# A Theory of Public Debt as a Macro-Financial Stability Tool


Guido Ascari[1] Jacopo Bonchi[2] Andrea Ferrero[3]

[1]University of Pavia and DNB

[2]University of Bologna

[3]University of Oxford


**New Challenges for Monetary-Fiscal Policy Interactions**

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# Motivation: ZLB, Asset Bubbles and Public Debt

- Before COVID-19 pandemic, **declining $r^*$** w/ the risk of
  1. **Binding ZLB**
  2. **Asset price bubbles**
- After COVID-19 pandemic, **large increase in public debt**
- **Research questions**
  1. can raising public debt prevent the ZLB from bind *and* the emergence of bubbles (= **Macro-Financial Stability**)?
  2. Is *safe* public debt sufficient?
  3. *Any* type of bubble can be prevented?

Graph Quotation

# What We Do

- We build a **a two-period OLG model** with
  - **Public debt**
  - **Non-neutral monetary policy costrained by the ZLB**
  - **Unleveraged and leveraged bubbles**
- We study under which conditions macro stability prevents the emergence of asset bubbles
- We study whether a safe level of public debt is sufficient for macro and/or financial stability

# Related Literature

- **Evidence on $r^*$ trend**
  Laubach and Williams (2003), Holston, Laubach and Williams (2017, 2023), Reis (2022, 2025), Christensen and Mouabbi (2025)
- **Secular Stagnation and persistent ZLB**
  Caballero and Fahri (2018), Rachel and Summers (2019), Eggertsson et al. (2019), Reis (2021), Ascari and Bonchi (2022)
- **Rational Bubbles in OLG models**
  Samuelson (1958), Tirole (1985), Kraay and Ventura (2007), Aoki and Nikolov (2014), Bengui and Phan (2018)

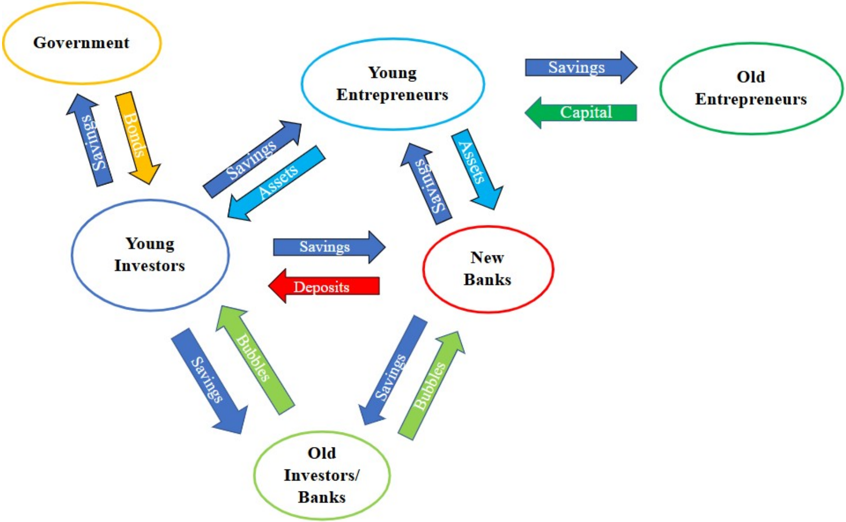
# Our Model

A two-period OLG Economy consisting of:

- Investors
- Entrepreneurs
- Banks
- Firms
- Government
- Central Bank

and with **2 key frictions**:

1. ZLB
2. DNWR


Government
Savings
Bonds
Young
Entrepreneurs
Savings
Capital
Old
Entrepreneurs
Savings
Assets
Savings
Assets
Young
Investors
Savings
Deposits
New
Banks
Savings
Bubbles
Savings
Bubbles
Old
Investors/
Banks

# Equilibrium (1)

1. **Real Markets**

$$Y_t = C_t^e + C_t^b + C_t^i + I_t + G_t$$
$$h \leq \bar{h}$$

where

$$I_t = K_t - (1-\delta^k)K_{t-1}$$

2. **Financial Markets**

$$L_t^e = L_t^i + L_t^b$$
$$D_t^i = D_t^b$$
$$B_t^i = B_t^g$$
$$Q_t^{B,b} + Q_t^{B,i} = Q_{t-1}^{B,b} + Q_{t-1}^{B,i} = 1$$

where the last equation holds only if $\widehat{P}_t^B = P_t^B > 0$

# Equilibrium (2)

- **Real (Natural) Interest Rate**

$$1 + r_t = \frac{\phi^e (1 - \alpha) Y_t}{(1 - \tau) \alpha Y_t - B_t^g - P_t^B}.$$

which amounts to the natural interest rate $1 + r_t^f$ for $Y_t = Y_t^f$

- **Overall financial market-clearing condition**

$$(1 - \tau) \alpha Y_t = L_t^e + B_t^g + P_t^B$$

SS

# Steady State Analysis

We focus on steady state equilibria where:

- **Binding borrowing constraint** for entrepreneurs;
- **Public debt is set**, $B^g = \bar{B}^g$, by adjusting government spending.

We start from the **simplest version of our model**:

- **Inflation equal to the target** ($\delta = 1$ in the DNWR);
- **Fixed capital** ($K_t = \bar{K}$) without depreciation ($\delta^k = 0$).

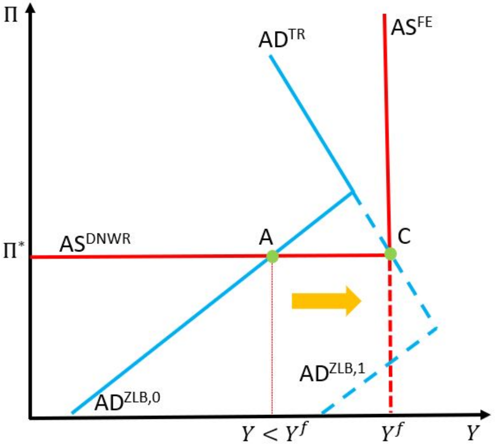
Parametric restrictions

# Steady State Equilibria

The equilibria are differentiated along two dimensions:

1. **Output/demand and policy rate**
   - **ZLB-U** (ZLB+Unemployment)
   - **TR-FE** (Taylor Rule+Full Employment)
2. **Bubble price**
   - **Bubbly** equilibrium ($P^B > 0$)
   - **Bubble-less** equilibrium ($P^B = 0$)

# Public Debt and Macro Stability




**ZLB-U** equilibrium at $A$:

$$1 + r^f_{NB} < 1 + r_{NB} < 1$$

$$i = 0$$

$$\bar{B}^g < \bar{B}^g_{ZLB}$$

**TR-FE** equilibrium at $C$:

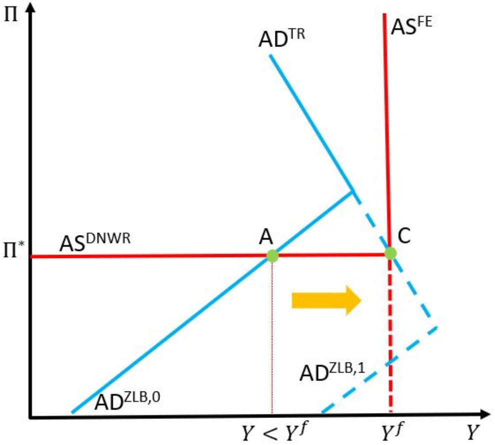
$$1 + r^f_{NB} = 1 + r_{NB}$$

$$i > 0$$

$$\bar{B}^g > \bar{B}^g_{ZLB}$$

natural rate

# Public Debt and Financial Stability




**ZLB-U** and **TR-FE** equilibria:

- **Unleveraged** bubbly
- **Leveraged** bubbly
- **Partially leveraged** bubbly
- **Bubble-less**

# Unleveraged Bubbly Equilibrium

- **Condition for the existence**

$$\frac{\phi^e (1-\alpha) Y}{(1-\tau)\alpha Y - \bar{B}^g} = (1 + r_{NB}) < (1-\rho)$$

or

$$\bar{B}^g < \bar{B}^g_{NB,U}(Y) \equiv \left[(1-\tau)\alpha - \frac{\phi^e (1-\alpha)}{(1-\rho)}\right] Y$$

# Leveraged Bubbly Equilibrium

- **Condition for the existence**

$$\frac{\phi^{e}\left(1-\alpha\right)Y}{\left(1-\tau\right)\alpha Y-\bar{B}^{g}}=\left(1+r_{NB}\right)<\left(1-\rho\right)+\frac{\left(\phi^{B}-\phi^{L}\right)}{\left(\phi^{D}-\phi^{L}\right)}\mu_{NB}$$

or

$$\bar{B}<\bar{B}^{g}_{NB,L}\left(Y\right)\equiv\left[\overline{q}-\left(\frac{\phi^{L}}{\phi^{D}-\phi^{L}}\right)\gamma\left(1-\tau\right)\alpha\right]Y$$

where

$$\mu_{NB}\equiv\overline{q}-\left(\frac{\phi^{L}}{\phi^{D}-\phi^{L}}\right)\gamma\left(1-\tau\right)\alpha-\frac{\bar{B}^{g}}{Y}\geq 0$$

# Partially Leveraged Bubbly Equilibrium

- **Condition for the existence**

$$(1 + r_{NB}) < (1 - \rho) \leq (1 - \rho) + \frac{\left(\phi^B - \phi^L\right)}{\left(\phi^D - \phi^L\right)} \mu_{NB}$$

or

$$\bar{B}^g \leq \bar{B}^g_{NB,L}(Y) < \bar{B}^g_{NB,U}(Y)$$

# Just a Recap

- Public debt can affect
  1. output/demand at the ZLB (**macro stability**)
  2. the existence conditions of different asset bubbles and thus prevent them (**financial stability**)

- **Public debt** as a **Macro-Financial stability tool** = equilibrium selection device among different equilibria in terms of
  - **bubble price** (and type)
  - **output/demand**

# Proposition 1

**Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debt**, bcz **TR-FE equilibrium** can be:

1. **Partially leveraged bubbly**

$$\bar{B}^g_{ZLB} < \bar{B}^g < \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right) < \bar{B}^g_{NB,U}\left(Y^f\right)$$

2. **Unleveraged bubbly**

$$\bar{B}^g_{ZLB} < \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right) \leq \bar{B}^g < \bar{B}^g_{NB,U}\left(Y^f\right)$$

3. **Bubble-less**

$$\bar{B}^g_{ZLB} < \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right) < \bar{B}^g_{NB,U}\left(Y^f\right) \leq \bar{B}^g$$

Prop 1 App

# Extensions

The previous proposition holds also w/:

- Inflation rate different from the target ( $\delta < 1$ in the DNWR)
- Endogenous capital with depreciation $(0 < \delta^k < 1)$
- Leveraged bubbles in the initial $ZLB - U$ equilibrium A (Prop 2)

# To Sum Up

- **Results**

  1. **Raising public debt NOT NECESSARILY prevents asset bubbles** by delivering **macro stability** (NO ZLB, $\Pi = \Pi^*$ and $Y = Y^f$) and **hurts potential output** (w/ endogenous K)

  2. **Safe public debt CAN** deliver **macro stability** but **NOT financial stability** (NO bubbles) under general conditions

- **Further extensions**

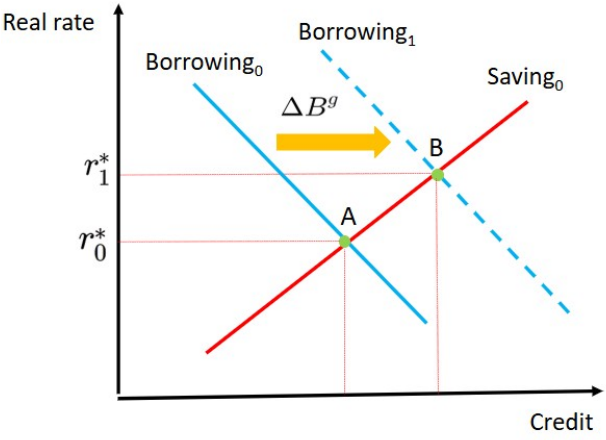
  1. **Risky** public debt

  2. **Harmful leveraged bubbles** (Jordá et al., 2015)

  3. **Debt maturity transformation** and risk management

*Thank you for your attention.*

# Public debt and $r^*$




Motivation

"*This* ***decline in the long-run neutral real interest rate*** *increases the future likelihood that* ***the FOMC will be unable to achieve its objectives because of financial instability or because of a binding lower bound on the nominal interest rate****....* ***the fiscal authority can mitigate this problem by issuing more public debt****, although such issuance is not without cost. It is, of course, the province of the fiscal authority to determine whether those costs are worth the benefits...*"

-*Narayana Kocherlakota, President of the Minneapolis FED, Bank of Korea Conference, 19 August 2015*

Back

# Investors (1)

- **Preferences**

$$U_t^i = E_t\left[v\left(\frac{D_t^i + B_t^i}{Y_t}\right)Y_t + C_{t+1}^i\right]$$

where

1. $v(.) = -\frac{1}{2}\left[\overline{q} - \frac{\left(D_t^i + B_t^i\right)}{Y_t}\right]^2$ if $0 \leq \frac{D_t^i + B_t^i}{Y_t} \leq \overline{q}$

2. $v(.) = 0$ if $\frac{D_t^i + B_t^i}{Y_t} > \overline{q}$

- **Budget constraints**

$$(1-\tau)\frac{W_t}{P_t}h_t = \widehat{P}_t^B Q_t^{B,i} + L_t^i + D_t^i + B_t^i + N_t^b$$

$$C_{t+1}^i = \widehat{P}_{t+1}^B Q_t^{B,i} + \left(1 + r_t^L\right)L_t^i + \left(1 + r_t^D\right)\left(D_t^i + B_t^i\right) + Z_{t+1}^b$$

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# Investors (2)

- **FOCs**

$$(1-\rho)\,P^B_{t+1} \leq \left(1+r^L_t\right)P^B_t$$

$$\left(1+r^D_t\right) = \left(1+r^L_t\right) - \mu_t$$

where the "safety premium" is

$$\mu_t = \left(\overline{q} - \frac{D^i_t + B^i_t}{Y_t}\right) \geq 0$$

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# Bubbly Assets

- Intrinsically worthless
- **Risky** bcz their future price $\widehat{P}^B_{t+1}$ can collapse to zero

$$\widehat{P}^B_{t+1} = \begin{cases} P_{t+1} > 0 & with \quad probability \quad 1-\rho \\ 0 & with \quad probability \quad \rho \end{cases}$$

- Once the bubble bursts, it does not re-emerge again

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# Entrepreneurs

- **Preferences**

$$U_t^e = E_t C_{t+1}^e$$

- **Budget constraints**

$$\left(1 + r_t^L\right) L_t^e \leq \phi^e E_t r_{t+1}^k K_t \tag{1}$$

$$C_{t+1}^e = \left[(1-\tau)\, r_{t+1}^k + (1-\delta^k) p_{t+1}^k\right] K_t - \left(1 + r_t^L\right) L_t^e$$

where

$$L_t^e = p_t^k K_t$$

with

$$K_t = (1-\delta) K_{t-1} + I_t$$

Back

# Entrepreneurs (2)

- **FOC**

$$E_t\left[\frac{(1-\tau)r^k_{t+1}+(1-\delta)p^k_{t+1}}{p^k_t}\right]=(1+\theta_t)\left(1+r^L_t\right)$$

where

$$\theta_t \geq 0$$

is the Lagrange multiplier on the collateral constraint (1)

Back

# Banks

- **Profit**

$$Z_{t+1}^{b}\left(\widehat{P}_{t+1}^{B}\right)=\left(1+r_{t}^{L}\right)L_{t}^{b}+\widehat{P}_{t+1}^{B}Q_{t}^{B,b}-\left(1+r_{t}^{D}\right)D_{t}^{b}$$

- **Balance sheet**

$$L_{t}^{b}+\widehat{P}_{t}^{B}Q_{t}^{B,b}=N_{t}^{b}+D_{t}^{b}$$

- **Collateral constraint**

$$\phi^{D}D_{t}^{b}=\phi^{L}L_{t}^{b}+\phi^{B}\widehat{P}_{t}^{B}Q_{t}^{B,b}$$

Back

# Banks (2)

- **FOCs**

$$(1-\rho)\,P^B_{t+1}+\left(\frac{\phi^B-\phi^L}{\phi^D-\phi^L}\right)\left[\left(1+r^L_t\right)-\left(1+r^D_t\right)\right]P^B_t \leq \left(1+r^L_t\right)P^B_t$$

Back

# Firms

- **Production function**

$$Y_t = K_{t-1}^{1-\alpha} h_t^{\alpha}$$

- **FOCs**

$$\frac{W_t}{P_t} = \alpha \frac{Y_t}{h_t}$$

$$r_t^k = (1-\alpha)\,\frac{Y_t}{K_{t-1}}$$

Back

# Government

- **Government budget constraint**

$$Y_t + B_t^g = G_t + \left(1 + r_{t-1}^D\right) B_{t-1}^g$$

where

$$Y_t = \frac{W_t}{P_t} h_t + r_t^k K_{t-1}$$

Back

# Central Bank

- **Interest rate rule** with **ZLB**

$$1 + i_t = \max\left[1, \left(1 + r_t^f\right)\Pi^*\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right],$$

where $\phi_\pi > 1$

- **Fisher equation**

$$1 + r_t = (1 + i_t)\, E_t\Pi_{t+1}^{-1}.$$

Back

# DNWR

$$W_t = \max\left(\delta\Pi^* W_{t-1}, W_t^{flex}\right),$$

where

$$W_t^{flex} \equiv \alpha P_t K_{t-1}^{1-\alpha} \bar{h}^{\alpha-1}$$

with $\delta \in (0, 1]$

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# Parametric Restrictions

- $\gamma\,(1-\tau)\,\alpha\phi^L/(\phi^D-\phi^L) < \overline{q} < (1-\gamma)\,(1-\tau)\,\alpha$
- $(1-\gamma)\,(1-\tau)\,\alpha > \frac{D_t^i+B_t^i}{Y_t} > \overline{q}$
- $0 < \phi^L < (1-\gamma)\phi^D < \phi^D < \phi^B < 1$

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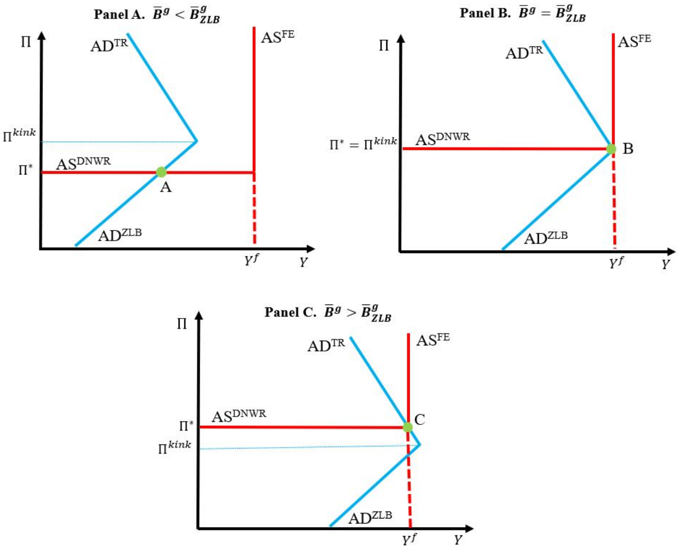
# Steady State Equilibria (Bubbly or Bubbleless)




Prop 1 Prop 2

# Proposition 1 and 2: General Assumptions

- Negative natural interest rate $\left(1 + r^{f}_{NB}\right) < 1$
- Maximum "safe" public debt-to-GDP ratio (just a definition)

$$\left(\frac{\bar{B}^{g}}{Y}\right)_{safe} \equiv \left[\bar{q} - \left(\frac{\phi^{L}}{\phi^{D} - \phi^{L}}\right)\gamma\left(1-\tau\right)\alpha\right].$$

- Some "safe" fiscal space

$$\frac{\bar{B}^{g}}{Y_{ZLB-U}} = \left(1-\tau\right)\alpha - \delta\Pi^{*}\phi^{e}\left(1-\alpha\right) < \left(\frac{\bar{B}^{g}}{Y}\right)_{safe}.$$

- Given $\bar{B}^{g}_{ZLB}$, the previous implies

$$\bar{B}^{g}_{ZLB} < \bar{B}^{g}_{NB,L}\left(Y^{f}\right) = \bar{B}^{g}_{safe}\left(Y^{f}\right)$$

- Finally

$$\bar{B}^{g}_{NB,L}\left(Y_{ZLB-U}\right) = \bar{B}^{g}_{safe}\left(Y_{ZLB-U}\right) < \bar{B}^{g}_{ZLB}$$

Back Prop 1 Back Prop 2

# Proposition 1: Specific Assumptions

- **Partially leveraged bubbles** are possible in the initial $ZLB-U$ equilibrium A (Panel A) because we impose:

  1. $\bar{B}^{g}_{NB,U}(Y) > \bar{B}^{g}_{NB,L}(Y)$ for any given $Y$
  2. $\frac{1}{\delta\Pi^{*}} = (1 + r_{NB,ZLB-U}) < (1-\rho)$

  so that $\bar{B}^{g} < \bar{B}^{g}_{NB,L}(Y) < \bar{B}^{g}_{NB,U}(Y)$ is associated with:

$$(1 + r_{NB,ZLB-U}) < (1-\rho) < (1-\rho) + \frac{\left(\phi^{B} - \phi^{L}\right)}{\left(\phi^{D} - \phi^{L}\right)}\mu_{NB}$$

- We define the maximum "safe" public debt

$$\bar{B}^{g}_{safe}(Y) = \bar{B}^{g}_{NB,L}(Y) \equiv \left[\bar{q} - \left(\frac{\phi^{L}}{\phi^{D} - \phi^{L}}\right)\gamma(1-\tau)\alpha\right]$$

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# Proposition 1

Figure | General Assumptions | Specific Assumptions

**Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debt**

*Given* $\bar{B}^g_{NB,L}(Y) < \bar{B}^g_{NB,U}(Y)$ *for any* $Y$ *and* $\frac{1}{\delta\Pi^*} < (1-\rho)$, *the government can achieve full macro stability, moving the economy from the initial* $ZLB-U$ *equilibrium (point A, Panel A) to the TR-FE equilibrium (point C, Panel C), even for a public debt level* $\bar{B}^g$ *below the maximum safe one,* $\bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right)$. *However, only if macro stability is obtained through* $\bar{B}^g > \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right)$ *bubbles can also be prevented. Instead, partially leveraged or unleveraged bubbles can still occur in the final* $TR-FE$ *equilibrium* ($P^{B,PL} > 0$ *or* $P^{B,U} > 0$) *for* $\bar{B}^g \leq \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right)$.

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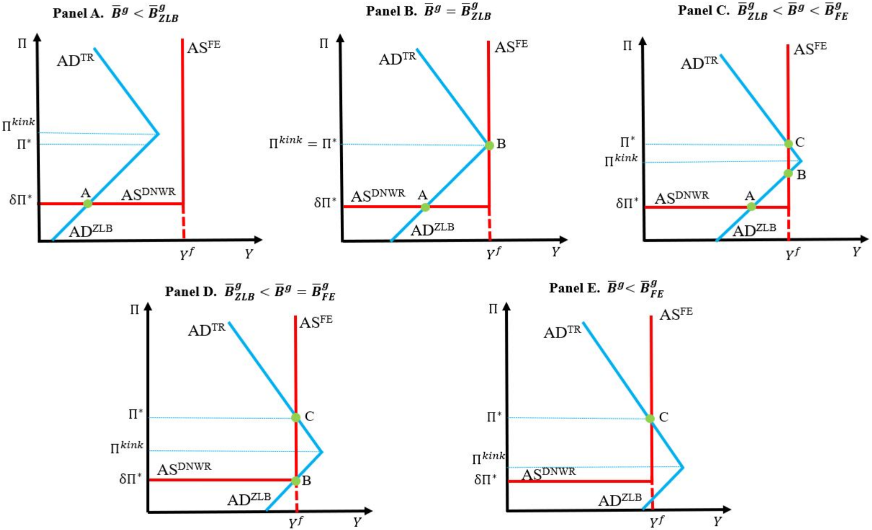
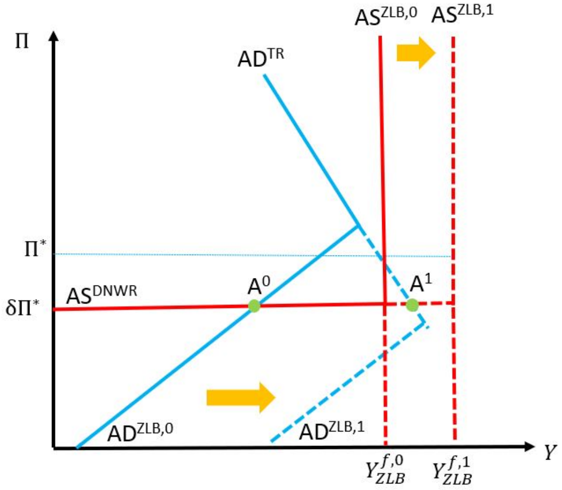
# Endogenous Inflation: Steady State Equilibria (Bubbly or Bubbleless)



# Endogenous Capital: ZLB-U Equilibrium (1)

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# Endogenous Capital: ZLB-U Equilibrium (2)

$\bar{B}^g$ affects also $AS$ bcz

$$K_{ZLB-U} = L^e_{ZLB-U} = (1-\tau)\,\alpha Y_{ZLB-U} - \bar{B}^g$$
$$= \left[\frac{(1-\tau)\,\alpha}{(1-\tau)\,\alpha - \Pi^*\phi^e\,(1-\alpha)} - 1\right]\bar{B}^g$$

$$Y^f_{ZLB-U} = K^{1-\alpha}_{ZLB-U}\bar{h}^\alpha = \left[(1-\tau)\,\alpha Y_{ZLB-U} - \bar{B}^g\right]^{1-\alpha}\bar{h}^\alpha$$
$$= \left\{\left[\frac{\Pi^*\phi^e\,(1-\alpha)}{(1-\tau)\,\alpha - \Pi^*\phi^e\,(1-\alpha)}\right]\bar{B}^g\right\}^{1-\alpha}\bar{h}^\alpha$$

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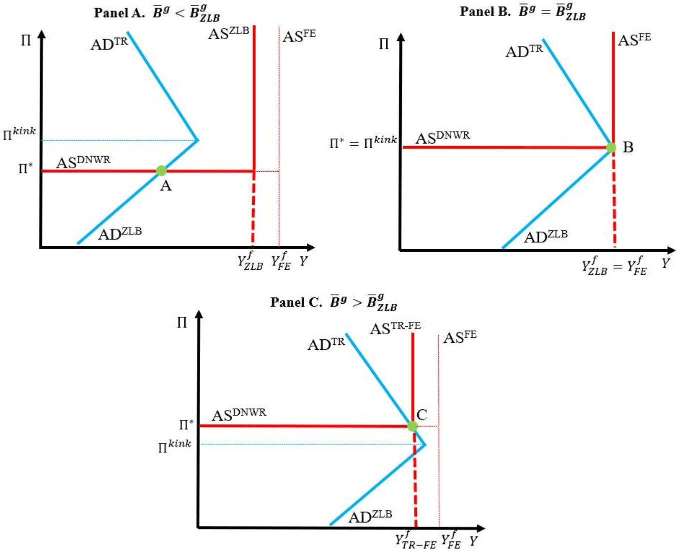
# Endogenous Capital:
# Steady State Equilibria (Bubbly or Bubbleless)

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# Endogenous Capital: Proposition 1 (Revisited)

**Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debts,**...AND **hurts potential output**

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# Proposition 2: Specific Assumptions

- **Leveraged bubbles** are possible in the initial $ZLB-U$ equilibrium A (Panel A) because we impose:

  1. $\bar{B}^{g}_{NB,U}(Y) < \bar{B}^{g}_{NB,L}(Y)$ for any given $Y$
  2. $\frac{1}{\delta\Pi^*} = (1 + r_{NB,ZLB-U}) > (1-\rho)$

  so that $\bar{B}^{g}_{NB,U}(Y) < \bar{B}^{g} < \bar{B}^{g}_{NB,L}(Y)$ is associated with:

$$(1-\rho) < (1 + r_{NB,ZLB-U}) < (1-\rho) + \frac{\left(\phi^B - \phi^L\right)}{\left(\phi^D - \phi^L\right)}\mu_{NB}$$

- We define the maximum "safe" public debt

$$\bar{B}^{g}_{safe}(Y) = \bar{B}^{g}_{NB,L}(Y) \equiv \left[\bar{q} - \left(\frac{\phi^L}{\phi^D - \phi^L}\right)\gamma(1-\tau)\alpha\right]$$

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# Proposition 2

Figure General Assumptions Specific Assumptions

**Macro stability does not necessarily prevent asset bubbles, though it can be achieved through safe public debt**

*Given* $\bar{B}^g_{NB,U}(Y) < \bar{B}^g_{NB,L}(Y)$ *for any* $Y$ *and* $(1-\rho) < \frac{1}{\delta\Pi^*}$, *the government can achieve full macro stability, moving the economy from the initial* $ZLB - U$ *equilibrium (point A, Panel A) to the TR-FE equilibrium (point C, Panel E), even for a public debt level* $\bar{B}^g$ *below the maximum safe one,* $\bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right)$. *However, only if macro stability is obtained through* $\bar{B}^g \geq \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right)$ *bubbles can also be prevented. Instead, leveraged bubbles can still occur in the final* $TR - FE$ *equilibrium* ($P^{B,L} > 0$) *for* $\bar{B}^g < \bar{B}^g_{safe}\left(Y^f\right) = \bar{B}^g_{NB,L}\left(Y^f\right)$.

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