some of the reserves accumulated by banks may be unavailable to pay out in stressed states. Two reasons for reserves encumbrance are speculation and regulation. A bank holding highly liquid reserves, with the reserves being required only in situations of liquidity stress, will want to try and "sell" liquidity in all the states it does not need it, for instance, by offering contingent lines of credit[6] or guaranteeing margin calls on speculation for a fee (see, for example, Anderson, Du, and Schlusche (2021)). Unfortunately, such commitments typically are also called upon in states where the economy is liquidity-stressed. The amount of free liquidity in such states will therefore shrink relative to the ex-ante size of the reserves.

Regulators may also step in to add to the encumbrances on reserves. Centralized clearing houses may require dealer banks to encumber a portion of the liquid assets as guarantee funds for the settlement of defaulted trades; similarly, regulation may require non-centrally-cleared positions of dealer banks to be backed by liquid assets. Commercial banks may also need to meet liquidity coverage ratios given the plethora of risks on their balance sheet. Regulators could, of course, suspend an ex-ante-imposed liquidity requirement in the face of ex-post stress so that more reserves are available to alleviate market illiquidity. Diamond and Kashyap (2016) explain why the regulator may not want to do this, for fear that localized stress morphs into a full-blown panic.[7] Furthermore, ratchet effects whereby supervisors scrutinize reductions in reserves closely no matter what the prior level held (see Nelson (2019, 2022)) would also lead to banks seeing higher implicit regulatory reserve requirements. Indeed, a recent discussion paper by

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deposit insurance to non-interest-bearing transaction accounts. The overall trend of an increase in uninsured deposits of banks with an increase in reserves is robust to removing this mechanical increase in both Exhibits.

[6] Yankov (2020) examines the changes in the liquidity management at banks and nonbank financial firms in the United States that occurred following the initial proposal of the liquidity coverage ratio (LCR) requirement in 2010 and its finalization in 2014. He concludes that "*While banks increased their liquid assets to meet the new regulatory liquidity requirements, nonbank financial institutions–such as insurance companies, finance companies, real estate investment trusts, pension funds, asset managers, mutual funds, and others–decreased their liquid assets and increased their reliance on bank credit lines to manage their liquidity risks.*" Thus banks effectively sold claims that would be collectively called upon in states where liquidity would likely be scarce.

[7] They argue that one function of a liquidity requirement is to prevent a panic by assuring depositors that the regulated bank has plenty of liquidity to meet both expected and unexpected needs. Depositors may hold off on running on a bank even when other banks are being run if it is convinced regulators will force the bank to hold on to liquidity under almost all circumstances.

the Bank of England (2022) flags such behavior during stress episodes in the pandemic, and seeks as part of its prudential liquidity framework to induce banks to use their surplus liquidity under stress.

The third effect of running central bank reserve expansion through commercial bank balance sheets, and perhaps most novel, is that in times of liquidity stress healthy banks may see a valuable *convenience yield* to liquid reserves – for instance, because it is dry powder in case conditions worsen. Consequently, a fraction of healthy banks may hoard liquidity and maintain unimpeachable balance sheets, in order to be perceived as safe and attract more deposit flows, rather than lending reserves out to stressed banks.

The financing of partially encumbered reserves with short-term deposits, coupled with reserve hoarding by some of the healthy banks that are recipients of flight-to-safety deposits, sets up an interesting dynamic in episodes of liquidity stress: loan rates in the interbank market can shoot up as stressed banks try and attract liquidity from healthy banks (see, Acharya and Mora (2015) for empirical documentation of such a dynamic during 2007-08). Importantly, the extent of illiquidity, and therefore the premium paid on borrowing in this situation (which will also affect the fire-sale prices of illiquid financial assets), need not fall in the reserves the central bank issues ex ante. Indeed, under plausible circumstances, every additional dollar of reserves the central bank issues up front can increase the net demand for liquidity in situations of liquidity stress, and can increase the interbank borrowing premium.

A higher anticipated bank borrowing rate in the future then cascades up front into a higher rate for term loans made by banks (as in Diamond and Rajan (2011), Shleifer and Vishny (2010), or Stein (2012)), lower investment by firms, and lower aggregate activity. Somewhat perversely, therefore, higher central bank reserve issuance can create more headwinds even to current activity by increasing future, and thus current, borrowing premia.

In sum, the key problem is that central bank reserves are placed with commercial banks, whence the expansion in central bank reserve assets can be outweighed by claims created on them. *The ex-ante supply of reserves affects the ex-post demand for them.* If, instead, central bank reserves were placed directly with households, or with financial intermediaries that did not issue claims on liquidity, the effects we hypothesize would be mitigated.

Given current practice, our paper suggests that under certain circumstances, there is a threshold size of the central bank balance sheet beyond which further expansion will increase the severity of future liquidity problems. Consequently, the balance sheet size that is optimal from a purely monetary perspective may be excessive from a financial stability perspective. More generally, even though the

central bank has no direct cost of creating additional fiat money (Friedman (1969)), our paper proposes a social cost stemming from the reactions of market participants with consequences for financial stability.

A number of theoretical papers (see, for example, Caballero and Krishnamurthy (2003), Lorenzoni (2008) and Stein (2012)) studying liquidity conclude that the problem lies with the commercial bank relying excessively on short-term financing. Essentially, individual banks take the expected rates in the interbank market as given, and do not take account of the effects of their financing or activity choices on those rates. Yet, as these papers recognize, this is a pecuniary externality, and absent any other amplification mechanism, should not make private bank financing choices differ from the social optimal. In our model, given the quantum of reserves that need to be held in our framework, banks make the socially optimal financing choice (between capital and deposit financing) so long as the degree of reserve hoarding is exogenous. When we endogenize reserve hoarding, however, it turns out that banks issue too much capital relative to the social optimal, a departure from the previous literature. Interestingly, this is because higher capital issuance up front (by each bank) reduces the ex-post inter-bank rate, increasing the degree of wasteful liquidity hoarding, and thus increasing the degree of liquidity shortfall that has to be met by dissipative capital issues then (by all liquidity-stressed banks). The last effect is an externality that individual bank's capital issuance does not internalize. This stark result stems partly from our focus on the costs of illiquidity rather than insolvency, but does highlight channels different from the literature.

The empirical literature on recent price spikes in usually liquid money markets has attributed them to frictions such as market segmentation, capital regulation, and timing mismatches (from intraday payments and Treasury sales). To alleviate these spikes, a number of commentators suggest that the central bank provide more liquidity in stressed times to a wider array of market participants (see the G30 Report on US Treasury Markets 2021), that it permanently expand the quantum of reserves (Copeland, Duffie, and Yang (2021)), or that it reduce or eliminate capital requirements against reserves (Liang and Parkinson (2020)). While these proposals will likely reduce stress ex post, we also need an ex-ante analysis of why the system is so fragile despite seemingly abundant reserves to understand the full consequences of the proposed policies.

For instance, the central bank can certainly flood the market with reserves ex post. Such intervention is not without cost. Ex post, it crowds out lending by healthy banks, increasing the scale of the needed intervention. Ex ante, market participants are even more inclined to write future claims on liquidity and ever more reliant on the central bank backstop. Consequently, we should expect escalating and more frequent central bank interventions over time, with broader categories of assets accepted as collateral for the central bank intervention, and potential distortions creeping into asset prices as well as asset allocations (see Acharya, Shin and Yorulmazer (2011), Diamond and Rajan (2012) or Farhi and

Tirole (2012)). Our analysis also suggests that because the demand for reserves expands with supply, the system is prone to hysteresis. The central bank's ability to shrink its balance sheet quickly and without incident after a period of expansion is significantly lower than might be expected if the forces we examine were absent. Central banks therefore have to gently ease the system into a regime with lower liquidity.

Closely related to our paper is Diamond, Jiang, and Ma (2021), who ask how the reserve build-up by the Federal Reserve could affect bank lending. While they too emphasize the need to finance reserves, their focus is on the crowding-out effects of such reserve holdings on corporate loans. Our focus instead is on the effects of reserves on ex-post liquidity, and how that would consequently impact corporate lending.

Another recent paper whose empirical finding relates closely to our main insight is Lopez-Salido and Vissing-Jorgensen (2022), who show that the opportunity value of reserves, measured as the effective federal funds rate minus the central-bank-paid interest on excess reserves, is (negatively) related to the quantity of outstanding reserves in a stable manner *only if* the relationship controls for the outstanding stock of commercial banking deposits. Viewed through the lens of our model, the marginal value of additional reserves should be related to the supply of reserves minus encumbrances on these reserves, the most significant one being the risk of deposit drawdowns (and, in practice, associated prudential regulations). Indeed, our model implies that other liquidity promises made by banks such as committed lines of credit to non-banks should also potentially enter such an adjustment. At any rate, our key observation, which is supported by the authors' empirical findings, is that the price of liquidity depends upon reserves that are free to move around in the financial system (Lopez-Salido and Vissing-Jorgensen's "deposit-adjusted reserves"), which may be far smaller in quantity than the central-bank-supplied stock.

In section I, we lay out a simple model of central bank balance sheet expansion; in section II we analyze the benchmark model which assumes a fraction of reserves are encumbered and takes as exogenous the fraction of healthy banks that choose to hoard reserves; in section III we examine the central bank or social planner's problem; in section IV we endogenize the level of reserve hoarding, and in section V the encumbrances on reserves (we also show robustness to assuming a fixed quantity of encumbered reserves). In section VI, we examine robustness and extensions, and then conclude.

I. The Model

Consider an economy with three dates, 0, 1, and 2. Subscripts denote the date in what follows and Greek letters are parameters. There are four sets of agents in the economy: firms, banks, risk-averse savers, and risk-neutral savers (with the central bank playing a cameo role in determining reserves). The state of the economy $\tilde{y}$ is revealed at date 1. It can be healthy ($y = 0$) or liquidity stressed ($y = 1$). Firms and banks maximize expected profits.

1.1. Firms

Each firm could be thought of as representing an entire sector of the real economy. The firm has access to an investment opportunity at date 0. The state of the firm $\tilde{z}$ is revealed at date 1. It is always healthy ($z=0$) when the economy is healthy. However, the firm can be hit by an independent and identically distributed shock that makes it stressed ($z=1$) with probability θ when the economy is liquidity stressed, which occurs with probability $\frac{q}{\theta}$. So the date-0 probability of a firm getting stressed at date 1 is q. The time line for the state space of economic outcomes is in Figure 1 (we will explain shortly the bank-level outcomes illustrated therein).
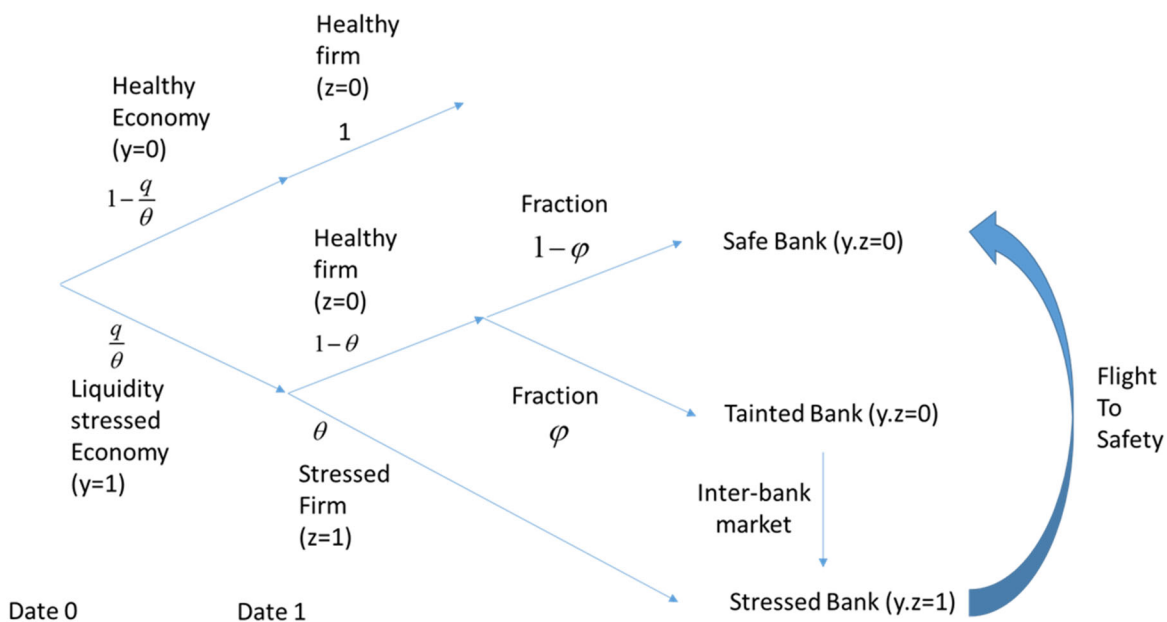
Figure 1: The state space of economic and bank-level outcomes

An investment of I_0 at date 0 produces $g_0(I_0)$ at date 2 if the firm is healthy. Liquidity stress in our model stems from real needs for spending at date 1, which in turn precipitate larger financial demands for liquidity. If stressed, the firm produces nothing at date 2 from its original investment. However, it has the possibility of "rescuing" some of its earlier investment at date 1 by investing an additional amount I_1. The expected output from such investment is $g_1(I_1)$. This output of the rescue investment is high enough in expectation to allow the firm to repay the expected value of its loans, both for the initial investment and the rescue investment. There is, however, a non-zero probability that nothing is produced from the rescue investment also and the entire sequence of investments is a write-off. Both g_0 and g_1 are

increasing and concave, and obey Inada conditions. We focus on real investment but a model where losses on financial investment precipitate margin calls, which necessitate new funding to avoid distressed selling, would have similar effects.

The firm starts out with own funds of W_0^F, and will supplement it with L_0^F of long-term borrowing from the bank. Apart from the real investment at date 0, it can also place deposits of D_0^F in the bank. We can think of this as the firm's precautionary liquidity holdings, and is isomorphic (up to the fees charged) to pre-contracted credit lines from the bank. At date 1, the stressed firm can withdraw its deposit, as well as borrow from the bank, in order to make its rescue investment, I_1.

1.2. Banks

Each bank lends to a firm (or in the alternative interpretation, an entire sector). So a bank and a firm constitute a pair. The analysis will be conducted on a per bank-firm pair basis. We will refer to a bank that has lent to a firm that has become stressed also as "stressed". At date 0, the bank can make a two-period loan of amount L_0^B (think of L_0^F as loan demand and L_0^B as loan supply, and in equilibrium, the two will be equal at L_0) at a cumulative gross interest rate of R_0^L. The bank incurs a cost of $\frac{1}{2}\lambda\left(L_0^B\right)^2$ in making the loan – the cost is increasing and convex because the bank has to manage, and lay off, an increasing amount of risk. At date 0, each bank also has to hold S_0 of reserves that the central bank has issued. For now, we assume it has no choice about the size of reserves it holds, these flow automatically from its (symmetric) share of financial activity, which is given.

1.3. Bank Financing

To finance its asset holdings, a bank can raise deposits at date 0 from the risk-averse saver, whose rate of time preference is 1. So if D_0 is the quantum of overall deposits it raises, then $(D_0 - D_0^F)$ is what it raises from the public, receiving the rest from the firm. Implicit here is the assumption that there are only a limited number of risk-neutral savers in the economy so deposits cannot be financed by them.

The risk-averse saver has log utility over consumption at date 2. We assume that if the low probability event that the stressed firm repays nothing on the rescue loan materializes at date 2, the bank will have to default on deposits at date 2. Anticipating their deposits to be haircut, risk-averse depositors will certainly run on the bank at date 2 to avoid being the one at the back of the line that gets nothing. In turn, anticipating a run at date 2 and thus possible zero consumption even with small probability, risk-

averse depositors will ask for their money back from a stressed bank at date 1. Put differently, even though the bank is solvent at date 1, as in Stein (2012) it will have to repay its risk-averse depositors immediately if stressed. Think of risk-averse depositors as institutions such as companies, hedge funds, and pension funds where their CFO loses their job if they have inadvertently left low-yielding transaction deposits in a bank that is risky or fails – this induces extreme risk-aversion about transaction deposit accounts.[8] The firm also withdraws some or all of its bank deposits to make the date-1 rescue investment.

The bank can raise long-term funding (that is, it can raise bank capital consisting of long-term bonds or equity) from the risk-neutral investor (an investor like Warren Buffet or a sovereign wealth fund), at both date 0 and date 1. The bank faces a repayment cost of $e_t + \dfrac{\alpha_t}{2} e_t^2$ at date 2 when it raises amount e_t at date t. The quadratic term could be composed of a variety of costs associated with long-term capital relative to short-term deposits, including higher illiquidity premia, higher term premia, higher borrower moral hazard, and due diligence costs. These costs could be significantly higher if the bank has to raise capital at date 1 when the economy is liquidity stressed rather than at date 0.

1.4. Firm Financing in the Stressed State at Date 1

To make its date-1 rescue investment, the stressed firm can borrow l_1^F from its bank at date 1 to supplement the deposits it withdraws. The bank will have to do significant due diligence and monitoring, given the stressed state of the firm, so the interest rate charged will be $(1 + r_1 + \gamma)$ where γ is the bank's deadweight due-diligence and monitoring costs which are passed on to the firm. For simplicity, we assume that all interest rates reflect expected values (so that face values are set to deliver that rate after accounting for any default risk). This reduces notation and lets us focus on liquidity.

1.5. Reserves Encumbrance and Interbank Market

We assume that a fraction τ of the reserves a bank has at date 1 is encumbered, *that is*, it cannot be used or lent, either for regulatory reasons or because they have been pledged elsewhere – we will endogenize this fraction later. Therefore, a bank can use $(1 - \tau)$ fraction of its initial reserves to meet depositor/ lending needs at date 1.

A stressed bank can also borrow in the interbank market, where healthy banks with surplus reserves can lend. The gross interest rate over the second period in the inter-bank market is 1 if there is an

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excess of loanable funds relative to demand. If not, the gross interest rate will rise to $(1+r_1)$ to equalize the demand and supply for funds; when this is the case, stressed banks and healthy banks that are active in the interbank market will find it attractive to issue some capital at date 1.

1.6. Flow of Reserves due to Deposit Flight and Capital Issuance at Date 1

Where do deposits that flee the distressed banks (as well as the incremental deposits that are created by the payments on the date-1 rescue investment) go? This is a critical issue and will influence important results in the paper. We assume these deposits get parked in *safe* banks. But what is safe? Any healthy bank that lends in the interbank market bears some risk of not being repaid, raising concerns with risk-averse institutional depositors about how much risk the bank is taking; in particular, these institutional depositors may learn from interbank markets that the bank has run down its reserve balances. We therefore assume that to be seen as *safe*, a healthy bank should maintain an unimpeachable balance sheet and, in particular, not lend to distressed banks in the date-1 interbank market. It will then attract a proportional share (with other safe banks) of the flight-to-safety deposits that flee the distressed banks.

A fraction φ of healthy banks will choose to lend in the interbank market if the economy is liquidity stressed, and thus become *tainted*. Tainted banks will not attract any flight-to-safety deposits (though we assume that their existing deposits do not withdraw – alternative assumptions are easily analyzed). For now, we assume that at date 0, any bank will assume that conditional on being healthy, there is an exogenous probability φ it will become tainted. We endogenize φ in section IV.

We also assume that any bank capital issued at date 1 is bought by risk-neutral investors who first acquire deposits in safe banks (for instance, by selling risk-averse depositors their treasury bills), and then transfer the safe bank's reserves to the capital-issuing bank by writing the latter a check (alternative assumptions would worsen the date-1 illiquidity problem). Similarly, any payment by firms for the rescue investment such as the purchase of equipment or inventory goes as a check to sellers, who then deposit the check in the safe banks.

These detailed assumptions about payments are necessary to track the flow of reserves through the banking system. Note that the banking system does not gain or lose reserves as a result of payments or capital issuance; the latter moves reserves from safe banks to capital-issuing banks, whereas the former moves reserves to safe banks. Importantly, there is no shortage of payment media at date 1.

1.7. Central Bank

The central bank issues reserves S_0 per bank at date 0, which each bank has to hold at date 0. Further

i) The banking system as a whole has to hold reserves issued by the central bank. With no ex-ante differentiation, banks assume they will be held symmetrically. Put differently, no bank can avoid holding reserves without refusing to accept legal tender as payment. We also do not initially allow for central bank reserves to be held directly by the non-bank sector. We study incentives to hold reserves later.

ii) We net out the volume of deposit creation engendered by the issuance of high-powered reserves, looking only at final "reduced-form" balance sheets. The pyramiding of deposits via the money multiplier typically introduces complications as to how claims are run upon, netted and settled (see, for example, Kashyap (2020)) that would magnify the problems we examine.

II. Analysis

2.1. The Firm's Problem

To ease understanding of the calculations that follow, we present firm and bank balance sheets at date 0 and date 1 in Exhibit 3. With probability q the firm will be stressed at date 1, and it will be healthy with probability $(1-q)$. So its maximization problems at date 0 and date 1 are as follows:

Date 0: $\underset{L_0^F, D_0^F}{Max}\, (1-q)\left[g_0(I_0)+D_0^F\right]+q\left[g_1(I_1)-l_1^F\left(1+\gamma+r_1\right)\right]-R_0^L L_0^F$

Date 1: $\underset{l_1^F}{Max}\, g_1(I_1)-l_1^F\left(1+\gamma+r_1\right)$

s.t. $I_0 = L_0^F + W_0^F - D_0^F$ and $I_1 = l_1^F + D_0^F$

The constraints are just budget constraints at each date. The firm's first order conditions (FOC's) then are

w.r.t. firm long-term borrowing from bank L_0^F: $\qquad (1-q)g_0{'}-R_0^L=0$ $\qquad$ (1.1)

w.r.t. firm deposit in bank D_0^F: $\qquad (1-q)\left(-g_0{'}+1\right)+q\left(g_1{'}\right)=0$ $\qquad$ (1.2)

w.r.t. date-1 firm borrowing from bank l_1^F: $\qquad g_1{'}-\left(1+\gamma+r_1\right)=0$ $\qquad$ (1.3)

Substituting the value of $g_1{'}$ from (1.3) into (1.2), we get $(1-q)g_0{'}=(1+q\gamma+qr_1)$. Term the right-hand side of this expression R_0^{DF}: it is the expected opportunity return for the firm of holding an additional dollar of deposit, and thus avoiding borrowing from the bank at date 1 if stressed. Comparing with (1.1) where the firm's marginal expected return on date 0 investment is equal to the cost of long-term

borrowing from the bank, we get $R_0^L = R_0^{DF}$. In words, the cost of long-term borrowing is equal to the opportunity return on holding an additional dollar of deposit. Let us now turn to the bank's problem.

2.2. The Bank's Problem

The bank maximizes profits given constraints, that is,

$$\underset{L_0^B, e_0, e_1}{Max} \quad R_0^L L_0^B + S_0 - e_0 - \frac{\alpha_0}{2}e_0^2 - D_0$$

$$+ \frac{q}{\theta}\theta\left[-\frac{\alpha_1}{2}e_1^2 - r_1\left(b_1(y=1, z=1) - l_1^B\right)\right]$$

$$+ \frac{q}{\theta}(1-\theta)\varphi\left[-\frac{\alpha_1}{2}e_1^2 - r_1 b_1(y=1, z=0)\right]$$

s.t. $\quad D_0 + e_0 = L_0^B + \frac{1}{2}\lambda(L_0^B)^2 + S_0$ (1.4)

$\quad b_1(y=1, z=1) = l_1^B + D_0 - S_0(1-\tau) - e_1$ (1.5)

$\quad b_1(y=1, z=0) = -S_0(1-\tau) - e_1$ (1.6)

$\quad l_1^B = l_1^F = I_1 - D_0^F$ (1.7)

The first line of the maximization is the bank's expected profits at date 2 from date-0 financing and lending activities. The second line is the expected loss for a stressed bank, where the loss comes from meeting deposit outflows and the stressed firm's equilibrium loan demand l_1^B ($= l_1^F$), net of financing costs and net of the return on loan. The stressed bank's loan to the stressed firm and deposit repayments are funded by unencumbered reserves, date-1 capital raised, and interbank borrowing (of $b_1(y=1, z=1)$, see (1.5)). The third line of the maximization is the expected profits to a healthy bank from becoming tainted and lending $-b_1(y=1, z=0)$ into the interbank market when the economy is stressed at date 1, net of financing costs. It makes the loan out of its unencumbered reserves and the capital it raises (see (1.6)). Note that both the stressed bank and the tainted bank will raise the same amount of capital in equilibrium because they will both see the same marginal return of using that capital in the interbank market (for the stressed bank it reduces borrowing, and for the tainted bank it increases loans). The constraint (1.4) simply reflects the sources and uses of funds at date 0 (the bank raises money from deposits and long-

term capital, and invests in long-term loans, the cost of making loans, as well as forced reserve holdings). Finally, there is no incremental loss or gain to safe banks at date 1, a feature that we will relax in Section IV.[9] Then, the first order conditions are

w.r.t. long-term loans to the firm L_0^B: $\quad R_0^L - (1 + \lambda L_0^B)(1 + qr_1) = 0$

From the bank's perspective, the date-0 return from making another dollar of loan should equal the cost of funding that dollar (and the associated marginal cost of managing the risk of the additional loan, λL_0^B) via flighty deposits, which cost a net rate of r_1 per dollar if the bank gets stressed. Let us term as R_0^{DB} the expected cost of funding via deposits, which equals $(1 + qr_1)$. Hence, $R_0^L = (1 + \lambda L_0^B)R_0^{DB}$. Next, FOC

w.r.t. date-0 capital issuance e_0: $\quad -(1 + \alpha_0 e_0) + (1 + qr_1) = 0$

This implies the marginal cost of raising an additional dollar of long-term funding or capital at date 0 should equal the saving on funding via deposits. So $e_0 = \dfrac{(R_0^{DB} - 1)}{\alpha_0} = \dfrac{qr_1}{\alpha_0}$. In words, the bank raises more capital at date 0 the higher the expected premium it will pay in the interbank market in the stressed state. Finally, FOC w.r.t. date-1 capital issuance e_1: $\quad -(1 + \alpha_1 e_1) + (1 + r_1) = 0$

So at the margin, the bank's cost of raising an additional dollar of capital at date 1 equals the cost of borrowing in the interbank market. Simplifying,

$$r_1 = \alpha_1 e_1 \qquad\qquad (1.8)$$

Hence the prevailing premium in the interbank market drives capital-raising at date 1 by stressed and healthy tainted banks and vice versa. Importantly, the firm and bank's maximization decisions link the various interest rates to the date-1 premium in the interbank market r_1 . So

$$R_0^L = R_0^{DF} = (1 + q\gamma + qr_1) = (1 + \lambda L_0^B)R_0^{DB} \qquad (1.9)$$

We know that the inter-bank premium is necessary in order to equalize the date-1 demand and supply of funds when the economy is liquidity stressed – essentially the premium draws forth more date-1 issuance

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[9] Note that banks that remain safe and hoard reserves will receive a lump sum in flight-to-safety deposits that depends on choices of *other* banks, and therefore will not affect the date-0 optimization problem of a given bank. When we endogenize φ in Section IV, this lump sum will matter at date 1 but still not affect the date-0 problem.

in the capital market by stressed and tainted banks even while reducing rescue investment by stressed firms and the associated demand for funds that spills over into the interbank market. The net date-1 shortfall in the interbank market in the liquidity stressed economy is

$\theta\left[I_1 + (D_0 - D_0^F)\right] - \left[\varphi(1-\theta)+\theta\right]S_0(1-\tau)$. The first term in the first square bracket is the "rescue" investment by the stressed firms and the second term is the expected withdrawal by the risk-averse depositors from stressed banks (which is redeposited in safe banks). The sum is the call on liquidity by the system, which is reduced by the available shrunken reserves (the last term) with stressed and tainted healthy banks. This overall shortfall, when positive, exactly equals $\left[\varphi(1-\theta)+\theta\right]e_1$, the date-1 capital raised by the tainted and stressed banks (note that safe banks do not raise date-1 capital because they have no profitable way to deploy it). So when r_1 is positive, we have from (1.8),

$$\left[\varphi(1-\theta)+\theta\right]\alpha_1^{-1}r_1 = \theta\left[I_1 + (D_0 - D_0^F)\right] - \left[\varphi(1-\theta)+\theta\right]S_0(1-\tau)$$

or $r_1 \equiv \alpha_1 f(r_1, S_0)$ where $f(r_1, S_0) \equiv \dfrac{\theta}{\left[\varphi(1-\theta)+\theta\right]}\left[I_1 + (D_0 - D_0^F)\right] - S_0(1-\tau)$. \qquad (1.10)

We denote the equilibrium interbank rate premium as $\overline{r_1}$. If there are sufficient unencumbered reserves in the banking system to meet funding needs without date-1 capital issuance, then $\overline{r_1} = 0$.

2.3. Date-1 Interbank Rate

Let us now analyze the interbank rate and how it varies with reserves issued, S_0.

Lemma 1: The date-1 equilibrium interest rate in the inter-bank market is unique.

Proof: See Appendix I.

The proof essentially demonstrates that the net need for capital issuance, $f(r_1, S_0)$, falls in r_1. Consequently, since the left hand side of (1.10) is increasing in r_1 and the right hand side is decreasing, there is a unique positive solution when the right hand side is positive at $r_1 = 0$, and it is 0 otherwise.

How does the possible positive equilibrium rate, $\overline{r_1}$, implicitly determined by (1.10), vary with central bank reserves? Totally differentiating (1.10), we have $\dfrac{1}{\alpha_1}\dfrac{d\overline{r_1}}{dS_0} = \dfrac{\partial f}{\partial r_1}\cdot\dfrac{d\overline{r_1}}{dS_0} + \dfrac{\partial f}{\partial S_0}$. Therefore,

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$$\left(\frac{1}{\alpha_1} - \frac{\partial f}{\partial r_1}\right)\frac{d\bar{r}_1}{dS_0} = \frac{\partial f}{\partial S_0} = \frac{\theta}{\left[\varphi(1-\theta)+\theta\right]} - (1-\tau).^{10} \text{ Since } \frac{\partial f}{\partial r_1} \text{ is negative as shown in the proof of}$$

Lemma 1, $\left(\frac{1}{\alpha_1} - \frac{\partial f}{\partial r_1}\right)$ is positive, and $Sign\left(\frac{d\bar{r}_1}{dS_0}\right) = Sign\left(\frac{\theta}{\left[\varphi(1-\theta)+\theta\right]} - (1-\tau)\right)$. Consequently,

Lemma 2: If

$(i)\quad \theta > \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}, \dfrac{d\bar{r}_1}{dS_0} > 0;\quad (ii)\ \theta < \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}, \dfrac{d\bar{r}_1}{dS_0} < 0;\quad (iii)\ \theta = \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}, \dfrac{d\bar{r}_1}{dS_0} = 0$.

Proof: See Appendix I.

Lemma 2 (ii) is the more traditional view that more reserves injected at date 0 will reduce the date-1 interbank premium. Lemma 2 (i) is more novel, suggesting the stress in the interbank market, and the associated equilibrium rate for funds, can *increase* in the extent of reserves that the central bank injects into the system at date 0. Importantly, this will also reduce date-0 and date-1 real investments. At first pass, the result seems counterintuitive. How can more liquidity supply at date 0 increase liquidity stress at date 1? However, this result is counterintuitive only from a partial-equilibrium perspective. Recognize first that the marginal source of funding of the reserves is demand deposits, which potentially create their own demand for liquidity in the stressed state (in proportion to the fraction of stressed banks, θ). Moreover, only a fraction $\varphi(1-\theta)$ of healthy banks use their unencumbered reserves to meet the liquidity demands of stressed banks, and only $(1-\tau)$ of each dollar of their reserves is available at date 1. Put differently, $\dfrac{d\bar{r}_1}{dS_0} > 0$ whenever the marginal liquidity provided by each dollar of reserves, $(1-\tau)$ $\left[\varphi(1-\theta)+\theta\right]$, is lower than the marginal call on liquidity when demand deposits are withdrawn from stressed banks, θ . Simplifying, the required condition for more date-0 reserves to constrain date-1 liquidity further and raise the equilibrium interbank rate is $\theta > \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}$ as in Lemma 2.

Note that a fraction $(1-\varphi)$ of healthy banks not only hoard *all* of their reserves, but also any flight-to-safety reserves they obtain. With all healthy banks hoarding ($\varphi = 0$), the condition for Lemma

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2 (i) is always met as long as $\tau > 0$; conversely, when $\varphi = 1$ so that all healthy banks lend in the interbank market, then Lemma 2 (i) holds whenever $\theta > (1-\tau)$.

2.4. Threshold Reserve Levels

Because $f(r_1, S_0)$ is decreasing in r_1, it must be that $\overline{r_1}$ is positive iff $f(0, S_0) > 0$. We have

$$f(r_1, S_0) = \frac{\theta}{[\varphi(1-\theta)+\theta]}\left(g_1'^{-1}(1+\gamma+r_1)+g_0'^{-1}\left(\frac{1+q\gamma+qr_1}{(1-q)}\right)-\frac{qr_1}{\alpha_0}-W_0^F+\frac{1}{2}\frac{q^2}{\lambda}\left(\frac{\gamma}{1+qr_1}\right)^2\right)$$
$$+ S_0\left(\frac{\theta}{[\varphi(1-\theta)+\theta]}-(1-\tau)\right)$$

So, for $f(0, S_0) > 0$, it must be that

$$S_0\left(\frac{\theta}{[\varphi(1-\theta)+\theta]}-(1-\tau)\right) > \frac{\theta}{[\varphi(1-\theta)+\theta]}\left(-g_1'^{-1}(1+\gamma)-g_0'^{-1}\left(\frac{1+q\gamma}{(1-q)}\right)+W_0^F-\frac{1}{2}\frac{q^2}{\lambda}(\gamma)^2\right) \equiv NLS$$

(1.11)

The left hand side is the net liquidity demand created by reserves. The right hand side is the net liquidity supplied (NLS) by the corporate sector anticipating a date-1 interbank premium of zero (and adjusting for any cost to the bank of long term lending). NLS is high when the corporate sector has a high level of starting internal funds W_0^F and a relatively low demand for funds for investment and loans. The equilibrium interbank rate (and date-1 capital market rate) is positive if the net liquidity demand exceeds supply at a rate of zero. Since $f(r_1, S_0)$ decreases in r_1, and changes in S_0 and τ only shift the term containing S_0 and not the slope of $f(r_1, S_0)$ with respect to r_1, using Lemma 2 we can describe the level of date-0 central bank reserves $\hat{S}_0$, at which the interbank rate turns positive. We can also describe how the interbank rate moves with reserves around that threshold. We have

Theorem 1:

(i)   If $\theta > \dfrac{\varphi(1-\tau)}{\tau+\varphi(1-\tau)}$, then $\overline{r_1} > 0$ is the unique equilibrium for $S_0 > \hat{S}_0$ with $\overline{r_1}$ increasing in

S_0; and, $\overline{r_1} = 0$ for $S_0 \le \hat{S}_0$. Note that $\hat{S}_0 \le 0$ if $NLS \le 0$.

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(ii)   If $\theta \leq \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$, then $\overline{r_1} > 0$ is the unique equilibrium for $S_0 < \hat{S}_0$ with $\overline{r_1}$ decreasing in

S_0; and, $\overline{r_1} = 0$ for $S_0 \geq \hat{S}_0$. Note that $\hat{S}_0 \leq 0$ if $NLS \geq 0$.

Proof: Follows from the discussion above.

2.5. Discussion

Theorem 1 (ii) is the traditional view of reserves. An increase in reserves should alleviate future illiquidity, reduce the interbank rate, and increase current (and future) real investment. A preponderance of reserves, $S_0 \geq \hat{S}_0$, ensure that the date-1 interbank interest rate premium will be zero.

Let us plot in Figure 2A the threshold value of reserves at which the date 1 interbank rate is zero , $\hat{S}_0$, for different values of θ (the fraction of the banking sector that becomes liquidity stressed). We do this for the more plausible case that the corporate sector absorbs liquidity so $NLS < 0$ (in Figure 2B in the Appendix I we analyze $NLS > 0$). The size of reserves is on the vertical axis and θ is on the horizontal axis. When $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (to the left of the vertical axis), $\hat{S}_0$ is positive, and rises in θ. Because higher ex-ante reserves loosen liquidity conditions, $\overline{r_1}$ falls in S_0, and the unhatched region below the $\hat{S}_0$ curve is where $\overline{r_1}$ is positive.

Theorem 1 (i) is the alternative view our model also offers. When $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ (to the right of the vertical axis), $\hat{S}_0$ is negative, and increases in θ. Because higher ex-ante reserves tighten liquidity in the stressed state, $\overline{r_1}$ increases in S_0, and the unhatched region above the $\hat{S}_0$ curve is where $\overline{r_1}$ is positive. In this unconventional case, the central bank cannot provide the demanded date-1 liquidity via banks through asset purchases at date 0 – reserve issuance also tends to absorb liquidity on net.
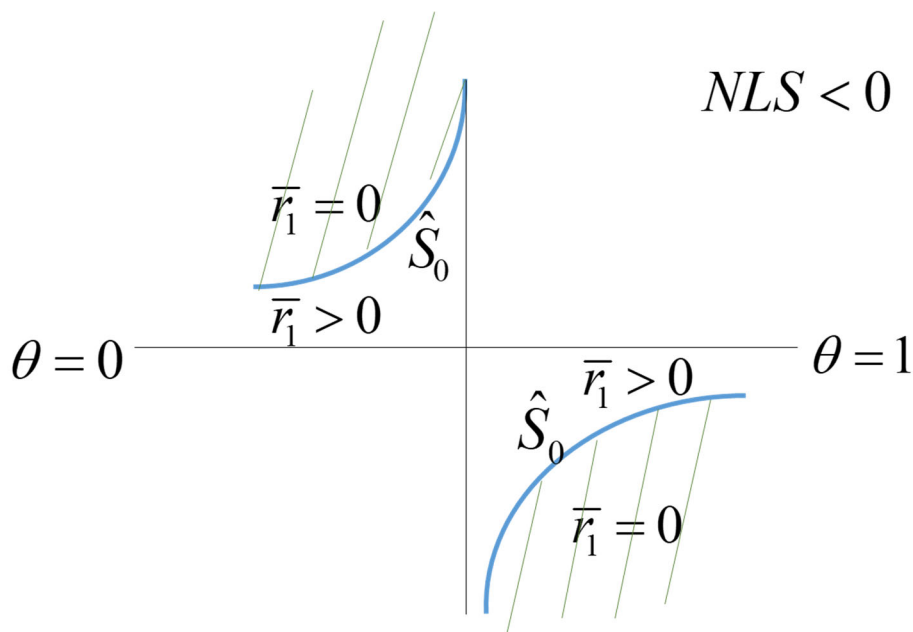
Figure 2A: Reserves are on the y axis, θ on x axis, with the axes intersecting at $\theta = \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$

To summarize, when the economy is healthy, the inter-bank premium is always zero. Each claimant's idiosyncratic liquidity demand is likely to be diversified away across a large set of diverse suppliers (see Kashyap, Rajan, and Stein (2002), for example). Central bank supplied liquidity is likely to be ample for such needs. We are focused on the net availability of liquidity in tail situations when liquidity demands become much more strongly positively correlated (for example, as witnessed by banks in the United States during 2007-08, see Acharya and Mora, 2015, or as witnessed by firms at onset of the pandemic in March 2020, see Kashyap, 2020, and Acharya, Engle and Steffen, 2021), or when the interbank market is tight due to the hoarding of liquidity by a significant proportion of surplus banks (as witnessed during the repo rate spike of September 2019, see Copeland, Duffie and Yang, 2021). In that case, the way the reserve holdings are financed matters, and the net demand for liquidity could increase in the size of reserves being financed. There is then a threshold central bank reserve issuance level (and balance sheet size) beyond which further issuance increases the net interbank rate r_1.

III. The Central Bank's Problem

We have taken the ex-ante level of reserves as given, and examined the consequences for the ex-post availability of liquidity as well as credit market rates and investment. What we have in mind thus far is the central bank may be setting reserves for monetary purposes, for instance to effect a target level of

quantitative easing.[11] What if the central bank/planner instead set reserves with the view of maximizing welfare in our framework, ignoring any other un-modeled effects of S_0?

3.1. The Central Bank/Planner 's Problem and Optimal Reserves

The central bank/planner wants to maximize output net of real costs, that is, maximize w.r.t. S_0

$$U \equiv \left((1-q)g_0(I_0) - I_0\right) + q\left(g_1(I_1) - I_1 - (I_1 - D_0^F)\gamma\right) - \tfrac{1}{2}\alpha_0 e_0^2 - \frac{q}{\theta}[(1-\theta)\varphi + \theta]\left(\tfrac{1}{2}\alpha_1 e_1^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2$$

(1.12)

where $(I_1 - D_0^F)$ is the firm's date-1 borrowing from the bank that is associated with a per unit deadweight monitoring cost γ. It follows that $\dfrac{dU}{dS_0} = \dfrac{\partial U}{\partial \bar{r}_1} \cdot \dfrac{d\bar{r}_1}{dS_0}$ since $\dfrac{\partial U}{\partial S_0} = 0$ (the central bank has no direct cost of supplying reserves as suggested by Friedman (1969)). It is easily shown (see Appendix I) that $\dfrac{\partial U}{\partial \bar{r}_1} < 0$. Consequently, the central bank wants to raise S_0 only if it brings down the date-1 interbank market rate premium, that is, $\dfrac{d\bar{r}_1}{dS_0} < 0$. Conversely, if $\dfrac{d\bar{r}_1}{dS_0} > 0$, the central bank wants to reduce reserve issuance. In the cases we have seen so far, the answer to the optimization is obvious: the central bank will set the reserves at any level such that the anticipated interbank rate premium $\bar{r}_1$ is zero.

3.2. Negative $\hat{S}_0$

When the threshold level of reserves, $\hat{S}_0$, is positive, it is clear that the central bank will set reserves at any level at or above $\hat{S}_0$ when $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$ and at or below $\hat{S}_0$ when $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$.

When the threshold level of reserves, $\hat{S}_0$, is negative, matters are equally easy when $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$. Essentially, the corporate sector is in liquidity surplus. If the intent is to set the interbank premium to zero, any positive level of reserves will also do since every dollar of reserves adds to date-1 liquidity.

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[11] Central banks have sought to expand reserves in order to implement quantitative easing, where the effects range from signaling monetary policy stance (see Krishnamurthy and Vissing-Jorgensen (2011)) to recapitalizing banks through the back door or repairing markets (see, for example, Acharya, Eisert, Eufinger, and Hirsch (2019)).

The problem arises when $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$. Additional reserves will exacerbate the liquidity shortage since every dollar of date-0 reserves subtracts from date-1 net liquidity. While "negative" reserves may be required in theory, it is unclear how this can be implemented. Perhaps it is best to recognize that when reserves do not add to net future liquidity, the central bank should find other instruments so that the banking sector can provide liquidity to the deficient corporate sector – for instance by making long-term loans to the banking sector to encourage the purchase of long-term corporate financial assets/loans, and then being prepared to lend against those assets in case the economy becomes liquidity stressed. Parenthetically, this may resemble the European Central Bank's Long-Term Refinancing Operation (LTRO) interventions.

3.3. Bank Capital

Could bank capital requirements be an added instrument for the central bank to alleviate the problems with setting reserves at a level that induces a positive r_1? The answer is no. While there is a pecuniary externality when a bank makes its financing choice between deposits and capital (the bank takes r_1 as given and does not internalize the fact that a choice of higher deposit financing will increase the net demand for liquidity at date 1 and raise r_1), it is well known that pecuniary externalities need not cause a divergence between the private optimal and social optimal choices. Indeed, we will show in Appendix I that this is indeed the case in our model. This is unlike Lorenzoni (2008) or Stein (2012), where the price of liquidity (or, equivalently, fire-sale prices) show up in bank collateral constraints, implying banks finance excessively with deposits. However, we will see a divergence between the private and social optimal when we turn to endogenizing φ in the next section.

3.4. Discussion

The monetary authorities may want to set reserves at a high level, for instance under a program of quantitative easing. It has been hard to discern the net positive macroeconomic effects of these interventions (see Fabo, Jancokova, Kempf, and Pastor (2021), Greenlaw et al. (2018) and Moreno (2019)). One reason is perhaps because so much else is going on over the term of the interventions. Our analysis suggests another possible reason – the possible offsetting effects of central bank balance sheet expansion on credit and liquidity through a positive interbank premium r_1, which may partly account for why the effects of unconventional monetary policy are hard to discern.

Recent papers have been more sanguine about the risks of central bank balance sheet expansion, indeed arguing it might reduce risks. A number of papers find the liquidity premium paid for near-money

assets such as T-bills (as measured by the spread they pay below illiquid assets of similar maturity and risk) falls with the quantum of issuance of such near-money assets (see, for example, Bansal, Coleman, and Lundblad (2010) and Krishnamurty and Vissing Jorgensen (2012)). Consequently, some economists (see Stein (2012) and Greenwood, Hanson, and Stein (2016)) have argued that the large-scale expansion of central bank reserves can similarly reduce the money premium in bank deposits. This will discourage short-term bank financing. Essentially, the argument is that central bank reserves will compete with short-term bank deposits for place on private investor portfolios. Being a better source of liquidity, the former will displace the latter, and make the financial system safer (by avoiding deposit-induced run risk).

However, Nagel (2016) questions the basic premise that an issuance of near-money assets (such as T-bills) reduces the implied money premium (by satiating some fixed demand for liquidity). He documents that the money premium is positively correlated with the level of interest rates, which in turn is positively correlated with the issuance of near-money assets like T-bills. When the level of interest rates is introduced as an explanatory variable, the correlation between the money premium and T-bill issuance loses significance. Indeed, given a targeted interest rate, any fall in near-money T-bills will lead to an undersupply of liquidity and a potential rise in interest rates, which the Fed will offset by expanding reserves. Since banks finance reserves with deposits in his model, this could lead to a negative correlation between T-bills and deposits, without the money premium actually changing.

We make a somewhat different point – that the demand for liquidity is affected by the issuance of reserves. Because of their very nature, banks will finance a central bank reserve expansion with short-term liabilities. Far from crowding out bank deposits, central bank reserve issuance may enhance them (as also pointed out by Nagel (2016)). Our focus, however is on stress situations. Because placing reserves with banks commingles them with other bank activities whose risks can precipitate depositor runs, freely available liquidity in stress situations may be much less than suggested by the level of reserve issuance. Indeed, we argue more reserve issuance may even reduce ex-post liquidity and raise financial fragility.

Greenwood, Hanson, and Stein (2016) argue that the optimal way for the central bank to crowd out the money premium in deposits is to do reverse repo transactions directly with a broader set of non-bank investors (which most central banks do not do today). This is right, of course. If, for instance, households could hold reserves directly (that is, hold accounts at the central bank), it would crowd out their need to hold bank deposits. If, however, the final resting place of reserves is on the balance sheet of some non-bank finance company, it is essential that they see reserves as substitutes for bank deposits. If they have to hold reserves in addition to deposits, we will argue in the Appendix III that the problems we have highlighted will not diminish significantly, since these intermediaries will also fund the incremental

short-term assets with short-term liabilities so as to avoid asset-liability mismatches and reduce interest-rate risk; these short-term liabilities can expose them to runs when their non-reserve assets turn risky.

IV. Flight to Safety and Liquidity Hoarding

We assumed so far that an *exogenous* fraction φ of healthy banks lend in interbank markets, and the remaining fraction receives the flighty deposits of stressed banks. To endogenize the fraction φ, we will allow healthy banks to choose between lending in the interbank markets and consequently being *tainted* by the stress, or staying clear of profitable albeit risky interbank lending and instead attracting flight-to-safety deposits in stressed situations. At the equilibrium value of φ and the equilibrium interbank rate, healthy banks must be indifferent between choices. Importantly, as we will see, the interbank market may remain endogenously shut, that is, $\varphi = 0$.

4.1. Convenience Yield on Reserves in Stressed State of the Economy

For this choice to be interesting, there should be some value to attracting flight-to-safety deposits and passing up the opportunity to earn a premium in lending to interbank markets. To this end, we assume that when the economy is liquidity stressed, each dollar of reserves has a *convenience yield* $\delta > 0$ to the final holder. This could be thought of as the precautionary value of reserves in case there is further un-modeled stress, their value in signaling a "fortress balance sheet" to other stakeholders looking for safety, or the franchise value of deposits associated with those reserves. Any interest on excess reserves that the central bank pays over and above the market rate would also add to δ. Since the convenience yield is enjoyed by the final holder, any movement in reserves results in a private wealth transfer that washes out in the aggregate. However, the convenience yield significantly affects banks' responses to a liquidity shock, and the incremental value of a bank at date 1.

An immediate question is why safe banks do not compete for flight-to-safety deposits by raising rates. Acharya and Mora (2015) show that safe banks did not raise deposit rates during the GFC, while distressed banks did. One explanation is that safe banks were trying to signal that they did not need funds in order to avoid the stigma associated with risky banks. Relatedly, the inflow from risk-averse flight-to-safety depositors may be driven by convenience and a desire for principal protection rather than to exploit small differences in rates. For instance, depositors may flee their stressed bank to the most proximate safe bank. Finally, a bank will have to pay any higher rate to all its depositors. If the flight-to-safety deposits are only a small fraction of a receiving bank's overall deposits, safe banks may be reluctant to compete for them. This is a similar effect to Drechsler, Savov and Schnabl (2017), who document that banks in concentrated banking areas are reluctant to pay depositors higher rates when the Fed raises rates, since

they have to also pay captive depositors that rate. Given these considerations, we assume safe banks cannot (or will not) raise rates to attract more flight-to-safety deposits.

4.2. Equilibrium as S_0 changes

In the benchmark model of Section III, the interbank market was always open so that r_1 was both the interbank rate as well as the lending rate to stressed firms net of bank monitoring cost γ (that is, firms borrow at $r_1 + \gamma$ but the bank earns the net rate r_1). Now, the interbank market may be shut, but stressed firms will still borrow from their banks; so we will denote as r_1 the bank lending rate to the firm net of the monitoring cost and it will always exist. As before, we will use the notation $\overline{r_1}$ for the equilibrium interbank lending rate when the interbank market is open (in which case it will equal r_1). For emphasis, when there is liquidity stress but the interbank market is closed, we will denote r_1 as the *autarky* rate r_1^A; this is simply the rate that equilibrates liquidity demand and supply when the interbank market is closed. Finally, the presence of a convenience yield for reserves implies that the interbank market will be open only if the interbank rate exceeds a "breakeven rate" which we will denote as r_1^φ.

Now, in the liquidity stressed state, there are three cases to consider:

Case 1: *Stressed banks have enough liquidity (while raising date-1 capital commensurate with the convenience yield) to meet the needs of deposit outflows and to fund rescue investment without accessing the inter-bank market.*

This first case arises when the level of reserves, and in turn of demandable deposits, is adequately low.

Case 2: *The liquidity needs of each stressed bank can be entirely met by its raising date-1 capital (beyond that warranted by the convenience yield).*

In this second case, the level of reserves is moderately high and the interbank market remains shut ($\varphi = 0$) even though stressed banks are liquidity-deficient; formally, this occurs because the autarky rate r_1^A is below the breakeven rate r_1^φ that healthy banks require to enter the interbank market.

Case 3: *The liquidity needs of the stressed banks are high enough that at the equilibrium autarkic interest rate, some of the healthy banks are willing to lend in the interbank market and become tainted. The equilibrium rate then is lower than the (now counterfactual) autarkic rate.*

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The third case arises when the level of reserves is high enough that the autarky rate r_1^A rises above the breakeven rate r_1^φ and the inter-bank market opens up ($\varphi > 0$). Some surplus banks are induced by the high inter-bank premium to provide liquidity to deficient banks. [12]

Characterizing this third case is key to understanding when in equilibrium the interbank market opens up or remains shut. To see this, let a share φ of the healthy banks choose to lend in the interbank market to stressed banks. They will lend all their unencumbered reserves as well as the capital raised at date 1 into the interbank market at rate r_1. The date-1 profits from doing so are $V_1^\varphi =$

$$\left[(r_1 - \delta) S_0 (1-\tau) + \frac{r_1^2}{2\alpha_1} \right],$$ where the first term is the incremental value from lending out own

unencumbered reserves, and the second term is the profit from raising capital ($e_1 = \frac{r_1}{\alpha_1}$) and lending the proceeds. The reserve outflows from the stressed and now tainted banks amount to

$S_0 (1-\tau)(\theta + (1-\theta)\varphi)$ and these go to the $(1-\theta)(1-\varphi)$ banks that choose to be seen as safe. So the

profit from being seen as safe and attracting the flight-to-safety deposits is $V_1^{1-\varphi} =$

$$\frac{\delta S_0 (1-\tau)(\theta + (1-\theta)\varphi)}{(1-\theta)(1-\varphi)} = \delta S_0 (1-\tau)\left(\frac{1}{(1-\theta)(1-\varphi)} - 1 \right).$$ In equilibrium, healthy banks should be

indifferent between choosing to become tainted or stay safe. So $V_1^\varphi = V_1^{1-\varphi}$, and rearranging terms

$$(1-\varphi) = \frac{\delta S_0 (1-\tau)}{(1-\theta)\left(r_1 S_0 (1-\tau) + \frac{r_1^2}{2\alpha_1} \right)} \qquad (1.13)$$

Inspecting (1.13), it is clear that $\frac{\partial \varphi}{\partial S_0} < 0, \frac{\partial \varphi}{\partial \delta} < 0, \frac{\partial \varphi}{\partial r_1} > 0$. In words, the share of healthy banks lending

in the interbank market falls in the ex-ante level of reserves (because, as S_0 increases, the relative profits

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[12] We assume that δ is sufficiently large for these cases to arise. The condition is formally stated in Appendix II, Proof of Theorems 2-3. When the convenience yield δ is small, it is possible that only Cases 1 and 3 arise since the breakeven rate r_1^φ may be lower than δ at the level of reserves that requires a switch out of Case 1.

from raising and lending capital fall relative to attracting the flight-to-safety deposits) as well as in the convenience yield, and increases in the available interbank rate.[13]

Then, requiring that $\varphi > 0$ yields the "breakeven interbank rate" r_1^φ, that induces some banks to lend in the interbank market. When $\varphi > 0$, the equilibrium interest rate $\overline{r_1}$ and φ are now jointly determined as a fixed-point by equations (1.10) and (1.13). Finally, comparing the breakeven interbank rate r_1^φ and the autarky rate r_1^A determines when Case 2 versus Case 3 arise. These details are worked out in Appendix II where we show formally that

Theorem 2: For $\delta > 0$ and $\tau > 0$, there exist critical thresholds for the level of reserves, S_0^* and S_0^{**}, where $S_0^{**} > S_0^* > 0$, such that the inter-bank market is open, that is, $\varphi > 0$, only for $S_0 > S_0^{**}$, and

(i) for $S_0 \leq S_0^*$, stressed banks are not liquidity-deficient (taking into account the capital raise dictated by the convenience yield), and the equilibrium lending rate to firms r_1 (net of monitoring cost) equals δ;

(ii) for $S_0 \in \left(S_0^*, S_0^{**} \right]$, stressed banks are liquidity-deficient and raise more capital at date1 than dictated by the convenience yield, but the inter-bank market remains shut (autarky). Furthermore, the autarkic lending rate to firms r_1^A satisfies $r_1^A > \delta$, $\dfrac{dr_1^A}{dS_0} > 0$, and $r_1^A \left(S_0^{**} \right) = r_1^\varphi \left(S_0^{**} \right) > 0$; and,

(iii) for $S_0 > S_0^{**}$, stressed banks are liquidity-deficient and raise capital as well as borrow in the inter-bank market at date 1; the inter-bank rate satisfies $\overline{r_1}\left(S_0 \right) \geq r_1^\varphi \left(S_0 \right) > 0$, with $r_1^\varphi \left(S_0 \right)$ increasing in S_0.

We also show in Appendix II that a sufficient condition for the equilibrium r_1 to be increasing in S_0 is

$$\theta > \frac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$$ as in section II. It is only a sufficient condition, however, since with endogenous φ,

the incentive to hoard also increases in S_0, further increasing the equilibrium interest rate. Furthermore,

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[13] As an aside, as $\delta \to 0$, we have $\varphi \to 1$. That is, as the convenience yield of reserves falls to zero, virtually all healthy banks choose to lend in the interbank market. Only a sliver of the healthy banks prefers being seen as safe, and these attract all the flight-to-safety reserves, which carry an infinitesimal convenience yield δ.

since φ rises from zero at the breakeven interest rate r_1^φ at which the interbank market opens, there is always a region in Case 3 in which increases in S_0 will raise the ex-post interbank rate r_1.

Finally, as the convenience yield δ associated with the possession of reserves increases, the inter-bank market remains shut over a wider range of the level of reserves, and the level of the inter-bank rate increases with δ whenever the inter-bank market is open. Formally,

Theorem 3: (i) S_0^* and S_0^{**} are increasing in δ ; and, (ii) for $S_0 > S_0^{**}$, $\frac{d\overline{r_1}}{d\delta} > 0$.

4.3. Examples and Details

Figures 3A and 3B illustrate model outcomes for a specific parameterization where $\delta = 0.2, \tau = 0.2$. In 3A, $\theta = 0.8 = (1-\tau)$ and in 3B, $\theta = 0.6 < (1-\tau)$. Other parameters are $\lambda = 1, \gamma = 0.4, q = 0.1, \alpha_0 = \alpha_1 = 1, W_0^F = 2, g_0' = g_1' = 1/I.$ The green line is the breakeven interbank rate $r_1^\varphi (S_0)$, the yellow line the autarkic bank lending rate $r_1^A (S_0)$ (with its hypothetical value extrapolated if the inter-bank market were to remain shut even for $S_0 > S_0^{**}$), and blue line is the equilibrium interbank rate $\overline{r_1} (S_0)$ when some healthy banks choose to enter the inter-bank market. While the entry of some healthy banks pulls the inter-bank rate down (blue line relative to the yellow line), it nevertheless remains above $r_1^\varphi (S_0)$ and is increasing in S_0 for both parameter sets.

We also illustrate the effects of varying δ in Figures 4A and 4B, where θ is set at 0.6, less than $(1-\tau)$. In Figure 4C, δ takes values close to zero, whereas in Fig 4b, it takes significantly higher values (the range of equilibrium interest rates is commensurately higher in the latter case). In both cases, we see that as δ increases, the threshold level of reserves above which the inter-bank market opens up shifts to the right to a higher value though this shift is relatively modest at low values of δ ; we also observe that as δ increases, the inter-bank rate is higher whenever the inter-bank market is open. Finally, Figure 4C shows that as δ increases, the proportion φ of surplus banks that enter the inter-bank market decreases.

4.4. Bank Capital Choices.

If the central bank chooses a level of S_0 to achieve monetary objectives that induce a positive $\overline{r_1}$, is there any improvement that it could induce by altering bank financing choices? Prima facie, there could be since banks now don't internalize either the effect of their financing choice on $\overline{r_1}$ or its effect on the endogenous φ . Indeed, it is the latter that causes a divergence between the private and social optimal, but

interestingly in a different direction from Lorenzoni (2008) and Stein (2012). The social planner would finance with more deposits and less capital at date 0 than the bank privately would!

Lemma 3: If $\bar{r_1} > 0$, $\dfrac{dU}{de_0}\Big|_{e_0 = \frac{qr_1}{\alpha_0}} < 0$, so at the private bank's optimal financing choice, the central

bank/planner wants the bank to finance reserve holdings with less capital.

Proof: See Appendix II.

The reason is interesting. Higher capital issuance up front (by each bank) reduces the ex-post inter-bank rate, thereby increasing the degree of liquidity hoarding, and thus increases the degree of liquidity shortfall that has to be met by date-1 dissipative capital issues (by all stressed and tainted banks). This externality, it turns out, swamps the direct effect of higher date-0 capital issues reducing the liquidity shortfall, reducing the date-1 interest rate, reducing the need for lending by tainted banks, and thus for raising their date-1 capital issues.

Our result differs from Stein (2012) because the nature of the spillover differs. In Stein (2012), higher bank capital reduces the fire-sale discount, causing lending by non-banks at date 0 to increase (the source of spillover). In a sense, non-banks hoard less ex ante to invest in possible fire sales and that increases activity. In our framework, as we saw in the previous section, the bank makes all lending decisions. So the pecuniary externality embedded in r_1 (our measure of fire-sale discounts) does not distort private financing choices away from the social optimal – there are no non-banks to be influenced. However, when φ is endogenous, something the bank ignores in its maximization, the planner wants to achieve the lowest dissipative capital issues associated with a desired r_1. A higher date-0 capital issue directly reduces liquidity stress and hence r_1, but the higher induced hoarding indirectly increases the liquidity shortfall and increases dissipative date-1 equity issuance. The indirect effect swamps the direct effect, and implies the social planner would want lower date-0 capital held than the private optimal.

Our implications resemble Diamond, Jiang, and Ma (2021) and Liang and Parkinson (2020), who suggest the supplementary leverage ratio (requiring capital to be held against *all* assets including the relatively safe ones) should not apply to reserves. However, their reason to do away with capital requirements is to eliminate a regulatory encumbrance on the use of reserves (see section 5.2). Our point is that lower capital may reduce bank incentives to hoard liquidity ex post. Of course, our result stems from the effects of capital issue on liquidity. There is no bank insolvency in our model, and associated costs. Incorporating these would certainly alter implications, as would the spillovers of fire-sale prices to lending by others (see Diamond and Rajan (2011), Shleifer and Vishny (2010), and Stein (2012)).

4.5. Endogenous δ

Thus far, we have assumed the date-1 convenience yield is exogenous. It is reasonable, however, to assume that the convenience yield is a function of r_1, a measure of the date-1 stress. The higher the realized date-1 stress, the more worries the system may have about hidden prospective problems (and opportunities) emanating from the current stress, and the more it may value the convenience yield in holding reserves. If $\delta(r_1) = \delta^A + \delta^B r_1^2$ where $\delta^A \geq 0, \quad \delta^B \geq 0$, we show in Appendix II that depending on parameters, it is possible the interbank market never opens (in particular when δ^B is sufficiently high, because the convenience yield grows fast with the inter-bank premium, increasing the returns to hoarding more than lending), or that it opens only if the date-1 rate lies in a range and is closed otherwise, or that it opens above a certain rate as with a fixed δ. The point, however, is that date-1 stress could be magnified significantly with endogenous δ.

4.6. Discussion

Arguably, δ is higher in environments where bank assets other than reserves are very illiquid, and where the incidence of systemic stresses are positively serially correlated. This then leads to the possible desirability of ex-post central bank intervention. The central bank could try to bring down $\overline{r_1}$ by injecting reserves at date 1 if the economy becomes liquidity stressed. This may run up against similar frictions to the fear of taint we have documented. A bank may face "stigma" in interbank markets if it accesses central bank facilities (see Hu and Zhang (2020) or Nelson (2022) for example); tapping intraday into the central bank could be problematic if it prompts rumors of potential stress at the bank, which cause other banks to freeze lending and wholesale deposits to flee.

Assuming banks overcome stigma in extreme situations, there are still three important caveats here. First, the most effective way for the central bank to intervene ex post is for it to lend unsecured into the interbank market. However, this entails significant risk of central bank loan losses. If it does lend against high-quality securities, though, the financial sector will have to hold those high-quality securities ex ante. If they are financed with deposit issuance, they add no extra liquidity to a stressed bank, even allowing for central bank lending against them. Furthermore, we show in section 6.3 and appendix III that bank incentives to voluntarily hold reserves (or equivalently, other high-quality safe assets that could be used to borrow reserves at date 1) can be lower than the socially desirable level. Of course, the central bank can broaden the range of assets it will lend against (for example, lend against corporate securities) even while increasing the size of the haircut it levies on collateral value. The larger the quantum of

intervention, the more the central bank is likely to depart from alleviating just liquidity risk, instead taking on other risks such as credit risk. [14]

But this leads to the second concern. Central bank intervention at date 1 will reduce the interbank rate. But then fewer healthy banks will want to lend into that interbank market. So the act of intervention ex post will crowd out private lending, potentially keep interbank markets shut over a wider range of ex-ante reserves, and increase the ex-post quantum of needed central bank intervention. Another subtle effect is also at play. Note that in our model, the incentive to hoard reserves is diminished by the capital issuance of stressed and tainted banks, which reduces the reserves hoarded by safe banks. If, however, capital issuances fall because the central bank adds new reserves to the system (which eventually find their way to the safe banks), it further increases the incentive to hoard over and above any effect on the interbank rate because a greater stock of reserves flows to the safe banks. Of course, central bank intervention may also reduce the convenience yield on reserves (assuming the private sector, in response, does not build up illiquidity again – see below), especially if the central bank commits to lending freely in the foreseeable future without much concern for security. This can reduce the incentive to hoard. The net quantum of required ex-post central bank intervention – whether it crowds in or crowds out – then depends on the interaction between the augmented quantity of reserves, the interbank rate, and the convenience yield. Once again, though, the central bank will have to take on balance sheet risk.

And finally, there are the better-known ex-ante consequences. If leveraged illiquid banks expect to receive central bank funds ex post, they may not reduce their illiquid assets in a timely manner by transferring assets to less leveraged, more liquid banks, taking on further liquidity risk in the process (Acharya and Tuckman, 2014). Similarly, the more the financial sector expects central bank intervention, the more it will increase the ex-ante issuance of claims on liquidity, effectively reducing liquidity holdings net of liquidity promises (see Acharya, Shin and Yorulmazer (2011), Diamond and Rajan (2012) or Farhi and Tirole (2012)), and necessitating intervention of yet greater magnitude.[15]

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[14] The central bank could offer secured lending to the entire financial sector against high quality assets rather than just to the banking sector (see Liang and Parkinson (2020), and the recent move to Standing Repurchase Facilities with a wider set of market participants). This will prevent the banking sector from becoming an impediment to the transfer of liquidity to stressed firms in the financial sector. It will not necessarily ensure that financial firms with surplus liquidity will recirculate it to stressed firms in the real economy or in the banking sector.

[15] Indeed, some central banks recognize that their provision of liquidity on demand creates dependence for more. Nelson (2019) cites a Norges Bank statement in 2010 justifying its move to a deficit reserves position in the system thus: *"When Norges Bank keeps reserves relatively high for a period, it appears that banks gradually adjust to this level…With ever increasing reserves in the banking system, there is a risk that Norges Bank assumes functions that should be left to the market. It is not Norges Bank's role to provide funding for banks…If a bank has a deficit of reserves towards the end of the day, banks must be able to deal with this by trading in the interbank market."*

In sum, illiquidity premia can be brought down ex post through central bank intervention – indeed, some see this as the fundamental purpose of a central bank. Yet repeated central bank intervention is not without cost. The central bank can distort the pricing and quantum of liquidity in the market considerably by displacing the market – it will tend to overdo intervention and underprice it. Participants will become extremely dependent on a fallible and not always predictable central bank, and will even game it into intervening. That too has costs. The scale of central bank liquidity interventions may only get larger, as it has over the past three decades (in 2020, the Fed rolled out many of the programs it had created during the Great Recession plus some new ones). Greater liquidity dependence, which we have argued can portend greater future liquidity stress, would be an important unintended consequence of central bank balance sheet expansion.

V. Encumbrance on Reserves

We have assumed an encumbrance τ on reserves. We initially show that speculation and regulation are two channels through which reserves encumbrance can be endogenized. Next, we argue our results are robust to assuming a fixed *level* of encumbrance on reserves (instead of assuming a fixed encumbrance *share* of reserves).

5.1. Endogenizing Encumbrance Share τ: Speculation

Reserves, as we argued in the introduction, have an optionality embedded in them. Ideally, banks would like to sell that option when they do not need it (when the economy is healthy), and retain it when the economy is liquidity stressed. Unfortunately, such selective sales of liquidity may be difficult.

Consider, for example, the prime brokerage services that banks offer. Let each bank serve one speculator. Let the speculator put on trades at date 0 of size x. In normal economic times, the bets pay off and return ηx to the speculator and fees of ρx to the bank. Conditional on the economy getting liquidity stressed (with probability $\frac{q}{\theta}$), the bank has to meet margin calls on the speculator, putting up reserves of κx. These calls have priority over all other claims on the bank (else it will have to default on exchanges, and see its brokerage business shut down). Alternatively, if the trades are centrally cleared to reduce any risk of contagion from such speculative positions, the clearinghouse would require the clearing members (banks) to over-collateralize their positions and contribute initial and variation margins and guarantee fund contributions. The resulting funds with clearinghouses are typically not allowed to be rehypothecated (or face significant limits on rehypothecation), and a large fraction of it is in the form of reserves deposited with the central bank, thereby being unavailable for further private use.

Finally, assume each speculator's search costs of putting on a profitable trade is increasing in the size of a trade (that is, there are fewer remaining low hanging fruit as they trade more) and decreasing in the unencumbered liquidity of the system, so it is $\dfrac{v}{2}\dfrac{x^2}{(S_0 - \kappa \overline{x})}$, where v is a parameter and $\overline{x}$ is the equilibrium level of trade per bank. This captures the notion that liquidity facilitates speculation, but speculators are aware that liquidity gets tied up as there is more speculative trade. Assume that $\eta > \rho$ which ensures that speculation is profitable net of fees. For simplicity, we focus on the model of Section 2 with an exogenously given share of surplus banks in the interbank market (in the sub-section with fixed encumbrance, we will examine endogenous shares). The speculator's maximization problem is then:

$$\underset{x}{Max}\quad (1 - \frac{q}{\theta})[\eta - \rho]x - \frac{v}{2}\frac{x^2}{(S_0 - \kappa \overline{x})}$$

The first order condition is $(1 - \frac{q}{\theta})[\eta - \rho] = \dfrac{vx}{(S_0 - \kappa \overline{x})}$. Recognizing that $x = \overline{x}$ in equilibrium, we have

$$\kappa \overline{x} = \frac{S_0 \kappa \left(1 - \dfrac{q}{\theta}\right)(\eta - \rho)}{v + \kappa \left(1 - \dfrac{q}{\theta}\right)(\eta - \rho)} = \tau S_0 \text{ where } \tau = \frac{\kappa \left(1 - \dfrac{q}{\theta}\right)(\eta - \rho)}{v + \kappa \left(1 - \dfrac{q}{\theta}\right)(\eta - \rho)}.$$

Assuming that the market for provision of prime-brokerage services to speculators is competitive among banks at date 0, the fee ρ per unit of speculative activity is set such that in expectation banks are compensated for the cost of providing the per-unit margin call κ. This zero-profit condition implies then that $\left(1 - \dfrac{q}{\theta}\right)\rho = \dfrac{q}{\theta}[\varphi(1 - \theta) + \theta]\overline{r_1}\kappa$. Substituting above for the implied $\rho(\overline{r_1})$, we obtain that the encumbrance per unit of reserves is a function of the date-1 interbank rate premium; it is $\tau(\overline{r_1})$, such that $\tau'(\overline{r_1}) \le 0$. This implies then that at low levels of the expected rate, there is greater speculation, and if liquidity needs in the stressed state rise, then speculative activity is tempered by the expectation of a rising interbank rate, creating an additional equilibrating force that clears the market for reserves. Using $\tau'(\overline{r_1}) \le 0$ and logic analogous to the proof of Theorem 1, it follows that

Theorem 4:

(i)   If $\theta > \dfrac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, then $\overline{r_1}>0$ is the unique equilibrium for $S_0 > \hat{S}_0$; $\overline{r_1}$ increases with

S_0 over a range $[\hat{S}_0, \hat{S}_0^{\,*}]$ till it reaches r_1^* where $\dfrac{\varphi(1-\tau(r_1^*))}{\tau(r_1^*)+\varphi(1-\tau(r_1^*))} = \theta$, after which $\overline{r_1}$ does

not increase with further increases in S_0. Also $\overline{r_1}=0$ for $S_0 \le \hat{S}_0$. Note that $\hat{S}_0 \le 0$ if $NLS \le 0$.

(ii)   If $\theta \le \dfrac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, then $\overline{r_1}>0$ is the unique equilibrium for $S_0 < \hat{S}_0$; $\overline{r_1}$ increases as S_0

falls till it reaches r_1^{**} at $S_0 = \hat{S}_0^{\,**}$ where $\dfrac{\varphi(1-\tau(r_1^{**}))}{\tau(r_1^{**})+\varphi(1-\tau(r_1^{**}))} = \theta$, after which $\overline{r_1}$ does not

increase with further decreases in S_0. Also $\overline{r_1}=0$ for $S_0 \ge \hat{S}_0$. Note that $\hat{S}_0 \le 0$ if $NLS \ge 0$.

In essence, case (i) which formalizes our novel insight continues to hold with the endogenous modeling for reserves encumbrance. As long as additional reserves create a net demand for liquidity when interbank rate is zero, that is, $\theta > \dfrac{\varphi(1-\tau(0))}{\tau(0)+\varphi(1-\tau(0))}$, increasing reserves leads to an interbank rate that is greater than zero, and which rises with reserves until the speculative encumbrance τ falls to the point that an incremental increase in reserves does not change the net demand for liquidity (per dollar of reserve), and in turn, $\overline{r_1}$ or $\tau(\overline{r_1})$. Interestingly, therefore, $\tau(\overline{r_1})$ never falls to zero. Hence, our starting assumption that prime-brokerage fee is lower than the speculative return, $\eta > \rho$, always holds in equilibrium. Importantly, the lower is the average or expected margin requirement κ on speculative activity, the greater the ex-ante speculative activity (all else equal), and in turn, the range of parameters for which more reserves can tighten interbank markets. In a richer model with multiple states of liquidity stress, margins may rise in a state-contingent – procyclical – manner when the liquidity stress is most severe due to the attendant increase in counterparty risk (see Aramonte, Schrimpf, and Shin (2021)).[16]

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[16] Barth and Kahn (2021) provide evidence, for instance, that speculative hedge fund positions in relative value (cash-futures) trades in Treasury markets grew from under $200 billion in 2013 to $800 billion in January 2020, rising sharply during 2018 and 2019, with required margins on their futures position rising sharply with the rise of VIX in March 2020. Schnabel (2020) also observes that initial and variation margins collected by the four European central counterparties rose immediately around the outbreak of the pandemic, with variation margins often growing more than fivefold and exceeding pre-pandemic cash positions for several of the derivatives counterparties.

5.2. Endogenizing Encumbrance Share τ : Regulation

To offset speculation, regulators may place their own encumbrances on reserves. Farhi, Golosov and Tsyvinski (2009) suggest a floor on liquidity holdings to prevent a bank from free riding on other banks *a la* Jacklin (1987). Calomiris, Heider, and Hoerova (2014) suggest a minimum level of cash reserves to limit risk shifting. Such regulations are likely to be insufficiently contingent. The most obvious such regulation is a requirement that a certain fraction of assets have to be held at all times in the most liquid form (see, for example, Diamond and Kashyap (2016)) or a capital requirement that binds precisely when a bank ought to lend out its excess reserves (see, for example, Vanderweyer (2019)).

Why cannot such requirements be dropped in times of stress? As Goodhart (2008) emphasizes, a policy of having at least one taxi at the station is of little benefit to the late-arriving traveler if it cannot be used. Diamond and Kashyap (2016) argue, however, that it may make sense for the regulator to prevent a bank from using up liquid reserves in stressed times if the anticipation of use causes the stress to spread – if savers believes healthy banks have no mandated liquid assets and might lend them all to stressed banks, they may run on all banks. We incorporate such regulatory requirements in Appendix III.

Some bank actions in response to uncertain regulation could also amplify encumbrances. D'Avernas and Vanderweyer (2021) attribute enhanced volatility and fragility in repo markets to regulations on intra-day bank liquidity holdings. They cite Jamie Dimon, CEO of JP Morgan *"[...] we have $120 billion in our checking account at the Fed, and it goes down to $60 billion and then back to $120 billion during the average day. But we believe the requirement under CLAR and resolution and recovery is that we need enough in that account, so if there's extreme stress during the course of the day, it doesn't go below zero."* In other words, regulations appear to have forced JP Morgan to hold a portion of reserves back for really extreme market events – since no one really knows what these might be, some portion of the reserves might be permanently encumbered.

Furthermore, Nelson (2019) documents that in a Bank Policy Institute (BPI) survey conducted in January 2019, bank examiner expectations about liquidity holdings were mentioned overwhelmingly as "important" or "very important" reasons for reserve demand by banks. Indeed, Nelson points out that in times of abundant reserves, bank supervisors scrutinize any drawdowns carefully, creating a ratchet effect (higher the held reserves, higher the reserves supervisor expect) limiting the ability of healthy banks to redeploy reserves when needed.

5.3. Fixed Encumbrance on Reserves ($\tau S_0 \equiv \overline{E}$)

Suppose that instead of a constant fraction, the regulatory encumbrance is a fixed amount $\overline{E}$ of required reserves, independent of total reserves, S_0. Our analysis carries over to this case even though with a fixed encumbrance, an increase in reserves cannot shrink ex-post liquidity simply because it is financed with deposits. However, the novelty here is that as long as there is a convenience yield on reserves in the stressed state, an increase in reserves increases the returns to hoarding and staying safe to attract flight-to-quality deposits; this reduces in turn the fraction of surplus banks in the interbank market (which may remain shut altogether). For reasons of space, we leave the formal statement of results and analysis to Appendix II (Theorem 5 and Figure 5).

VI. Robustness

We now elaborate on some of the assumptions we have made so far and discuss their robustness.

6.1. Nature of Liquidity Shock

We have assumed the liquidity shock at date 1 is a shock to firm fundamentals – and thereby affects both sides of the funding bank's balance sheet (in terms of loan demands and deposit withdrawals). Yet to the extent that liquidity shocks precipitate solvency concerns, any large-scale flow out of the financial system such as a significant buildup of Treasury balances would trigger similar effects, as would other forms of contagion such as firms drawing down credit lines (following the collapse of Lehman Brothers, see Ivashina and Scharfstein (2010), and at onset of the pandemic, see Acharya, Engle and Steffen (2021)).[17]

An important simplification is that the share of stressed banks, θ, is invariant to the build-up in reserves. It might seem that the risks to commercial bank balance sheets should fall as the share of reserves composing those balance sheets increases. If so, our model would hold for only the range of reserve expansion where commercial bank credit risk is not swamped by reserve expansion. Yet this neglects three possible sources of risk. First, beyond a certain point, incremental reserves are entirely funded by demandable deposits (as implied by Exhibits 1 and 2). So absent commercial bank capital-raising, even small amounts of credit risk relative to the size of the commercial bank's assets can have large consequences. Second, the monetary effects of central bank balance sheet expansion, if sizeable

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[17] There is some evidence that credit line "promises" by banks have risen along with the supply of reserves. Undrawn credit lines issued by banks in the United States expanded by $452,367 million during QE III (between September 2012 and October 2014) and $457,539 million post the pandemic (between March 2020 and Dec 2020). We are grateful to Sascha Steffen for sharing with us these calculations based on FDIC Call Reports.

(though see the discussion in section 3.4), should expand the size of corporate borrowing and increase the risk thereof (that is, it should increase I_0). Third, we have assumed the encumbrance on reserves from speculative activity to be risk-free. In practice, some of it will be risky (speculators will go bust). Our model in Section 5.1 of the speculative elements forcing the reserves encumbrance share τ to be higher at low interbank rates also suggests that a prolonged period of easy money could cause the encumbrance rate to rise (and outside of our model, possibly also because the optimistic speculators get richer and put on bigger bets as in Geanakopolos (2008)). The same factors causing greater speculation could also cause θ to rise. Prolonged easy conditions may then switch ex-ante reserves from alleviating future liquidity stress to exacerbating it. A deeper analysis of the underpinnings of θ and τ, and their interconnected dynamics, offers an interesting avenue for future research.

6.2. Ex-ante Reserves and Activity

We have looked at economic activity after central-bank-issued reserves find their way to bank balance sheets. However, the act of issuing reserves may itself propel activity at date 0, setting up the monetary policy/financial stability tradeoff we referred to earlier. Quantitative easing was one, another could be if some kind of mandated reserve requirement held back bank deposit creation and thus lending (see, for example, Stein (1998)). Suppose $D_0 \leq \varsigma S_0$ so that deposits cannot be more than ς (greater than one) times bank reserve holdings. On the one hand, a binding reserve constraint on deposit issuance will limit ex-ante lending, as well as increase the use of capital in financing. On the other, it will limit the extent of liquidity stress ex post, and thus reduce the interbank premium, with attendant positive effects on ex-ante lending. Since reserve requirements are still in place in a number of countries, working out the consequences is an interesting area of future research.

6.3. Other Extensions

Our simple model allows for many other possible extensions and explorations. Two are worth sketching. First, what if banks were not forced to hold S_0? It turns out, not surprisingly, that banks will privately not have the same incentives as the planner/central bank and will want to optimally hold different levels of reserves in a variety of circumstances (see Appendix III).

Finally, what if the central bank issues reserves directly to the non-bank financial sector? Here again (see Appendix III), a desire to match the duration of liabilities with assets to reduce risk will result in non-banks financing with short-term liabilities. Many of the consequences we have documented will follow. In April 2021, the Federal Reserve reinstated the supplementary leverage ratio (SLR) for

commercial banks. This is a regulatory capital requirement that was suspended in April 2020 in the wake of the pandemic (see Covas, 2021). Given the increased cost to banks of funding reserves with long-term capital, they released reserves. Interestingly, money market funds, themselves funded with short-term liabilities, took on the reserves, redepositing them at Fed through reverse-repo facilities. This suggests the natural way for intermediaries to fund reserves is short-term even in the non-bank financial sector and will likely lead to concerns of financial fragility akin to the ones we have analyzed for the banking sector.

Conclusion

The significant expansion of central bank balance sheets in recent years should have reduced liquidity stress, and even perhaps increased real activity. We propose reasons why central bank balance sheet expansion may be less helpful in stress situations than one might think a priori. In particular, the financing of reserves on bank balance-sheets creates deposit claims on future liquidity that offset the reserves; furthermore, the encumbrance of reserves due to speculation and regulation, and reserves hoarding by healthy banks in times of liquidity stress, may prevent liquidity flowing to stressed banks. Ex ante, these effects may partly explain why central bank balance sheet expansion has less effect on real activity than one might anticipate. We have likely only scratched the surface in modeling and sketching out implications of the phenomenon that the ex-ante supply of reserves affects the ex-post demand for them. There is clearly more work to be done in understanding and mitigating liquidity stress implied by this phenomenon.

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Exhibit 1

Incremental Depository Institution balance sheets (obtained from Flow of Funds data Z1.111 – Level Data: U.S.-Chartered Depository Institutions)

All entries under Assets and Liabilities are increments, that is, changes, for that entry in Millions of Dollars; all ratios are increment or change in the numerator divided by that for the denominator.

QE II (between November 2010 and June 2011)

Assets		**Liabilities**	
Cash	60	Bonds	-62,773
Debt Securities	94,351	Holding company investment	42,870
Loans	-103,791	Commercial paper	-46,539
Misc	18,748	Loans	-80,632
Repos	-11,639	Miscellaneous	-315,306
Reserves	194,070	Insured deposits	1,264,014
		Uninsured deposits	-534,919

|	
Deposits/Total Liabilities	2.73361
Deposits/(Cash+Securities+ Repos+ Reserves)	2.63361
Deposits/(Repos+Reserves)	3.99655
Uninsured deposits/(Repos+ Reserves)	-2.93217
Uninsured deposits/(Uninsured+ insured deposits)	-0.73368

QE III (between September 2012 and October 2014)

Assets		**Liabilities**	
Cash	11,191	Bonds	-112,030
Debt Securities	504,642	Holding company investment	332,381
Loans	804,170	Commercial paper	-86,743
Misc	-64,076	Loans	108,019
Repos	-29,398	Miscellaneous	184,540
Reserves	713,351	Insured deposits	-810,496
		Uninsured deposits	2,528,429

Deposits/Total Liabilities	0.80124
Deposits/(Cash+Securities+ Repos+ Reserves)	1.43187
Deposits/(Repos+Reserves)	2.51177
Uninsured deposits/(Repos+ Reserves)	3.69679
Uninsured deposits/(Uninsured+ insured deposits)	1.47179

Pandemic (between March 2020 to end 2020)

Assets		Liabilities	
Cash	15,843	Bonds	26,083
Debt Securities	1,041,056	Holding company investment	202,606
Loans	289,404	Commercial paper	26,651
misc	272,661	Loans	-227,272
Repos	179,821	Miscellaneous	-125,790
Reserves	1,282,417	Insured deposits	1,317,938
		Uninsured deposits	1,719,650

Deposits/Total Liabilities	1.03324
Deposits/(Cash+Securities+ Repos+ Reserves)	1.20581
Deposits/(Repos+Reserves)	2.07736
Uninsured deposits/(Repos+ Reserves)	1.17604
Uninsured deposits/(Uninsured+ insured deposits)	0.56612

Exhibit 2
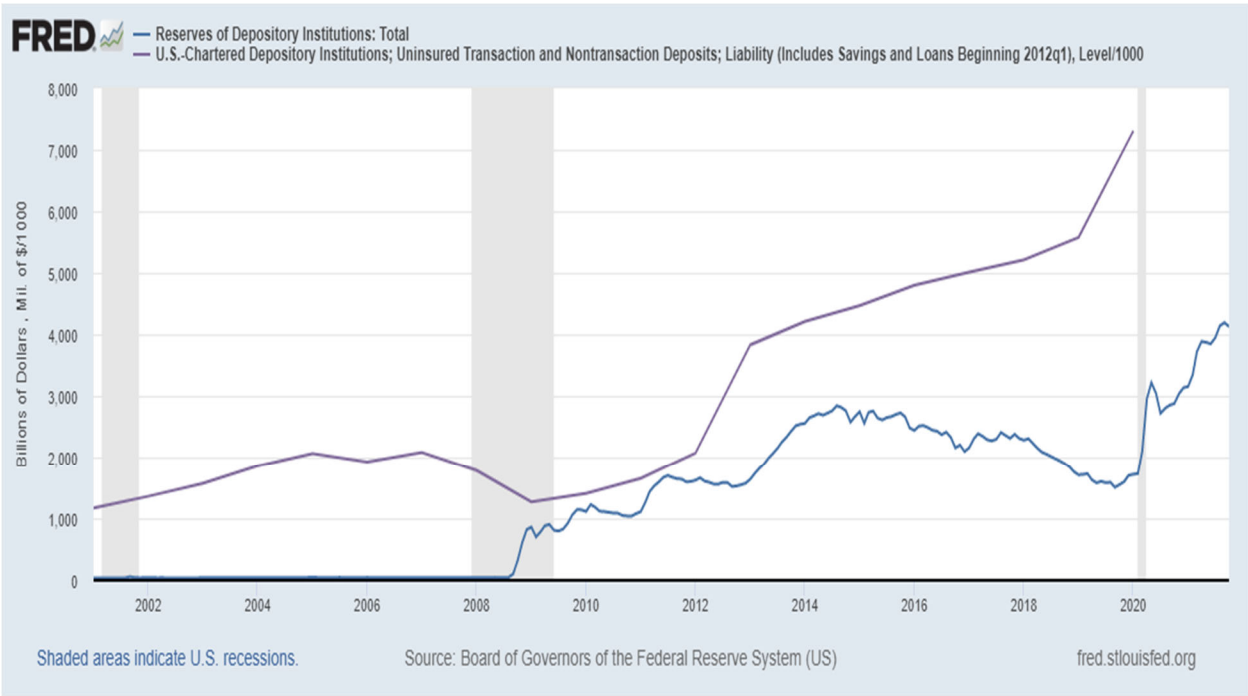
Reserves of depository institutions and uninsured deposit liabilities. The data are from the Federal Reserve Bank of St Louis database (FRED).

Exhibit 3: Bank and Firm Balance Sheets

Firm Balance Sheet at Date 0	
Assets	**Liabilities**
I_0	$L_0^F \quad (=L_0^B)$
D_0^F	W_0^F
	Net worth

Bank Balance Sheet at Date 0	
Assets	**Liabilities**
$L_0^B + \frac{1}{2}\lambda(L_0^B)^2$	D_0
S_0	e_0
	Net worth

Firm Balance Sheet at Date 1 if stressed	
Assets	**Liabilities**
I_1	l_1^F
	L_0^F
	Net worth

Bank Balance Sheet at Date 1 if bank stressed	
Assets	**Liabilities**
$L_0^B + \frac{1}{2}\lambda(L_0^B)^2$	Possible interbank borrowing = b_1
τS_0	e_1
$l_1^B \quad (=l_1^F)$	e_0
	Net worth

Firm Balance Sheet at Date 1 if healthy	
Assets	**Liabilities**
I_0	L_0^F
D_0^F	W_0^F
	Net worth

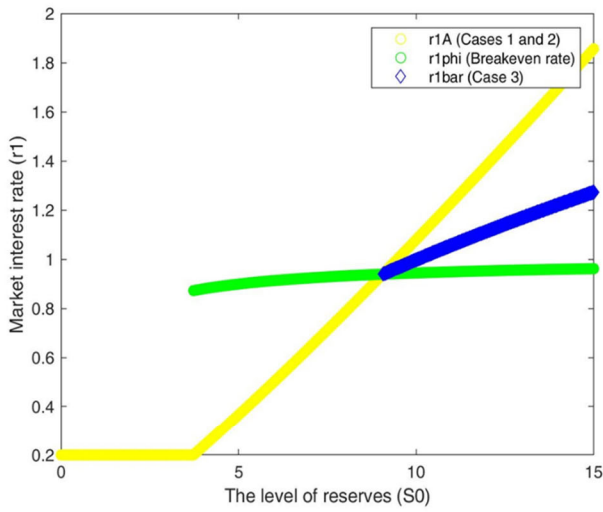| Bank Balance Sheet at Date 1 if economy stressed, bank healthy but "tainted" (makes interbank loans)	
Assets	**Liabilities**
$L_0^B + \frac{1}{2}\lambda(L_0^B)^2$	D_0
Interbank loans of up to $e_1 + (1-\tau)S_0$	e_1
Reserves of $(S_0 + e_1$-interbank loans)	e_0
	Net worth

Figure 3A: Market Rate and Reserves

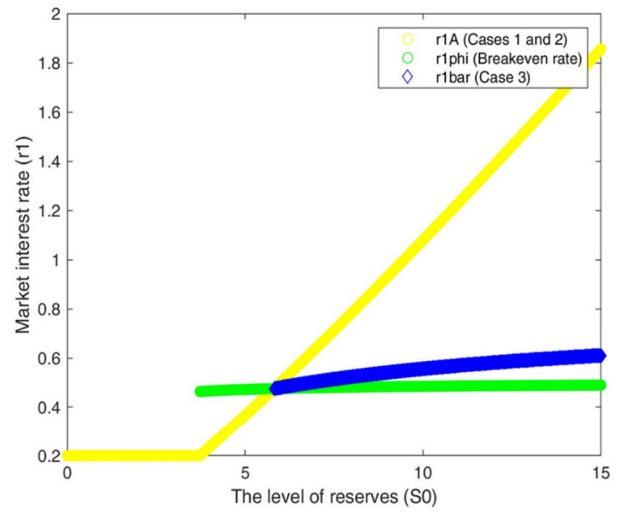$\delta = 0.2,\ \tau = 0.2\ ,\theta = 0.8$

Figure 3B: Market Rate and Reserves

$\delta = 0.2,\ \tau = 0.2\ ,\theta = 0.6$

Figure 3: Numerical example for the extended model of section IV with endogenous interbank entry: The effect of varying the size of the shock θ

Figure 4A: Market Rate and Reserves for low δ
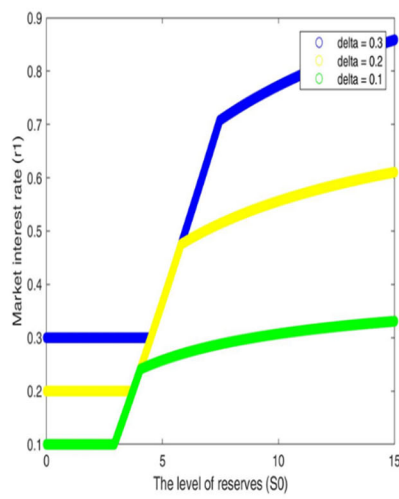
Figure 4B: Market Rate and Reserves for high δ

Figure 4C: φ and reserves for various δ

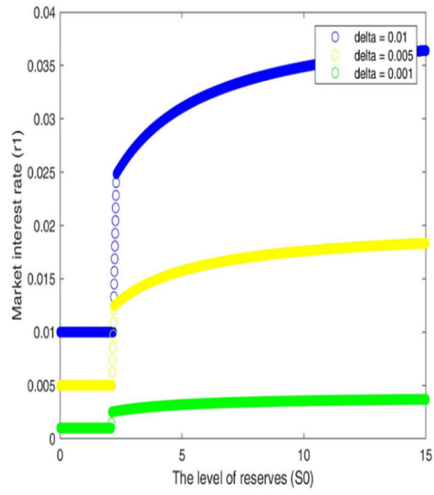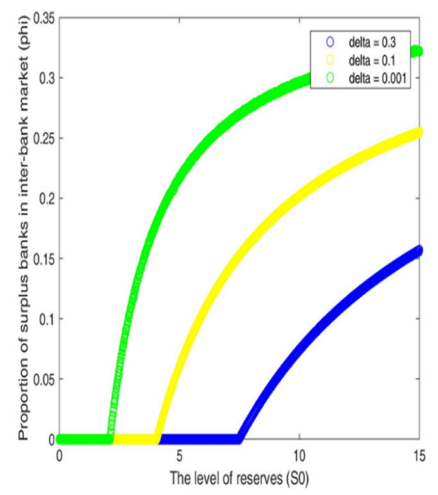**Figure 4: Numerical example for the extended model of section IV with endogenous interbank entry: The effect of varying the convenience yield on reserves** δ

Appendix I – Proofs for Sections II and III

Proof of Lemma 1: The right hand side of (1.10) is decreasing in r_1 (we will see this shortly). The left hand side is obviously increasing in r_1. If so, if the right hand side of (1.10) is positive when $r_1 = 0$ then there is excess demand for funds in the inter-bank market when the premium is zero, and hence there is an unique positive crossing point, the equilibrium r_1. If the right hand side is non-positive when $r_1 = 0$, there is (weakly) excess supply, and r_1 is zero. So it remains to show the right hand side of (1.10) is decreasing in r_1. From (1.3), $I_1 = g_1'^{-1}(1 + \gamma + r_1)$, which is decreasing in r_1. Turn next to the second term in the square brackets, $(D_0 - D_0^F)$. This equals $\left[I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2}\lambda(L_0^B)^2 \right]$. We know

$$I_0 = g_0'^{-1}\left(\frac{1 + q\gamma + qr_1}{(1-q)} \right)$$ which is decreasing in r_1. Also, $-e_0 = -\dfrac{qr_1}{\alpha_0}$, which is decreasing in r_1. The

next term, $(S_0 - W_0^F)$, is a constant. That leaves the last term, in the expression for $(D_0 - D_0^F)$,

$\frac{1}{2}\lambda(L_0^B)^2$. From (1.9), $L_0^B = \dfrac{1}{\lambda}\left(\dfrac{R_0^{DF} - R_0^{DB}}{R_0^{DB}} \right) = \dfrac{q}{\lambda}\left(\dfrac{\gamma}{1 + qr_1} \right)$, which decreases in r_1 whence given L_0^B is

positive, $\frac{1}{2}\lambda(L_0^B)^2$ also decreases in r_1. So we have $(D_0 - D_0^F)$, the deposits the bank raises from the public, decreasing in r_1. Finally, the last term on the right hand side of (1.10), $-\alpha_1 S_0(1-\tau)$, is a constant. So the right hand side of (1.10) is decreasing in r_1 and the equilibrium r_1 is unique. Q.E.D.

Threshold level of reserves when *NLS* > 0: Recall that *NLS* is the net liquidity supplied by the corporate sector anticipating a date-1 interbank premium of zero (and adjusting for any cost to lending). When $NLS > 0$ and the risk of liquidity stress in the economy is high, *that is, $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$* (this is

the region to the right of the vertical axis in Figure 2B), $\hat{S}_0$ is positive, and falls in θ. Intuitively, because higher ex-ante reserves tighten liquidity in the stressed state, and a higher θ consumes more liquidity per dollar of reserves, the reserve threshold at which the net liquidity supplied by the corporate sector is fully consumed is positive and falls in θ. Furthermore, $\bar{r}_1$ increases in S_0, and the unhatched region above the $\hat{S}_0$ curve is where $\bar{r}_1$ is positive. When $NLS > 0$ and the risk of liquidity stress in the economy is low, *that is, $\theta < \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$* (that is, in the region to the left of the vertical axis in Figure 2B), $\hat{S}_0$ is negative,

and falls in θ. Because higher ex-ante reserves loosen liquidity conditions, $\overline{r_1}$ falls in S_0, and the unhatched region below the $\hat{S}_0$ curve is where $\overline{r_1}$ is positive. The hatched area is where $\overline{r_1}$ is zero.
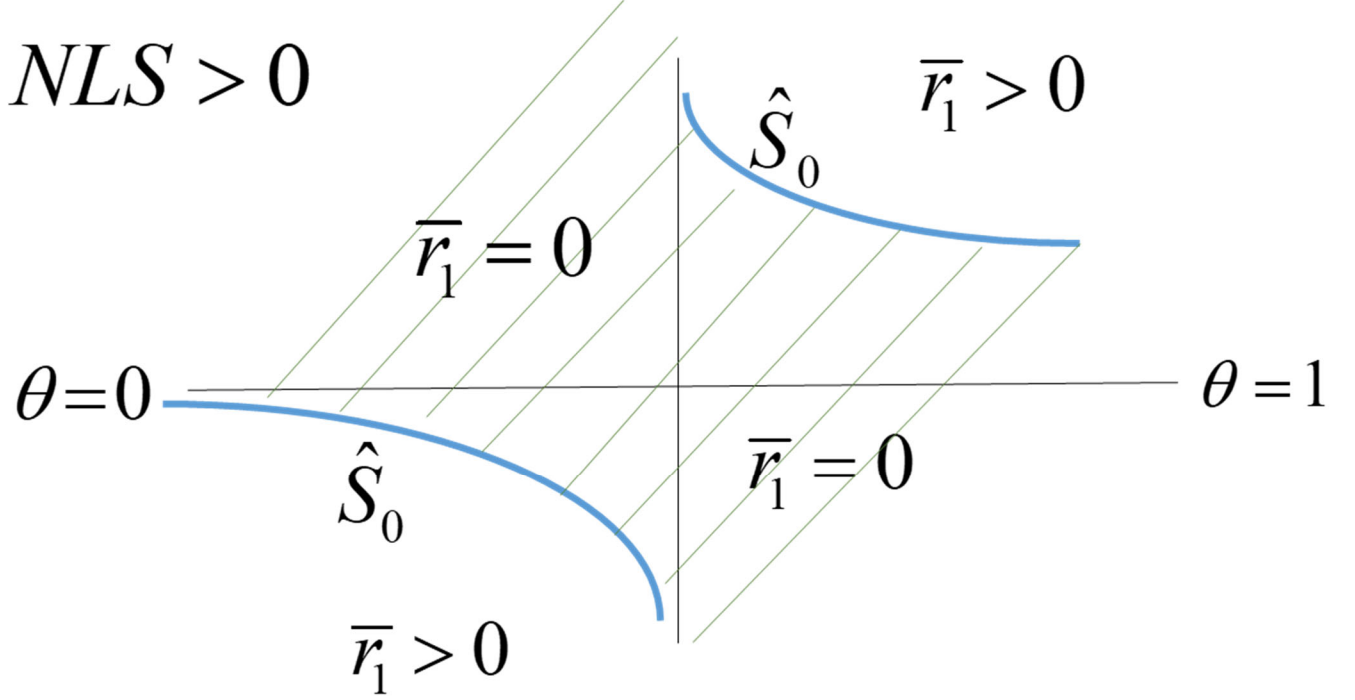

Figure 2B: Reserves are on the y axis, θ on x axis, with the two axes intersecting at $\theta = \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$

Proof that $\dfrac{\partial U}{\partial r_1} < 0$ **in section III:** Substituting $D_0^F = \left(L_0 + W_0^F - I_0\right)$, the planner maximizes

$$\left((1-q)g_0(I_0) - I_0(1+q\gamma)\right) + q\left(g_1(I_1) - I_1(1+\gamma)\right) - \tfrac{1}{2}\alpha_0 e_0{}^2 - \frac{q}{\theta}\left[\varphi(1-\theta)+\theta\right]\left(\tfrac{1}{2}\alpha_1 e_1{}^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2 + q(L_0 + W_0^F)\gamma$$

Differentiating U w.r.t. r_1, we get $\dfrac{\partial U}{\partial r_1} =$

$$\left((1-q)g_0{}' - (1+q\gamma)\right)\frac{dI_0}{dr_1} + q\left(g_1{}' - (1+\gamma)\right)\frac{dI_1}{dr_1} - \alpha_0 e_0\frac{de_0}{dr_1} - \frac{q}{\theta}\left[\varphi(1-\theta)+\theta\right]\alpha_1 e_1\frac{de_1}{dr_1} + (q\gamma - \lambda L_0)\frac{dL_0}{dr_1}$$

(1.12)

Substituting from the firm's FOC, that is, $(1-q)g_0' = (1+q(\gamma+r_1))$, $g_1' = (1+\gamma+r_1)$, and

$\lambda L_0 = \dfrac{q\gamma}{1+qr_1}$, inspection reveals that the first 4 elements are all negative, so $\dfrac{\partial U}{\partial r_1} < 0$ if

$(q\gamma - \lambda L_0)\dfrac{dL_0}{dr_1} \le 0$. But $L_0 = \dfrac{q}{\lambda}\left(\dfrac{\gamma}{1+qr_1}\right)$. So $(q\gamma - \lambda L_0) \ge 0$. Since $\dfrac{dL_0}{dr_1} < 0$, $\dfrac{\partial U}{\partial r_1} < 0$. Q.E.D.

Proof that with exogenous fixed φ, $\dfrac{dU}{de_0} = 0$ at the privately optimal $e_0 = \dfrac{qr_1}{\alpha_0}$

$$U = \left((1-q)g_0(I_0) - I_0(1+q\gamma)\right) + q\left(g_1(I_1) - I_1(1+\gamma)\right) - \tfrac{1}{2}\alpha_0 e_0^2$$
$$- \dfrac{q}{\theta}[\varphi(1-\theta)+\theta]\left(\tfrac{1}{2}\alpha_1 e_1^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2 + q(L_0 + W_0^F)\gamma$$

Assume e_0 is set exogenously (so it does not respond to the interest rate). Now we know that

$$\dfrac{dU}{de_0} = \dfrac{\partial U}{\partial e_0} + \dfrac{\partial U}{\partial r_1}\dfrac{dr_1}{de_0} = -\alpha_0 e_0 + \dfrac{\partial U}{\partial r_1}\dfrac{dr_1}{de_0}.$$

$$\dfrac{\partial U}{\partial r_1} = [(1-q)g_0' - (1+q\gamma)]\dfrac{\partial I_0}{\partial r_1} + q[g_1' - (1+\gamma)]\dfrac{\partial I_1}{\partial r_1} - \dfrac{q}{\theta}[\varphi(1-\theta)+\theta]\alpha_1 e_1 \dfrac{\partial e_1}{\partial r_1} + [q\gamma - \lambda L_0]\dfrac{\partial L_0}{\partial r_1}.$$

Substituting from the firm's FOC, that is, $(1-q)g_0' = (1+q(\gamma+r_1))$, $g_1' = (1+\gamma+r_1)$, and

$\lambda L_0 = \dfrac{q\gamma}{1+qr_1}$, as well as recognizing that $e_1 = \dfrac{r_1}{\alpha_1}$, we get

$$\dfrac{\partial U}{\partial r_1} = qr_1\left[\dfrac{\partial I_0}{\partial r_1} + \dfrac{\partial I_1}{\partial r_1} + \dfrac{q\gamma}{1+qr_1}\dfrac{\partial L_0}{\partial r_1} + \dfrac{\varphi(1-\theta)+\theta}{\alpha_1\theta}\right].$$

From (1.10), $r_1 = \dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[I_1 + (D_0 - D_0^F)\right] - S_0(1-\tau)$.

Therefore, $\dfrac{dr_1}{de_0} = \dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[\lambda L_0 \cdot \dfrac{dL_0}{dr_1}\cdot\dfrac{dr_1}{de_0} - 1 + \dfrac{dI_0}{dr_1}\cdot\dfrac{dr_1}{de_0} + \dfrac{dI_1}{dr_1}\cdot\dfrac{dr_1}{de_0}\right].$

Rearranging, and substituting $\lambda L_0 = \dfrac{q\gamma}{1+qr_1}$, $\dfrac{dr_1}{de_0} = \dfrac{-\dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}}{1 - \dfrac{\alpha_1\theta}{[\varphi(1-\theta)+\theta]}\left[\dfrac{q\gamma}{1+qr_1}\dfrac{dL_0}{dr_1} + \dfrac{dI_0}{dr_1} + \dfrac{dI_1}{dr_1}\right]}.$

Returning to the expression, we obtain that $\frac{dU}{de_0} = -\alpha_0 e_0 + \frac{\partial U}{\partial r_1} \frac{dr_1}{de_0}$. We know $\alpha_0 e_0 = q r_1$ at the private

optimal. Also, multiplying the earlier expressions, we get $\frac{\partial U}{\partial r_1} \frac{dr_1}{de_0} = q r_1$, so $\frac{dU}{de_0}\bigg|_{e_0 = \frac{q r_1}{\alpha_0}} = 0$.   Q.E.D.

Appendix II – Proofs and Analysis for Sections IV and V

Section 4.2-4.3: Bank Choices at Date 1 (in the presence of a convenience yield δ)

Consider the three cases for the aggregate liquidity condition at date 1. We will denote the incremental

value of a bank at date 1 as $V_1(y,z)$ where recall that $y = 1$ if the economy is liquidity stressed and zero

otherwise, while $z=1$ if the bank is stressed and zero otherwise.

Case 1: *Stressed banks have enough liquidity to meet the needs of deposit outflows and to fund rescue investment without accessing the inter-bank market.*

Since reserves have a convenience yield δ , stressed banks will issue some capital e_1 to add to

reserves even if they do not need to use it for loans or deposit outflows. Furthermore, no bank will loan

out reserves without earning at least the convenience yield. Finally, since liquidity is in surplus, any

competition to make bank loans would push the bank lending rate down to the convenience yield. The

bank solves $\quad \max_{e_1} V_1(y=1, z=1) \equiv \left[r_1 \left(I_1(r_1) - D_0^F \right) - \delta \left(D_0 - D_0^F + I_1(r_1) - e_1 \right) - \frac{\alpha_1}{2} e_1^2 \right]$ where the

first term of the maximization is the return on loans, the second term the cost of the reserve outflow

reduced by the inflow of capital, while the last term is the incremental cost of raising capital over and

above the gross cost of 1. Since $r_1 = \delta$, the stressed bank makes no profit from the rescue loan. Solving,

$e_1 = \delta/\alpha_1 > 0$ even if the stressed bank has no need to use the funds to meet depositor outflows or loan

demand (details are in proofs of Theorems 2-3 below but the analysis follows the structure in Section 3).

Safe banks will not issue capital since they know capital issuance will not alter their reserves on

net – any investor in capital will first acquire reserves from the safe banks to buy the capital.[18] Since there

is no need for interbank loans, no healthy bank will become tainted. This means that the reserve outflows

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[18] Of course, safe banks may issue capital assuming it will come from reserve flows from other safe banks. If everyone does this, no one will have any additional reserves, but everyone will have issued capital commensurate with the size of the convenience yield and incurred the associated costs. Allowing for this adds little to the analysis.

from the stressed banks are spread across all the healthy banks, and their incremental date-1 value is

$$V_1(y=1, z=0) = \frac{\delta\theta(D_0 - D_0^F + I_1 - e_1)}{(1-\theta)}$$ where the numerator is the value of flight-to-safety deposit

outflows (plus new deposits created by purchases less capital issued) to the healthy banks, and the

denominator is the measure of healthy banks. It also follows then at date 0, $e_0 = \frac{q\delta}{\alpha_o}$.

Finally, this case arises when the stressed bank's reserves are enough to meet the demands on it,

that is, $S_0(1-\tau) \geq (D_0 - D_0^F + I_1 - e_1)$. Substituting for the endogenous $(D_0 - D_0^F)$, we see this case

arises when $\tau S_0 \leq \left[\left(\frac{q\delta}{\alpha_0} + \frac{\delta}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right]$ where I_1 is the optimized value

evaluated at $r_1 = \delta$, I_0 at $R_0^L = (1+q\gamma+q\delta)$ and $L_0 = \frac{q\gamma}{\lambda(1+q\delta)}$ (see proofs of Theorems 2-3 for all

steps). Note that since r_1 is a constant, as S_0 increases deposits increase dollar for dollar since no

additional capital is issued. Since a fraction τ of the reserves will be encumbered, the distressed bank's

net need for date-1 funds grows as S_0 grows. Eventually, it will exhaust available own funds at date 1,

and have to issue more capital (compared to the amount that would be optimal considering only the

convenience yield). This is when the economy moves into Case 2.

<u>Case 2:</u> *The liquidity needs of each stressed bank can be entirely met by its raising date-1 capital (beyond that warranted by the convenience yield).*

Now, the rate at which the stressed bank lends to the firm, r_1, rises above δ to incentivize

further date-1 capital-raising. However, the rate stays too low for any of the healthy banks to lend in the

interbank market. Essentially, the stressed bank is in *autarky* and has to issue costly capital even though

there is plentiful lending capacity in the system. Let the equilibrium bank lending rate in autarky be r_1^A.

The stressed bank maximizes

$$V_1(y=1, z=1) = \max_{e_1}\left[r_1^A\left(I_1(r_1^A) - D_0^F\right) - \delta S_0(1-\tau) - \frac{\alpha_1}{2}e_1^2\right]$$ such that

$e_1 = \left(D_0 + I_1(r_1^A) - D_0^F - S_0(1-\tau)\right)$. It follows that $e_1 = \frac{r_1^A}{\alpha_1}$ (and $e_0 = \frac{qr_1^A}{\alpha_0}$). As before, a rise in

the date-1 interest rate equilibrates the demand and supply of liquidity by decreasing the size of the rescue

investment and increasing the capital raised. Expanding the constraint for the maximization, we get

$$\left. r_1^A \middle/ \alpha_1 \right. = \left(I_0 + I_1 + \tfrac{1}{2}\lambda L_0^{\,2} - W_0^F - \left. qr_1^A \middle/ \alpha_0 \right. + \tau S_0 \right).$$ Furthermore, because the stressed banks are on their

own, once again an increase in ex-ante reserves S_0 always raises r_1^A, regardless of the size of τ (so long

as $\tau > 0$).[19]   Since the stressed banks just meet liquidity demand using all their unencumbered

reserves, the healthy banks get all of it. So $V_1(y=1, z=0) = \dfrac{\delta \theta S_0 (1-\tau)}{(1-\theta)}$.

<u>Case 3:</u> *The liquidity needs of the stressed banks are high enough that at the equilibrium autarkic interest rate, some of the healthy banks are willing to lend in the interbank market and become tainted. The equilibrium rate then is lower than the (now counterfactual) autarkic rate.*

Given the analysis in Section 4.2 for this case, for Case 3 to occur, it must be that $\varphi > 0$, that is,

$$\frac{\delta S_0(1-\tau)}{(1-\theta)\left(r_1 S_0(1-\tau) + \left. r_1^2 \middle/ 2\alpha_1 \right. \right)} < 1.$$ Rearranging, this requires $\left[\left. r_1^2 \middle/ 2\alpha_1 \right. + r_1 S_0(1-\tau) - \dfrac{\delta S_0(1-\tau)}{(1-\theta)} \right] > 0$.

Since the expression on the left hand side of the inequality is increasing in r_1, it must be that the threshold

value or the "breakeven interbank rate" r_1^φ that induces banks to lend in the interbank market is the

positive root of the quadratic equation obtained by setting the expression to zero. So

$$r_1^\varphi = \alpha_1 S_0 (1-\tau) \left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)S_0(1-\tau)}} - 1 \right].$$ Since this increases in S_0, we know that in Case 2, an

increase in S_0 expands both the autarky rate r_1^A as well as the rate r_1^φ necessary for the system to move

into Case 3. However, under reasonable assumptions, we show in proofs of Theorems 2-3 that r_1^φ

increases at a decreasing rate while r_1^A does not, so at a high enough S_0, $r_1^A > r_1^\varphi$ and the interbank

market will open.

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[19] Since all the endogenous variables on the right hand side are decreasing in r_1^A while the left hand side is

increasing in r_1^A, there is a unique equilibrium r_1^A, and S_0 shifts it up whenever $\tau > 0$.

Proofs of Theorems 2-3: We now provide remaining steps of the proofs by first detailing the date-0 maximization problems in the presence of a convenience yield on reserves at date 1. The firm's maximization problem remains unchanged. The bank's maximization problem is

$$\underset{L_0^B, e_0}{Max}\ R_0^L L_0^B + S_0 - e_0 - \frac{\alpha_0}{2} e_0^{\ 2} - D_0 + E_0\left[V(y,z)\big|L_0^{\ B}, e_0\right]$$

s.t. $D_0 + e_0 = L_0^B + \frac{1}{2}\lambda(L_0^B)^2 + S_0$

Case 1: The convenience yield associated with reserves in the stressed state, δ , is an opportunity cost for stressed banks, and they pass it on while lending to their firm at date 1. So they lend at rate $(1 + \gamma + \delta)$ where γ is their monitoring cost. Therefore,

$$V(y=1, z=1) = \underset{e_1}{Max}\left[-\delta(D_0 - e_1) - \frac{\alpha_1}{2} e_1^{\ 2}\right]$$

In turn, $e_1 = \delta/\alpha_1, e_0 = q\delta/\alpha_0$. The $(1-\theta)$ healthy banks divide the deposit outflows from the stressed banks so

$$V(y=1, z=0) = \frac{\theta\delta(D_0 - D_0^F + I_1 - \delta/\alpha_1)}{(1-\theta)}$$. Note that in making decisions at date 0, the inflows that come into the bank if it were healthy at date 1 are unrelated to any decision it takes at date 0 – it stems from decisions (on the size of loans, capital raise, and deposit funding) taken by other banks.[20] So, maximizing at date 0 w.r.t. L_0^B , we get $R_0^L = (1+q\delta)(1+\lambda L_0^B)$. From the firm's maximization, we know $R_0^D = (1+q\gamma+q\delta) = R_0^L$, so $L_0^B = \frac{q\gamma}{\lambda(1+q\delta)}$. We now derive when

$(D_0 - D_0^F + I_1 - e_1) < S_0(1-\tau)$. Since $(D_0 - D_0^F) = (S_0 + L_0^B + \frac{1}{2}\lambda(L_0^B)^2 - e_0) - (W_0^F + L_0^F - I_0)$

$= I_0 - e_0 + (S_0 - W_0^F) + \frac{1}{2}\lambda(L_0^B)^2$, where the second equality uses $L_0^B = L_0^F$, the condition simplifies to

$\tau S_0 \leq \left[\left(q\delta/\alpha_0 + \delta/\alpha_1\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right]$.

Case 2: Here the opportunity cost of lending at date 1 is r_1 (since this is the marginal cost of raising capital, the source of incremental funding at date 1), and it replaces δ in the bank's maximization in Case 1. The stressed bank sees

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[20] Put differently, all the variables in this expression should have a superscript O to signify they are decisions made by other banks. In the symmetric equilibrium, however, they will be equal to the values chosen by the bank whose maximization decisions we are studying.

$V(y=1,z=1)=\underset{e_1}{Max}\left[-r_1(D_0-e_1)-\dfrac{\alpha_1}{2}e_1^{2}\right]$. The healthy banks receive

$V(y=1,z=0)=\dfrac{\theta\delta S_0(1-\tau)}{(1-\theta)}$. Furthermore, $\tau S_0=\left[\left(\dfrac{qr_1}{\alpha_0}+\dfrac{r_1}{\alpha_1}\right)+W_0^{F}-I_0-I_1-\dfrac{1}{2}\lambda L_0^{2}\right]$ for

liquidity demand to equal liquidity supply. Since the right hand side increases in r_1, a higher S_0 always induces a higher r_1, whatever the level of τ so long as it is positive.

Case 3: For the bank, the date-0 maximization is similar to the one in Case 2. In this case, if healthy, the bank may use its reserves to lend at date 1. However, this will not enter its maximization since it takes the reserves as given. The bank's maximization problem at date 0, and the stressed bank's problem at date 1 then is as in case 2, where it takes r_1 as given.

Now, recall that S_0^{*} is the level of reserves at which the stressed bank can just meet liquidity needs with the (shadow) rate δ and reserves having a convenience yield δ . That is,

$S_0^{*}=\dfrac{1}{\tau}\left[\left(\dfrac{q\delta}{\alpha_0}+\dfrac{\delta}{\alpha_1}\right)+W_0^{F}-I_0-I_1-\dfrac{1}{2}\lambda L_0^{2}\right]$ where $g_0{}'(I_0)=\dfrac{1+q(\gamma+\delta)}{1-q}$,

$g_1{}'(I_1)=(1+\gamma+\delta)$, and $L_0=\dfrac{q\gamma}{\lambda(1+q\delta)}$. Note the right hand side is increasing in δ so S_0^{*} is

increasing in δ . Furthermore, the net rate the stressed banks charge firms is $(\gamma+\delta)$ for $S_0<S_0^{*}$.

When S_0 rises from S_0^{*}, the (shadow autarky) rate r_1^{A} rises from δ . It solves

$\tau S_0=\left[\left(\dfrac{qr_1}{\alpha_0}+\dfrac{r_1}{\alpha_1}\right)+W_0^{F}-I_0-I_1-\dfrac{1}{2}\lambda L_0^{2}\right]$ where $g_0{}'(I_0)=\dfrac{1+q(\gamma+r_1)}{1-q}$,

$g_1{}'(I_1)=(1+\gamma+r_1)$, and $L_0=\dfrac{q\gamma}{\lambda(1+qr_1)}$. Once again, since the right hand side increases in r_1, r_1^{A} is

increasing in S_0. If we further assume $g_0{}''',g_1{}'''$ are both positive, then it is convex.

Also $r_1^{\varphi}=\alpha_1 S_0(1-\tau)\left[\sqrt{1+\dfrac{2\delta}{\alpha_1(1-\theta)S_0(1-\tau)}}-1\right]$. So $r_1^{\varphi}>0$ for $S_0>0$. Furthermore, it is

straightforward to show that r_1^{φ} is increasing in S_0 and it is concave. Assume for now, and we will revisit

later, that at S_0^{*}, $r_1^{\varphi}>r_1^{A}$. Since both rates are increasing in S_0, and r_1^{A} is convex in S_0 while r_1^{φ} is

concave, they can intersect only once at S_0^{**}. So the (shadow) rate is r_1^{A} as S_0 increases from S_0^{*} to S_0^{**}

after which it becomes the rate dictated by the interbank market. Finally, r_1^{φ} increases in δ (as does r_1^{A},

see above). So S_0^{**} increases in δ . Finally, since the equilibrium φ falls in δ , the required

equilibrating interbank rate also increases in δ . Now, it can be shown using the 2nd order Taylor-series

expansion of $\sqrt{(1+x)}$ in r_1^{φ} , that at S_0^* , $r_1^{\varphi} - r_1^A > \dfrac{\delta\theta}{(1-\theta)} - \dfrac{\delta^2}{2\alpha_1(1-\theta)^2 S_0^*(1-\tau)} > 0$, if

$\delta < 2\alpha_1\theta(1-\theta)(1-\tau)S_0^*$. Substituting for S_0^* , and doing some algebra, it can be shown that a sufficient

condition for this is that $\theta(1-\theta) > \dfrac{\tau}{2(1-\tau)}$ and $\delta > \delta^*$ where δ^* satisfies

$$\dfrac{\tau}{2\alpha_1\theta(1-\theta)(1-\tau)}\delta^* = \left[\left(q\delta^*\Big/\alpha_0 + \delta^*\Big/\alpha_1\right) + W_0^F - I_0(\delta^*) - I_1(\delta^*) - \tfrac{1}{2}\lambda\left[L_0(\delta^*)\right]^2\right].$$ This

guarantees that there exists a unique $S_0^{**} > S_0^*$ such that $r_1^{\varphi} < r_1^A$ (interbank market is open) if and only

if $S_0 > S_0^{**}$.Q.E.D.

Condition for r_1 to be increasing in S_0 in Case 3: Recognize that in this region, r_1 is determined by
equating the demand by stressed banks for loans in the inter-bank market to the supply by tainted banks of
those loans. So $\theta\left[D_0 - D_0^F + I_1 - S_0(1-\tau) - e_1\right] = \varphi(1-\theta)\left[S_0(1-\tau) + e_1\right]$. Substituting

$\left(D_0 - D_0^F\right) = \left[I_0 - e_0 + (S_0 - W_0^F) + \tfrac{1}{2}\lambda(L_0^B)^2\right]$ and $e_1 = \dfrac{r_1}{\alpha_1}$ and rearranging, we get

$\dfrac{r_1}{\alpha_1} = \dfrac{\theta}{\left[\varphi(1-\theta) + \theta\right]}\left[I_0 + I_1 - e_0 + \tfrac{1}{2}\lambda(L_0^B)^2 - W_0^F\right] + \dfrac{\left[\theta\tau - \varphi(1-\theta)(1-\tau)\right]}{\left[\varphi(1-\theta) + \theta\right]}S_0$. Denoting the right hand

side of this equality as f as before and totally differentiating, we get $\left[\dfrac{1}{\alpha_1} - \dfrac{\partial f}{\partial r_1}\right]\dfrac{dr_1}{dS_0} = \dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial S_0} + \dfrac{\partial f}{\partial S_0}$.

Since $\dfrac{\partial f}{\partial r_1} < 0$, $\dfrac{dr_1}{dS_0} > 0$ if $\dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial S_0} + \dfrac{\partial f}{\partial S_0} > 0$. But $\dfrac{\partial f}{\partial \varphi} < 0$ by inspection, and we argued in the text that

$\dfrac{\partial \varphi}{\partial S_0} < 0$. So $\dfrac{\partial f}{\partial \varphi}\dfrac{\partial \varphi}{\partial S_0} > 0$ and a sufficient condition for $\dfrac{dr_1}{dS_0} > 0$ is that $\dfrac{\partial f}{\partial S_0} > 0$. This then requires

$\left[\theta\tau - \varphi(1-\theta)(1-\tau)\right] > 0$, which on simplifying requires $\theta > \dfrac{\varphi(1-\tau)}{\tau + \varphi(1-\tau)}$. Note that this is only

sufficient, since even if it does not hold, it may still be that $\dfrac{dr_1}{dS_0} > 0$. Intuitively, there is now a new

channel through which a higher S_0 leads to a higher r_1 : a higher S_0 leads to a lower φ ceteris paribus,
since healthy banks have more reason to stay on the sideline given the larger flight to safety flows, which
in turn leads to a greater net need for liquidity from capital-raising, and hence a higher r_1 .

Section 4.4. Proof of Lemma 3.

$$U = \left((1-q)g_0(I_0) - I_0(1+q\gamma)\right) + q\left(g_1(I_1) - I_1(1+\gamma)\right) - \tfrac{1}{2}\alpha_0 e_0{}^2$$

$$- \frac{q}{\theta}\left[\varphi(1-\theta) + \theta\right]\left(\tfrac{1}{2}\alpha_1 e_1{}^2\right) - \tfrac{1}{2}\lambda\left(L_0\right)^2 + q(L_0 + W_0^F)\gamma$$

Since φ is not influenced directly by e_0 but only indirectly via r_1, we have

$$\frac{dU}{de_0} = \frac{\partial U}{\partial e_0} + \left(\frac{\partial U}{\partial r_1}\Big|_{\varphi=const}\right)\frac{dr_1}{de_0} + \frac{\partial U}{\partial \varphi}\frac{\partial \varphi}{\partial r_1}\frac{dr_1}{de_0} = -\alpha_0 e_0 + \left(\left(\frac{\partial U}{\partial r_1}\Big|_{\varphi=const}\right) + \frac{\partial U}{\partial \varphi}\frac{\partial \varphi}{\partial r_1}\right)\frac{dr_1}{de_0}.$$

Also $r_1 = \alpha_1 f(r_1, \varphi, e_0)$. Therefore, $\dfrac{dr_1}{de_0} = \alpha_1\left[\dfrac{\partial f}{\partial e_0} + \dfrac{\partial f}{\partial r_1}\cdot\dfrac{dr_1}{de_0} + \dfrac{\partial f}{\partial \varphi}\cdot\dfrac{\partial \varphi}{\partial r_1}\dfrac{dr_1}{de_0}\right].$

Rearranging, $\dfrac{dr_1}{de_0} = \dfrac{\alpha_1\dfrac{\partial f}{\partial e_0}}{\left[1 - \alpha_1\left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial \varphi}\cdot\dfrac{\partial \varphi}{\partial r_1}\right)\right]}$. Substituting in the earlier expression, we get

$$\frac{dU}{de_0} = -\alpha_0 e_0 + \left(\left(\frac{\partial U}{\partial r_1}\Big|_{\varphi=const}\right) + \frac{\partial U}{\partial \varphi}\frac{\partial \varphi}{\partial r_1}\right)\frac{\alpha_1\dfrac{\partial f}{\partial e_0}}{\left[1 - \alpha_1\left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial \varphi}\cdot\dfrac{\partial \varphi}{\partial r_1}\right)\right]} \qquad (A)$$

Now we can see that

$$\frac{\partial U}{\partial \varphi} = -\frac{q}{\theta}(1-\theta)\frac{1}{2}\alpha_1 e_1{}^2 < 0 \qquad \text{(the direct effect of a higher } \varphi \text{ is more dissipative date-1 equity}$$

issuance by tainted banks). Also from (1.13), $(1-\varphi) = \dfrac{\delta S_0(1-\tau)}{(1-\theta)\left(r_1 S_0(1-\tau) + {r_1^2}\big/{2\alpha_1}\right)}.$

So $\dfrac{\partial \varphi}{\partial r_1} = \dfrac{\delta S_0(1-\tau)}{(1-\theta)}\dfrac{\left(S_0(1-\tau) + {r_1}\big/{\alpha_1}\right)}{\left(r_1 S_0(1-\tau) + {r_1^2}\big/{2\alpha_1}\right)^2} > 0$, and $\dfrac{\partial f}{\partial \varphi} = \dfrac{-\theta(1-\theta)}{\left(\varphi(1-\theta)+\theta\right)^2}\left[I_1 + \left(D_0 - D_0^F\right)\right] < 0.$

We can then understand the effect of internalizing φ on how the social planner would set e_0 from

scrutinizing the terms containing φ on the right hand side of (A) above. In the numerator,

$\dfrac{\partial U}{\partial \varphi}\dfrac{\partial \varphi}{\partial r_1}\alpha_1\dfrac{\partial f}{\partial e_0} > 0$. An increase in e_0 reduces the liquidity shortfall f, reducing the rate r_1, reducing the

mass of tainted banks φ, and reducing dissipative date-1 equity issues by tainted banks, thus increasing

welfare. However, there is an offsetting dampening effect from the term in the denominator,

$-\alpha_1 \dfrac{\partial f}{\partial \varphi} \cdot \dfrac{\partial \varphi}{\partial r_1} > 0$, because a fall in the ex post inter-bank rate increases the degree of liquidity hoarding,

and thus increases the degree of liquidity shortfall that has to be met by date-1 dissipative capital issues.

To check whether the social planner's capital choice is higher than the private optimal (which ignores the dependence of φ on the bank's capital issuance), we need to see whether

$$\frac{dU}{de_0} = -\alpha_0 e_0 + \left(\left(\frac{\partial U}{\partial r_1} \bigg|_{\varphi=const} \right) + \frac{\partial U}{\partial \varphi} \frac{\partial \varphi}{\partial r_1} \right) \frac{\alpha_1 \dfrac{\partial f}{\partial e_0}}{\left[1 - \alpha_1 \left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial \varphi} \cdot \dfrac{\partial \varphi}{\partial r_1} \right) \right]}$$ is greater or less than zero at the

private optimal $e_0 = \dfrac{qr_1}{\alpha_0}$. In other words, we need to check whether

$$\left(\left(\frac{\partial U}{\partial r_1} \bigg|_{\varphi=const} \right) + \frac{\partial U}{\partial \varphi} \frac{\partial \varphi}{\partial r_1} \right) \frac{\alpha_1 \dfrac{\partial f}{\partial e_0}}{\left[1 - \alpha_1 \left(\dfrac{\partial f}{\partial r_1} + \dfrac{\partial f}{\partial \varphi} \cdot \dfrac{\partial \varphi}{\partial r_1} \right) \right]} > qr_1.$$ Multiplying both sides by the denominator on

the left hand side, and recognizing from our earlier analysis with exogenous φ that

$$\left(\frac{\partial U}{\partial r_1} \bigg|_{\varphi=const} \right) \alpha_1 \frac{\partial f}{\partial e_0} = qr_1 \left(1 - \alpha_1 \frac{\partial f}{\partial r_1} \right),$$ we require that $\dfrac{\partial U}{\partial \varphi} \dfrac{\partial f}{\partial e_0} > -qr_1 \dfrac{\partial f}{\partial \varphi}.$

Now $\dfrac{\partial U}{\partial \varphi} \dfrac{\partial f}{\partial e_0} = \left(-\dfrac{q}{\theta} (1-\theta) \dfrac{1}{2} \alpha_1 e_1^{\,2} \right) \cdot \left(\dfrac{-\theta}{\varphi(1-\theta)+\theta} \right).$ We substitute $e_1 = \dfrac{r_1}{\alpha_1}.$

Note also that $-qr_1 \dfrac{\partial f}{\partial \varphi} = -qr_1 \cdot \left(\dfrac{-\theta(1-\theta)}{\left(\varphi(1-\theta)+\theta \right)^2} \left(I_1 + \left(D_0 - D_0^F \right) \right) \right).$ Substituting on both sides of the

inequality and recognizing that $r_1 = \dfrac{\alpha_1 \theta}{\left[\varphi(1-\theta)+\theta \right]} \left[I_1 + (D_0 - D_0^F) \right] - \alpha_1 S_0 (1-\tau),$ we get

$\dfrac{\partial U}{\partial \varphi} \dfrac{\partial f}{\partial e_0} > -qr_1 \dfrac{\partial f}{\partial \varphi}$ iff $\dfrac{r_1}{2} > r_1 + \alpha_1 S_0 (1-\tau)$ which is impossible. Indeed, the inequality goes the other

way, so $\dfrac{dU}{de_0} \bigg|_{e_0 = \frac{qr_1}{\alpha_0}} < 0.$ The social planner prefers lower capital than the privately optimal level. Q.E.D.

Section 4.5: Endogenous δ.

If $\delta(r_1) = \delta^A + \delta^B r_1^2$ where $\delta^A \geq 0$, $\delta^B \geq 0$, then it follows from (1.13) that

$$(1-\varphi) = \frac{(\delta^A + \delta^B r_1^2)S_0(1-\tau)}{(1-\theta)\left(r_1 S_0(1-\tau) + r_1^2 \Big/ 2\alpha_1\right)}. \text{ Let } A = \frac{\delta^B}{(1-\theta)}, \; B = \frac{1}{2\alpha_1 S_0(1-\tau)}, \; C = \frac{\delta^A}{(1-\theta)}. \text{ Then the}$$

previous expression is $(1-\varphi) = \dfrac{C + A r_1^2}{r_1 + B r_1^2}$. Note that A, B, C are all positive.

We want to see when the RHS ≥ 1 so that there is no r_1 for which $\varphi > 0$. This requires

$(A-B)r_1^2 - r_1 + C \geq 0$. The roots of the quadratic are $r_1 = \dfrac{1 \pm \sqrt{1 - 4C(A-B)}}{2(A-B)}$.

Case 1: If $4C(A-B) > 1$, there are no real roots to the quadratic. So $C + A r_1^2$ always lies above $r_1 + B r_1^2$ (as at $r_1 = 0$), which means φ is always zero. The interbank market never opens in this case.

Case 2: If $4C(A-B) \leq 1$ and $A > B$, there are two positive real roots, $r_1^+ > r_1^-$, and $\varphi > 0$ if and only if $r_1 \in (r_1^-, r_1^+)$. Essentially, the two curves $C + A r_1^2$ and $r_1 + B r_1^2$ intersect at two points, and $\varphi > 0$ in between. The interbank market opens only in a range of rates and is closed both above and below.

Case 3: If $A < B$, then there is only one positive root, which is $r_1^+ = \dfrac{-1 + \sqrt{1 + 4C(B-A)}}{2(B-A)}$, and $\varphi > 0$ iff $r_1 > r_1^+$. Essentially, the slope of $r_1 + B r_1^2$ is higher than $C + A r_1^2$, so it intersects once from below, after which $\varphi > 0$. The interbank market opens only above a specific rate.

Fixed Encumbrance on Reserves ($\tau S_0 \equiv \overline{E}$) (section 5.3)

Consider the full model of Section IV with the endogenized share of surplus banks in the interbank market. Case 1 in which each stressed bank is self-sufficient in liquidity at the convenience yield δ arises whenever $\overline{E} \leq \overline{E}^* \equiv \left[\left(q\delta \Big/ \alpha_0 + \delta \Big/ \alpha_1\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right]$, a condition that is independent of the level of reserves; note that I_1 is the optimized value evaluated at $r_1 = \delta$, I_0 at

$R_0^L = (1 + q\gamma + q\delta)$ and $L_0 = \dfrac{q\gamma}{\lambda(1 + q\delta)}$. For $\overline{E} > \overline{E}^*$, the interbank market may be shut (autarky) or open; when shut, the autarkic rate $r_1^A(\overline{E})$ satisfies $\overline{E} = \left[\left(q r_1^A \Big/ \alpha_0 + r_1^A \Big/ \alpha_1\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right]$,

and is now a function of the fixed level of encumbrance and not of the level of reserves, with I_1, I_0 and L_0 set accordingly. Naturally, $r_1^A(\overline{E})$ is increasing in the encumbrance $\overline{E}$, and $r_1^A(\overline{E}) > \delta$ for $\overline{E} > \overline{E}^*$.

A key question then is when is the autarkic rate above or below the breakeven rate r_1^φ that induces banks to lend in the interbank market. With fixed encumbrance, this rate is given by

$$r_1^\varphi = \alpha_1 \left(S_0 - \overline{E}\right)\left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)\left(S_0 - \overline{E}\right)}} - 1\right],$$ which as before is increasing and concave in S_0. Note

also that the endogenous share φ of surplus banks that lend in the interbank market for a given rate r_1

satisfies $(1-\varphi) = \dfrac{\delta\left(S_0 - \overline{E}\right)}{(1-\theta)\left[r_1\left(S_0 - \overline{E}\right) + {r_1^2}\big/{2\alpha_1}\right]}$ and the equilibrium interbank rate $\overline{r_1}$ is given by the

usual market-clearing condition adjusted for encumbrance being now at a fixed level:

$$\frac{r_1}{\alpha_1} = \frac{\theta}{\left[\varphi(1-\theta) + \theta\right]}\left[I_0 + I_1 - e_0 + \frac{1}{2}\lambda(L_0^B)^2 - W_0^F\right] - \frac{\varphi(1-\theta)}{\left[\varphi(1-\theta) + \theta\right]}S_0 + \overline{E}.$$ It can then be shown that

Theorem 5: For $S_0 > \overline{E} > \overline{E}^*$, there exists a critical threshold $\overline{S}_0 > \overline{E}$ such that

(i)   For $S_0 \in \left(\overline{E}, \overline{S}_0\right)$, the autarkic rate $r_1^A\left(\overline{E}\right)$ exceeds the breakeven rate $r_1^\varphi\left(S_0\right)$, the interbank

        market is open $(\varphi > 0)$, and the equilibrium interbank rate $\overline{r_1} \in \left(r_1^\varphi(S_0), r_1^A\left(\overline{E}\right)\right)$.

(ii)   For $S_0 \geq \overline{S}_0$, the autarkic rate $r_1^A\left(\overline{E}\right)$ is at or below the breakeven rate $r_1^\varphi\left(S_0\right)$, the

        interbank market is shut $(\varphi = 0)$, and the equilibrium interbank rate $\overline{r_1}$ equals the autarkic

        rate $r_1^A\left(\overline{E}\right) > \delta$.

(iii)   When fixed encumbrance $\overline{E}$ is sufficiently small such that $r_1^A\left(\overline{E}\right) < r_1^\varphi(S_0 \to \infty) = \dfrac{\delta}{(1-\theta)}$,

        then both cases (i) and (ii) arise and $\overline{r_1}$ is strictly increasing in S_0 for at least some range of

        S_0 in $\left(\overline{E}, \overline{S}_0\right]$; otherwise, when $r_1^A\left(\overline{E}\right) \geq \dfrac{\delta}{(1-\theta)}$, only case (i) arises and $\overline{S}_0 \to \infty$.

Proof of Theorem 5: Following earlier derivations, but with a fixed encumbrance, we know

$$r_1^\varphi = \alpha_1\left(S_0 - \overline{E}\right)\left[\sqrt{1 + \frac{2\delta}{\alpha_1(1-\theta)\left(S_0 - \overline{E}\right)}} - 1\right].$$ Clearly $r_1^\varphi \to 0$ as $S_0 \to \overline{E}$. Also, $r_1^\varphi \to \frac{\delta}{(1-\theta)}$ as

$S_0 \to \infty$. Finally, r_1^φ is increasing in S_0. We also know that r_1^A is the value of r_1 that solves

$$\overline{E} = \left[\left(\frac{qr_1}{\alpha_0} + \frac{r_1}{\alpha_1}\right) + W_0^F - I_0 - I_1 - \frac{1}{2}\lambda L_0^2\right],$$ where I_0, I_1, L_0 depend on r_1 in the usual manner.

Since none of the elements on the right hand side change with S_0, r_1^A does not change with S_0.

Therefore, if $r_1^A < \frac{\delta}{(1-\theta)}$ because $\overline{E}$ is small, there is an $\overline{S}_0 > \overline{E}$ such that $r_1^\varphi = r_1^A$ at $S_0 = \overline{S}_0$, and

$r_1^\varphi > r_1^A$ for $S_0 > \overline{S}_0$. So the equilibrium interbank rate $\overline{r}_1 \in \left(r_1^\varphi(S_0), r_1^A\left(\overline{E}\right)\right)$ for $\overline{S}_0 > S_0 > \overline{E} > \overline{E}^*$ and

$\overline{r}_1 = r_1^A\left(\overline{E}\right)$ for $S_0 \geq \overline{S}_0$. If, however, $r_1^A \geq \frac{\delta}{(1-\theta)}$, then $r_1^\varphi < r_1^A$ for all finite S_0, and the equilibrium

interbank rate $\overline{r}_1 \in \left(r_1^\varphi(S_0), r_1^A\left(\overline{E}\right)\right)$ for all finite S_0. Q.E.D.

We illustratin the result with examples in Figures 5A and 5B, with the same parameters as in

Figures 3-4 ($\theta = 0.6$). When $\overline{E}$ is low, the interbank market is open for low levels of S_0 but shuts down

at high levels (Panel A); when $\overline{E}$ is high, the interbank market is always open regardless of the level of

S_0. In both parameterizations, $\overline{r}_1$ is strictly increasing in S_0. It is therefore a robust feature of the

equilibrium that the interbank market may remain shut and the interbank rate can increase in the level of

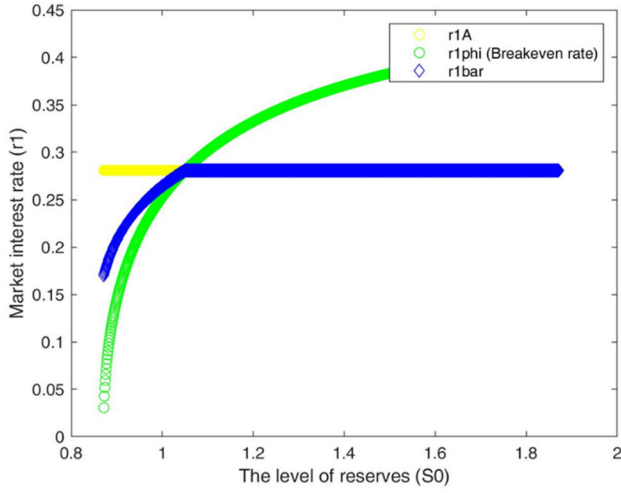reserves when the interbank market is open.

Figure 5A: Market Rate and Reserves
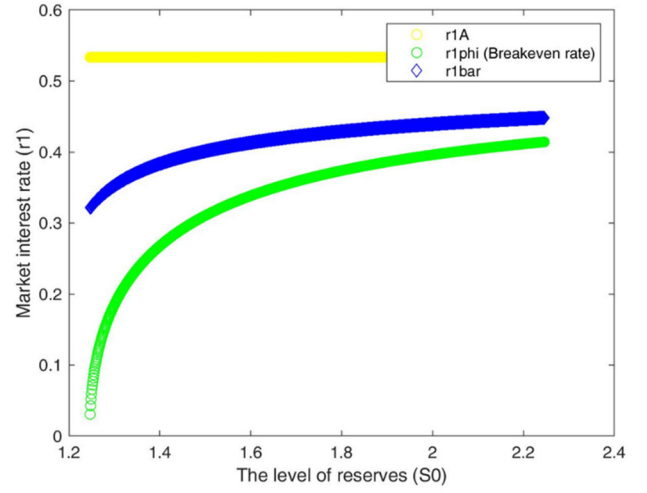
$\bar{E} = 0.87$, $\theta = 0.6$

Figure 5B: Market Rate and Reserves

$\bar{E} = 1.25$, $\theta = 0.6$

Figure 5: Numerical example for the effect of varying the fixed encumbrance on reserves $\bar{E}$ in the extended model of section 5.3 [21]

Appendix III – Extensions not in the main body of the paper

Embedding Liquidity Regulations (section 5.2)

In the context of our framework, suppose that after reserves are set and speculation is under way, regulators can affect overall τ $(= \tau^{Spec} + \tau^{Reg})$ by setting τ^{Reg}. Let the fraction of banks that suffer withdrawals at date 1 be $K(\tau^{Reg})\theta$ instead of θ, with $K' < 0$, $K'' > 0$ and $K(0) = 1$. This means the share of banks that are stressed falls in mandatory regulatory reserve holdings (in part because that also curbs the effects of speculation). However, this also hampers the liquidity available from healthy banks in times of liquidity stress. Hence, if regulators are narrowly focused on maximizing overall liquidity

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[21] Note that in Figures 5A-B, the equilibrium interbank rate is rising in the level of reserves; in general, this holds when the systemic extent of liquidity shock θ is sufficiently small and the convenience yield δ is sufficiently high; when this is not the case, it can be shown that an increase in the supply of reserves can cause the rate to decrease as it starts out high, close to the autarkic rate, when reserves are close to the fixed encumbrance, and then decreases towards the breakeven rate.

available per dollar of reserves ex post, given the central bank has set reserves, they would maximize

$(1-\tau)-K(\tau^{\text{Reg}})\theta.$ So they would optimally choose $\tau^{\text{Reg}*}=K'^{-1}(-\frac{1}{\theta})$. On inspection, and bearing in

mind that risk reduction has diminishing returns so that $K''>0$, the higher is θ the greater will be the

regulatory encumbrance $\tau^{\text{Reg}*}$. Depending on functional forms, that is, how effectively a higher τ^{Reg}

reduces the share of banks that are stressed, it can be shown that all the cases we have discussed earlier

could still be possible with optimal regulation. Our model easily allows for an analysis of alternative

formulations of the regulatory requirement. For instance, if banks are required to maintain τD_0 of

deposits as reserves at all times (that is, a traditional reserve requirement), we can show easily that once

again $\tau^{\text{Reg}*}=K'^{-1}(-\frac{1}{\theta})$ since deposit issuance moves one for one with reserves.

Private Incentives to Hold Reserves (section 6.3)

What if banks were not forced to hold S_0, that is, what if banks can choose S_0 in addition to

term lending and financing at date 0 (see representative bank's objective function in section 2.2)? To

make the problem relevant, we assume there is a cost to carrying liquidity due to agency problems or a

capital cost of holding excess reserves of $C(S_0)$ such that $C'(S_0)>0$, $C''(S_0)>0$. Focusing only on

the bank's optimal choice of reserve holdings, and assuming that $\delta\to0$ so that $\varphi\to1$ (all surplus banks

lend in the interbank market in the stressed state of the economy), this choice – based on bank's objective

function in Section 3 – boils down to

$$\underset{S_0}{Max}\quad S_0-D_0-C(S_0)-\frac{q}{\theta}\theta(r_1\left(D_0-S_0(1-\tau)\right))+\frac{q}{\theta}(1-\theta)r_1S_0(1-\tau)$$

Recognizing that deposits D_0 increase one for one in reserves S_0, the FOC w.r.t. S_0 is

$-C'(S_0)-qr_1\tau+\frac{q}{\theta}(1-\theta)r_1(1-\tau)=0$. Simplifying

$$S_0^{pvt}=C'^{-1}\left[qr_1\left(\frac{(1-\tau)}{\theta}-1\right)\right]$$

There is thus no guarantee that the level of (per bank) reserves that the central bank wants to issue (given

its other concerns for conducting unconventional monetary policy) are at, or below, the level that

commercial banks want to optimally hold. Suppose for instance that the central bank wants to initially place higher level of reserves than S_0^{pvt}. If $\theta > (1 - \tau)$, banks privately do not wish to hold any reserves; furthermore, an increase in anticipated interest rates reduces the S_0^{pvt} that the commercial bank would optimally like to hold. Since under this same condition, we know that anticipated interbank rates are rising in S_0, the divergence, between what commercial banks are willing to hold and what the central bank has to place, grows with S_0. Put differently, the prospect of greater liquidity shortages need not increase the commercial banks' private incentives to hold more reserves; indeed, recognizing that the source of the shortage is the financing of reserves, they may want to hold less or none at all.

Maturity Matching or Short-term Financing of Reserves by Shadow Banks (section 6.3)

We have assumed that the reserves end up on bank balance sheets. What if the central bank departs from normal practice and allows non-bank financial firms to hold reserves directly? Unless the central bank buys money-like assets from the non-bank private sector, we may not get significantly different outcomes; if the central bank buys long-term financial assets and pays with reserves, for standard risk management reasons the non-bank private sector may want to match the maturity of their liability structure to their shorter-maturity asset holdings.

To see this, let us focus on the healthy state (that is, assume $q = 0$), and assume that economy-wide date-1 short-term (gross) interest rates in the healthy state are $(1 + r)$ with probability p and $(1 - r)$ with probability $(1 - p)$. The net rates (+r and –r) represent the state-contingent cost of rolling over each bank's liquidity shortfall given by $(D_0 - S_0)$. Further, assume the financial firm holding reserves wants to finance it so as to minimize costs, but it also dislikes the variability of its date-2 profits given by the variance of profits, $p(1-p) 4r^2 (D_0 - S_0)^2$, with aversion parameter $\psi / 2$. Finally, the cost of capital issuance at date 0 is $R_0^E = \left[p(1+r) + (1-p)(1-r) + \Delta_0^E \right]$, where Δ_0^E is a capital risk premium. So ignoring the other activities of the financial firm, its objective function for choosing the maturity structure of its liabilities, given the need to finance reserve holdings, is as follows (where variables have their earlier connotation):

$$\underset{D_0}{Max} \quad \left[-R_0^E e_0 - p(1+r)(D_0 - S_0) - (1-p)(1-r)(D_0 - S_0) - \frac{\psi}{2} p(1-p)4r^2 (D_0 - S_0)^2 \right]$$

$$s.t. \quad e_0 = S_0 - D_0$$

It is straightforward from the maximization that $D_0 = \left[S_0 + \dfrac{\Delta_0^E}{\psi p(1-p)4r^2} \right]$. So deposits increase one

for one with reserves and also increase with the capital premium – the point is that longer term financing for reserves can increase the variability of profits by locking in financing costs while leaving returns on reserves variable. Financial firms will match maturity to avoid this variability. Put differently, so long as central-bank-issued reserves have to be financed somewhere in the economy rather than resting in household balance sheets, there will be some offsetting short-term liabilities.