INSIDE AND OUT: FINAL DEMAND PROXIMITY AND AGGREGATE SUPPLY IN PRODUCTION NETWORKS *

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October 2026

Abstract

In a networked economy, firms can sell their output either inside the production network, as an intermediate input to other firms, or outside it, as a final good to consumers. This paper introduces final demand proximity, the extent to which a sector's output reaches final consumers directly or through other sectors, as a general equilibrium force. Final demand proximity affects firms' pricing decisions and how demand is reallocated between the consumption market and the production network. As a result, marginal revenues and costs become endogenous and jointly depend on the network structure. We illustrate this mechanism in a simple framework and formalize it in a multi-sector model. Final demand proximity shapes the slope of sectoral supply curves, and therefore of aggregate supply, in a non-monotonic way: supply curves are flatter at both high and low levels of final demand proximity. We test and confirm this prediction using country-sector variation within the European Monetary Union.

JEL: E31, E32, E52, F41

KEYWORDS: production networks, supply curve, final demand, real rigidities, inflation dynamics.

Corresponding author: Daniele Siena, dansiena@gmail.com. We thank participants at seminars at Banque de France, Bocconi University, Catholic University of Sacred Heart, Federal Reserve Board of Governors, International Association for Applied Econometrics Annual Conference 2026 (Nova School of Business and Economics), Networks, Inflation, and Monetary Policy Transmission conference (Sveriges Riskbank Annual Research Conference 2026), and 2nd Macroeconometrics in Salerno workshop (University of Salerno) for helpful comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Banque de France. All remaining errors are ours. *Preliminary draft, please do not circulate without permission*.

1. INTRODUCTION

Production networks are central to understanding aggregate outcomes (*e.g.*, Long and Plosser 1983; Acemoglu *et al.* 2012). Firms are interconnected through their purchases of intermediate inputs, and these linkages transmit costs and shocks across the economy. When prices are sticky, the network structure also shapes aggregate supply (*e.g.*, Basu 1995; Rubbo 2023). Yet, a production network describes not only whom firms buy from, but also whom they sell to. Some sell mainly to other firms within the network, so their output reaches final demand only after several stages of production. Others sell directly to final consumers outside the network. This distinction matters: whether output is absorbed inside or outside the network affects both price-setting behaviour and quantities.

Motivated by this observation, in this paper we study to what extent the final destination of output shapes sectoral and aggregate supply in a networked economy. This dimension has been largely overlooked, yet it is key to understanding how sectoral characteristics beyond Input-Output linkages matter for the propagation of shocks and for aggregate fluctuations.

We argue that *final demand proximity* – the extent to which a sector's output reaches final consumers directly or only indirectly through other firms – matters for sectoral and aggregate supply. We proceed in three steps. First, using a general framework, we show that the destination of output is an equilibrium object of the network and explain why it matters for price setting. Second, we formalize the mechanism in a multi-sector New Keynesian model. Relative to the standard production network mechanism, the key difference is simple. Firms are both buyers and sellers within the network. Consistently with the existing literature, when they are buyers, firms' marginal costs depend on the prices of intermediate inputs. However, when they are sellers, their marginal revenues depend on whether their output is sold outside the network, to final consumers, or inside it, to other firms. By effecting both sides of the price setting problem, production networks shape the slope of the sectoral and aggregate supply curve. Finally, we test the model's predictions empirically, exploiting country-sector variation within the European Monetary Union.

Final demand proximity is neither a new concept nor an additional assumption: it is a feature that every production network model already contains. Every such model imposes the same sectoral market clearing condition, namely that a sector's output is bought either by households or by other sectors. The literature typically writes this condition in terms of the output of the *buying* sector, which delivers the Leontief inverse and summarizes how intensely sectors trade with one another. By constrast, writing the very same condition in terms of the output of the *selling* sector delivers something different. It states that each unit of output either leaves the network immediately, sold to consumers, or stays inside it and is sold again. The buying sector faces the same condition, and so does the next one, so applying it recursively tracks output until it is finally consumed. Final demand, in other words, is the only way out of the network.

The result is a distribution: the share of a sector's output that reaches consumers directly, after one further stage of production, after two, and so on. Two facts follow. First, every sector is exposed to final demand, even one that never sells a single unit to households, because its output is eventually consumed embodied in other sectors' products. Second, how intensely a

sector trades within the network and how far it stands from consumers are distinct: two sectors that sell the same share of their output to other firms can be at very different distances from final demand, depending on whom their buyers sell to in turn. This explains why the standard framework has been silent on proximity: summing the distribution across stages yields one for every sector, so the information it contains disappears once the model is closed. The information was always there; closing the model is precisely the step that sums it away.

As mentioned, the core intuition is that each sector serves two markets at once: final consumers outside the network and other producers inside it. Consequently, when a firm changes its price, demand moves across these two margins, and firms internalize this effect and adjust accordingly. While this mechanism is generalizable, we illustrated it using a tractable Kimball demand structure. The strength of the proximity effect depends on the composition of sectoral sales: a sector that sells almost entirely to final consumers sets prices differently from one that sells mainly within the production network, and both differ from a sector for which the two destinations are important. Price setting therefore depends not only on marginal cost, but also on how a price change shifts the composition of revenues.

This implies that there is also a general equilibrium force at play, because intermediate sales are never fully disconnected from final demand. In fact, even if an intermediate input is sold entirely inside the network, it will eventually reach final consumers. This is exactly what is accounted by the Input-Output structure and the Leontief inverse, tracking successive production stages. In this way, the destination of sectoral output becomes a network object and final demand proximity measures how the output of a sector reaches final consumers across all production chains in which the sector participates. A sector can therefore have little direct exposure to final demand and still have large indirect exposure to it.

We derive three main theoretical results. First, final demand proximity directly affects the slope of sectoral supply curves. While in a standard multi-sector New Keynesian model the production network affects inflation mainly through the propagation of marginal costs across intermediate input linkages, we show that the network also affects the demand elasticity perceived by firms and, therefore, their marginal revenues. The slope of the sectoral Phillips curve becomes jointly determined by three forces: *(i)* nominal rigidities; *(ii)* the standard network cost channel; and *(iii)* the destination of production.

Second, the effect of final demand proximity is non-monotonic. Sectors located at the two extremes of the production structure are affected less, as their sales operate mainly through a single demand margin. The effect is stronger for sectors in intermediate positions, where both final and intermediate demand matter and price changes operate simultaneously through both margins at once. As a result, sectoral supply curves are flatter at the two extremes of the production structure and steeper for sectors in intermediate positions. In the middle of the network, firms serve a more balanced mix of final and intermediate customers, so a change in their price can be better absorbed through reallocation of demand between these two types of users. Importantly, the relevant demand reallocation is within sector rather than across sectors: a price change does not shift demand from one sector to another, but reallocates demand for a given sector's output between its intermediate and final users. This reallocation does not reflect competition across sectors: it concerns how demand for a given sector's output is distributed

between its different uses. Because both margins are active, firms in the middle of the network pass changes in marginal cost more strongly into prices, which steepens their supply curves. The interaction between the reallocation of demand and the propagation of output across production stages is therefore strongest in the middle of the network. As a result, the model generates an inverted U-shaped relationship between final demand proximity and the slope of sectoral supply.

Third, sectoral heterogeneity matters for aggregate supply. In our framework, sectors can no longer be aggregated using only their direct importance in final consumption. A sector that represents a small fraction of household expenditure can still be important because its output is used repeatedly as an intermediate input by other sectors. The relevant aggregation weights therefore depend on the Input-Output structure, and especially on the Leontief inverse , which captures both the direct and indirect use of each sector's output. Hence, final demand proximity shapes not only the slope of individual sectoral supply curves, but also the slope of the aggregate supply curve.

In the last part of the paper, we empirically test these predictions. We use country/sector data for the countries that first joined the European Monetary Union, excluding Luxembourg because of data availability, and focus on NACE two digit sectors. We combine producer prices and sectoral production from Eurostat with Input-Output measures describing two different dimensions of the production structure. Following Antràs and Chor (2013), upstreamness and downstreamness capture the position of sectors along production chains and their distance from final demand (final demand proximity). Forward and backward linkages, from Mancini *et al.* (2024), instead capture the extent and direction of sectoral participation in the production network. This distinction is key and allows us to distinguish both features of the network: how much a sector participates and where its output is located relative to final demand.

The data show that these are indeed very different dimensions of production networks. Their correlation is weak and, more importantly, they have different implications for the sectoral supply. We first estimate the slope of sectoral supply curves and provide graphical evidence on their relationship with the network measures. Final demand proximity displays a clear non-monotonic relation with the slope: sectors very close and very far from final demand have flatter supply curves, while sectors in intermediate positions have steeper ones. Network participation, instead, has a monotonic relation with the slope. In particular, stronger backward linkages are associated with flatter supply curves, whereas stronger forward linkages are associated with steeper ones.

We then test these relationships exploiting the panel dimension of the data. We estimate sectoral Phillips curves relating producer price inflation to sectoral output growth and interact output growth with the different network measures. The EMU provides a useful environment for this exercise because the countries in the sample share the same monetary authority. Following the identification logic of Hazell *et al.* (2022), we use time and country/time fixed effects to absorb common and country level factors affecting inflation, like inflation expectations and economic activity. We also control for country/sector fixed effects and lagged inflation, and instrument output growth and its interactions with their first lags.

The econometric evidence confirms the graphical patterns. Both upstreamness and down-

streamness matter for the slope of sectoral supply, generating the non monotonic relation associated with final demand proximity. Forward and backward linkages have instead opposite effects: forwardness steepens the supply curve, while backwardness flattens it. These results are robust to allowing for a structural break after the Great Financial Crisis, when using Bartik IVs instead of internal lags, when muting the time dimension of network characteristics, and to an alternative definition of sectoral slack.

The slope of aggregate supply is often taken as a structural characteristic of the economy, determined by nominal rigidities, technology and market structure. Our results show that it also depends on how production is organized across sectors and on where sectoral output ultimately goes, if inside the network or out or both. This is first order to understand how sectoral shocks, but also common shocks, are propagated through the economy.

The remainder of the paper is organized as follows. Section 2 develops the main intuition and introduces the Input-Output measures. Section 3 formalizes the mechanism in a multi-sector New Keynesian framework. Section 4 presents the data and the descriptive evidence. Section 5 presents the empirical analysis. Section 6 concludes.

Literature review.– This paper relates to three strands of literature. The first strand studies production networks and the macroeconomic effects of Input-Output linkages. A central insight of this literature is that sectoral shocks can have aggregate effects because sectors are connected through production chains (*e.g.*, Acemoglu *et al.* 2012; Oberfield 2018; Baqaee and Farhi 2019a; Dionisi 2026). Related works study price setting behaviours (*e.g.*, Ghassibe and Nakov 2025) and monetary policy (*e.g.*, Ghassibe 2021; La'O and Tahbaz-Salehi 2022; Ghassibe 2024; Qiu *et al.* 2026) in multi-sector economies with an Input-Output network. Across these papers, the central network object is the Leontief inverse, namely the degree of interconnectivity across network participants. The paper contributes by adding the destination of production to the standard Input-Output mechanism. In particular, firms/sectors are not only directly and indirectly connected through network linkages, but are also directly and indirectly exposed to the final demand, since any intermediate input sold must eventually end up in consumption. This is not an additional hypothesis, but a structural feature of any Input-Output economy. We further show that this distance to final demand is not only a description of the organization of production, but it is relevant for the way prices are set and therefore for aggregate supply. The Input-Output system modifies the aggregate supply curve (*e.g.*, Basu 1995; Christiano 2016), and Rubbo (2023) shows that production networks modify the New Keynesian Phillips Curve (NKPC) because nominal rigidities accumulate along intermediate input chains. Our paper builds directly on this insight, but sizes a different and complementary margin. Where a firm/sector is located relative to final demand matters because it determines how many stages of production are before and after a sector. We show that the vertical position of production gives birth to a sectoral NKPC which is inverted-U shaped: closer and further sectors to final demand exhibit flatter NKPCs, while those in the middle of the supply chain have a larger pass-through from marginal costs to prices.

The second strand studies global value chains and the measurement of sectoral position in production. The empirical literature has developed measures of upstreamness, downstreamness,

backwardness and forwardness to describe how sectors participate in domestic and international production chains (*e.g.*, Fally 2012; Antràs *et al.* 2012; Antràs and Chor 2013; Mancini *et al.* 2024). Our paper builds on these different measures by showing that these directly map into the market clearing equilibrium condition of an Input-Output economy. We theoretically characterize a destination matrix that accounts for how a sector is located far from final demand conditional on the fact that the intermediate input it sold does not reach only its final demand, but also the final consumers of the other sectors at which it is connected. Moreover, we show that these measures have empirically sizeable different implications for the Phillips curve.

The third strand studies the flattening and heterogeneity of the Phillips curve (*e.g.*, Altissimo *et al.* 2006; Imbs *et al.* 2011; Mavroeidis *et al.* 2014; Hottman and Reyes-Heroles 2023; Furlanetto and Lepetit 2024; Gagliardone *et al.* 2024). A large literature documents that the relation between inflation and real activity has weakened in many advanced economies, while remaining difficult to estimate precisely. Stock and Watson (2019) provide a useful empirical benchmark for measuring the relation between inflation and activity. Hazell *et al.* (2022) show how regional variation inside a monetary union can be used to discipline the slope of the Phillips curve while controlling for common inflation expectations. We follow this identification logic in a country sector setting. Since the countries in our sample share a monetary authority, time fixed effects absorb common monetary shocks and common expectation movements, while country time fixed effects absorb country level disturbances. Our paper also relates to work linking globalization and inflation dynamics. Contributions such as Sbordone (2007), Guerrieri *et al.* (2010), Benigno and Faia (2016) and Guilloux-Nefussi (2020) show that openness, foreign competition and international production affect markups and the sensitivity of inflation to domestic slack. Florio *et al.* (2025) studies how participation and position in global value chains affect the Phillips curve at the country level. We differ by moving the analysis to the country sector level and by separating two mechanisms that are hard to identify in aggregate data. The first is the propagation of nominal rigidities through production stages. The second is the role of final demand proximity in shaping the demand elasticity faced by firms. The mechanism in the model is connected to the literature on real rigidities. Kimball (1995) shows that non constant demand elasticity generates strategic complementarities in price setting. We use this logic inside a production network. The relevant strategic complementarity is within a sector, between sales to final consumers and sales to other producers. This within sector margin transforms the distance to final demand into a determinant of the Phillips curve slope. Relative to these literatures, the paper makes three contributions. First, it introduces proximity to final demand as a separate network dimension for inflation dynamics. Second, it develops a model in which proximity affects the Phillips curve through a Kimball demand system and vertical integration. Third, it documents that proximity and participation have different empirical relations with sectoral Phillips curve slopes. The joint message is that production networks affect inflation through both cost propagation and demand composition. Ignoring either margin gives an incomplete account of why some sectors adjust prices more strongly to economic activity than others.

2. Networks and the Destination of Production

Production networks describe not only how sectors are interconnected, but also how production ultimately reaches final demand. The existing literature has predominantly focused on the first dimension, studying how inter-sectoral linkages shape the propagation of shocks and aggregate outcomes (*e.g.*, Acemoglu *et al.* 2012; Baqaee and Farhi 2019b; La'O and Tahbaz-Salehi 2022; Rubbo 2023). In this view, the production network can be thought of as a sphere of interconnected sectors, where these buy and sell intermediate inputs and also sell directly to final consumers. This representation naturally captures inter-sectoral trade, but it does not distinguish how far a sector's production is from final demand. We argue that this distinction is important, as production can reach final consumers both directly and indirectly: when a sector sells its output as an intermediate input, it becomes part of a supply chain that may pass through several production stages before reaching final demand. Thus, the same network of inter-sectoral connections can generate supply chains of different lengths, so that sectors with similar degrees of participation in the network may lie at very different distances from final consumers. We capture this dimension by defining *final proximity* as the average number of production steps a product travels before reaching overall final demand. Final proximity therefore reveals a dimension of the network that is implicit in its inter-sectoral structure, but distinct from sectoral participation. The purpose of this section is to show this mechanism and make its intuition transparent using a general framework. In the next section, we embed the same idea into a standard New Keynesian model and study its implications for price setting and aggregate supply.

Consider an economy made of $\Phi = \{1, \ldots, i, j, \ldots, m\}$ sectors, each characterized by a representative firm producing output y_i, and where aggregate output Y is just a collection of sectoral production levels, $Y = f^y\left[y_1, \ldots, y_i, \ldots, y_m\right]$. Each sector-$i$ produces using labour l_i and a set of intermediate inputs from other sectors

$$y_i = f_i^y\left[l_i, x_{1i}, x_{2i}, \ldots, x_{ji}, \ldots, x_{mi}\right]$$

where x_{ji} represents the intermediate input that sector-j sells to sector-i at price p_j^x, and $f_i^y\left[\cdot\right]$ is assumed to be continuous, differentiable, increasing in its argument, and homogeneous of degree one. Note that the first index is the seller, while the second is the buyer, identifying Input-Output matrix's row and column, respectively.[1]

In a classical way (*e.g.*, Acemoglu *et al.* 2012; Baqaee and Farhi 2019b), each unit of output has two possible destinations. A fraction is sold as an intermediate input to downstream sectors, $x_{ij}, \forall j \in \Phi$, while the remaining fraction is sold to final consumers, c_i. Sectoral market clearing

[1] We adopt throughout the standard Input-Output convention, in which the row index of a transaction denotes the supplying sector and the column index the purchasing one, so that $\boldsymbol{\mathcal{A}}$ is the conventional matrix of technical coefficients (*e.g.*, Antràs *et al.* 2012), associated to its Leontief inverse representation, $\boldsymbol{\mathcal{L}}$ (see Subsection 2.1). The production network literature (*e.g.*, Carvalho 2014 adopts the transposed convention, indexing by the purchasing sector first, so that the matrices used here are the transpose of those employed there. Nothing of substance depends on this choice, but the canonical orientation is the natural one for our purposes: the central object of the paper, the destination matrix $\boldsymbol{\Theta}$ introduced in Definition 1, is built from the same transaction as $\boldsymbol{\mathcal{L}}$ and differs from it only in the normalisation, the seller's output rather than the buyer's. Keeping the row index aligned with the supplying sector makes this common structure transparent and dispenses with transpositions throughout.

therefore requires

$$y_i = c_i + x_i^{sup} \qquad \text{with} \quad x_i^{sup} = \sum_{j \in \Phi} x_{ij} \tag{1}$$

where x_i^{sup} is the total network-based demand satisfied by sector-i. We further define $\phi_i^c = \frac{c_i}{y_i}$, so that it holds $1 - \phi_i^c = \frac{x_i^{sup}}{y_i} \equiv \phi_i^x$, to be the share of sector-i's output directly devoted to final consumption. Since $0 \leq c_i \leq y_i$, then $\phi_i^c \in [0,1]$. This object summarizes the destination of sectoral output: for $\phi_i^c \to 1$, the sector only sells to the final consumer; for $\phi_i^c \to 0$, the sector sells entirely within the production network.

Households are represented by a continuous and differentiable utility function

$$f^u\Big[C, \{l\}_{i \in \Phi}\Big] \qquad \text{with} \quad C = f^c\big[c_1, \ldots, c_i, \ldots, c_m\big]$$

which is increasing and concave in consumption, $f^u_{c_i} > 0$ and $f^u_{c_i c_i} < 0$. Demand theory implies that final consumption depends on aggregate production Y and on the $m \times 1$ vector of final good prices, $\boldsymbol{p}^c = \big[p_1^c, \ldots, p_i^c, \ldots, p_m^c\big] \mapsto P^c$, that is $c_i = f_i^c\big(Y, \boldsymbol{p}^c\big), \forall i \in \Phi$.

2.1. Same identity, two normalisations

The condition in eq. (1) is the fundamental Input-Output identity that guides the Leontief inverse result: identifying with $\alpha_{ij} = \frac{x_{ij}}{y_j}$ the direct technical coefficient representing the amount of good-i required to produce one unit of good-j, if plugged into the sectoral equilibrium expressed in vectorial form one gets $\boldsymbol{y} = \boldsymbol{c} + \boldsymbol{A}\boldsymbol{y}$, where $\boldsymbol{A} = \big[\alpha_{ij}\big] \in \mathbb{R}^{m \times m}$ is the $m \times m$ Input-Output matrix, associated to the $m \times 1$ vectors of sectoral production $\boldsymbol{y}$ and sectoral final demand $\boldsymbol{c}$; provided that $\text{Radius}\,[\boldsymbol{A}] < 1$, then the matrix is invertible, reaching the benchmark Leontief inverse operator $\boldsymbol{\mathcal{L}} = (\boldsymbol{I} - \boldsymbol{A})^{-1}$, with $\boldsymbol{I}$ being an $m \times m$ identity matrix, since matrix inversion leads to $\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{c}$. This is the classical network representation, and its central object $\boldsymbol{\mathcal{L}}$ measures the intensity with which sectors are directly and indirectly connected in buying/selling to one another, that is the extent to which a sector is *participating* to the production network.

The Leontief accounting captures the conventional interpretation of production networks. Since sectors are technologically connected through intermediate input purchases, any variation propagates across sectors according to the network topology summarized by $\boldsymbol{\mathcal{L}}$. However, it leaves implicit another dimension. A network structure describes not only where sectors purchase their intermediate inputs: from the sectoral clearing condition, it pins down also where they sell their output. Every sector simultaneously serves two markets: its final consumers, and downstream producers through intermediate sales, which indirectly serve another part of the final demand, that of its buyers. Indeed, how far a sector stands from final consumption is an intrinsic object of an Input-Output system: production networks determine not only how sectors are related, but also how sectoral production ultimately reaches final demand. We call this object *final demand proximity*.

To infer this, divide the same Input-Output transaction x_{ij} by the output of the selling sector-i – in other words, normalize eq. (1) in terms of one unit of y_i.

Definition 1 (Destination shares) *For every pair $\{i,j\} \in \Phi$, the destination share of sector-i towards sector-j is $\theta_{ij} = \frac{x_{ij}}{y_i}$, collected into the $m \times m$ destination matrix $\boldsymbol{\Theta} = \big[\theta_{ij}\big] \in \mathbb{R}^{m \times m}$.*

While α_{ij} is a technological object, describing input requirements through inter-sectoral trade activities, θ_{ij} is a commercial object, describing where sectoral output goes. The former measures the amount of sector-i input required to produce one unit of output in sector-j; the latter, instead, measures the share of sector-i's sales that is directed to sector-j. This distinction has important implications. Starting from the sectoral market clearing condition, the production side yields the familiar Leontief representation, in which final demand determines gross output through the technological requirements of the Leontief inverse. The same market clearing condition can, however, be viewed from the commercial side, yielding a "destination representation" in which final demand is characterized by the recursive paths of sectoral output across the network ending in final consumption. Our main result formalizes this relationship: the Leontief inverse and the destination matrix provide dual representations of the same market clearing system, linking the technological structure of production to the commercial structure of demand.

We now make this duality explicit by reconsidering the sectoral equilibrium from both sides of the market. By Definition 1, the clearing condition becomes

$$1 = \phi_i^c + \sum_{j \in \Phi} \theta_{ij} \qquad \text{that is} \quad \mathbf{1} = \boldsymbol{\phi}^c + \boldsymbol{\Theta}\mathbf{1} \tag{2}$$

where $\mathbf{1}$ is the $m \times 1$ vector of one unit of output, and $\boldsymbol{\phi}^c = \left[\phi_1^c, \ldots, \phi_i^c, \ldots, \phi_m^c\right]'$.

LEMMA 1 (Destination matrix) *Matrix $\boldsymbol{\Theta}$ is non-negative and sub-stochastic, and the row-deficiency of $\boldsymbol{\Theta}$ relative to unity is exactly the vector of direct final demand shares:*

$$\left(\boldsymbol{I} - \boldsymbol{\Theta}\right)\mathbf{1} \;=\; \boldsymbol{\phi}^c \;\geq\; \mathbf{0}$$

Proof: non-negativity is immediate from $x_{ij} \geq 0$ and $y_i > 0$. The displayed identity is a rearrangement of eq. (2), which is eq. (1) divided by y_i. Sub-stochasticity follows by $\sum_{j \in \Phi} \theta_{ij} = 1 - \phi_i^c \leq 1$ since $\phi_i^c \geq 0$. Intuitively, sub-stochasticity means that each row of $\boldsymbol{\Theta}$ sums to at most one: the output of each sector can be either passed on to other sectors as an intermediate input or absorbed by final demand. The row-deficiency $1 - \sum_{j \in \Phi} \theta_{ij} = \phi_i^c$ therefore measures exactly the share that exits the production network as final demand. The upper bound is attained when $\phi_i^c \to 0$, in which case $\sum_{j \in \Phi} \theta_{ij} \to 1$ and the entire output of sector-i remains within the production network. ■

To build intuition, inspect how one unit of sector-i's output travels through the economy. If one unit of sector-i's output is directly allocated to final demand with share ϕ_i^c, its journey ends immediately. Differently, with share θ_{ij}, it is purchased by sector-j, embodied in another intermediate input, and its journey continues through the production network. Thus, the fact that $\boldsymbol{\Theta}$ is sub-stochastic has a direct economic interpretation: the fraction of output that is not passed on to another sector is precisely the fraction absorbed by final demand. Final demand is therefore the leak of the production network: the only way for output to exit the Input-Output system. The row-deficiency of $\boldsymbol{\Theta}$ is not merely a mathematical residual, but exactly measures this leakage. Lemma 1 makes this point explicit, since the same market clearing condition that determines how output is allocated across sectors also determines how much of it leaves the network. In this sense, final demand is not an afterthought imposed on the production network from outside, but it is written into the market clearing condition itself as the exact amount by which the network fails to reproduce itself.

Crucially, this journey does not stop after one step, because sector-j faces the same identity in eq. (1). The recursive structure of the market clearing condition is what generates final demand

proximity. Since eq. (2) holds for sector-j as well, the unity term $\mathbf{1}$ displays one unit of every other sectors' output. Considering sector-j, it can be replaced by $\phi_j^c + \sum_{m\in\Phi}\theta_{jm}$. Substituting,

$$1 = \phi_i^c + \sum_{j\in\Phi}\theta_{ij}\,\phi_j^c + \sum_{j\in\Phi}\sum_{m\in\Phi}\theta_{ij}\,\theta_{jm} = \phi_i^c + \left[\boldsymbol{\Theta}\,\boldsymbol{\phi}^c\right]_i + \left[\boldsymbol{\Theta}^2\,\mathbf{1}\right]_i$$

The three terms now read: output sold directly to households; output sold to sectors that directly sell to households; and output still circulating inside the network after two transactions. Repeating the substitution $n = \{0,\dots,N\}$ times on the inside-the-network residual term:

$$1 = \sum_{n=0}^{N}\left[\boldsymbol{\Theta}^n\,\boldsymbol{\phi}^c\right]_i + \left[\boldsymbol{\Theta}^{N+1}\,\mathbf{1}\right]_i$$

an identity that holds for every finite N stages; the residual denotes the exact amount of sectoral output that recursively circulates across sectors at $N+1$ iterations, and it vanishes as $N\to\infty$, that is, if and only if sectoral output eventually leaves the production network.

2.2. Network participation and network position

The clearing condition in eq. (1) states that the output of sector-i is split between two destinations, and the network transaction x_{ij} appears as an absolute quantity: to make it comparable across sectors it must be normalised. In the previous subsection we have seen that there are exactly two natural ways of doing so, because every intermediate transaction involves exactly two sectors: one that buys and one that sells. Normalising by the output of the buyer yields the conventional Input-Output matrix and its Leontief inverse representation. Normalising by the output of the seller delivers a second, different object, the destination matrix.

The two normalisations are not two different assumptions about the economy: they are the same matrix expressed in two different sets of units, the buyer's and the seller's.

Lemma 2 (Duality of the operators) *Let* $\widehat{\boldsymbol{y}} = \mathrm{Diag}\,[\boldsymbol{y}]$ *with* $y_i > 0,\ \forall i\in\Phi$. *Then*

$$\boldsymbol{\Theta} = \widehat{\boldsymbol{y}}^{-1}\boldsymbol{\mathcal{A}}\,\widehat{\boldsymbol{y}} \qquad \textit{so that} \quad \mathrm{Radius}\,[\boldsymbol{\Theta}] = \mathrm{Radius}\,[\boldsymbol{\mathcal{A}}]$$

Proof: evaluate the $\{i,j\}$ element of the right-hand side of the expression for $\boldsymbol{\Theta}$, recalling that $[\boldsymbol{\mathcal{A}}]_{ij} = \alpha_{ij}$. Then $[\widehat{\boldsymbol{y}}^{-1}\boldsymbol{\mathcal{A}}\widehat{\boldsymbol{y}}]_{ij} = y_i^{-1}\alpha_{ij}\,y_j = \frac{1}{y_i}\cdot\frac{x_{ij}}{y_j}\cdot y_j = \frac{x_{ij}}{y_i} = \theta_{ij}$, which is Definition 1. Since $\widehat{\boldsymbol{y}}$ is diagonal and invertible, $\boldsymbol{\Theta}$ and $\boldsymbol{\mathcal{A}}$ are similar matrices and therefore share the same spectrum, so that $\mathrm{Radius}\,[\boldsymbol{\Theta}] = \mathrm{Radius}\,[\boldsymbol{\mathcal{A}}]$. ■

The condition $\mathrm{Radius}\,[\boldsymbol{\mathcal{A}}] < 1$, necessary and sufficient to invert the Input-Output matrix and to have the convergence of the Neumann expansion over n stages that generates the Leontief system, is the same condition that guarantees that every sector's output eventually reaches final consumption. What is usually presented as a technical requirement for matrix inversion has, when read in the forward direction, an economic content: an economy is final-demand-absorbing if and only if every production chain terminates in households' final consumption. In other words, intermediate inputs may circulate through production stages, including through cycles, but no intermediate inputs remain indefinitely trapped in self-sustaining production loops without ultimately reaching final demand.

Indeed, since the Leontief system admits the expansion $\boldsymbol{I} + \boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{A}}^2 + \cdots + \boldsymbol{\mathcal{A}}^n$, that is

$\mathcal{L} = \sum_{n\geq 0} \mathcal{A}^n$ with each power-n corresponding to an additional step in the production network (direct inputs, inputs of inputs, and so on), define the forward counterpart of the Leontief inverse as the associated stage-specific and destination-specific shares

$$\Xi = (I - \Theta)^{-1} \equiv \sum_{n\geq 0} \Theta^n \tag{3}$$

which we refer to as the *final demand operator*. Combining this object with Lemma 1 delivers three more objects.

DEFINITION 2 (Final demand exposure) *For every sector-i, stage-n, and absorbing sector-j,*

$$\xi_{ij}(n) \equiv \left[\Theta^n\right]_{ij} \phi_j^c \quad \textit{and} \quad \varpi_i(n) \equiv \sum_{j\in\Phi} \xi_{ij}(n) = \left[\Theta^n \boldsymbol{\phi}^c\right]_i \quad \textit{and} \quad \xi_{ij} \equiv \sum_{n\geq 0} \xi_{ij}(n)$$

We call $\varpi_i(n)$ the final demand proximity distribution of sector-i at stage-n.

Expanding the matrix power, the quantity $\xi_{ij}(n)$ sums, over all production chains of length-n that begin in sector-i and terminate in sector-j, the product of the destination shares along the chain, weighted by the share of j's output that reaches households. In other words, it identifies the fraction of sector-i's output that is absorbed by final demand for good-j after exactly n intermediate transactions. Thus, it answers two questions simultaneously: where does sector-i's output ultimately end up, and how many production stages does it take to get there? The factor $\left[\Theta^n\right]_{ij}$ traces the share of output that is passed from sector to sector for n steps, while ϕ_j^c captures the share that, once it reaches sector-j is finally absorbed by households.

Summing over final destinations gives $\varpi_i(n)$, the fraction absorbed after exactly n stages regardless of which good absorbs it. This quantity abstracts from the identity of the final good and keeps only the distance from final demand. It therefore tells us what fraction of sector-i's output is absorbed by households after exactly n intermediate transactions. In this sense $\varpi_i(n)$ describes the distribution of sector-i's output over supply chain lengths before final absorption.

Finally, summing over stages gives ξ_{ij}, the fraction absorbed through good-j regardless of how many stages it took. This quantity abstracts from the number of intermediate transactions and keeps only the identity of the final absorbing sector. It therefore measures the fraction of sector-i's output that ultimately reaches final demand for good-j, regardless of how many production stages separate i from j.

In short, $\xi_{ij}(n)$ is a joint distribution over who absorbs the output and how far away that absorber is, while $\varpi_i(n)$ and ξ_{ij} are its two marginal distributions. The two aggregations thus preserve different dimensions of the same underlying distribution: $\varpi_i(n)$ preserves proximity to final demand, while ξ_{ij} preserves the identity of the final destination.

With these definitions in hand, we can now make explicit a synthetic definition of *sectoral proximity to final demand*: the average number of stages that it takes an intermediate input to reach final consumption. The next result summarizes the economic implication of the discussion.

THEOREM 1 (Double representation of sectoral market clearing) *Let* $\text{Radius}[\mathcal{A}] < 1$. *Then, the sectoral market clearing condition in eq. (1) admits two equivalent representations, obtained from the two normalisations of the same intermediate transaction:*

- *network participation, normalising by the buyer's output,* $\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{c}$ *with* $\boldsymbol{\mathcal{L}} = (\boldsymbol{I} - \boldsymbol{\mathcal{A}})^{-1}$*;*
- *network position, normalising by the seller's output,* $\mathbf{1} = \boldsymbol{\Xi}\boldsymbol{\phi}^{c}$ *with* $\boldsymbol{\Xi} = (\boldsymbol{I} - \boldsymbol{\Theta})^{-1}$.

Moreover, network participation is the aggregation of network position: $\sum_{j\in\Phi} \xi_{ij} = 1$ *implies* $\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{c}$*. Consequently, every sector is fully exposed to final demand, including sectors with* $\phi_i^c = 0$ *that sell nothing to households. Sectors differ not in whether they are exposed, but in the stage and destination at which that exposure materialises, as described by* $\left\{\xi_{ij}(n)\right\}_{i,j\in\Phi,n\geq 0}$.

Proof: let us prove each part separately. The claim for network participation follows from the Leontief system derived at the beginning of Section 2. That for network position follows a similar logic. By Lemma 2, Radius $[\boldsymbol{\Theta}]$ = Radius $[\boldsymbol{\mathcal{A}}] < 1$, so $\boldsymbol{I} - \boldsymbol{\Theta}$ is invertible and the Neumann series of eq. (3) converges. Applying $\boldsymbol{\Xi}$ to Lemma 1 delivers the result. If expanded row-by-row using eq. (3) and Definition 2, then $\sum_{n\geq 0} \varpi_i(n) = 1$. The claim for aggregation sparks from similarity, which follows from Lemma 2: $\boldsymbol{\Theta} = \widehat{\boldsymbol{y}}^{-1}\boldsymbol{\mathcal{A}}\widehat{\boldsymbol{y}}$ implies $\boldsymbol{I} - \boldsymbol{\Theta} = \widehat{\boldsymbol{y}}^{-1}(\boldsymbol{I} - \boldsymbol{\mathcal{A}})\widehat{\boldsymbol{y}}$ and, inverting both sides, $\boldsymbol{\Xi} = \widehat{\boldsymbol{y}}^{-1}(\boldsymbol{I} - \boldsymbol{\mathcal{A}})^{-1}\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{y}}^{-1}\boldsymbol{\mathcal{L}}\widehat{\boldsymbol{y}}$. Using similarity and $\phi_j^c = \frac{c_j}{y_j}$, then $\xi_{ij} = [\boldsymbol{\Xi}]_{ij}\phi_j^c = y_i^{-1}\mathcal{L}_{ij}y_j \cdot \frac{c_j}{y_j} = \frac{\mathcal{L}_{ij}c_j}{y_i}$. Summing over $j \in \Phi$ and multiplying by y_i, the statement $\sum_{j\in\Phi}\xi_{ij} = 1$ reads as $y_i = \sum_{j\in\Phi}\mathcal{L}_{ij}c_j = [\boldsymbol{\mathcal{L}}\boldsymbol{c}]_i$, which is precisely the network participation. Finally, the claim on full exposure readily applies. The double sum $\sum_{n\geq 0}\sum_{j\in\Phi}\xi_{ij}(n) = \sum_{n\geq 0}\varpi_i(n) = 1$ holds for every $i \in \Phi$, irrespective of the value of ϕ_i^c; if $\phi_i^c = 0$ then $\varpi_i(0) = 0$ and the entire unit mass is distributed over $n \geq 1$. ■

The theorem claims three separate statements. The first is one of existence. The market clearing condition contains a second operator, $\boldsymbol{\Xi}$, which is not an additional modelling ingredient but a re-normalisation of an identity that every production network model already imposes. Whereas $\boldsymbol{\mathcal{L}}$ answers the question of how much output is needed to sustain final demand, $\boldsymbol{\Xi}$ answers the question of how output finds its way to final demand. The first is a statement about *network participation*, the intensity of inter-sectoral trade; the second is a statement about *network position*, the location of a sector along its supply chains that terminate in consumption.

The second is one of completeness. The weights $\left\{\xi_{ij}(n)\right\}_{i,j\in\Phi,n\geq 0}$ form a joint distribution over who absorbs the output and how far away that absorber is, and sum to unity for every sector. No output is lost inside the network and no sector is insulated from households. A sector selling nothing directly to consumers is not weakly exposed to final demand: it is fully exposed, with the entire mass located at $n \geq 1$. This is the formal counterpart of the observation that a sector may supply an intermediate input which, travelling through many stages, ultimately reaches consumption. Final demand exposure is thus universal; what varies across sectors is its distribution over network stages.

The third is one of aggregation, and it explains why the destination channel has remained implicit. The Leontief representation $\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{c}$ is the statement that the destination shares of each sector sum to one. In fact, it arises after aggregating the different destinations of sectoral output into a single quantity: the total amount of final demand ultimately reached by each sector. In terms of the distribution $\left\{\xi_{ij}(n)\right\}_{i,j\in\Phi,n\geq 0}$, the Leontief representation therefore retains only its total mass. Since the destination shares of each sector sum to one, this mass is equal to unity for every sector and, by construction, contains no information about how that mass is distributed across destinations or across different degrees of proximity to final demand. The very step that closes the analysis therefore also integrates out the information on final demand proximity. The information was never absent from the classical framework: it was aggregated away. Recovering the destination structure amounts precisely to undoing this aggregation and retaining the distribution that lies behind the unit mass pinned down by $\boldsymbol{\mathcal{L}}$.

COROLLARY 1 (Participation and proximity are independent) *There exist economies in which two sectors have identical direct final demand shares $\phi_i^c = \phi_j^c$, so that $\varpi_i(0) = \varpi_j(0)$, and hence identical shares of direct intermediate sales $\phi_i^x = \phi_j^x$, but arbitrarily different final demand proximity, $\varpi_i(\cdot) \neq \varpi_j(\cdot)$.*

Proof: fix any $q \in (0,1)$, and consider $\{i,j\}$ with $\phi_i^c = \phi_j^c = q$. Let the remaining share $1-q$ of sector-i's sales be routed through a production chain of length n_i, ending in final demand, and the remaining share $1-q$ of sector-j's sales through a chain of length n_j. The resulting exposure distributions are $\varpi_i(n) = q \cdot 1_{n=0} + (1-q) \cdot 1_{n=n_i}$ and $\varpi_j(n) = q \cdot 1_{n=0} + (1-q) \cdot 1_{n=n_j}$. Hence, $\varpi_i(0) = \varpi_j(0) = q$, so the two sectors have identical network participation (*i.e.*, they sell the same fraction of their output within the network), while $\varpi_i(\cdot) \neq \varpi_j(\cdot)$ whenever $n_i \neq n_j$. Since n_i and n_j can be chosen arbitrarily, the two sectors can have identical network participation and arbitrarily different final-demand proximities. ■

Corollary 1 establishes that network participation and network position are genuinely separate dimensions of the same production network. Knowing how much of a sector stays inside the network doesn't say anything about how far it travels before leaving it for final uses. Sectors with the same degree of network participation may nevertheless occupy different positions along the production chain and, consequently, stand at different distances from final demand. Thus, network participation captures the extent to which a sector is embedded in the production network, while network position captures where those connections place the sector relative to final demand. A complete characterisation of a sector's location in the Input-Output space therefore requires both dimensions.

2.3. MEASURING THE INPUT-OUTPUT SPACE

The framework developed suggests that the economic relevance of a sector depends not only on the network of intermediate input transactions, but also on where its production is ultimately absorbed along the multiple supply chains it serves. This observation naturally motivates a discussion of how the position of sectors within the Input-Output space is conventionally located. In fact, once sectoral output can be allocated between intermediate uses and final consumption, sectors are naturally distinguished by their proximity to final demand.[2] The destination of production provides a natural framework to characterize the position of sectors along the production network chains at which the sector is connected.

It is therefore useful to briefly review the standard notions proxying network position and participation that have been developed in the Input-Output literature, namely upstreamness and downstreamness (*e.g.*, Antràs *et al.* 2012), forward and backward linkages (*e.g.*, Mancini *et al.* 2024). These are summarized in Table 1. *Upstreamness* measures the average distance separating a sector from final demand. Intuitively, it captures the expected number of production stages that the output of a sector must traverse before being ultimately embodied in final consumption. Sectors with high upstreamness predominantly supply intermediate goods to other producers, so that their output undergoes multiple rounds of transformation before reaching households. Conversely, sectors with low upstreamness produce goods that are absorbed relatively directly by final consumers and therefore lie closer to final demand. The dual concept is *downstream-*

[2] A sector whose output is directly absorbed by households is close to final demand, whereas a sector whose production is mainly used as an intermediate input by other producers lies further away, as its output must pass through additional stages of production before reaching final consumers.

TABLE 1: MEASURING THE NETWORK: CONCEPTUAL DEFINITIONS

Measure	**Economic Interpretation**	**Captured Dimension**
upstreamness	*distance from final demand*	position along supply chains
downstreamness	*distance from primary inputs*	position relative to production origins
backward linkages	*dependence on suppliers*	input-side participation
forward linkages	*importance as supplier*	output-side participation

Different definitions for measuring the Input-Output space. Source: Antràs et al. *(2012) and Mancini* et al. *(2024).*

ness, which instead characterizes the position of a sector relative to the origin of the production process. It measures the extent to which production remains embedded throughout successive downstream stages before reaching final demand, identifying sectors whose output continues to circulate along long supply chains. Together, upstreamness and downstreamness provide complementary descriptions of a sector's location within the sequence of production.

A different perspective is provided by backward and forward linkages, which focus on production interdependencies rather than on the position of sectors along supply chains. *Backward linkages* measure the extent to which a sector relies on intermediate inputs purchased from upstream suppliers and therefore summarize its exposure to shocks originating elsewhere in the network. *Forward linkages* instead quantify the importance of a sector as a supplier of intermediate inputs to downstream nodes, capturing its potential to transmit disturbances throughout the production network. Forward and backward linkages therefore describe how strongly sectors are connected to the rest of the economy through their input and output relationships.

Figure 1 provides a graphical illustration of the complementary measures discussed above.

EXAMPLE 1 (Measuring the network) *Left panel focuses on network participation. The highlighted sector-*s *is connected to other sectors through both its immediate suppliers (red) and its customers (blue). Backward linkages capture the intensity of the sector's dependence on upstream nodes, namely the amount of intermediate inputs required from other sectors to sustain production. Forward linkages, in contrast, capture the importance of the sector as an intermediate supplier to downstream nodes, reflecting the extent to which its output is used in the production processes of other sectors. These measures therefore characterize the degree of participation of a sector within the production network. Direct sales to final demand are not part of either measure and are therefore represented by dashed links, as forward and backward linkages are captured exclusively by inter-sectoral production relationships from the Input-Output matrix,* $\boldsymbol{\mathcal{A}}$*, and therefore abstract from sectoral sales to final consumers. Right panel instead illustrates proximity to final demand-*C*. It emphasizes the location of the same sector within the sequence of production stages (green) leading to final demand, as identified by* $\boldsymbol{\Theta}$*. The highlighted paths represent alternative supply chains through which the sector's output ultimately reaches households, with the numbered markers identifying the remaining production stages separating the sector from final demand along each chain. Importantly, a sector generally belongs to many supply chains simultaneously. It may be relatively close to final demand along some production paths while remaining considerably further away along others. Proximity to final demand should therefore be interpreted as a network-wide characteristic rather than*

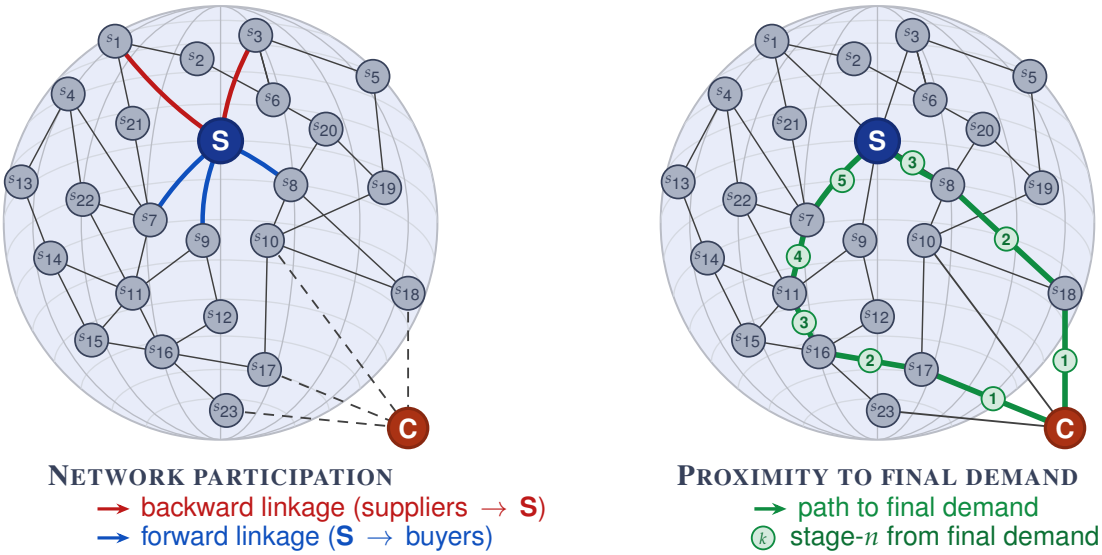


FIGURE 1: MEASURING THE NETWORK: GRAPHICAL VISUALIZATION

Note: stylized measures of the Input-Output network. Left panel highlights the reference sector's backward (red) and forward (blue) immediate linkages, corresponding to the immediate inter-sectoral transactions captured by the Input-Output matrix $\boldsymbol{\mathcal{A}}$; recursively extending also the direct connections of the reached sectors yields the complete network interactions summarized by the Leontief inverse $\boldsymbol{\mathcal{L}}$. Right panel, instead, highlights two representative paths (green) from the reference sector to final demand, as captured by the destination matrix $\boldsymbol{\Theta}$; stage labels denote the remaining production stages. For graphical clarity, only a subset of the links to final demand is shown, although every sector supplies final demand through its own final-demand component.

the distance associated with any individual supply chain.

Consistent with the discussion in Subsections 2.1-2.2, proximity to final demand reflects the destination of sectoral production. Sectors differ not only in the number of production relationships they maintain with the rest of the economy, but also in how many stages separate their output from its ultimate absorption by final consumers. These measures are particularly useful in the present context because they provide alternative ways of characterizing the notion of proximity to final demand emerging from the simple framework above. If sectoral output is allocated between intermediate transactions and final absorption, then the economic relevance of a sector depends not only on the network through which production propagates, but also on where the sector is located along that sequence of transformations.

COROLLARY 2 (Upstreamness and the stage-distance distribution) *Let the average number of production stages separating sector-i from final demand be given by* $\mathcal{U}_i = \sum_{n\geq 0}(n+1)\,\varpi_i(n)$. *Then* $\boldsymbol{\mathcal{U}} = \boldsymbol{\Xi}\mathbf{1}$, *which coincides with the upstreamness measure of Antràs et al. (2012).*

Proof: the recursive definition of upstreamness states that a sector's distance from final demand equals one stage plus the sales-weighted average distance of its customers, $\mathcal{U}_i = 1 + \sum_{j\in\Phi}\theta_{ij}\mathcal{U}_j$, that is $\boldsymbol{\mathcal{U}} = \mathbf{1} + \boldsymbol{\Theta}\boldsymbol{\mathcal{U}}$, whose solution is $\boldsymbol{\mathcal{U}} = \boldsymbol{\Xi}\mathbf{1}$. To verify that this equals the first moment of $\varpi_i(\cdot)$, use $\sum_{n\geq 0} n\boldsymbol{\Theta}^n = \boldsymbol{\Theta}(\boldsymbol{I}-\boldsymbol{\Theta})^{-2} = \boldsymbol{\Theta}\boldsymbol{\Xi}^2$, so that $\sum_{n\geq 0} n\,\varpi_i(n) = \left[\boldsymbol{\Theta}\boldsymbol{\Xi}^2\boldsymbol{\phi}^c\right]_i$ which, given Lemma 1, becomes $\left[\boldsymbol{\Theta}\boldsymbol{\Xi}^2(\boldsymbol{I}-\boldsymbol{\Theta})\mathbf{1}\right]_i = \left[\boldsymbol{\Theta}\boldsymbol{\Xi}\mathbf{1}\right]_i = \left[(\boldsymbol{\Xi}-\boldsymbol{I})\mathbf{1}\right]_i$, where the last equality uses $\boldsymbol{\Theta}\boldsymbol{\Xi} = \boldsymbol{\Xi}-\boldsymbol{I}$, itself obtained by premultiplying $\boldsymbol{\Xi} = \boldsymbol{I}+\boldsymbol{\Theta}\boldsymbol{\Xi}$. Adding $\sum_{n\geq 0}\varpi_i(n) = 1$ from Theorem 1 gives $\mathcal{U}_i = \left[\boldsymbol{\Xi}\mathbf{1}\right]_i$. ■

This result gives a direct interpretation of upstreamness in terms of the distribution $\varpi_i(n)$. While the latter describes the entire distribution of production stages separating sector-i from final demand, upstreamness summarises this distribution through its first moment. Specifi-

cally, upstreamness corresponds to the average production-stage distance from sector-i to final demand, with $\varpi_i(n)$ assigning the weight of production associated with paths involving n intermediate stages. The equivalence with the measure of Antràs *et al.* (2012) therefore shows that the recursive upstreamness measure admits a distributional interpretation: it is not only a recursive network statistic, but also the mean of the sector's stage-distance distribution.

The same distributional logic admits a dual representation: tracing the same production structure from primary inputs rather than from final demand yields a complementary measure of sectoral position.

COROLLARY 3 (The dual distribution from primary inputs) *Suppose the value of sectoral output exhausts payments to intermediate inputs and primary factors, and let* $\omega_{ij} = \frac{p_j^x x_{ji}}{p_i y_i}$ *denote cost shares and* $\phi_i^v = 1 - \sum_{j\in\Phi}\omega_{ij}$ *the share of primary factors in the value of sector-i's output. Then* $\boldsymbol{\Omega} = \widehat{\boldsymbol{p}}^{-1}\boldsymbol{\mathcal{A}}'\widehat{\boldsymbol{p}}$*, with* $\widehat{\boldsymbol{p}} = \text{Diag}[\boldsymbol{p}]$*, so that* $\text{Radius}[\boldsymbol{\Omega}] = \text{Radius}[\boldsymbol{\mathcal{A}}] < 1$.[3] *Consequently, by the same iteration outlined for Theorem 1 now applied to the cost identity,* $\sum_{n\geq 0}\varpi_i^v(n) = 1$ *with* $\varpi_i^v(n) \equiv \left[\boldsymbol{\Omega}^n\boldsymbol{\phi}^v\right]_i$*, and* $\mathcal{D}_i \equiv \sum_{n\geq 0}(n+1)\varpi_i^v(n)$ *coincides with downstreamness, the average distance of sector-i from primary inputs.*

Proof: by definition of the technical coefficients, $\alpha_{ji} = \frac{x_{ji}}{y_i}$, so that $\omega_{ij} = \frac{p_j^x x_{ji}}{p_i y_i} = \frac{p_j}{p_i}\cdot\frac{x_{ji}}{y_i} = \frac{p_j}{p_i}\alpha_{ji} = \left[\widehat{\boldsymbol{p}}^{-1}\boldsymbol{\mathcal{A}}'\widehat{\boldsymbol{p}}\right]_{ij}$. Since $\widehat{\boldsymbol{p}}$ is diagonal and invertible, $\boldsymbol{\Omega}$ and $\boldsymbol{\mathcal{A}}'$ are similar matrices and therefore share the same spectrum; transposition leaves the spectrum unchanged, so that $\text{Radius}[\boldsymbol{\Omega}] = \text{Radius}[\boldsymbol{\mathcal{A}}]$. For what concerns iteration, the cost identity $1 = \phi_i^v + \sum_{j\in\Phi}\omega_{ij}$ is formally identical to eq. (2) with $\boldsymbol{\Omega}$ in place of $\boldsymbol{\Theta}$ and $\boldsymbol{\phi}^v$ in place of $\boldsymbol{\phi}^c$: primary factors playing on the cost side the role that final demand plays on the sales side. Note that the row index of $\boldsymbol{\Omega}$ denotes the purchasing sector, decomposing its costs by origin, whereas the row index of $\boldsymbol{\Theta}$ denotes the selling sector, decomposing its output by destination; both matrices are therefore row-normalised and row-sub-stochastic, and in both the row-deficiency measures the leakage out of the network. The proof of Theorem 1 then applies verbatim. ■

Each sector is therefore characterised by two distributions, generated by the two sides of the same accounting identity: $\varpi_i(\cdot)$ describes how far it stands from final consumption, $\varpi_i^v(\cdot)$ how far it stands from primary inputs. Both are generated by the same technology matrix under a different rescaling, in quantities on the sales side and in prices on the cost side: it is the choice of scale, and not of technology, that determines which of the two geometries is observed. The pair $\{\mathcal{U}_i, \mathcal{D}_i\}$ locates the sector between the two ends of the production process, and sectors for which $\mathcal{U}_i \simeq \mathcal{D}_i$ occupy the centre of the chains.

By Corollary 3, the cost side of the same sector is governed by a dual distribution. A sector's supply schedule therefore inherits two geometries: its marginal cost responds to shocks according to its distance from primary inputs, and its marginal revenue according to its distance from final demand. This observation changes the economic role of production networks. Standard Input-Output models emphasize the cost side of production: network linkages determine how marginal costs compound across sectors (*e.g.*, Rubbo 2023). Once sectors are allowed to simultaneously sell to downstream producers and final consumers, price adjustments also reallocate demand across these two destinations. This distinction is central in environments with

[3] The similarity requires that the price at which a sector sells its output as an intermediate input coincides with the price valuing its output, $p_j^x = p_j$. Under destination-specific pricing, $\boldsymbol{\Omega} = \widehat{\boldsymbol{p}}^{-1}\boldsymbol{\mathcal{A}}'\widehat{\boldsymbol{p}^x}$ is a diagonal equivalence rather than a similarity, the spectrum need no longer be preserved, and $\text{Radius}[\boldsymbol{\Omega}] < 1$ has to be assumed separately. Destination-specific markups are therefore precisely what breaks the symmetry between the two sides of the identity, which is the mechanism studied in Section 3.

endogenous price setting, as sectors choose prices taking into account both production costs and the demand they face. Section 3 formalizes this intuition as a structural network object within a multi-sector New Keynesian framework, where network propagation shapes marginal costs through intermediate input purchases, while the destination of output shapes marginal revenues through the composition of sectoral sales.

3. Inside and Out in a Networked New Keynesian Framework

This section embeds the destination of sectoral output into a multi-sector New Keynesian economy. Every sector operates on both sides of the production network. As a *network-buyer*, it sources labour and intermediate inputs from its suppliers, and its marginal cost is priced by the cost-side geometry of Corollary 3. As a *network-seller*, it sells the same good to final consumers and to downstream producers, and its customer base is described by the sales-side geometry of Theorem 1. Price setting is where the two sides meet: a sector resetting its price weighs the marginal cost inherited from its suppliers against the marginal revenue earned from its customers. Subsection 3.1 describes technologies and preferences, which pin down the network-buyer block. Subsection 3.2 characterizes the network-seller. A Kimball (1995) aggregator makes the composition of sales an argument of the elasticity of demand, eq. (8), and the recursive Input-Output structure turns this composition into proximity to final demand (Definition 3 and Proposition 1). Subsection 3.3 introduces Calvo pricing, defines the equilibrium, and derives sectoral Phillips curves whose slope is an inverted-U function of proximity to final demand (Proposition 2); Subsection 3.4 illustrate these results in three simple networks. Subsection 3.5 aggregates the sectoral curves.

3.1. Functional relationships

Sectoral output y_i is produced according to a Cobb-Douglas technology

$$y_i = z_i \left(l_i\right)^{\alpha_i} \left(x_i^{dem}\right)^{1-\alpha_i}$$

where z_i is the Hicks-neutral productivity of sector-i, l_i its labour input with output elasticity α_i, and x_i^{dem} the bundle of intermediate inputs purchased within the production network at prices $\boldsymbol{p}^x = \left[p_1^x, \ldots, p_i^x, p_j^x, \ldots, p_m^x\right]$, which map into the price index P_i^x faced by the sector. The bundle is assembled through a Constant Elasticity of Substitution (CES) technology, $x_i^{dem} = \left(\sum_{j\in\Phi} \alpha_{ji}^{1/\epsilon}\ x_{ji}^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)}$, with elasticity of substitution $\epsilon > 1$, where x_{ji} is the intermediate input that sector-i buys from sector-j at intensity α_{ji}, consistently with the canonical indexing of Section 2. Whenever $x_{ji} = 0 = \alpha_{ji}$, sector-i does not use the commodity produced by sector-j. The collection of the α's delivers the $m \times m$ Input-Output matrix $\boldsymbol{\mathcal{A}} = \left[\alpha_{ij} \geq 0\right]$, and Constant Returns to Scale (CRS) imply $1 - \alpha_i = \sum_{j\in\Phi} \alpha_{ji}$, a column sum of $\boldsymbol{\mathcal{A}}$: the input requirements of sector-i are read along the column of the buyer, whereas its sales are read along the row of the seller.[4]

[4] This is the model counterpart of the two normalisations of Subsection 2.1. The column of $\boldsymbol{\mathcal{A}}$ collects what sector-i buys and generates its marginal cost; the row of $\boldsymbol{\mathcal{A}}$, once normalised by the seller's output, generates the destination matrix $\boldsymbol{\Theta}$ of Definition 1 and therefore its marginal revenue.

Taking wages and intermediate input prices as given, sector-i minimizes total costs, $\mathcal{C}_i = w_i\, l_i + P_i^x\, x_i^{dem}$, per unit of output,

$$mc_i = \min_{l_i,\, x_i^{dem}} \quad \mathcal{C}_i \qquad \text{s.t.} \qquad 1 = z_i\, (l_i)^{\alpha_i} \left(x_i^{dem}\right)^{1-\alpha_i}$$

which delivers the demand system for intermediate inputs together with the nominal marginal cost of the sector. To a first order around the steady state,

$$\widehat{mc}_i = -\hat{z}_i + \alpha_i\, \hat{w}_i + \sum_{j\in\Phi} \omega_{ij}\, \hat{p}_j^x \tag{4}$$

where hats denote log-deviations and the weights are steady-state cost shares. Eq. (4) tells us that the marginal cost of sector-i responds to its wage in proportion to its labour share, and to the price of each supplier in proportion to the corresponding cost share. In value terms, these shares coincide with the matrix $\mathbf{\Omega}$ of Corollary 3, $\omega_{ij} = \frac{p_j^x x_{ji}}{p_i y_i}$, while the labour share coincides with the primary-factor leakage $\phi_i^v = \alpha_i$. The network-buyer block therefore inherits, without further assumptions, the cost-side distribution $\omega_i^v(\cdot)$ and its mean $\mathcal{D}_i$.

A competitive final consumer combines sectoral consumption levels $\{c_i\}_{i\in\Phi}$ into aggregate consumption through the CES aggregator $C = \left(\sum_{i\in\Phi} \omega_i^{1/\epsilon}\, c_i^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)}$, where ω_i sizes the weight of sector-i in final consumption, with $\sum_{i\in\Phi}\omega_i = 1$.[5] Expenditure minimization delivers the conditional final demand, $c_i = \omega_i \left(p_i^c / P^c\right)^{-\epsilon} C$, and the consumer price index, $P^c = \left(\sum_{i\in\Phi} \omega_i\, p_i^{c,1-\epsilon}\right)^{\frac{1}{1-\epsilon}}$.

The household gets utility from consumption and dis-utility from labour in all sectors,

$$\frac{C^{1-\gamma}}{1-\gamma} - \sum_{i\in\Phi} \frac{l_i^{1+\psi}}{1+\psi}$$

where $\gamma \geq 0$ is the coefficient of relative risk aversion and $\psi \in [0,1]$ the inverse Frisch elasticity of labour supply, with the wage w serving as the numeraire. Time is discrete and infinite, $t+k$ for $k \in [0,\infty)$. Given a discount factor β^k, the representative household chooses sequences of consumption, labour supply, and one-period nominal bond holdings to maximize lifetime utility subject to

$$P_{t+k}^c\, C_{t+k} + B_{t+k+1} = \sum_{i\in\Phi} w_{i,t+k}\, l_{i,t+k} + (1 + r_{t+k})\, B_{t+k} + D_{t+k}$$

where B_{t+k} is the risk-free bond paying off in period $t+k$ at the nominal interest rate r_{t+k}, and D_{t+k} the rebated nominal profits of sectors. The optimal consumption-leisure trade-off in each sector and the inter-temporal allocation of consumption satisfy

$$\frac{w_{i,t}}{P_t^c} = C_t^{\gamma}\, l_{i,t}^{\psi} \qquad \text{and} \qquad 1 = \beta\,(1 + r_{t+1})\, \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_t^c}{P_{t+1}^c}\right]$$

Technologies and preferences determine how each sector sources its inputs. The same unit of output must also be absorbed: it is sold either to the representative household or to downstream

[5] At a symmetric steady state these coincide with the final expenditure shares b_i of Corollary 5.

producers, which in turn sell their own output along the chain. By Theorem 1, this second dimension is encoded in the market clearing condition, eq. (1), and is integrated out once output is expressed through the Leontief inverse. The network-seller block characterizes it explicitly and makes it an argument of the pricing decision.

3.2. Static and recursive destination of sales

This subsection characterizes the network-seller in two steps. The first step is static: a Kimball (1995) frontier between the two destinations of output makes the composition of sales an equilibrium object, and the elasticity of demand faced by the seller an increasing function of how dispersed that composition is, eq. (8). The second step is recursive: following intermediate sales along the supply chains replaces the static composition with proximity to final demand, a summary of the entire stage-distance distribution $\varpi_i(\cdot)$, as in Definition 3 and Proposition 1.

At the equilibrium described by eq. (1), every unit of sectoral output has two destinations: final consumption, c_i, and intermediate sales to downstream producers, $x_i^{sup} = \sum_{j\in\Phi} x_{ij}$. Per unit of output, they define the shares $\phi_i^c = \frac{c_i}{y_i}$ and $\phi_i^x = \frac{x_i^{sup}}{y_i} = \sum_{j\in\Phi}\theta_{ij}$, with $\phi_i^c + \phi_i^x = 1$ by Lemma 1. Together with the i-th row of the destination matrix $\boldsymbol{\Theta}$ of Definition 1, these shares describe the customer base of sector-i.

The two destinations are linked through a Kimball (1995) aggregator. Per unit of output, the feasible allocations of sales satisfy

$$\left[1-\varsigma_i\right]\mathcal{K}\left[\phi_i^c\right] + \varsigma_i\,\mathcal{K}\left[\phi_i^x\right] = 1$$

where $\varsigma_i \in (0,1)$ is the within-sector weight on intermediate sales, and $\mathcal{K}:\mathbb{R}\to\mathbb{R}$ is smooth, increasing and convex, $\mathcal{K}'[\cdot]>0$ and $\mathcal{K}''[\cdot]>0$. The constraint therefore describes a concave frontier along which output can be transformed from one destination into the other at an increasing marginal rate. We adopt the parametric form

$$\mathcal{K}\left[\phi_i^u\right] = \frac{\exp\left(\eta_i^{prim}\left[\phi_i^u - 1\right]\right) - 1}{\eta_i^{prim}} \qquad \text{for} \quad u = \{c, x\}$$

with $\mathcal{K}[1]=0$ and $\mathcal{K}'\left[\phi_i^u\right] = \exp\left(\eta_i^{prim}\left[\phi_i^u - 1\right]\right)$. The primitive curvature $\eta_i^{prim} > 0$ governs the shape of the frontier, and therefore how costly it is to reallocate output across destinations. Given the destination-specific prices p_i^c and p_i^x, the network-seller allocates its output so as to maximize the nominal value of sales, $\mathcal{R}_i = p_i^c\,c_i + p_i^x\,x_i^{sup}$, that is

$$\max_{c_i,\,x_i^{sup}} \quad \mathcal{R}_i \quad \text{s.t.} \quad 1 = \left[1-\varsigma_i\right]\mathcal{K}\left[\phi_i^c\right] + \varsigma_i\mathcal{K}\left[\phi_i^x\right]$$

The first-order conditions equate the relative price of the two destinations to the marginal rate of transformation along the frontier, $\frac{p_i^c}{p_i^x} = \frac{1-\varsigma_i}{\varsigma_i}\exp\left(\eta_i^{prim}\left[\phi_i^c - \phi_i^x\right]\right)$. Using $\phi_i^c + \phi_i^x = 1$ and solving for the final-demand share,

$$\phi_i^c = \frac{c_i}{y_i} = \frac{1}{2} + \frac{1}{2\,\eta_i^{prim}}\log\left[\frac{\varsigma_i}{1-\varsigma_i}\frac{p_i^c}{p_i^x}\right] \tag{5}$$

Eq. (5) tells us that the composition of sales is an equilibrium object, which responds to the relative price of the two destinations with a constant semi-elasticity,

$$\sigma_i = \frac{\partial \phi_i^c}{\partial \log\left[p_i^c / p_i^x\right]} = \frac{1}{2\,\eta_i^{prim}} \tag{6}$$

which we label the *reallocation sensitivity* of sector-i. A flatter frontier makes the composition more responsive. As $\eta_i^{prim} \to 0$, $\mathcal{K}[\cdot]$ becomes linear, the two destinations are perfect substitutes, and $\sigma_i \to \infty$; as $\eta_i^{prim} \to \infty$, the frontier becomes kinked and the composition is fixed, $\sigma_i \to 0$. At a symmetric steady state, $p_i^c = p_i^x \equiv p_i$, the composition is constant, which is also the condition under which the similarity of Corollary 3 holds exactly.

The elasticity of a network-seller.– A change in the price of sector-i affects the quantity it sells along two margins. By the chain rule on eq. (5),

$$\frac{\partial y_i}{\partial p_i^c} = -\epsilon\, \mathcal{S}_i^c \left(\frac{p_i^c}{P^c}\right)^{-\epsilon} \frac{1}{p_i^c} + \sum_u \left[\frac{\partial \mathcal{S}_i^u}{\partial p_i^c} \left(\frac{p_i^u}{P^u}\right)^{-\epsilon}\right]$$

where P^u denotes either the consumer price index, P^c, or the intermediate input price index, P_i^x, and the demand shifters are $\mathcal{S}_i^c = \omega_i C$ and $\mathcal{S}_i^x = \sum_{j\in\Phi} \alpha_{ij}\, x_j^{dem}$, the latter running along the i-th row of $\boldsymbol{\mathcal{A}}$, that is over the sectors that buy from i. The first term is the *outer* margin: substitution between the good of sector-i and the goods of other sectors. The second is the *inner* margin: the reallocation of sales across the customer base of sector-i, operating through the demand shifters. The elasticity of demand faced by the network-seller is then

$$\epsilon_i^{kim} = \epsilon\left(1 + \eta_i^{kim}\right) \tag{7}$$

where $\eta_i^{kim} = \overline{\eta}_i\, \eta_i^{eff}$, with $\overline{\eta}_i = \frac{1}{2\,\eta_i^{prim}\,\epsilon}$ and $\eta_i^{eff} = \phi_i^c\, \phi_i^x$. Since $\epsilon\,\overline{\eta}_i = \sigma_i$ by eq. (6), the elasticity separates additively into its two margins,

$$\epsilon_i^{kim} = \epsilon + \sigma_i\, \phi_i^c\, \phi_i^x \tag{8}$$

The second is the inner premium, that is the product of two objects with distinct economic content. The reallocation sensitivity σ_i is a property of the frontier: it measures how strongly the composition of sales responds to relative prices. The product $\phi_i^c\, \phi_i^x$ is a property of the customer base: it measures how much composition there is to reallocate. To see this, consider the destination of a randomly selected unit of sectoral output. It is sold to final consumers with probability ϕ_i^c and to downstream producers with probability $\phi_i^x = 1 - \phi_i^c$, so that the destination of output is a Bernoulli random variable with variance $\phi_i^c\, \phi_i^x$.[6] By Corollary 4, this variance is built entirely from the mass that $\varpi_i(\cdot)$ places at the origin and from its complement.

REMARK 1 *Under the Kimball aggregator, the destination of output is an argument of the demand schedule faced by sector-i. The inner premium in eq. (8) vanishes, and* $\epsilon_i^{kim} = \epsilon$*, whenever the composition of sales is fixed,* $\sigma_i \to 0$*, or the customer base is concentrated on a*

[6] The variance satisfies $0 \le \phi_i^c\, \phi_i^x \le \frac{1}{4}$, attaining its maximum when the two destinations are equally represented, with slope $\frac{\partial \phi_i^c\, \phi_i^x}{\partial \phi_i^x} = 1 - 2\phi_i^x > 0$ for $\phi_i^x < \frac{1}{2}$.

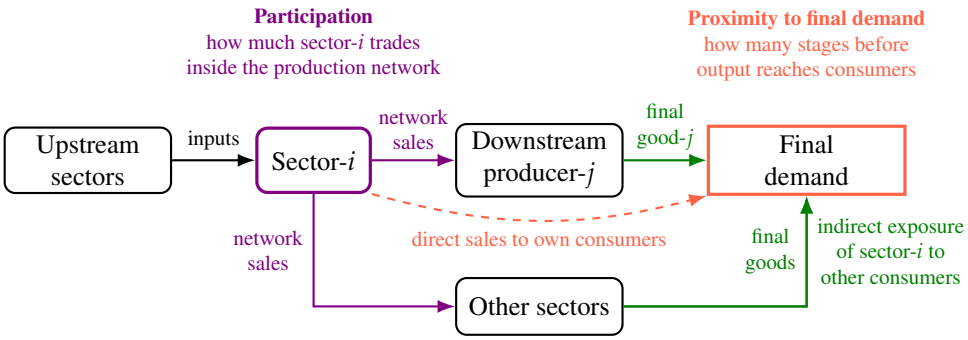


FIGURE 2: RECURSIVE PRODUCTION CHAINS AND FINAL DEMAND

Note: stylized representation of the destination of sectoral sales and its recursion throughout the production network. Purple arrows denote network participation through intermediate transactions, green arrows capture proximity to final demand through flows toward final demand, and the dashed line represents direct sales to own final consumers.

single destination. If $\phi_i^c \to 1$*, output is sold entirely to final consumers, as in a multi-sector economy without network interactions; if* $\phi_i^x \to 1$*, all output is absorbed by downstream producers, which is the upper bound of Lemma 1. Between these two poles, the premium increases with the balance of the customer base and is maximal at* $\phi_i^c = \phi_i^x = \frac{1}{2}$.

Remark 1 tells us that the elasticity of demand faced by a network-seller is an endogenous property of the production network: two sectors facing the same households and the same frontier face different elasticities whenever their customer bases differ.

Recursive destinations.– The dispersion $\phi_i^c \, \phi_i^x$ is static. In the language of Definition 2, it equals $\varpi_i(0)\,[1 - \varpi_i(0)]$: it only distinguishes output absorbed by final demand at the origin from output that is not, and treats every intermediate sale as one stage away from final consumers. Intermediate sales, however, are inputs of downstream producers, whose own output is again split between final consumers and further producers, and so on along every chain until it is absorbed by final demand. Sector-i is therefore exposed not only to its own final consumers, but to all the final demand that its output contributes to satisfy, which is the full-exposure statement of Theorem 1.

Following intermediate sales through the successive stages moves the network from a Leontief interpretation – how much output of sector-i is required, directly and indirectly, by the other sectors – to a consumption-adjusted version of the vertically-integrated sectors of Pasinetti (1973) – how much of it is ultimately embodied in final demand, and after how many stages. This is the *vertically-integrated node* concept, which pins down the destination of sectoral sales by interacting final demand with the recursive structure of the network.

EXAMPLE 2 (Recursivity until final demand) *Figure 2 illustrates how recursion reshapes the destination of sectoral sales. Output of sector-i is sold either to downstream producers or directly to final demand. Once Input-Output linkages are taken into account, these destinations are no longer mutually exclusive: output sold as an intermediate input propagates through downstream production stages before being absorbed by final demand, so that intermediate sales are themselves a channel through which sectoral production reaches consumers. Sector-i is thus exposed both to its own final consumers and to the final demand that its output helps satisfy through production linkages.*

Definition 3 summarizes the recursion in a single scalar per sector.

DEFINITION 3 (Final demand proximity) *For a per-stage discount factor $\delta \in (0,1)$, the proximity to final demand of sector-i is*

$$v_i(\delta) = \sum_{n\geq 0} \delta^n \varpi_i(n) = \left[(\boldsymbol{I} - \delta\boldsymbol{\Theta})^{-1}\boldsymbol{\phi}^c\right]_i \tag{9}$$

where $\varpi_i(n) = \left[\boldsymbol{\Theta}^n\boldsymbol{\phi}^c\right]_i$ is the share of output of sector-i absorbed by final demand after exactly n intermediate transactions.

Proximity weights each unit of output of sector-i by the number of production stages it crosses before reaching final demand, discounting each stage at rate δ: the part of output embodied in final goods after few stages contributes nearly one-for-one, while the part transformed again and again into new intermediate inputs contributes progressively less. In other words, the discount factor δ is the per-stage attenuation of the inner margin along the chain.[7]

We refer to $v_i(\cdot)$ as the degree of vertical integration of sector-i, and write it as v_i whenever no notational ambiguity arises. Replacing the static dispersion with its recursive counterpart delivers the network-consistent effective curvature

$$\eta_i^{eff} = v_i\left[1 - v_i\right] \tag{10}$$

which enters eqs. (7)-(8) in place of $\phi_i^c\,\phi_i^x$. Proposition 1 states the properties of proximity and connects it to the network objects of Section 2.

PROPOSITION 1 (Proximity as a generating function) *For every sector-$i \in \Phi$, $v_i(\delta)$ is the probability generating function of the stage-distance distribution $\varpi_i(\cdot)$ of Definition 2. Hence:*

(i) $v_i(0) = \phi_i^c$, *so that eq. (10) nests the static dispersion,* $\eta_i^{eff} = \phi_i^c\,\phi_i^x$;

(ii) $v_i(1) = 1$, *which is Theorem 1;*

(iii) $v_i'(1) = \sum_{n\geq 0} n\,\varpi_i(n)$, *the mean stage distance of sector-i from final demand, which coincides with the upstreamness $\mathcal{U}_i$ of Corollary 2 up to its normalisation;*

(iv) for $\delta \in (0,1)$ and $\phi_i^c < 1$, $v_i(\delta) \in \left[\phi_i^c, 1\right)$, and v_i is strictly decreasing in the distance from final demand: if $\varpi_i(\cdot)$ first-order stochastically dominates $\varpi_j(\cdot)$ and $\varpi_i \neq \varpi_j$, then $v_i(\delta) < v_j(\delta)$.

Proof: by Theorem 1, $\varpi_i(\cdot)$ is a probability distribution over $n = \{0,\ldots,N\}$ stages, so the series in eq. (9) converges for $\delta \in [0,1]$ and equals $\mathbb{E}\left[\delta^{N_i}\right]$, with $N_i \sim \varpi_i$. The first three claims simply follow by evaluating the series and its (left) derivative at $\delta \in \{0,1\}$. For the last claim, $v_i \geq \varpi_i(0) = \phi_i^c$, and $v_i < 1$ because $\delta^n < 1$ for $n \geq 1$ and ϖ_i places mass $\phi_i^x > 0$ on $n \geq 1$. Since δ^n is strictly decreasing in n for $\delta \in (0,1)$, its expectation is strictly lower under a distribution that first-order stochastically dominates, and differs from, another. ■

Proposition 1 places eq. (10) on the map of Section 2. The static dispersion is its evaluation at $\delta = 0$, where only the mass of $\varpi_i(\cdot)$ at the origin matters; full exposure is its evaluation at $\delta = 1$, where all sectors are identical; for $\delta \in (0,1)$, v_i orders sectors by their distance from

[7] Without discounting, every sector would be fully exposed to final demand and therefore identical in the cross-section; see Proposition 1.

TABLE 2: NETWORK AND PRICING OBJECTS

	Network-buyer (column of $\boldsymbol{\mathcal{A}}$)	***Network-seller*** (row of $\boldsymbol{\Theta}$)
Direct exposure	cost shares $\boldsymbol{\Omega}$, labour shares $\phi_i^v = \alpha_i$	destination shares $\boldsymbol{\Theta}$, final shares $\boldsymbol{\phi}^c$
Recursive operator	$\boldsymbol{\mathcal{L}} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$	$\boldsymbol{\Xi} = (\boldsymbol{I} - \boldsymbol{\Theta})^{-1}$
Stage distribution	none	$\varpi_i(\cdot)$, Definition 2
Summary statistic	none	proximity $v_i(\delta)$, upstreamness $\mathcal{U}_i$
Enters the NKPC through	marginal cost $mc_{i,t}$, eq. (4)	slope κ_i, eq. (14)
Pricing objects		
Calvo term	$\varrho_i = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i}$	
Reallocation sensitivity	$\sigma_i = \frac{1}{2\eta_i^{prim}} = \epsilon\,\overline{\eta}_i$, eq. (6)	
Seller elasticity	$\epsilon_i^{kim} = \epsilon + \sigma_i\, v_i\,(1 - v_i)$, eqs. (8)-(10)	
Aggregation weights	$\boldsymbol{\lambda} = \boldsymbol{\mathcal{L}}\boldsymbol{\omega}$, influence vector of Corollary 5	

Summary of the salient element of the theory.

final demand. As a result, $\eta_i^{eff} = v_i\,(1 - v_i)$ is maximal for sectors whose discounted exposure to final demand equals one half, $v_i = \frac{1}{2}$, and vanishes at both ends of the supply chain. The elasticity of a network-seller,

$$\epsilon_i^{kim} = \epsilon + \sigma_i\, v_i\,(1 - v_i)$$

therefore depends on where the sector sits along its supply chains rather than only on how it splits its sales: two sectors with identical direct exposure ϕ_i^c face different elasticities whenever their intermediate sales travel along chains of different length. Strategic complementarity between the two destinations is no longer a static feature of the sales split; it is a property of how sectoral output is prolonged along the successive steps of each supply chain.

3.3. NEW KEYNESIAN PHILLIPS CURVES

This subsection introduces nominal rigidities and characterizes sectoral Phillips curves. Table 2 collects the network objects that enter the pricing problem, organized by the side of the network they describe. Eq. (14) gives the slope of sectoral Phillips curves, and Proposition 2 characterizes it as a function of proximity to final demand. Throughout, hats denote log-deviations from a symmetric steady state in which $p_i^c = p_i^x$ for every sector, and bold symbols denote vectors. The two blocks of Table 2 are read along different dimensions of the network and enter the Phillips curve through different objects: the network-buyer block through the marginal cost, the network-seller block through the slope.

Nominal rigidities take the form of Calvo pricing. Within each sector, a randomly selected fraction $1 - \chi_i$ of producers resets the intermediate input price p_i^x every period, while the remaining fraction χ_i keeps it unchanged. A resetting producer internalizes both sides of the production network: as a network-buyer, it bears the marginal cost of eq. (4); as a network-

seller, it earns marginal revenues that depend on the allocation of its sales between downstream producers and final consumers. The optimal reset price solves

$$\max_{\left\{p^{x,*}_{i,t+k}\right\}_{k=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \chi_i^k \Lambda_{t,t+k} \left[\left(mr_{i,t+k} - mc_{i,t+k} \right) \; y_{i,t+k} \right] \right\}$$

subject to $y_{i,t+k} = \left(\frac{p^{x,*}_{i,t}}{P^x_{t+k}} \right)^{-\epsilon_i^{kim}} Y_{t+k}$, where $\Lambda_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\gamma} \frac{P^c_t}{P^c_{t+k}}$ is the stochastic discount factor through which the household discounts future profit flows. Marginal revenues follow from the revenue function of the network-seller. Using the composition of sales, then $\mathcal{R}_i = \left(p_i^c \, \phi_i^c + p_i^x \, \phi_i^x \right) y_i$. Along the demand system, selling more output requires a change in prices and therefore, through eq. (5), a change in the composition of sales. Marginal revenues are accordingly given by

$$mr_i = \frac{\partial \mathcal{R}_i}{\partial y_i} = \underbrace{p_i^c \, \phi_i^c + p_i^x \, \phi_i^x}_{\text{average price}} + \underbrace{y_i \left[p_i^c - p_i^x \right] \frac{\partial \phi_i^c}{\partial y_i}}_{\text{composition}} \tag{11}$$

where we use $\frac{\partial \phi_i^x}{\partial y_i} = -\frac{\partial \phi_i^c}{\partial y_i}$. The first component is the price of a unit of output averaged across destinations. The second is the composition component: an additional unit of output shifts the split of sales between destinations, and this matters for revenues whenever the two destinations are priced differently. The first-order condition of the pricing problem is

$$\mathbb{E}_t \sum_{k=0}^{\infty} \chi_i^k \Lambda_{t,t+k} y_{i,t+k} \left(mr_{i,t+k} - mc_{i,t+k} \right) \epsilon_i^{kim} \frac{1}{p^{x,*}_{i,t}} = \mathbb{E}_t \sum_{k=0}^{\infty} \chi_i^k \Lambda_{t,t+k} y_{i,t+k} \frac{\partial \, mr_{i,t+k}}{\partial \, p^{x,*}_{i,t}} \tag{12}$$

The left-hand side is the Calvo trade-off between the margin of marginal revenues over marginal costs and the elasticity of demand. The right-hand side collects the response of marginal revenues to the reset price. At the symmetric steady state, the composition component of eq. (11) vanishes and marginal revenues reduce to the price; their response to the reset price does not vanish, because resetting p_i^x changes the relative price of the two destinations and, through eq. (5), the composition of sales. The destination of output therefore enters the pricing condition twice: through the elasticity ϵ_i^{kim} on the left-hand side, and through the composition of marginal revenues on the right-hand side.

Sectoral Phillips curves.– Log-linearising eq. (12) around the symmetric steady state, together with the Calvo price index $p^{x,1-\epsilon_i^{kim}}_{i,t} = \chi_i \, p^{x,1-\epsilon_i^{kim}}_{i,t-1} + \left[1 - \chi_i \right] p^{x,*,1-\epsilon_i^{kim}}_{i,t}$, yields the sectoral New Keynesian Phillips Curves (NKPCs) for the price at which each sector sells within the production network,

$$\pi_{i,t} = \beta \, \mathbb{E}_t \left[\pi_{i,t+1} \right] + \kappa_i \, mc_{i,t} \tag{13}$$

whose slope is[8]

[8] The slope is obtained log-linearising eq. (12) and the Calvo price index, where the composition component of eq. (11) is evaluated at the vertically-integrated shares of eq. (9), so that the sensitivity of the composition to the scale of sales, $y_i \frac{\partial \phi_i^c}{\partial y_i}$, equals $v_i \left(1 - v_i \right)$. The derivation is reported in Supplemental Appendix A.

$$\kappa_i = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i} \frac{v_i(1-v_i)}{\epsilon_i^{kim}} \tag{14}$$

The slope of a sectoral Phillips curve is the elasticity of sectoral inflation with respect to the real marginal cost, holding expected inflation constant. Denoting by $\varrho_i = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i}$ the Calvo frequency term and using eqs. (7) and (10), it has three components:

$$\kappa_i = \underbrace{\varrho_i}_{\text{nominal rigidity}} \cdot \underbrace{\frac{v_i(1-v_i)}{\epsilon}}_{\text{composition}} \cdot \underbrace{\frac{1}{1+\eta_i^{kim}}}_{\text{real rigidity}} \quad \text{with} \quad \eta_i^{kim} = \overline{\eta}_i\, v_i(1-v_i) \tag{15}$$

We discuss each in turn. The first is the frequency of price adjustment, and it does not depend on the network. The other two depend on the position of sector-i through the same object, the dispersion of its customer base along the chain, $\eta_i^{eff} = v_i(1-v_i)$, but they move the slope in opposite directions.

Real rigidity. The last component is a real rigidity generated by the inner margin. A resetting producer anticipates that a change in its price reallocates sales across its customer base, on top of shifting demand towards other sectors: by eq. (8), its demand is more price-sensitive than under the outer margin alone, by the inner premium $\sigma_i\,\eta_i^{eff}$. The reset price then passes through a smaller fraction of marginal-cost movements, and the Phillips curve flattens. This is a within-sector counterpart of the strategic complementarities of Kimball (1995) – as in Guerrieri *et al.* (2010) –, and it is strongest where the customer base is most dispersed.

Composition of marginal revenues. The middle component operates on marginal revenues, eq. (11). When the customer base is dispersed, a change in the reset price moves both the level and the composition of revenues. A producer whose price is misaligned with its marginal cost bears the revenue consequences on both destinations and, through the recursive chain, over all the downstream stages its output reaches. The incentive to keep the price aligned with marginal costs is correspondingly stronger, and inflation responds more to marginal costs. At either end of the chain the customer base is concentrated on a single destination, the composition component shuts down, and a price change operates through one margin only.

Proposition 2 characterizes how the two forces combine.

PROPOSITION 2 (Proximity and the slope of the Phillips curve) *Let* $\eta_i^{eff} = v_i(1-v_i) \in \left[0, \frac{1}{4}\right]$. *For every* $\overline{\eta}_i \geq 0$*:*

(i) κ_i *is strictly increasing in* η_i^{eff}*, with elasticity* $\frac{\partial \log[\kappa_i]}{\partial \log\left[\eta_i^{eff}\right]} = \frac{1}{1+\eta_i^{kim}} \in (0,1]$*;*

(ii) κ_i *is strictly increasing in* v_i *for* $v_i < \frac{1}{2}$ *and strictly decreasing for* $v_i > \frac{1}{2}$*; it attains its maximum,* $\kappa_i^{max} = \frac{\varrho_i}{\epsilon(4+\overline{\eta}_i)}$*, at* $v_i = \frac{1}{2}$*, and converges to zero as* $v_i \to 0$ *or* $v_i \to 1$*;*

(iii) for $\eta_i^{eff} > 0$*,* κ_i *is strictly decreasing in* $\overline{\eta}_i$*, and hence in the reallocation sensitivity* σ_i*.*

Proof: by eq. (15), $\log[\kappa_i] = \log[\varrho_i/\epsilon] + \log\left[\eta_i^{eff}\right] - \log\left[1+\overline{\eta}_i\,\eta_i^{eff}\right]$. Differentiating with respect to $\log\left[\eta_i^{eff}\right]$ gives $1 - \overline{\eta}_i \frac{\eta_i^{eff}}{1+\overline{\eta}_i\,\eta_i^{eff}} = \frac{1}{1+\eta_i^{kim}}$, which proves the first claim. The second claim follows from the first and $\frac{\partial \eta_i^{eff}}{\partial v_i} = 1-2v_i$, evaluating κ_i at $\eta_i^{eff} = \frac{1}{4}$ and as $\eta_i^{eff} \to 0$. Finally, the last claim follows from $\frac{\partial \log[\kappa_i]}{\partial \overline{\eta}_i} = -\frac{\eta_i^{eff}}{1+\overline{\eta}_i\,\eta_i^{eff}} < 0$ and $\overline{\eta}_i = \frac{\sigma_i}{\epsilon}$. ■

Proposition 2 tells us that the two forces, although both driven by the dispersion of the customer base and both peaking at the same point of the chain, do not offset one-for-one: a one per cent increase in η_i^{eff} steepens the Phillips curve by $\frac{1}{1+\eta_i^{kim}}$ per cent. The real rigidity absorbs a fraction $\frac{\eta_i^{kim}}{1+\eta_i^{kim}}$ of the composition force without ever reversing it. The inverted-U is therefore generated by the composition of marginal revenues and shaped by the Kimball real rigidity, which lowers its peak and makes the slope concave in the dispersion of the customer base; the more responsive the composition of sales to relative prices, the stronger this attenuation. Combined with Proposition 1, the slope is an inverted-U function of the distance from final demand along any family of sectors ordered by first-order stochastic dominance: it is maximal for sectors whose discounted exposure to final demand equals one half, and it vanishes for sectors at either end of their supply chains.

REMARK 2 (Position, not participation) *The slope in eq. (14) depends on the whole stage-distance distribution $\varpi_i(\cdot)$ through v_i, not only on the share of output sold directly to final consumers. Two sectors with the same direct exposure ϕ_i^c and the same frequency of price adjustment χ_i generically have different slopes when their intermediate sales travel along chains of different length, consistently with Corollary 1. Conditional on the frequency term ϱ_i and on the curvature of the frontier, cross-sectional heterogeneity in slopes is indexed by proximity to final demand alone.*

3.4. STYLIZED EXAMPLES

Three simple networks illustrate how proximity to final demand shapes the slope of sectoral Phillips curves. In all of them, prices are equally sticky across sectors, $\chi_i = \chi$, and the frontier has a common curvature, $\eta_i^{prim} = \eta^{prim}$, so that the cross-section of slopes is generated by the network alone.

Roundabout economy.– Every sector sells a common share $\phi \in (0,1)$ of its output to final consumers and spreads the remaining share across downstream buyers according to an arbitrary row-stochastic matrix $\boldsymbol{G}$, so that $\boldsymbol{\Theta} = (1-\phi)\boldsymbol{G}$ and $\boldsymbol{\phi}^c = \phi \mathbf{1}$. The stage-distance distribution is geometric and common to all sectors, $\varpi_i(n) = \phi(1-\phi)^n$, with mean distance $\frac{1-\phi}{\phi}$, and proximity to final demand is, $\forall\, i \in \Phi$,

$$v_i = \frac{\phi}{1-\delta(1-\phi)}$$

All sectors share the same proximity and therefore the same slope: in the roundabout economy the network shapes the level of the Phillips curve slope, not its cross-section. By Proposition 2, the level is maximal when $v_i = \frac{1}{2}$, that is when the share of final sales equals $\phi^* = \frac{1-\delta}{2-\delta}$. Since $\phi^* < \frac{1}{2}$ for every $\delta \in (0,1)$, once recursion is accounted for the slope is maximal in economies whose sectors sell most of their output to other producers.

Vertical chain.– Sectors are ordered along a single chain, $i = 1, \ldots, m$: each sector-$i < m$ sells its entire output to sector-$i+1$, and the most downstream sector-m sells its entire output to final consumers. No sector splits its sales between the two destinations, so that the static disper-

sion is zero for all of them, $\phi_i^c \, \phi_i^x = 0$. Proximity to final demand is nevertheless heterogeneous: $\varpi_i(\cdot)$ places unit mass at the distance $m - i$, and

$$v_i = \delta^{m-i}$$

By Proposition 2, the slope of sector-i is an inverted-U function of its distance from final demand. It is zero for the producer of the final good ($v_m = 1$), it declines towards zero for stages further upstream than the peak, and it peaks at the stage whose proximity δ^{m-i} is closest to one half, that is at a distance from final demand of approximately $n^* = \frac{\log[2]}{\log[1/\delta]}$. The entire cross-section of slopes is generated by the position of sectors along the chain.

Same participation, different position.– Sectors $\{a, b\}$ sell the same share ϕ of their output to final consumers and the remaining share to a single downstream buyer. sector-a sells to a producer of final goods, which sells its entire output to consumers; sector-b sells to a producer of intermediate goods, which sells its entire output to a producer of final goods. The two sectors have the same direct exposure to final demand and the same static dispersion, $\phi(1-\phi)$, but

$$v_a = \phi + \delta(1-\phi) \; > \; \phi + \delta^2(1-\phi) = v_b$$

By Proposition 2, sector-b has the steeper Phillips curve if and only if v_b is closer to one half than v_a. With $\phi = 0.4$ and $\delta = 0.6$, for instance, $v_a = 0.76$ and $v_b \approx 0.62$, so that the sector further away from final demand responds more strongly to its marginal cost. Identical participation in the network is compatible with different slopes, which are pinned down by the position of each sector's sales along the chain (Remark 2).

The production network therefore enters the sectoral Phillips curve on both sides of the pricing decision. Through the marginal cost, eq. (4), it loads on the prices of suppliers along the columns of $\boldsymbol{\mathcal{A}}$. Through the slope, eq. (15), it depends on where output is absorbed along the rows of $\boldsymbol{\Theta}$. The position of a sector relative to final demand is a direct determinant of how strongly its inflation responds to its marginal cost.

3.5. Slope of the aggregate NKPC

Aggregate inflation is an average of sectoral inflation rates. In a production network, a change in the price of sector-i reaches aggregate inflation both directly, through household consumption of its good, and indirectly, through the marginal costs of the downstream sectors that use it as an input, eq. (4). The aggregation weights therefore account for direct and indirect contributions to final demand. From the equilibrium condition $\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{c}$ of Theorem 1 and the household demand system $\boldsymbol{c} = \boldsymbol{\omega}\, C$, sectoral output satisfies $\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{\omega}\, C$, which defines the vector of network-adjusted weights $\boldsymbol{\lambda} = \boldsymbol{\mathcal{L}}\boldsymbol{\omega}$ whose i-th element measures the total contribution of sector-i to aggregate consumption once all recursive production stages are accounted for. This is the influence vector of Corollary 5, and therefore the destination marginal ξ_{ij} of the joint distribution: a sector deeply embedded in production chains receives a weight larger than its direct consumption share ω_i. At the symmetric steady state, $\lambda_i = \frac{y_i}{C}$, so that $\sum_{i\in\Phi} \lambda_i$ is the ratio of total sales to final consumption. Aggregate inflation is $\pi_t = \boldsymbol{\lambda}'\boldsymbol{\pi}_t$, with $\boldsymbol{\pi}_t = [\pi_{1,t}, \ldots, \pi_{i,t}, \ldots, \pi_{m,t}]'$, so that the propagation of sectoral price changes through intermediate production is fully inter-

nalized.

REMARK 3 *The aggregation adopted relies on the Leontief inverse representation of production networks, whereby aggregate inflation is obtained using network-adjusted weights. Alternative aggregation schemes depend on the object of interest and on further model primitives; see Rubbo (2023).*

Substituting the sectoral Phillips curves into the aggregation equation delivers the aggregate New Keynesian Phillips Curve,

$$\pi_t = \beta\, \mathbb{E}_t\left[\pi_{t+1}\right] + \boldsymbol{\lambda}'\left(\boldsymbol{\kappa} \odot \boldsymbol{mc}_t\right) = \beta\, \mathbb{E}_t\left[\pi_{t+1}\right] + \sum_{i\in\Phi} \lambda_i\, \kappa_i\, mc_{i,t} \tag{16}$$

Eq. (16) tells us that aggregate inflation is driven by sectoral marginal costs weighted by two conceptually distinct objects. The first is the network relevance of the sector, λ_i, which measures how strongly its price propagates through production chains before reaching final demand. The second is its pricing responsiveness, κ_i, which by Proposition 2 is governed by proximity to final demand. These are the two marginals of Definition 2: the first weights sectors by destination, the second by proximity.

Aggregate NKPC in terms of output gap.– Sectoral marginal costs map into measures of real slack through the optimality conditions of sectors and households. When sectoral output exceeds its efficient (flexible-price) level, inputs become relatively scarce, wages increase, and marginal costs rise, which yields $mc_{i,t} = \zeta_i\, g_{i,t}$, where $g_{i,t}$ denotes the sectoral output gap and $\zeta_i > 0$ summarizes the mapping between real activity and marginal costs. The sectoral NKPCs become $\pi_{i,t} = \beta\ \mathbb{E}_t\left[\pi_{i,t+1}\right] + \tilde{\kappa}_i\, g_{i,t}$, with $\tilde{\kappa}_i = \kappa_i\, \zeta_i$. Aggregating with the same network-consistent weights,

$$\pi_t = \beta\, \mathbb{E}_t\left[\pi_{t+1}\right] + \kappa^{agg}\, g_t^{agg} \tag{17}$$

with $\kappa^{agg} = \sum_{i\in\Phi} \lambda_i\, \tilde{\kappa}_i$ and $g_t^{agg} = \frac{\sum_{i\in\Phi} \lambda_i\, \tilde{\kappa}_i\, g_{i,t}}{\sum_{i\in\Phi} \lambda_i\, \tilde{\kappa}_i}$. The relevant notion of aggregate slack is itself endogenous to the production network: both its weights and the sectoral sensitivities depend on the production structure, so that inflation responds to a network-weighted notion of slack that reflects how shocks propagate along chains that ultimately reach final consumers.

The aggregate slope compounds the two marginals. Denoting cross-sectional means over Φ by a bar and the cross-sectional covariance by $\mathrm{Cov}_\Phi\left[\cdot,\cdot\right]$,

$$\kappa^{agg} = m \cdot \left(\overline{\lambda}\ \overline{\tilde{\kappa}}\ +\ \mathrm{Cov}_\Phi\left[\lambda_i, \tilde{\kappa}_i\right]\right) \tag{18}$$

The first term is the slope of an economy in which every sector had the average network weight and the average responsiveness. The second is positive when the sectors that matter most for final demand through the network are also those whose prices respond most to marginal costs, that is, for given ζ_i and by Proposition 2, when network influence is concentrated at intermediate distances from final demand. Since λ_i is governed by the destination marginal and $\tilde{\kappa}_i$ by proximity, the sign of the covariance is a property of the joint distribution of Definition 2, not of either marginal in isolation. In the roundabout economy of Subsection 3.4, with a common $\zeta_i = \zeta$, the covariance vanishes and every sector sells a share ϕ of its output to final consumers, so that $\sum_{i\in\Phi} \lambda_i = \frac{1}{\phi}$ and $\kappa^{agg} = \frac{\tilde{\kappa}}{\phi}$: a larger share of intermediate sales raises

the aggregate weight of sectoral slopes and, through v_i, moves their common level along the inverted-U.

Taking stocks

Every sector operates on both sides of the production network, and the two sides meet in its pricing decision. As a network-buyer, it inherits marginal costs from its suppliers along the columns of $\boldsymbol{\mathcal{A}}$, eq. (4). As a network-seller, it allocates its output along the rows of $\boldsymbol{\Theta}$, and the Kimball frontier turns the composition of this allocation into an argument of the elasticity of demand, eq. (8). Following intermediate sales through the recursive Input-Output system converts the composition of sales into proximity to final demand, a summary of the entire stage-distance distribution (Definition 3 and Proposition 1). Proximity then shapes the response of inflation to marginal costs through two forces that depend on the same object, the dispersion of the customer base along the chain. The composition of marginal revenues steepens the Phillips curve; the real rigidity generated by the inner margin flattens it. The former dominates (Proposition 2), so that responsiveness is maximal for sectors at intermediate distances from final demand and vanishes at both ends of the chain. The examples show that this cross-section is generated by position rather than participation: a vertical chain, in which no sector splits its sales, displays the full inverted-U, while a roundabout economy, in which all sectors share the same position, displays none. Aggregate inflation weights these heterogeneous slopes by network influence, and its slope depends on how influence and proximity covary across sectors, eq. (18). The destination of production is therefore a determinant of inflation dynamics in its own right, alongside the propagation of production costs.

4. Data, Variables Definition and Descriptives

4.1. Data

Our focus is on the European Monetary Union (EMU) and in particular those countries that joined the union from the introduction of the common currency in 1999: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. This sample selection has several advantages. First, countries that joined the union more recently exhibit very peculiar convergence trajectories in terms of inflation, economic activity, and sectoral integration in both the European single market and the global economy. These potentially are sources of bias in our identification strategy and in the estimation of the Phillips curve (PC) at sectoral level. This is especially critical when considering the role played by inflation expectations when some countries transition from a national monetary authority to a supranational one. Second, late entrants have unreliable or missing sectoral data. Unfortunately, Luxembourg suffers the same problem and therefore we exclude it from the analysis despite being a member of the EMU since the beginning. We refer to the group of first entrants as EMU11. Here we describe data and provide variable definitions.

Sectoral prices and output.– Eurostat provides data on sectoral production and producer prices at sectoral level. The data are available at the NACE 2-digit level and cover 42 sectors in

total across mining and quarrying (NACE B), manufacturing (NACE C), electricity, gas, steam and air conditioning supply (NACE D) and water collection, treatment and supply (NACE E). Given this, we define producer price inflation (π^{PPI}) as the year-on-year percentage change in the sectoral producer price index (PPI), and sectoral output growth (Δy) as the year-on-year percentage change of sectoral output. Both series are jointly available at quarterly frequency from 2005Q1 to 2024Q4.

Sectoral proximity to the final consumer.– We use data from Antràs and Chor (2013) to identify which sectors supply goods directly to the final consumer or indirectly, *i.e.*, through a series of passages within the production network. In fact, a sector can sell its output either to final consumers as a consumption good (*e.g.*, a textile factory can sell part of its fabrics directly to the consumer) or to the production network as an intermediate good. In the latter case, the original intermediate good is transformed over successive production stages –together with other intermediate inputs– into a more elaborate final consumption good (*e.g.* the fabric is eventually turned into clothing).

Antràs and Chor (2013) provides two distinct measures that capture to what extent the value added at the sectoral level comes from the final good demand or from other (prior) passages of the production network. In particular, *upstreamness* captures how much of the sectoral value added is passed to the network. The higher its value, the larger the share of sectoral output that enters the network as an intermediate input and is further processed before reaching final demand, and thus the farther the sector is from the final consumer. Conversely, a lower value of *upstreamness* indicates that the sector is closer to the final consumer and that a larger share of its output is absorbed directly by final consumption. The second measure, *downstreamness*, captures the extent to which a sector's value added has already been generated at earlier stages of the production network. The higher its value, the closer the sector is to the final consumer: most of the value has already been created upstream, and the sector performs the final stages of production and product refinement. Conversely, a lower value of *downstreamness* indicates that the sector is farther from final demand.

To sum up, both measures capture how far, on average, a sector's production is from the final consumer. While *upstreamness* is defined with respect to final demand, *downstreamness* is defined with respect to the sourcing of intermediate inputs. Final demand therefore affects *downstreamness* only indirectly, through the structure of the production network that links primary inputs to final goods.

Both measures are bounded between 1 and infinity, but one is not complementary to the other, *i.e.*, $downstreamness \neq 1 - upstreamness$, as a sector can be contemporaneously upstream for the supply of some intermediary goods and downstream for the supply of others to the final consumer. In light of this, similarly to Antràs and Chor (2013), we introduce a measure capturing the (net) proximity of a sector to the final consumer. In particular, we define

$$Proximity = Downstreamness - Upstreamness \tag{19}$$

such that the higher is the value of $Proximity$ the closer is the sector to final demand overall. In particular, if $Proximity > 0$, the sector is "downstream-skewed": its inputs have travelled

farther than its output will. If $Proximity < 0$, the sector is "upstream-skewed": it is farther from final consumption than its inputs are from their origin. If $Proximity = 0$ the sector is "in the middle" of the chain: similar distance from/to the consumer in both directions.

This data is available from 1995 until 2022 at yearly frequency and covers 22 sectors at the NACE 2-digit level. In light of this, we interpolate this data with a linear trend such that the true observed data point equates the value in the first quarter of the year.

Sectoral participation in the value chain.– While the previous measures describe each sector's proximity to final demand, they say little about the topology of the network, and in particular about the number of production stages and nodes that intermediate goods pass through before reaching a given sector for further transformation. To capture this dimension, we use data from Mancini *et al.* (2024), who propose measures of the *intensity* of trade within the network: *backwardness* and *forwardness*. the higher a sector's *backwardness*, the larger the number of sectors it buys from and the larger the volume of its purchases. Conversely, the higher its *forwardness*, the larger the number of sectors it sells to and the larger the volume of its sales. Both values are expressed as share of sectoral output and bounded between 0 and 1.

Yet, these variables alone do not reveal the extent to which a sector is deeply integrated in the network, or whether it is central or peripheral. For this reason, similarly to Mancini *et al.* (2024), we build a measure of sectoral (net) position within the network:

$$Participation = Forwardness - Backwardness \tag{20}$$

such that the higher is the value of $Position$, the more important the sector is as a supplier to a large number of nodes. In particular, if $Participation > 0$, the sector is strategic for many other nodes and most of the production flows and linkages run from the center of the network outward only. Conversely, if $Participation < 0$, the sector is dependent on many nodes and intermediary linkages running from the periphery of the network inward. Finally, if $Participation = 0$ the sector lies somewhere between the center and the periphery. This data is available from 1995 until 2020 at yearly frequency and covers 22 sectors at the NACE 2-digit level. Also in this case, we interpolate this data with a linear trend such that the true observed data point equates the value in the first quarter of the corresponding year.

The Stock & Watson Phillips Curve.– We use PPI inflation and sectoral production to estimate a simple correlation between economic activity and prices. Following Stock and Watson (2019), for each sector and country, we estimate the following model:

$$\Delta\pi^{PPI}_{i,j,t} = \alpha_{i,j} + \kappa_{i,j}\Delta y_{i,j,t} + \varepsilon_{i,j,t} \tag{21}$$

where $\Delta\pi^{PPI}_{i,j,t}$ is the change in the 4-quarter moving average of PPI inflation in sector-i of country-j, $\alpha_{i,j}$ is the country-sector constant and $\varepsilon_{i,j,t}$ is the error term. Hence, for each country and sector we collect the estimate $\kappa_{i,j}$, a (biased) Ols correlation between PPI inflation and economic activity.

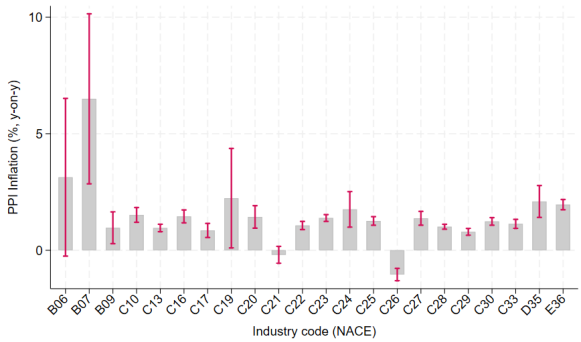
4.2. FINAL SAMPLE AND DESCRIPTIVES

By merging these different datasets, our final sample covers 22 sectors of the EMU11 between 2005Q1 and 2020Q4. The sectors are: extraction of crude petroleum and natural gas (B06); mining of metal ores (B07); mining support service activities (B09); manufacture of food products (C10); manufacture of textiles (C13); manufacture of wood and of products of wood and cork (except furniture); manufacture of articles of straw and plaiting materials (C16); manufacture of paper and paper products (C17); manufacture of coke and refined petroleum products (C19); manufacture of chemicals and chemical products (C20); manufacture of basic pharmaceutical products and pharmaceutical preparations (C21); manufacture of rubber and plastic products (C22); manufacture of other non-metallic mineral products (C23); manufacture of basic metals (C24); manufacture of fabricated metal products (except machinery and equipment) (C25); manufacture of computer, electronic and optical products (C26); manufacture of electrical equipment (C27); manufacture of machinery and equipment (C28); manufacture of motor vehicles, trailers and semi-trailers (C29); manufacture of other transport equipment (C30); repair and installation of machinery and equipment (C33); electricity, gas, steam and air conditioning supply (D35); water collection, treatment and supply (E36).

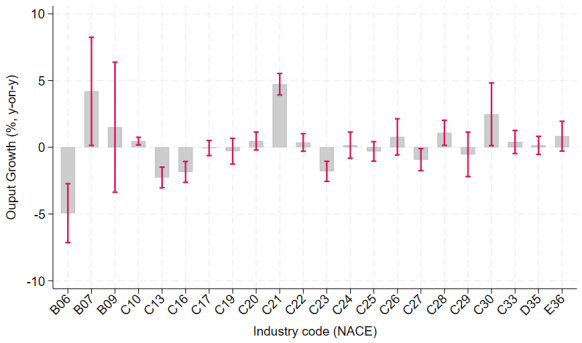
Figure 3 provides descriptives on the five variables under analysis: sectoral PPI inflation, output growth, proximity to the final consumer, participation and the slope of the Stock and Watson PC. In Figure 3a, the petroleum and extraction sector (B06), mining (B07), and coke and refined petroleum products (C19) exhibit the highest average year-on-year PPI inflation, respectively equal to 3.5% and 7% and 3%. Also, for these sectors, the cross-country dispersion is the largest, as highlighted by the 95% confidence interval around the cross-country means. For all other sectors, there is substantial heterogeneity despite a smaller dispersion within the sector. As shown in Figure 3b, the heterogeneity across sectors of the year-on-year output growth rate is even more marked. Also in this case, cross-country dispersion is higher in sectors like B06 and B07 and B09 (mining support activities).

In Figure 3c and 3d, we plot the mean values of the variables *Proximity* and *Participation*. For better comparability, (only) here both variables are rescaled between -1 and +1. As from Figure 3c sectors like mining of metal ores (B07), mining support service activities (B09), manufacture of food products (C10), manufacture of textiles (C13), manufacture of chemicals and chemical products (C20), manufacture of basic pharmaceutical products and pharmaceutical preparations (C21), manufacture of other non-metallic mineral products (C23), manufacture of fabricated metal products (C25), manufacture of computer, electronic and optical products (C26), manufacture of electrical equipment (C27) and manufacture of machinery and equipment (C28) are all sectors with negative proximity. In other words, they do not supply directly the final consumer as most of their output is sold to the network and arrives to the final consumer only after being modified or used as an intermediary input in several passages of productions. For these sectors $Downstreamness < Upstreamness$. All other sectors – manufacture of paper and paper products (C17), manufacture of coke and refined petroleum products (C19), manufacture of basic metals (C24), manufacture of other transport equipment (C30), repair and installation of machinery and equipment (C33), electricity, gas, steam and air con-

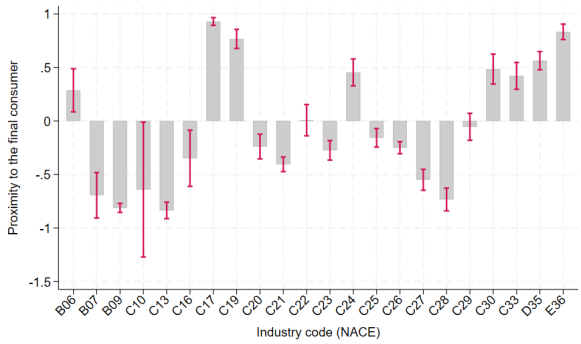
FIGURE 3: DESCRIPTIVE STATISTICS



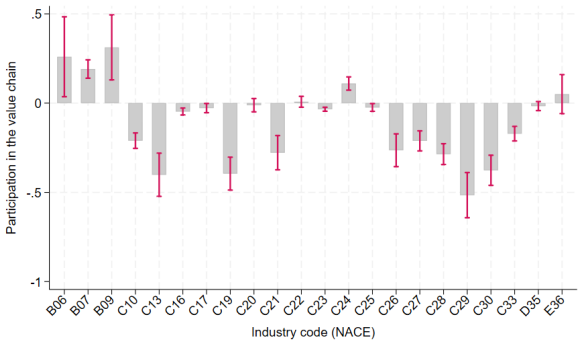
(A) PPI INFLATION



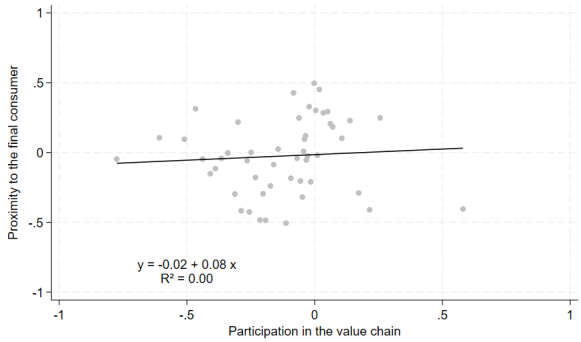
(B) OUTPUT GROWTH



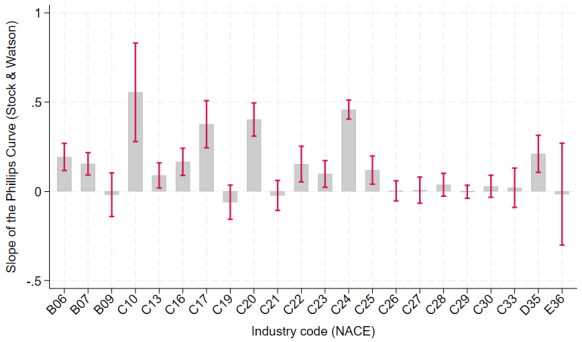
(C) PROXIMITY



(D) PARTICIPATION



(E) PROXIMITY VS. PARTICIPATION



(F) SECTORAL SLOPES

Notes: Figure 3a-3f plot respectively year-on-year PPI inflation; year-on-year sectoral output growth; a measure of sectoral *proximity* to the final consumer from Antràs and Chor (2013); a measure of *participation* to the value chain from Mancini *et al.* (2024); the cross-sectional relationship of these two measures; the slope of the Phillips Curve as evaluated as in Stock and Watson (2019). For variable description and sample definition see Section 4.

ditioning supply (D35); water collection, treatment and supply (E36) – are relatively closer to the final consumer $Downstreamness > Upstreamness$. Sectors like manufacture of rubber and plastic products (C22) and manufacture of motor vehicles, trailers and semi-trailers (C29) are in the middle as the serve both the network and the final consumer in similar way $Downstreamness \simeq Upstreamness$.

As from Figure 3d, only four sectors sell (relatively) more of their output to other nodes, *i.e.*, from the center to the periphery of the network (*i.e.*, $Forwardness > Backwardness$). These sectors are: extraction of crude petroleum and natural gas (B06); mining of metal ores (B07); mining support service activities (B09) and manufacture of basic metals (C24). Manufacture of paper and paper products (C17), of chemicals and chemical products (C20), of fabricated metal products (C25), electricity, gas, steam and air conditioning supply (D35), water collection, treatment and supply (E36) are on the other hands in between the periphery and the center of the network (*i.e.*, $Forwardness \simeq Backwardness$). All other sectors are more peripheral as most of their added value comes from previous passages inside the network (*i.e.*, $Forwardness < Backwardness$).

In Figure 3e, we (bin)scatter[9] each country-sector mean of the variable *Proximity* on *Participation*. These two variables are only weakly (and insignificantly) correlated as they capture two different concept: *Proximity* relates to the distance of a sector to the final demand, whereas *Participation* relates to the topology of the network and the number of links before or after a specific node. This is consistent with Corollary 1.

Finally, in Figure 3f we plot the Stock and Watson correlations. Manufacture of food products (C10), manufacture of paper and paper products (C17), of chemicals and chemical products (C20) and of basic metals (C24) exhibit the steepest PC.

5. Proximity, Participation and the Slope of the Phillips Curve

5.1. Non linearities in the Phillips curve

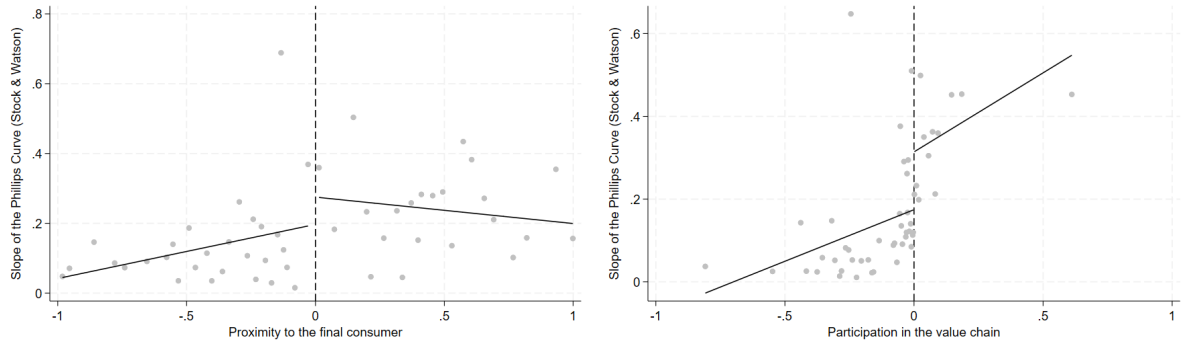
In this section, we provide a first-hand evidence that the proximity of a sector to the final consumer and its participation to the network matter for the slope of the PC and –consequently– for the transmission of shocks and the update of prices from one node to another. To do so, we consider the within country-sector means of the variable *Proximity* and *Participation* and plot them against the Stock and Watson slope of the PC. To better educate each relation, we exclude the few negative slopes estimated using eq. (21), and we keep the scaling of *Proximity* and *Participation* between -1 and +1 for better comparability.

Figure 5a (bin)scatter[10] the variable *Proximity* against sectoral slopes along with fitting lines for values above and below the level of *Proximity* equal to zero. Overall, The figure reveals an inverted V-shaped relationship between *Proximity* and the slope of the sectoral PC. In particular, the closer is a sector to final demand ($Proximity > 0$), the smaller is the structural relationship

[9] We sort observations into 50 bins.

[10] We sort observations into 50 bins to avoid that the correlations will be biased by few outliers.

FIGURE 5: SECTORAL PROXIMITY, PARTICIPATION AND THE SLOPE OF THE PHILLIPS CURVE



(A) PROXIMITY (B) PARTICIPATION

Notes: Figure 5a and 5b (bin)scatter the *Proximity* and *Participation* measures on the sectoral slope of Phillips Curve evaluated as in Stock and Watson (2019). The number of bins is 50. For variable description and sample definition see Section 4.

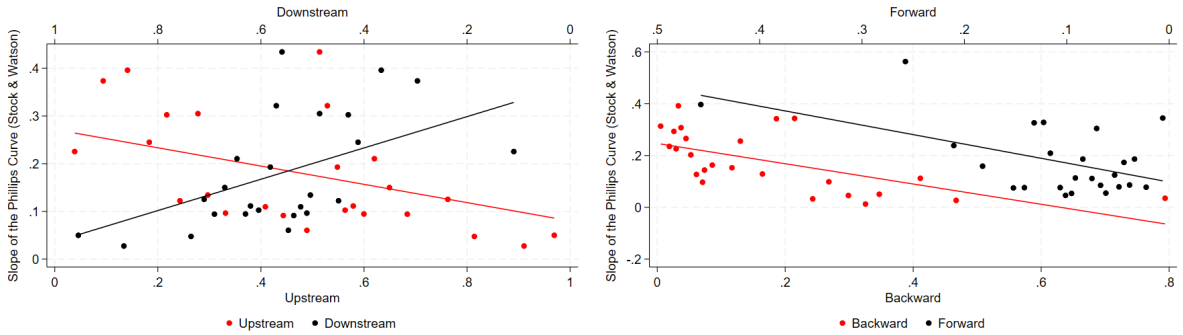
between PPI inflation and output growth. Seemingly, the more a sector is distant from the final consumer and mostly supply other nodes in the network (*Proximity* < 0), the flatter is the PC. Conversely, the PC is the steepest if the sector supplies equally to both final demand and the rest of the network, *i.e.*, when *Proximity* is close to zero.

The relevance of the final demand for the sectoral PC is the novelty of this paper, which is the first showing that the strong dependence of a sector from demand (whether it comes from final consumers or other sectors in the network) has non-linear implications for the relationship between prices and output. On the other hand, other papers focus on how the topology of the network influences the PC. For example Rubbo (2023) focuses on how the number of passages in front or before a certain node of the network influences the PC. In particular, the bigger the number of nodes prior to a specific sector, the flatter is the PC at that level, as the compounding of real rigidities directly affects the slope. The variable *Participation* captures this dimension. In line with the predictions from Rubbo (2023), Figure 5b shows a monotonic relationship between the sectoral slope and *Participation*: if a sector passes its product to a large number of (more peripheral) nodes (*i.e.*, $Forwardness > Backwardness$), then its PC will be steeper. Conversely, if a sector is peripheral and its value added is explained by a large number of passages occurred within the network, then (*i.e.*, $Forwardness < Backwardness$), then its PC is flatter.

As highlighted in Section 4, the components of both *Proximity* and *Participation* are not complementary or mutually exclusive. Therefore, in order to corroborate the evidence of Figure 5a-b, we consider every single component separately and check whether the main intuition is still valid. Starting from Figure 7a, we plot the slope of the PC respectively on the variable *Upstreamness* and *Downstreamness* (2nd x-axis).[11] The more a sector is upstream, *i.e.*, the more it supplies to the rest of the network, the more its PC is flat. Similarly, the more a sector is downstream and sells to the final consumer, the more its PC is flat. As from Figure 7b, the more a sector is characterized by high *Backwardness*, *i.e.*, the more the value of its output is generated by previous passages within the network, the more its slope is flat. Conversely, the

[11] For comparability, we scale both variables between 0 and +1.

FIGURE 7: DECOMPOSITION OF SECTORAL PROXIMITY AND PARTICIPATION



(A) DOWNSTREAM/UPSTREAM (B) BACKWARD/FORWARD

Notes: Figure 7a and 7b (bin)scatter each component of *Proximity* and *Participation* measures – *i.e.*, *Downstreamness*, *Upstreamness*, *Forwardness* and *Backwardness* – on the sectoral slope of Phillips Curve evaluated as in Stock and Watson (2019). The number of bins is 50. For variable description and sample definition see Section 4.

more a sector is characterized by high *Forwardness* (2nd x-axis) and the value of its output is passed to more peripheral nodes, the more its slope is steeper. In Appendix C.1, we corroborate these results by showing that the components of *Proximity* and *Participation* also matter for the frequency of price adjustment at sectoral level. In fact, as the slope of the PC is generally steeper if the frequency of price adjustment is high, sectors in which the level of *Upstreamness* and *Downstreamness* is high are characterized by less frequent price adjustment and a flatter PC. For the same principle, sectors with high level of *Backwardness* and *Forwardness* respectively exhibit low and high frequency of price adjustments, along with flatter and steeper sectoral PCs.

OLS evidence.– To support of our graphical analysis, here we exploit the panel-dimension of our dataset to estimate the following "naive" Phillips curve defined at the country-sector level:

$$\pi^{PPI}_{i,j,t} = \alpha_{i,j} + \kappa_1 \Delta y_{i,j,t} + \kappa_2 \Delta y_{i,j,t} \times x_{i,j,t} + W'_{i,j,t-1}\gamma + \varepsilon_{i,j,t} \tag{22}$$

where the variable $x_{i,j,t}$ can take different values:

- $x_{i,j,t} = \{Proximity; Proximity \times I(Prox. \geq 0); Downstreamness; Upstreamness\}$;
- $x_{i,j,t} = \{Participation; Participation \times I(Part. \geq 0); Forwardness; Backwardness\}$.

The dummy $I(Prox. \geq 0)$ and $I(Part. \geq 0)$ value one when the variable within brackets is weakly positive. $W_{i,t,t}$ is a vector of controls containing the lag of PPI inflation and x_i, capturing the level effect of the variable of interest. By estimating this eq. (22), we can go in-depth into the graphical evidence of Figure 5 and 7, and check that the slope of the PC changes as we expect. Table 3 shows Ols estimates.

In panel A, we consider the variable *Proximity* alone, around the 0-threshold and its single components. From column 1, the slope of the PC (κ_1) is roughly 0.04 and is not influenced by the level of *Proximity*. When decomposing the effect of *Proximity* around the 0-threshold (column 2), first of all we find a slightly larger estimate of the PC ($\kappa_1 = 0.05$), but now *Proximity* has an impact on the slope depending whether it takes negative or positive values. When below zero, a marginal increase of the *Proximity* increases the slope of the PC. Finally in column 3 we explore the two margins: *Upstreamness* and *Upstreamness*: both measures lead to a flattening.

TABLE 3: NON-LINEARITIES IN THE PHILLIPS CURVE - FIGURE 2 AND 3 IN DEPTH

A. Proximity and its components

	(1) π^{ppi}	(2) π^{ppi}	(3) π^{ppi}
Δy	0.0369***	0.0516***	0.2462***
	(0.0035)	(0.0061)	(0.0245)
$\Delta y \times Proximity$	0.0057		
	(0.0055)		
$\Delta y \times Proximity \times I(Prox. < 0)$		0.0295***	
		(0.0098)	
$\Delta y \times Proximity \times I(Prox. \geq 0)$		-0.0279**	
		(0.0127)	
$\Delta y \times Downstream$			-0.0816***
			(0.0115)
$\Delta y \times Upstream$			-0.0091*
			(0.0055)
Observations	9908	9908	9908
R^2	0.6322	0.6326	0.6352
Country-Industry Fe	Yes	Yes	Yes
Time Fe	No	No	No
Country-Time Fe	No	No	No

B. Participation and its components

	(1) π^{ppi}	(2) π^{ppi}	(3) π^{ppi}
Δy	0.0577***	0.0528***	0.0689***
	(0.0039)	(0.0054)	(0.0075)
$\Delta y \times Participation$	0.0050***		
	(0.0004)		
$\Delta y \times Participation \times I(Part. < 0)$		0.0044***	
		(0.0006)	
$\Delta y \times Participation \times I(Part. \geq 0)$		0.0061***	
		(0.0010)	
$\Delta y \times Forward$			0.0036***
			(0.0010)
$\Delta y \times Backward$			-0.0055***
			(0.0005)
Observations	10026	10026	10026
R^2	0.6380	0.6381	0.6388
Country-Industry Fe	Yes	Yes	Yes
Time Fe	No	No	No
Country-Time Fe	No	No	No

Notes: Panel A of Table 3 shows results for (22) when the dependent variable is PPI (year-on-year) inflation, and the explanatory variable is the (year-on-year) growth rate of sectoral output. The latter is interacted with *Proximity* and its components. In Panel B the explanatory variable is interacted with *Participation* and its components. The controls are a country-sector fixed effect and the lag of PPI inflation. For variable description and sample definition see Section 4.

In panel B, we consider the variable *Participation* alone, around the 0-threshold and its single components. From column 1, we find that the slope of the PC (κ_1) is roughly 0.06 and significantly increases for for larger values of *Participation*. When decomposing the effect of *Participation* around the 0-threshold (column 2), $\kappa_1 = 0.05$ remain similar while higher *Participation* increases the slope on both sides of the 0-threshold (although the effect is slightly stronger for $Participation > 0$). In Column 3, we look at each component and find that the bigger is the number of passages in front of a sector (*i.e.*, high *Forwardness*) the steeper is the PC. Conversely, the bigger is the number of passages occurred before a specific node (*i.e.*, high *Backwardness*), the flatter is the PC.

5.2. THE SECTORAL PHILLIPS CURVE

Overall, these results confirm the evidence from Figure 5a. Yet, the estimates can be biased due to omitted variable bias and the correlation of the error term with both dependent and independent variables. For this reason, in this section we apply to our sectoral data the methodology of Hazell *et al.* (2022) for the identification of the regional Phillips Curve. This identification insight leans on the fact that – within a monetary union – sectors and regions are subject to the same monetary authority, same aggregate (monetary or supply) shocks and consequently have the same aggregate inflation expectations. For this reason, using time fixed effects along with lags of endogenous variables as instruments allow to reduce the Ols bias. In light of methodology, by considering the single components of our *Proximity* and *Participation* measures, we rewrite our model as follows:

$$\pi^{PPI}_{i,j,t} = \alpha_{i,j} + \theta_{(t)} + \theta_{(t\times j)} + \kappa_1 \Delta y_{i,j,t} + \sum_{x\in\Omega} \kappa_x \Delta y_{i,j,t} \times x_{i,j,t} + W'_{i,j,t-1}\gamma + \varepsilon_{i,j,t} \qquad (23)$$

TABLE 4: THE SECTORAL PHILLIPS CURVE

	(1) π^{ppi}	(2) π^{ppi}	(3) π^{ppi}	(4) π^{ppi}	(5) π^{ppi}	(6) π^{ppi}
Δy	0.0363***	0.0365***	0.3419***	0.0317**	0.2368***	0.2148***
	(0.0074)	(0.0076)	(0.0361)	(0.0143)	(0.0378)	(0.0453)
$\Delta y \times Downstreamness$			-0.0921***		-0.0678***	-0.0678***
			(0.0191)		(0.0204)	(0.0204)
$\Delta y \times Upstreamness$			-0.0426***		-0.0338***	-0.0329***
			(0.0106)		(0.0107)	(0.0109)
$\Delta y \times Forwardness$				0.0160***	0.0164***	0.0165***
				(0.0018)	(0.0018)	(0.0018)
$\Delta y \times Backwardness$				-0.0072***	-0.0047***	-0.0048***
				(0.0009)	(0.0010)	(0.0010)
Observations	9907	9907	9907	9907	9907	9907
R^2	0.5889	0.5877	0.5942	0.5808	0.5870	0.5869
Country-Industry Fe	Yes	Yes	Yes	Yes	Yes	Yes
Time Fe	Yes	Yes	Yes	Yes	Yes	Yes
Country-Time Fe	No	Yes	Yes	Yes	Yes	Yes
Other Controls	No	No	No	No	No	Yes
IV	Lags	Lags	Lags	Lags	Lags	Lags

Notes: Table 4 show results of the estimation of eq. (23). The dependent variable is PPI (year-on-year) inflation, the explanatory variable is the (year-on-year) growth rate of sectoral output. The latter is interacted with the components of *Proximity* and *Participation*. The controls are a country-sector fixed effect, a time fixed effect, a time-country fixed effect, the lag of PPI inflation and a structural break in the Phillips curve following the Great Financial Crisis. Endogenous variables are instrumented with their own first lag. For variable description and sample definition see Section 4. See also Appendix C.2 for first-stage statistics.

where $\theta_{(t)}$ is a time fixed effect netting out the role of all shocks and expectations common to all countries and sectors in the EMU11; $\theta_{(t\times j)}$ is a time-country fixed effect that nets out the role of regional shocks; x is an element/variable of the vector $\Omega = \{Upstreamness; Downstreamness; Forwardness; Backwardness\}$; $W'_{i,j,t-1}$ contains the lag of the dependent variable along with all components the vector Ω, which controls for changes in the characteristics of the network. Also we control for a structural break in the Phillips curve detected in several papers on the flattening of the PC in EMU countries after the Great Financial Crisis (GFC), *e.g.*, Florio *et al.* (2025) and Siena and Zago (2024). Formally, we control for $\Delta y_{i,j,t} \times I(Post\ GFC)$, where $I(Post\ GFC)$ is a dummy variable taking value one for periods after the GFC. The variable $\Delta y_{i,j,t}$ and all its possible interactions – $\Delta y_{i,j,t} \times x_{i,j,t}$ and $\Delta y_{i,j,t} \times I(Post\ GFC)$ – are instrumented with their own first lag.[12]

Table 4 shows results. In column 1, when controlling only for time fixed effects, we find a slope of 0.036 which does not change when controlling also for country-time fixed effects (column 2). In column 3 we consider the role of *Upstreamness* and *Downstreamness*. Relative to the Ols estimate of Table 3, the estimated slope is now bigger and equal to 0.34. If the role played by *Upstreamness* on the slope of the PC is similar in size to the Ols estimate, the IV estimate is larger when looking at the effect of *Downstreamness*. Yet, also under this identification, both margins operate on the slope as expected: the more a sector is upstream (downstream), the flatter is the PC, *i.e.*, there is an inverted V-shaped relationship between *Proximity* to the final consumer and the sensitivity of prices to economic conditions.

In column 4 we consider *Frowardness* and *Backwardness* alone. The IV estimate of κ_1 is smaller relative to what found in panel B of table 3. However, the role played by *Frowardness*

[12] In Appendix C.2, we show first stage statistics and validity of the instruments.

and *Backwardness* under IV estimation is relatively stronger. Also in this case the results hold: the more passages occur after (before) a specific node, the steeper (flatter) is the PC, *i.e.*, there is a positive and monotonic relationship between *Participation* and the sensitivity of prices to economic conditions.

In column 5, we run a horse race including all network margins jointly, to check whether their significance changes and whether any network characteristic becomes irrelevant once the others are controlled for. The results are similar to those in columns 3 and 4, although the slope κ_1 is now closer to its counterpart in column 3 than to that in column 4. In column 6, we allow for a structural break in the PC following the Global Financial Crisis (GFC). The results remain robust in this case as well.

All in all, these results confirm the graphical intuition from Figures 5 and 7: while *Participation* has a monotonic effect on the slope of the PC, *Proximity* has a non-linear effect resembling an inverted V-shaped curve. In appendix, we perform several robustness checks. In Appendix C.3, we instrument the endogenous variables in eq. (23) with two alternative sector-level Bartik shocks, which use either cross-country sectoral growth or within-country aggregate growth as shifters of country-sector demand. Under this alternative instrumentation, the main results are confirmed. In Appendix C.4, closer to the work of Hazell *et al.* (2022), we show that defining the sectoral output gap as the sum of four discounted growth rates does not change the results. Finally, in Appendix C.5, we show that the results also hold when we shut down the time variation in *Proximity* and *Participation*. This indicates that the results are driven by the structure of the network rather than by changes in sectoral network characteristics over time.

6. Conclusion

Production networks describe not only where firms buy their input, but also where they sell their output. This paper shows that this second dimension matters for price setting and aggregate supply. We introduce final demand proximity, the extent to which the output of a sector reaches final consumers directly or indirectly through other sectors. Once firms sell both inside the network, to other producers, and outside it, to final consumers, production networks affect both sides of their price setting problem. As buyers, firms inherit marginal costs from their suppliers. As sellers, their marginal revenues depend on the destination of their output.

We formalize this mechanism in a multi-sector New Keynesian model. Final demand proximity directly affects the slope of sectoral supply curves. Importantly, its effect is not monotonic. Sectors located at the two extremes of the production structure display flatter supply curves, while sectors in intermediate positions display steeper ones. The intuition is simple. At the extremes, firms mainly operate through one demand margin. In the middle, both final and intermediate demand are relevant: a price change reallocates sales across the two margins and propagates through a broader set of production relationships. These sectoral differences also matter in the aggregate, since the contribution of each sector depends not only on its direct importance for final consumption, but also on its role throughout the production network.

We test these predictions using country-sector variation within the EMU11. The data show that proximity to final demand and participation in the production network capture different

dimensions of production. Consistently with the model, final demand proximity has a non-monotonic relationship with sectoral supply slopes: sectors very close to or very far from final demand have flatter supply curves, while sectors in intermediate positions have steeper ones. Network participation has instead a monotonic effect: forwardness steepens the supply curve, whereas backwardness flattens it. These results are robust to alternative specifications accounting for common and country level factors and for changes in the Phillips curve after the Great Financial Crisis.

The main implication is that the slope of aggregate supply is not independent of how production is organized. Production networks matter not only because they propagate marginal costs, but also because they determine how sectoral output reaches final demand and, therefore, how firms set prices. Where a shock originates consequently matters not only for how strongly it propagates through the network, but also for how prices react along the way. Understanding aggregate supply in a network economy therefore requires looking at both sides of production relationships: whom firms buy from and whom they sell to, inside and out.

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SUPPLEMENTAL APPENDIX

(Outline) *In this appendix we report all the material complementary to the main text; it is made of additional tables and figures, and of discussions on further analytical results. Section A deepens the theoretical findings of Sections 2-3 by presenting further results (Subsection A.1) and re-deriving the slope without relying on constitutive assumptions (Subsection A.2). Section B shows the derivation of full NK-DSGE model presented in Section 3. Finally, Section C complements and enriches the exposition of the empirical findings in Sections 4-5.*

A. THEORETICAL RESULTS

A.1. More on the final destination of sales

Theorem 1 establishes that the destination of sectoral output is described by a distribution rather than by a number, and Corollaries 2-3 extract from it the two conventional measures of network position. This subsection completes the characterisation. We first show that the Leontief representation is just the initial point of the destination of sales. Then, we relate the framework to the influence vector of Acemoglu *et al.* (2012), showing precisely which dimension of the distribution that object retains and which one it integrates out.

COROLLARY 4 (Participation as the mass beyond the first stage) *For every sector-$i \in \Phi$,*

$$\varpi_i(0) = \phi_i^c \qquad \textit{and} \qquad 1 - \varpi_i(0) = \sum_{j \in \Phi} \theta_{ij} = \frac{x_i^{sup}}{y_i}$$

Proof: set $n = 0$ in Definition 2: $\varpi_i(0) = \left[\boldsymbol{\Theta}^0 \boldsymbol{\phi}^c\right]_i = \left[\boldsymbol{\phi}^c\right]_i$. The second equality is eq. (2) combined with ϕ_i^c. ■

Network participation, the object that the Leontief representation isolates, is thus a single number extracted from the full distribution $\varpi_i(\cdot)$: the complement of the mass located at the origin. The interpretation is immediate. In fact, the mass $\varpi_i(0)$ corresponds to the share of sector-i's output that does not enter the production network and is instead absorbed directly as final demand, while the complementary mass $1 - \varpi_i(0)$ is the share of output that enters the network through intermediate demand. Hence, the conventional measure of network participation can be understood as the mass of the destination distribution lying beyond its initial point. In this sense, the Leontief representation extracts from $\varpi_i(\cdot)$ only the extensive margin of network involvement: it records whether and in what share a sector's output enters the production network, but not how far that output subsequently travels through it. This distinction is important because two sectors can have the same degree of network participation while exhibiting very different distributions of destination across the network.[I] The corollary makes precise what the Leontief representation retains from the full distribution and, equally importantly,

[I] For example, two sectors may satisfy $1 - \varpi_i(0) = 1 - \varpi_j(0)$, and therefore have identical network participation, even though the mass of $\varpi_i(\cdot)$ is concentrated close to the initial stage whereas the mass of $\varpi_j(\cdot)$ is distributed over substantially more distant stages. The participation measure therefore collapses the entire distribution beyond the origin into a single number. It preserves the amount of output that enters the network, but integrates out its subsequent propagation across network stages.

what it leaves unexplained: participation captures the *extensive margin* of network involvement, whereas the distribution $\varpi_i(\cdot)$ also contains information about the *depth* and propagation of output within the network.

A natural objection to the framework is that final demand already appears in the standard production network literature, through the influence vector that weights the Leontief inverse by final expenditure shares (refer to Acemoglu *et al.* 2012). If that object already combines the network with final demand, one may ask what is left to add. The result below answers the objection by locating the influence vector inside the joint distribution $\{\xi_{ij}(n)\}$ of Definition 2, and by showing exactly which of its two dimensions it retains. The influence vector is shown to be the destination marginal ξ_{ij}: it records which final good absorbs a sector's output, and is by construction silent on how far that good is. This delivers a precise statement of the contribution: the literature has employed one of the two marginals of the distribution, and the mechanism of this paper operates through the other.

COROLLARY 5 (Influence vector as a destination marginal) *Let $b_j = \frac{p_j c_j}{P^c C}$ denote final expenditure shares and $\lambda_i = \frac{p_i y_i}{P^c C}$ sectoral Domar weights, with $\boldsymbol{\mathcal{L}}$ expressed in value terms. Then:*

$$\xi_{ij} = \frac{b_j \mathcal{L}_{ij}}{\lambda_i} \qquad \textit{so that} \quad \sum_{j\in\Phi} \xi_{ij} = 1 \quad \Longleftrightarrow \quad \boldsymbol{\lambda} = \boldsymbol{\mathcal{L}}\boldsymbol{b}$$

The influence vector $\boldsymbol{\lambda}$ therefore depends on the joint distribution $\{\xi_{ij}(n)\}_{i,j\in\Phi,n\geq 0}$ only through its destination marginal ξ_{ij}, assigning equal weight to every stage-n.[II] *Consequently, two economies in which the output of sector-i reaches the same final goods through chains of different length deliver the same ξ_{ij} and the same influence decomposition, but different $\varpi_i(\cdot)$.*

Proof: the aggregation step in the proof of Theorem 1 gives $\xi_{ij} = \mathcal{L}_{ij}\frac{c_j}{y_i}$ in physical units. Multiplying numerator and denominator by p_j and p_i respectively and dividing both by nominal final expenditure $P^c C$ converts the expression into value shares, $\xi_{ij} = \mathcal{L}_{ij}\frac{b_j}{\lambda_i}$, where $\boldsymbol{\mathcal{L}}$ is now built on value-based technical coefficients. Summing over $j \in \Phi$ and multiplying by λ_i, the statement $\sum_{j\in\Phi}\xi_{ij} = 1$ reads $\lambda_i = \sum_{j\in\Phi} b_j \mathcal{L}_{ij} = [\boldsymbol{\mathcal{L}}\boldsymbol{b}]_i$, which is the influence vector. For what concerns the equal weighting of stages, by Definition 2, $\xi_{ij} = \sum_{n\geq 0}\xi_{ij}(n)$, so ξ_{ij} is the linear functional of the joint distribution that assigns weight one to every stage-n. ■

The influence vector answers the question of whose consumption a sector ultimately serves. Its construction weights every production stage identically, which is appropriate for the purpose it was designed for: measuring how large a sector is once all indirect demand for its output has been traced. However, for the present paper, once the stages are weighted equally, the destination marginal sums to one for every sector, which is the aggregation result of Theorem 1, and the cross-sectional information about position disappears. The two marginals of $\{\xi_{ij}(n)\}_{i,j\in\Phi,n\geq 0}$ are thus complementary rather than substitutable summaries of the same underlying object: ξ_{ij} measures a sector's size once the network is fully unfolded, $\varpi_i(\cdot)$ measures its distance from the point at which that unfolding terminates. Standard production network models require the first and are therefore free to integrate out the second. As Section 3 shows, price setting requires

[II] Under the canonical Input-Output convention adopted here, in which the row index of $\boldsymbol{\mathcal{L}}$ is the supplying sector, the influence vector reads $\boldsymbol{\lambda} = \boldsymbol{\mathcal{L}}\boldsymbol{b}$; it coincides with the familiar expression $\boldsymbol{\lambda}' = \boldsymbol{b}'\boldsymbol{\mathcal{L}}$ of Acemoglu *et al.* (2012), written under the transposed convention.

both, because the elasticity of demand faced by a sector depends not only on how much final demand its output eventually serves, but also on how many stages of transformation separate the sector from it.

A.2. The inverted-U without the Kimball channel

The sectoral Phillips curve of the model has slope

$$\kappa_i = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i} \cdot \frac{1}{1+\Psi_i}$$

where the first factor is the standard Calvo term, which contains no network information, and the second is the pass-through of marginal costs into prices, governed by the degree of real rigidity Ψ_i. The economic content of the paper is entirely in the second factor: it is there that the position of a sector relative to final demand enters, and it is there that the inverted-U originates.

In the main text we specify real rigidity as the ratio of the two components of the effective demand elasticity, the one invariant to the composition of the customer base and the one determined by it. That specification is transparent and delivers $\kappa_i \propto \frac{v_i(1-v_i)}{\epsilon_i^{kim}}$ directly. It is, however, a specification: it assigns real rigidity to the *composition* of the elasticity, whereas the canonical definition assigns it to the *slope* of the elasticity in the own price. A natural concern is therefore that the central prediction of the model – that inflation responsiveness is highest for sectors in the middle of the supply chain – is an artefact of how real rigidity has been specified.

This appendix shows that it is not. We re-derive the sectoral Phillips curve under a different and independent source of variable demand elasticity, obtaining Ψ_i from the canonical superelasticity definition, without the Kimball aggregator and without any normalisation of its curvature parameter. The resulting slope has a *different functional form* from the one in the main text, but it is inverted-U shaped in exactly the same object, the proximity to final demand v_i, with a maximum at exactly the same point. The inverted-U is therefore a structural feature of the destination geometry of the production network, not a property of the demand aggregator used to represent it.

The assumption.– The starting point is that the demand faced by a sector is not homogeneous. Sector-i sells to buyers located at different distances from final consumption, and there is no reason for those buyers to share the same price sensitivity.

Step A.1 (Elasticity attenuates with stage-distance) *Let the demand for sector-i's output that is absorbed by final demand after exactly n intermediate transactions have own-price elasticity*

$$\epsilon(n) = \epsilon^{far} + \Delta\epsilon \cdot \delta^n \qquad with \quad \Delta\epsilon = \epsilon^{near} - \epsilon^{far} \quad and \quad \delta \in (0,1) \tag{A.1}$$

so that elasticity declines geometrically with the number of production stages separating the sector from final consumption, from ϵ^{near} at $n = 0$ to ϵ^{far} as $n \to \infty$.

The economic content of eq. (A.1) is cost-share dilution. The further a sector's output travels before reaching consumers, the smaller is its price as a share of the cost of the final good in which it is eventually embodied, and therefore the smaller is the quantity response induced

by a given percentage change in the sector's own price. Pass-through attenuates along supply chains, and δ is the per-stage attenuation factor. This is a documented empirical regularity, and it is a statement about the technology of the network rather than about preferences.

Two remarks are worth making before proceeding.

REMARK 4 (The discount factor acquires a structural meaning) *The parameter δ introduced in Remark 6 to render the proximity index non-degenerate was, in the main text, a device: without it the accounting identity forces $v_i = 1$ for every sector. Here it is not a device but a primitive: it is the rate at which own-price elasticity attenuates per production stage. The two roles coincide, which is reassuring, and this appendix therefore also supplies the interpretation of δ that the main text leaves open.*

REMARK 5 (No Kimball aggregator is used) *Everything that follows is a CES demand system, segment by segment. There is no Kimball schedule, no curvature parameter η_i^{prim}, and no revenue-maximisation problem allocating sales between destinations. The seller makes no reallocation decision at all: the composition of its customer base moves mechanically, because segments with different elasticities shrink at different rates when the price rises. This is what makes the exercise a genuine robustness check rather than a relabelling.*

The derivation.– Now, let us derive the slope of the sectoral NKPC.

Step A.2 (Effective elasticity is the proximity index) *Let $y_i(n)$ denote the part of sector-i's output absorbed by final demand after n stages, so that $\varpi_i(n) = \frac{y_i(n)}{y_i}$ is the stage-distance distribution of Definition 2, and let each segment have constant elasticity $\epsilon(n)$, that is $y_i(n) = A_i(n) p_i^{-\epsilon(n)}$. The elasticity of total demand is the share-weighted average of segment elasticities,*

$$-\frac{\partial \log[y_i]}{\partial \log[p_i]} = \frac{\sum_{n\geq 0} y_i(n)\epsilon(n)}{y_i} = \sum_{n\geq 0} \varpi_i(n)\epsilon(n) \equiv \epsilon_i^{kim}$$

and substituting eq. (A.1),

$$\epsilon_i^{kim} = \epsilon^{far} + \Delta\epsilon \sum_{n\geq 0} \delta^n \varpi_i(n) = \epsilon^{far} + \Delta\epsilon \cdot v_i(\delta) \tag{A.2}$$

where $v_i(\delta)$ is exactly the discounted proximity index of Remark 6.

The effective demand elasticity faced by a sector is an affine function of its proximity to final demand, with no aggregator, no curvature parameter and no allocation decision. Sectors close to final consumption face more elastic demand simply because more of their output is absorbed by buyers on whom their price weighs heavily. The generating function of the stage-distance distribution, which Section 2 derives as a pure accounting object, turns out to be the sufficient statistic for the demand elasticity.

Step A.3 (How the composition responds to the price) *Because segments differ in elasticity, a change in p_i changes their relative size. Differentiating $\varpi_i(n) = \frac{y_i(n)}{y_i}$, then*

$$\frac{\partial \log[\varpi_i](n)}{\partial \log[p_i]} = -\epsilon(n) + \epsilon_i^{kim} \qquad \Longrightarrow \qquad \frac{\partial \varpi_i(n)}{\partial \log[p_i]} = -\varpi_i(n)\left[\epsilon(n) - \epsilon_i^{kim}\right] \quad \text{(A.3)}$$

Segments more elastic than average shrink; segments less elastic than average expand. Applying eq. (A.3) *to the definition of the proximity index,*

$$\begin{aligned}\frac{\partial v_i}{\partial \log[p_i]} &= \sum_{n\geq 0} \delta^n \frac{\partial \varpi_i(n)}{\partial \log[p_i]} = -\sum_{n\geq 0} \delta^n \varpi_i(n)\left[\epsilon(n) - \epsilon_i^{kim}\right] \\ &= -\Delta\epsilon\left[\sum_{n\geq 0} \delta^{2n}\varpi_i(n) - \Big(\sum_{n\geq 0} \delta^n \varpi_i(n)\Big)^2\right]\end{aligned}$$

where the second line substitutes eq. (A.1) *and eq.* (A.2) *and collects terms. Defining the random variable* $Z_i = \delta^N$*, with* N *distributed according to* $\varpi_i(\cdot)$*, the bracket is* $\mathbb{E}\left[Z_i^2\right] - \left(\mathbb{E}[Z_i]\right)^2$*:*

$$\frac{\partial v_i}{\partial \log[p_i]} = -\Delta\epsilon \cdot \mathbb{V}_i, \qquad \textit{with} \quad \mathbb{V}_i = \mathrm{Var}\left[\delta^N\right] = v_i\left(\delta^2\right) - \left[v_i(\delta)\right]^2 \quad \text{(A.4)}$$

The responsiveness of the customer base to the sector's own price is the variance of its discounted stage-distance distribution. The object $\mathbb{V}_i$ is a pure statistic of $\varpi_i(\cdot)$, computable from the destination matrix alone, and it is the exact generalisation of the Bernoulli dispersion $v_i(1-v_i)$ used in the main text: when the distribution has mass at two stages only, $\mathbb{V}_i$ reduces to $v_i(1-v_i)$ up to the scale of the two support points. It vanishes when a sector's output reaches final demand at a single distance, and is largest when that output is split between chains of very different length.

Step A.4 (Real rigidity from the canonical definition) *The degree of real rigidity is the elasticity of the desired markup with respect to the own relative price. With* $\mathcal{M}_i = \frac{\epsilon_i^{kim}}{\epsilon_i^{kim}-1}$*:*

$$\begin{aligned}\frac{d\log[\mathcal{M}_i]}{d\,\epsilon_i^{kim}} &= \frac{1}{\epsilon_i^{kim}} - \frac{1}{\epsilon_i^{kim}-1} = -\frac{1}{\epsilon_i^{kim}\left(\epsilon_i^{kim}-1\right)} \\ \longrightarrow \Psi_i &= \frac{1}{\epsilon_i^{kim}\left(\epsilon_i^{kim}-1\right)}\frac{\partial \epsilon_i^{kim}}{\partial \log[p_i]}\end{aligned}$$

Differentiating eq. (A.2) *and substituting eq.* (A.4)*,*

$$\frac{\partial \epsilon_i^{kim}}{\partial \log[p_i]} = \Delta\epsilon \cdot \frac{\partial v_i}{\partial \log[p_i]} = -(\Delta\epsilon)^2 \mathbb{V}_i \;\leq\; 0 \quad \text{(A.5)}$$

and therefore

$$\Psi_i = -\frac{(\Delta\epsilon)^2 \mathbb{V}_i}{\epsilon_i^{kim}\left(\epsilon_i^{kim}-1\right)} \;\leq\; 0 \quad \text{(A.6)}$$

Note that eq. (A.5) is negative regardless of the sign of $\Delta\epsilon$: the elasticity gap enters squared. Whether near buyers are more or less price-sensitive than far ones, the heterogeneity itself

makes demand less elastic as the price rises, because sales tilt towards whichever segment is less sensitive.

Step A.5 (Sectoral Phillips curve) *Substituting eq.* (A.6) *into the Calvo block gives*

$$\pi_{i,t} = \beta\, \mathbb{E}_t\left[\pi_{i,t+1}\right] + \kappa_i^{new}\, mc_{i,t} \tag{A.7}$$

$$\textit{with } \kappa_i^{new} = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i}\left[1 - \frac{(\Delta\epsilon)^2 \mathbb{V}_i}{\epsilon_i^{kim}\left(\epsilon_i^{kim}-1\right)}\right]^{-1}.$$

The shape.– Finally, the interpretation os the slope.

PROPOSITION 3 (Inverted-U) *Provided* $(\Delta\epsilon)^2 < 4\,\epsilon_i^{kim}\left(\epsilon_i^{kim}-1\right)$, *the slope* κ_i^{new} *in eq.* (A.7) *is positive and strictly increasing in the dispersion* $\mathbb{V}_i$. *Since* $\mathbb{V}_i$ *vanishes when a sector's output reaches final demand at a single distance and is maximised when it is split evenly between near and far chains,* κ_i^{new} *is inverted-U shaped in proximity to final demand, with its maximum at the same interior point as in the main text.*

Proof: write $\kappa_i^{new} \propto (1-\gamma_i \mathbb{V}_i)^{-1}$ with $\gamma_i = \frac{(\Delta\epsilon)^2}{\epsilon_i^{kim}\left(\epsilon_i^{kim}-1\right)} > 0$. The denominator is decreasing in $\mathbb{V}_i$, hence the ratio is increasing. Positivity requires $\gamma_i \mathbb{V}_i < 1$; since $\delta^N \in (0,1]$, Popoviciu's inequality gives $\mathbb{V}_i \leq \frac{1}{4}$, so $\gamma_i < 4$ suffices, which is the stated condition. ■

The condition is not restrictive in practice: for $\epsilon_i^{kim} \simeq 4$ it requires $|\Delta\epsilon| < 6.9$, well beyond any plausible gap in price sensitivity between intermediate and final buyers. It should nonetheless be reported, since it is the condition under which the desired markup is well defined.

B. MODEL DERIVATION

B.1. Household block

Step B.1 (Intra-temporal allocation across sectoral goods) *The household minimises* $\sum_{i\in\Phi} p_i^c c_i$ *subject to* $C = \left(\sum_{i\in\Phi} \omega_i^{1/\epsilon} c_i^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)}$. *The first-order condition with multiplier* μ^c *reads*

$$p_i^c = \mu^c \frac{\partial C}{\partial c_i} = \mu^c\, \omega_i^{1/\epsilon}\, c_i^{-1/\epsilon}\, C^{1/\epsilon}$$

Solving for c_i *and using constant returns of the aggregator* ($\mu^c = P^c$) *yields the demand system and the associated price index*

$$c_i = \omega_i \left(\frac{p_i^c}{P^c}\right)^{-\epsilon} C \qquad \textit{with} \quad P^c = \left[\sum_{i\in\Phi} \omega_i \left(p_i^c\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{B.1}$$

Eq. (B.1) *pins down the vector* $\mathbf{c}$, *hence the direct final-demand shares* $\boldsymbol{\phi}^c$ *of Lemma 1: the leakage of the network is a preference object, everything else about the destination of output is not.*

Step B.2 (Inter-temporal problem) *The household's problem is to maximise* $\mathbb{E}_t \sum_{k\geq 0} \beta^k \left[\frac{C_{t+k}^{1-\gamma}}{1-\gamma} - \sum_{i\in\Phi} \frac{l_{i,t+k}^{1+\psi}}{1+\psi}\right]$ *subject to* $P_{t+k}^c C_{t+k} + B_{t+k+1} = \sum_{i\in\Phi} w_{i,t+k} l_{i,t+k} + (1+r_{t+k})\, B_{t+k} + D_{t+k}$. *With multiplier* Λ_{t+k} *on the period-*$t+k$ *constraint,*

$$C_{t+k}^{-\gamma} = \Lambda_{t+k} P_{t+k}^{c}, \qquad l_{i,t+k}^{\psi} = \Lambda_{t+k} w_{i,t+k}, \qquad \Lambda_{t+k} = \beta\,(1 + r_{t+k+1})\,\mathbb{E}_{t+k}\Lambda_{t+k+1}$$

Eliminating Λ delivers the Euler equation, the sectoral labour supply schedule and the stochastic discount factor used in the Calvo problem:

$$1 = \beta\,(1 + r_{t+1})\,\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t^c}{P_{t+1}^c}\right]$$

$$l_{i,t}^{\psi} = C_t^{-\gamma}\frac{w_{i,t}}{P_t^c} \tag{B.2}$$

$$\Lambda_{t,t+k} = \beta^k\left(\frac{C_{t+k}}{C_t}\right)^{-\gamma}\frac{P_t^c}{P_{t+k}^c} \tag{B.3}$$

B.2. Sectors as network-buyers: marginal costs

Step B.3 (Intermediate input bundle) *Minimising $\sum_{j\in\Phi} p_j^x x_{ji}$ subject to $x_i^{dem} = \left(\sum_{j\in\Phi}\alpha_{ji}^{1/\epsilon}x_{ji}^{(\epsilon-1)/\epsilon}\right)^{\epsilon/(\epsilon-1)}$ gives*

$$x_{ji} = \alpha_{ji}\left(\frac{p_j^x}{P_i^x}\right)^{-\epsilon}x_i^{dem} \qquad \text{with} \quad P_i^x = \left[\sum_{j\in\Phi}\alpha_{ji}\left(p_j^x\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{B.4}$$

Both sums run over the i-th column of $\boldsymbol{\mathcal{A}}$: the buyer block reads the network along columns, which is the normalisation by the buyer's output of Subsection 2.1.

Step B.4 (Nominal marginal cost) *Solve $mc_i = \min_{l_i, x_i^{dem}} w_i l_i + P_i^x x_i^{dem}$ subject to $1 = z_i l_i^{\alpha_i}\left(x_i^{dem}\right)^{1-\alpha_i}$. The ratio of the two first-order conditions gives the optimal input mix*

$$\frac{w_i}{P_i^x} = \frac{\alpha_i}{1-\alpha_i}\frac{x_i^{dem}}{l_i} \qquad \Longrightarrow \qquad x_i^{dem} = \frac{1-\alpha_i}{\alpha_i}\frac{w_i}{P_i^x}l_i$$

Substituting into the unit-output constraint and solving for labour,

$$1 = z_i l_i\left(\frac{1-\alpha_i}{\alpha_i}\frac{w_i}{P_i^x}\right)^{1-\alpha_i} \qquad \Longrightarrow \qquad l_i = \frac{1}{z_i}\left(\frac{\alpha_i}{1-\alpha_i}\frac{P_i^x}{w_i}\right)^{1-\alpha_i}$$

Since $P_i^x x_i^{dem} = \frac{1-\alpha_i}{\alpha_i} w_i l_i$, total unit cost is $C_i = \frac{w_i l_i}{\alpha_i}$, so that

$$mc_i = \frac{1}{z_i}\left(\frac{w_i}{\alpha_i}\right)^{\alpha_i}\left(\frac{P_i^x}{1-\alpha_i}\right)^{1-\alpha_i} \tag{B.5}$$

Constant returns to scale guarantee that mc_i is independent of the scale of production. In value terms the resulting shares are exactly the cost matrix $\boldsymbol{\Omega}$ of Corollary 3, with $\alpha_i = \phi_i^v$ playing the role of the primary-factor leakage: the cost block of the model is the cost side of the same accounting identity, not an analogy to it.

Step B.5 (Recursive cost propagation) *Log-linearising eq. (B.5) and eq. (B.4) around the symmetric steady state – where the expenditure weights of the price index are $\frac{\alpha_{ji}}{\sum_{j\in\Phi}\alpha_{ji}} = \frac{\alpha_{ji}}{1-\alpha_i}$ – gives*

$$\widehat{mc}_{i,t} = \alpha_i \widehat{w}_{i,t} + (1-\alpha_i)\,\widehat{P}^x_{i,t} - \widehat{z}_{i,t} \qquad \textit{and} \qquad (1-\alpha_i)\,\widehat{P}^x_{i,t} = \sum_{j\in\Phi} \alpha_{ji}\widehat{p}^x_{j,t}$$

so that, in matrix form,

$$\widehat{\boldsymbol{mc}}_t = \boldsymbol{\alpha} \odot \widehat{\boldsymbol{w}}_t + \boldsymbol{\mathcal{A}}'\widehat{\boldsymbol{p}}^x_t - \widehat{\boldsymbol{z}}_t$$

Under flexible prices $\widehat{\boldsymbol{p}}^x_t = \widehat{\boldsymbol{mc}}_t$*, the equation inverts into the familiar Leontief representation*

$$\widehat{\boldsymbol{p}}^x_t = \left(\boldsymbol{I} - \boldsymbol{\mathcal{A}}'\right)^{-1}\left(\boldsymbol{\alpha} \odot \widehat{\boldsymbol{w}}_t - \widehat{\boldsymbol{z}}_t\right) = \boldsymbol{\mathcal{L}}'\left(\boldsymbol{\alpha} \odot \widehat{\boldsymbol{w}}_t - \widehat{\boldsymbol{z}}_t\right) \tag{B.6}$$

Eq. (B.6) *is the vertical channel of the network: it is the only channel present in conventional models. The transpose is not cosmetic: it records that costs are cumulated along columns, from primary factors forward, which is the dual distribution* $\varpi^v_i(\cdot)$ *of Corollary 3. Everything that follows concerns the horizontal channel, absent from eq.* (B.6)*, which cumulates along rows towards final demand.*

B.3. Sectors as network-sellers: the Kimball allocation

Step B.6 (The seller's constraint) *Let* $\phi^c_i = \frac{c_i}{y_i}$ *and* $\phi^x_i = \frac{x^{sup}_i}{y_i} = \sum_{j\in\Phi}\theta_{ij}$*, with* $\phi^c_i + \phi^x_i = 1$ *by Lemma 1. The two destinations are aggregated by a Kimball schedule written in unit-of-output terms,*

$$[1-\varsigma_i]\,\mathcal{K}\left[\phi^c_i\right] + \varsigma_i \mathcal{K}\left[\phi^x_i\right] = 1 \tag{B.7}$$

with $\varsigma_i \in (0,1)$ *and* $\mathcal{K}:\mathbb{R}\to\mathbb{R}$ *smooth and increasing. The parametric form adopted is*

$$\mathcal{K}\left[\phi^u_i\right] = \frac{\exp\left(\eta^{prim}_i\left[\phi^u_i - 1\right]\right) - 1}{\eta^{prim}_i} \tag{B.8}$$

with $\mathcal{K}'\left[\phi^u_i\right] = e^{\eta^{prim}_i\left[\phi^u_i - 1\right]}$*, and for* $u = \{c, x\}$*. As* $\eta^{prim}_i \to 0$*, then* $\mathcal{K}\left[\phi^u_i\right] \to \phi^u_i - 1$*, and the aggregator collapses to the CES benchmark.*

Step B.7 (Revenue maximisation and the sales-share schedule) *Normalise* $y_i = 1$*, so that* $c_i = \phi^c_i$ *and* $x^{sup}_i = \phi^x_i$*. The network-seller solves*

$$\max_{\phi^c_i,\phi^x_i}\ \mathcal{R}_i = p^c_i\phi^c_i + p^x_i\phi^x_i \qquad \textit{s.t.} \qquad \textit{eq.}\ (B.7)$$

with Lagrangian $\mathcal{L} = p^c_i\phi^c_i + p^x_i\phi^x_i + \mu_i\left\{1 - [1-\varsigma_i]\,\mathcal{K}\left[\phi^c_i\right] - \varsigma_i\mathcal{K}\left[\phi^x_i\right]\right\}$*. The first-order conditions are* $p^c_i = \mu_i[1-\varsigma_i]\,\mathcal{K}'\left[\phi^c_i\right]$ *and* $p^x_i = \mu_i\varsigma_i\mathcal{K}'\left[\phi^x_i\right]$*, whose ratio eliminates the multiplier:*

$$\frac{p^c_i}{p^x_i} = \frac{1-\varsigma_i}{\varsigma_i}\frac{\mathcal{K}'\left[\phi^c_i\right]}{\mathcal{K}'\left[\phi^x_i\right]} \overset{\textit{eq. (B.8)}}{=} \frac{1-\varsigma_i}{\varsigma_i}\exp\left(\eta^{prim}_i\left[\phi^c_i - \phi^x_i\right]\right)$$

Taking logs, using $\phi^x_i = 1 - \phi^c_i$ *so that* $\phi^c_i - \phi^x_i = 2\phi^c_i - 1$*, and rearranging,*

$$2\phi^c_i - 1 = \frac{1}{\eta^{prim}_i}\log\left[\frac{\varsigma_i}{1-\varsigma_i}\frac{p^c_i}{p^x_i}\right]$$

which is exactly the sales-share schedule of the main text,

$$\phi_i^c = \frac{c_i}{y_i} = \frac{1}{2} + \frac{1}{2\eta_i^{prim}} \log\left[\frac{\varsigma_i}{1-\varsigma_i}\frac{p_i^c}{p_i^x}\right] \tag{B.9}$$

At the symmetric steady state $p_i^c = p_i^x$ the share is constant and equal to $\frac{1}{2} + \frac{1}{2\eta_i^{prim}} \log\left[\frac{\varsigma_i}{1-\varsigma_i}\right]$, so that ς_i pins down the steady-state composition of the customer base and η_i^{prim} its price sensitivity:

$$\frac{\partial \phi_i^c}{\partial \log\left[p_i^c\right]} = -\frac{\partial \phi_i^c}{\partial \log\left[p_i^x\right]} = \frac{1}{2\eta_i^{prim}} \tag{B.10}$$

This is also the state at which the similarity of Corollary 3 holds exactly, since $p_i^c = p_i^x$ rules out destination-specific wedges.

B.4. The effective Kimball elasticity

Step B.8 (Total demand faced by the seller) *Aggregating eq. (B.1) and eq. (B.4) across buyers, total demand for good-i is*

$$y_i = \mathcal{S}_i^c \left(\frac{p_i^c}{P^c}\right)^{-\epsilon} + \mathcal{S}_i^x \left(\frac{p_i^x}{P^x}\right)^{-\epsilon} \qquad \text{with} \quad \mathcal{S}_i^c = \omega_i C \quad \text{and} \quad \mathcal{S}_i^x = \sum_{j\in\Phi} \alpha_{ij} x_j^{dem}$$

where $\mathcal{S}_i^c$ and $\mathcal{S}_i^x$ are the two demand shifters. Note the index reversal relative to eq. (B.4): the seller's shifter runs along the i-th row of $\boldsymbol{\mathcal{A}}$, over the sectors that buy from i, whereas the buyer's price index ran along its column. Differentiating with respect to p_i^c, for $u = \{c, x\}$,

$$\frac{\partial y_i}{\partial p_i^c} = \underbrace{-\epsilon \mathcal{S}_i^c \left(\frac{p_i^c}{P^c}\right)^{-\epsilon} \frac{1}{p_i^c}}_{\text{outer margin: CES substitution}} + \underbrace{\sum_u \left[\frac{\partial \mathcal{S}_i^u}{\partial p_i^c}\left(\frac{p_i^u}{P^u}\right)^{-\epsilon}\right]}_{\text{inner margin: reallocation of the customer base}} \tag{B.11}$$

Step B.9 (Evaluating the inner margin) *Write the shifters as $\mathcal{S}_i^c = \phi_i^c \overline{\mathcal{S}}_i$ and $\mathcal{S}_i^x = \phi_i^x \overline{\mathcal{S}}_i$, where $\overline{\mathcal{S}}_i$ is the total customer base of sector-i and $\{\phi_i^c, \phi_i^x\}$ the Kimball allocation eq. (B.9). The reallocation term in eq. (B.11) is therefore governed by $\frac{\partial \phi_i^c}{\partial \log\left[p_i^c\right]}$. Two observations close the derivation.*

1. *a price change of good-i applies to the same physical good-sold to two destinations. The relevant experiment is therefore a proportional change of $\{p_i^c, p_i^x\}$, which delivers the CES term $-\epsilon\frac{y_i}{p_i}$ from the outer margin;*
2. *the reallocation is bounded by the size of both margins: sales can be shifted away from a destination only to the extent that the other destination is quantitatively present. Formally, eq. (B.9) is the first-order representation around the balanced point of the logistic reallocation rule $\phi_i^c = \left[1 + \exp\left\{-\frac{2}{\eta_i^{prim}} \log\left[\frac{\varsigma_i}{1-\varsigma_i}\frac{p_i^c}{p_i^x}\right]\right\}\right]^{-1}$, with exact semi-elasticity*

$$\frac{\partial \phi_i^c}{\partial \log\left[p_i^c\right]} = \frac{2\phi_i^c \phi_i^x}{\eta_i^{prim}}$$

which collapses to eq. (B.10) at $\phi_i^c = \phi_i^x = \frac{1}{2}$.

The logistic form is not an additional assumption but the only reallocation rule consistent with the two bounds of the problem: shares must remain in $[0,1]$*, and reallocation must vanish when either destination does. The linear schedule eq. (B.9) is its local representation, valid where the customer base is not too unbalanced. Substituting points 1-2 into eq. (B.11), evaluated at the symmetric steady state where* $\left(\frac{p_i^u}{P^u}\right)^{-\epsilon}$ *is common across* $u = \{c, x\}$*, and multiplying by* $-\frac{p_i}{y_i}$*, then*

$$\epsilon_i^{kim} = \epsilon + \frac{\phi_i^c \phi_i^x}{2\eta_i^{prim}} = \epsilon\left(1 + \eta_i^{kim}\right)$$

with $\eta_i^{kim} = \overline{\eta}_i \eta_i^{eff}$, $\overline{\eta}_i = \frac{1}{2\eta_i^{prim}\epsilon}$, *and* $\eta_i^{eff} = \phi_i^c \phi_i^x$.

Step B.10 (The multiplier as a Bernoulli variance) *Let* $d_i \in \{final, intermediate\}$ *be the destination of a randomly drawn unit of* y_i*, with* $\Pr(d_i = final) = \phi_i^c$*. Then* d_i *is Bernoulli and*

$$\operatorname{Var}[d_i] = \phi_i^c(1-\phi_i^c) = \phi_i^c \phi_i^x = \eta_i^{eff} \in \left[0, \tfrac{1}{4}\right] \qquad with \quad \frac{\partial \operatorname{Var}[d_i]}{\partial \phi_i^x} = 1 - 2\phi_i^x \tag{B.12}$$

so that η_i^{eff} *is maximised at* $\phi_i^c = \phi_i^x = \frac{1}{2}$ *and vanishes at either polar case. When one destination is negligible,* $\epsilon_i^{kim} \to \epsilon$ *and the CES benchmark is recovered. In the language of Definition 2, eq. (B.12) is a functional of the first two points of* $\varpi_i(\cdot)$ *only, since* $\phi_i^c = \varpi_i(0)$ *by Corollary 4.*

B.5. Recursive destinations: vertically-integrated nodes

The multiplier eq. (B.12) treats intermediate sales as a single destination, *i.e.*, it places every sector one production stage away from final demand. The recursive extension follows intermediate sales through successive stages.

Step B.11 (The destination matrix and its Neumann expansion) *Recall from Definition 1 that* $\theta_{ij} = x_{ij}/y_i$*, collected in* $\boldsymbol{\Theta}$*, and from Lemma 1 that the direct final-demand shares are* $\boldsymbol{\phi}^c = (\boldsymbol{I} - \boldsymbol{\Theta})\mathbf{1}$*. The share of sector-i output that reaches final demand after exactly n stages is the i-th element of* $\boldsymbol{\Theta}^n \boldsymbol{\phi}^c$*, which is the proximity distribution* $\varpi_i(n)$ *of Definition 2. Summing over all stages,*

$$\boldsymbol{v} = \sum_{n \geq 0} \boldsymbol{\Theta}^n \boldsymbol{\phi}^c = (\boldsymbol{I} - \boldsymbol{\Theta})^{-1} \boldsymbol{\phi}^c \equiv \boldsymbol{\Xi}\boldsymbol{\phi}^c \qquad with \quad \boldsymbol{\Xi} = (\boldsymbol{I} - \boldsymbol{\Theta})^{-1} \tag{B.13}$$

which is the consumption-adjusted Pasinetti (1973) transformation of the network: unlike the Leontief inverse of eq. (B.6), which cumulates production requirements along columns, $\boldsymbol{\Xi}\boldsymbol{\phi}^c$ *cumulates the final-demand content of each stage along rows. The two operators are the two normalisations of Theorem 1, and Remark 6 shows why the second cannot be used undiscounted.*

Step B.12 (Network-consistent Kimball multiplier) *Replacing the static split* $\{\phi_i^c, \phi_i^x\}$ *by the recursive one* $\{v_i, 1 - v_i\}$ *in eq. (B.12) gives*

$$\eta_i^{eff} = v_i \left[1 - v_i\right]$$

The interpretation is unchanged but the underlying random variable is different: v_i is now the probability that a unit of sector-i output is embodied in final demand along some production chain, rather than the probability that it is sold directly to consumers. Equivalently, eq. (B.12) is evaluated on a summary of the entire distribution $\varpi_i(\cdot)$ rather than on its mass at the origin alone.

REMARK 6 (Accounting degeneracy and the required normalisation) *The definition in eq. (B.13) is degenerate under the accounting identity $\boldsymbol{\phi}^c = \mathbf{1} - \boldsymbol{\Theta}\mathbf{1}$ established in Lemma 1. Indeed,*

$$\boldsymbol{v} = (\boldsymbol{I} - \boldsymbol{\Theta})^{-1}(\mathbf{1} - \boldsymbol{\Theta}\mathbf{1}) = (\boldsymbol{I} - \boldsymbol{\Theta})^{-1}(\boldsymbol{I} - \boldsymbol{\Theta})\,\mathbf{1} = \mathbf{1}$$

equivalently, the Neumann series telescopes: $\sum_{n\geq 0}\boldsymbol{\Theta}^n\mathbf{1} - \sum_{n\geq 0}\boldsymbol{\Theta}^{n+1}\mathbf{1} = \mathbf{1}$. Hence $v_i \equiv 1$ and $v_i(1 - v_i) \equiv 0$ for every sector: every unit of output eventually reaches final demand, which is precisely the full-exposure statement of Theorem 1. The degeneracy is therefore not a modelling error but the theorem itself: universal exposure carries no cross-sectional information, exactly as the aggregation step of Theorem 1 integrates the stage dimension out. To obtain a non-degenerate proximity index, the recursion must be discounted across stages,

$$\boldsymbol{v}(\delta) = \sum_{n\geq 0}\delta^n\boldsymbol{\Theta}^n\boldsymbol{\phi}^c = (\boldsymbol{I} - \delta\boldsymbol{\Theta})^{-1}\boldsymbol{\phi}^c \qquad \text{with} \quad \delta \in (0,1)$$

so that $v_i(\delta) = \sum_{n\geq 0}\delta^n\varpi_i(n) \in (\phi_i^c, 1)$, with $v_i(0) = \phi_i^c$ recovering the static multiplier of Corollary 4 and $v_i(1) = 1$ recovering the degenerate case of Theorem 1. The object is the generating function of the stage-distance distribution, and its derivative at $\delta = 1$ returns the upstreamness of Corollary 2: the three results of Section 2 are three evaluations of the single function that the model prices. The parameter δ measures how much one additional production stage dilutes a sector's proximity to final demand, and is the object that makes v_i a genuine position statistic rather than an accounting identity. All results below are stated for $v_i \equiv v_i(\delta)$ with $\delta < 1$.

B.6. Calvo pricing and sectoral NKPCs

Step B.13 (The reset-price problem) *With reset probability $1 - \chi_i$, a sector allowed to adjust chooses $p_{i,t}^{x,*}$ to solve*

$$\max_{p_{i,t}^{x,*}} \; \mathbb{E}_t\left\{\sum_{k=0}^{\infty}\chi_i^k\Lambda_{t,t+k}\left[\left(mr_{i,t+k} - mc_{i,t+k}\right)y_{i,t+k}\right]\right\} \quad s.t. \quad y_{i,t+k} = \left(\frac{p_{i,t}^{x,*}}{P_{t+k}^{x}}\right)^{-\epsilon_i^{kim}} Y_{t+k}$$

with $\Lambda_{t,t+k}$ from eq. (B.3). Using $\frac{\partial y_{i,t+k}}{\partial p_{i,t}^{x,*}} = -\epsilon_i^{kim}\frac{y_{i,t+k}}{p_{i,t}^{x,*}}$, the first-order condition is

$$\mathbb{E}_t\sum_{k\geq 0}\chi_i^k\Lambda_{t,t+k}y_{i,t+k}\left(mr_{i,t+k} - mc_{i,t+k}\right)\epsilon_i^{kim}\frac{1}{p_{i,t}^{x,*}} = \mathbb{E}_t\sum_{k\geq 0}\chi_i^k\Lambda_{t,t+k}y_{i,t+k}\frac{\partial\, mr_{i,t+k}}{\partial\, p_{i,t}^{x,*}}$$

When marginal revenues are constant, $mr_{i,t+k} = \frac{p_{i,t}^{x,*}}{P_{t+k}^{x}}$ and $\frac{\partial\, mr_{i,t+k}}{\partial\, p_{i,t}^{x,*}} = \frac{1}{P_{t+k}^{x}}$, the above equation collapses to the textbook reset price

$$p_{i,t}^{x,*} = \frac{\epsilon_i^{kim}}{\epsilon_i^{kim} - 1} \frac{\mathbb{E}_t \sum_{k\geq 0} \chi_i^k \Lambda_{t,t+k} y_{i,t+k} P_{t+k}^{x} mc_{i,t+k}}{\mathbb{E}_t \sum_{k\geq 0} \chi_i^k \Lambda_{t,t+k} y_{i,t+k}} \tag{B.14}$$

a discounted weighted average of future nominal marginal costs times the gross Kimball markup $\mathcal{M}_i^{kim} = \frac{\epsilon_i^{kim}}{\epsilon_i^{kim}-1}$. The price index is $\left(p_{i,t}^{x}\right)^{1-\epsilon_i^{kim}} = \chi_i \left(p_{i,t-1}^{x}\right)^{1-\epsilon_i^{kim}} + [1-\chi_i]\left(p_{i,t}^{x,*}\right)^{1-\epsilon_i^{kim}}$.

Step B.14 (The network-seller correction to marginal revenue) *Under CES demand the desired markup is constant and the reset price tracks marginal cost one-for-one. Under the Kimball structure it does not: raising the reset price reallocates sales across destinations and therefore moves marginal revenue. Log-linearising $mr_{i,t+k}$ around the steady state,*

$$\widehat{mr}_{i,t+k} = -\Psi_i \left(\widehat{p}_{i,t}^{x,*} - \widehat{P}_{t+k}^{x}\right) \qquad with \quad \Psi_i \equiv -\frac{\partial \log\left[\mathcal{M}_i\right]}{\partial \log\left[p_i / P\right]}\bigg|_{ss} \tag{B.15}$$

The object Ψ_i is the degree of real rigidity: it measures how much the desired markup falls when the sector prices above its competitors, and therefore how reluctant the sector is to move its own price away from the pack. It is conceptually distinct from the nominal rigidity χ_i, which governs how often prices change; Ψ_i governs by how much they change when they do. Substituting eq. (B.15) into the reset-price condition and collecting the terms in $\widehat{p}_{i,t}^{x,*}$, which do not depend on the horizon-k, delivers a pass-through factor $\frac{1}{1+\Psi_i}$ on marginal costs and a unit coefficient on the price index: real rigidity attenuates the response to the sector's own costs, not its tracking of the aggregate.

What remains is to pin down Ψ_i in this environment. The effective demand elasticity derived above decomposes into two additive components,

$$\epsilon_i^{kim} = \underbrace{\epsilon}_{\text{substitution across goods}} + \underbrace{\frac{v_i\left(1-v_i\right)}{2\,\eta_i^{prim}}}_{\text{reallocation across destinations}}$$

the first invariant to the composition of the customer base, the second entirely determined by it. We measure the primitive curvature in the units in which one unit of dispersion produces one unit of elasticity, $2\,\eta_i^{prim} = 1$ (in other words, the primitive curvature parameter is homogeneous across sectors), and define real rigidity as the ratio of the two components,

$$\Psi_i = \frac{\epsilon}{v_i\left(1-v_i\right)} \qquad \Longrightarrow \qquad \frac{1}{1+\Psi_i} = \frac{v_i\left(1-v_i\right)}{\epsilon + v_i\left(1-v_i\right)} = \frac{v_i\left(1-v_i\right)}{\epsilon_i^{kim}}$$

so that the pass-through of marginal costs into prices equals the share of the total demand response attributable to reallocation along the network rather than to substitution towards competing goods. A sector whose demand responds mainly by shifting between its own destinations retains pricing power over its customer base and passes costs through; a sector whose demand responds mainly by disappearing towards other goods does not, and holds its markup instead.

The numerator is the network-diversification force, which makes price adjustment more effective as the customer base widens; the denominator is the strategic-complementarity force, which makes it more costly as demand becomes more elastic.[1]

Step B.15 (Sectoral Phillips curve) *Log-linearising eq. (B.14) together with eq. (B.15),*

$$\widehat{p}_{i,t}^{x,*} = \frac{1}{1+\Psi_i}\left(1-\beta\chi_i\right)\mathbb{E}_t\sum_{k\geq 0}\left(\beta\chi_i\right)^k\left[\widehat{mc}_{i,t+k}+\widehat{P}_{t+k}^{x}\right]+\frac{\Psi_i}{1+\Psi_i}\widehat{P}_t^{x}$$

and combining with the log-linear price index $\widehat{p}_{i,t}^{x} = \chi_i\widehat{p}_{i,t-1}^{x}+\left[1-\chi_i\right]\widehat{p}_{i,t}^{x,*}$*, the sectoral inflation rate* $\pi_{i,t} = \widehat{p}_{i,t}^{x}-\widehat{p}_{i,t-1}^{x}$ *satisfies*

$$\pi_{i,t} = \beta\mathbb{E}_t\left[\pi_{i,t+1}\right]+\kappa_i mc_{i,t} \tag{B.16}$$

with $\kappa_i = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i}\frac{1}{1+\Psi_i} = \frac{(1-\chi_i)(1-\beta\chi_i)}{\chi_i}\frac{v_i[1-v_i]}{\epsilon_i^{kim}}$.

REMARK 7 *(Inverted-U)* *By the same algebra as eq. (B.12),* $v_i\left[1-v_i\right]$ *attains its maximum at* $v_i = \frac{1}{2}$ *and vanishes as* $v_i \to 0$ *or* $v_i \to 1$*. Since* $v_i\left(\delta\right)$ *is decreasing in the distance from final demand, intermediate values of* v_i *correspond to intermediate values of the upstreamness* $\mathcal{U}_i$ *of Corollary 2. Sectors at either extreme of the supply chain interact with a single demand margin and a narrow customer base; sectors in intermediate positions face both the largest scope for reallocation and the broadest downstream exposure. The slope* κ_i *is therefore inverted-U shaped in proximity to final demand.*

B.7. Equilibrium definition

Step B.16 (Equilibrium) *The equilibrium concept combines the two blocks: given sectoral prices, all markets clear, and the evolution of prices is consistent with Calvo pricing on the network-seller side and with cost minimization on the network-buyer side.*

DEFINITION 4 *(Equilibrium)* *Given sectoral probabilities of price stickiness* $\left\{\chi_i\right\}_{i\in\Phi}$*, sectoral productivities* $\left\{z_{i,t}\right\}_{i\in\Phi}$ *and the nominal interest rate* r_{t+1}*, an equilibrium is a sequence of sectoral prices* $\left\{p_{i,t}^{c}, p_{i,t}^{x}\right\}$*, wages* $\left\{w_{i,t}\right\}$*, labour inputs* $\left\{l_{i,t}\right\}$*, intermediate quantities* $\left\{x_{ij,t}\right\}$*, final demands* $\left\{c_{i,t}\right\}$*, sectoral outputs* $\left\{y_{i,t}\right\}$ *and compositions of sales* $\left\{\phi_{i,t}^{c}\right\}$ *such that: (i) the household maximizes lifetime utility subject to its budget constraint; (ii) network-buyers minimize costs per unit of output; (iii) network-sellers allocate output across destinations according to eq. (5); (iv) a fraction* $1-\chi_i$ *of producers in each sector sets the price that solves eq. (12), while the remaining producers keep their price unchanged; and (v) goods markets clear according to eq. (1) and labour markets clear.*

[1] Two properties follow without further assumptions: $\Psi_i \geq 0$, since $\epsilon > 0$ and $v_i\left(1-v_i\right) > 0$; and $\Psi_i \to \infty$ at both $v_i \to 0$ and $v_i \to 1$, which drives $\kappa_i \to 0$ at the two ends of the supply chain. The normalisation $2\eta_i^{prim} = 1$ is a choice of units: η_i^{prim} is a curvature and converts a share into a semi-elasticity, whereas $v_i\left(1-v_i\right)$ is a pure number bounded by $\frac{1}{4}$, so a conversion factor must be fixed before the two can be combined. It restricts the price sensitivity of the sales split but not its steady-state composition, which remains governed by ς_i. Subsection A.2 shows that the resulting inverted-U survives when real rigidity is specified in the canonical way instead.

The equilibrium concept therefore nests the flexible-price equilibrium, obtained as $\chi_i = 0$ for every sector, and the equilibrium with a fixed composition of sales, obtained as $\sigma_i \to 0$ for every sector, in which condition (iii) pins ϕ_i^c at its steady-state value.

B.8. Derivations for stylized economies

Step B.17 (Preliminaries) *Two facts are used throughout. First, by Definition 3, the stage-distance distribution and proximity to final demand are*

$$\varpi_i(n) = \left[\boldsymbol{\Theta}^n \boldsymbol{\phi}^c\right]_i \qquad \text{and} \qquad v_i = \sum_{n\geq 0} \delta^n \varpi_i(n) = \left[(\boldsymbol{I} - \delta\boldsymbol{\Theta})^{-1}\boldsymbol{\phi}^c\right]_i \tag{B.17}$$

and every network below satisfies Lemma 1, $\boldsymbol{\Theta}\mathbf{1} + \boldsymbol{\phi}^c = \mathbf{1}$. Second, with common Calvo probabilities, $\chi_i = \chi$, and common curvature, $\eta_i^{prim} = \eta^{prim}$, eq. (15) gives $\kappa_i = \mathcal{H}(\eta_i^{eff})$, with $\mathcal{H}(\eta) = \frac{\varrho}{\epsilon}\frac{\eta}{1+\overline{\eta}\,\eta}$, which is strictly increasing by Proposition 2. Since $\eta_i^{eff} = v_i(1-v_i) = \frac{1}{4} - \left(v_i - \frac{1}{2}\right)^2$, comparing slopes across sectors reduces to comparing distances of proximity from one half:

$$\kappa_i > \kappa_j \iff \left|v_i - \tfrac{1}{2}\right| < \left|v_j - \tfrac{1}{2}\right| \tag{B.18}$$

Step B.18 (Roundabout economy) *Let $\boldsymbol{G}$ be row-stochastic, $\boldsymbol{G}\mathbf{1} = \mathbf{1}$, and set $\boldsymbol{\Theta} = (1-\phi)\boldsymbol{G}$ and $\boldsymbol{\phi}^c = \phi\mathbf{1}$. Then $\boldsymbol{\Theta}\mathbf{1} + \boldsymbol{\phi}^c = (1-\phi)\mathbf{1} + \phi\mathbf{1} = \mathbf{1}$, so Lemma 1 holds. Since $\boldsymbol{G}^n\mathbf{1} = \mathbf{1}$ for every stage-n,*

$$\varpi_i(n) = (1-\phi)^n \phi \left[\boldsymbol{G}^n\mathbf{1}\right]_i = \phi(1-\phi)^n$$

a geometric distribution on $n = \{0,\ldots,N\}$ stages, common to all sectors, with mean $\sum_{n\geq 0} n\,\phi(1-\phi)^n = \frac{1-\phi}{\phi}$. This is also $v_i'(1)$ by Proposition 1. Summing the geometric series, which converges because $\delta(1-\phi) < 1$, proximity is, $\forall\, i \in \Phi$,

$$v_i = \phi \sum_{n\geq 0} \left[\delta(1-\phi)\right]^n = \frac{\phi}{1-\delta(1-\phi)}$$

Equivalently, $\mathbf{1}$ is an eigenvector of $\boldsymbol{\Theta}$ with eigenvalue $1-\phi$, so that $(\boldsymbol{I} - \delta\boldsymbol{\Theta})^{-1}\mathbf{1}$ equals $\frac{\mathbf{1}}{1-\delta(1-\phi)}$. Since v_i, χ and η^{prim} are common, η_i^{eff} and $\kappa_i = \mathcal{H}(\eta_i^{eff})$ are common to all sectors.

For what concerns the level of the slope, proximity is strictly increasing in the final share,

$$\frac{\partial v_i}{\partial \phi} = \frac{1-\delta}{\left[1-\delta(1-\phi)\right]^2} > 0$$

so, by eq. (B.18), the common slope is strictly increasing in ϕ as long as $v_i < \frac{1}{2}$ and strictly decreasing afterwards. It is maximal at $v_i = \frac{1}{2}$, that is $2\phi = 1-\delta(1-\phi)$, or

$$\phi^* = \frac{1-\delta}{2-\delta} \qquad \text{and} \qquad \phi^* < \frac{1}{2} \iff 2(1-\delta) < 2-\delta \iff \delta > 0$$

Finally, let us look at the aggregate slope. By $\boldsymbol{y} = \boldsymbol{\lambda} C$, then $\lambda_i = \frac{y_i}{C}$. At the symmetric steady state, $c_i = \phi y_i$ for every sector and $\sum_{i\in\Phi} c_i = C$, because $c_i = \omega_i C$ and $\sum_{i\in\Phi}\omega_i = 1$. Hence $\sum_{i\in\Phi}\lambda_i = \frac{\sum_{i\in\Phi} y_i}{\sum_{i\in\Phi} c_i} = \frac{1}{\phi}$. With a common $\zeta_i = \zeta$, $\tilde{\kappa}_i = \tilde{\kappa}$ is common, $\mathrm{Cov}_\Phi[\lambda_i, \tilde{\kappa}_i] = 0$,

and eq. (18) gives $\kappa^{agg} = \tilde{\kappa} \sum_{i \in \Phi} \lambda_i = \frac{\tilde{\kappa}}{\phi}$.

Step B.19 (Vertical chain) *Let* $\theta_{i,i+1} = 1$ *for* $i < m$*, with all other entries of* $\boldsymbol{\Theta}$ *equal to zero, and* $\boldsymbol{\phi}^c = \boldsymbol{e}_m$*, the* m*-th unit vector. Each row of* $\boldsymbol{\Theta}$ *sums to one for* $i < m$ *and to zero for* $i = m$*, so Lemma 1 holds. Since* $\phi_i^c \in \{0,1\}$ *for every sector, the static dispersion is* $\phi_i^c \phi_i^x = 0$ *for all* i*; by Proposition 1(i), the static evaluation* $\delta = 0$ *gives* $v_i(0) \in \{0,1\}$ *and* $\kappa_i = 0$ *for every sector.*

The matrix $\boldsymbol{\Theta}$ *shifts the unit vectors upstream,* $\boldsymbol{\Theta} \boldsymbol{e}_j = \boldsymbol{e}_{j-1}$ *for* $j > 1$ *and* $\boldsymbol{\Theta} \boldsymbol{e}_1 = \mathbf{0}$*. By induction,* $\boldsymbol{\Theta}^n \boldsymbol{e}_m = \boldsymbol{e}_{m-n}$ *for* $n \leq m-1$ *and* $\boldsymbol{\Theta}^n = \mathbf{0}$ *for* $n \geq m$*, so that*

$$\varpi_i(n) = \begin{cases} 1 & n = m - i \\ 0 & n \neq m - i \end{cases} \qquad \text{and} \qquad v_i = \sum_{n=0}^{m-1} \delta^n \varpi_i(n) = \delta^{m-i}$$

where the series is finite because $\boldsymbol{\Theta}$ *is nilpotent,* $(\boldsymbol{I} - \delta \boldsymbol{\Theta})^{-1} = \sum_{n=0}^{m-1} \delta^n \boldsymbol{\Theta}^n$*. It turns out that the shape along the chain is as follows. Let* $d_i = m - i$ *denote the distance from final demand and treat it as continuous. With* $v = \delta^d$*, then*

$$\frac{\partial \eta^{eff}}{\partial d} = (1 - 2v) \frac{\partial v}{\partial d} = \left(1 - 2\delta^d\right) \delta^d \log[\delta]$$

which is positive for $\delta^d > \frac{1}{2}$ *and negative for* $\delta^d < \frac{1}{2}$*, because* $\log[\delta] < 0$*. Moving upstream from the final-good producer, the slope therefore rises from* $\kappa_m = \mathcal{H}(0) = 0$ *up to the distance*

$$n^* = \frac{\log[2]}{\log[1/\delta]} \tag{B.19}$$

and declines thereafter, converging to zero as $d_i \to \infty$*. On the integer grid of stages, eq. (B.18) implies that the steepest Phillips curve belongs to the sector whose proximity* δ^{m-i} *is closest to one half, which lies at* $\lfloor n^* \rfloor$ *or* $\lceil n^* \rceil$ *stages from final demand.*

Step B.20 (Same participation, different position) *Consider four sectors: a and b, an intermediate-good producer g, and a final-good producer f. The non-zero entries of the network are*

$$\theta_{af} = \theta_{bg} = 1 - \phi \quad , \quad \theta_{gf} = 1 \quad , \quad \phi_a^c = \phi_b^c = \phi \quad , \quad \phi_g^c = 0 \quad , \quad \phi_f^c = 1$$

so every row satisfies Lemma 1. The two sectors have the same direct exposure and the same static dispersion, $\phi_a^c \phi_a^x = \phi_b^c \phi_b^x = \phi(1 - \phi)$*. Applying eq. (B.17):*

$$[\boldsymbol{\Theta} \boldsymbol{\phi}^c]_a = \theta_{af} \phi_f^c = 1 - \phi \qquad [\boldsymbol{\Theta} \boldsymbol{\phi}^c]_b = \theta_{bg} \phi_g^c = 0$$
$$\left[\boldsymbol{\Theta}^2 \boldsymbol{\phi}^c\right]_a = \theta_{af} [\boldsymbol{\Theta} \boldsymbol{\phi}^c]_f = 0 \qquad \left[\boldsymbol{\Theta}^2 \boldsymbol{\phi}^c\right]_b = \theta_{bg} [\boldsymbol{\Theta} \boldsymbol{\phi}^c]_g = (1 - \phi) \theta_{gf} \phi_f^c = 1 - \phi$$

and all higher powers vanish. Hence, ϖ_a *places mass* ϕ *at* $n = 0$ *and* $1 - \phi$ *at* $n = 1$*, while* ϖ_b *places mass* ϕ *at* $n = 0$ *and* $1 - \phi$ *at* $n = 2$*, so that* ϖ_b *first-order stochastically dominates* ϖ_a *and* $v_a \equiv \phi + \delta(1 - \phi) > \phi + \delta^2(1 - \phi) \equiv v_b$*, consistently with Proposition 1. For what concerns the ranking of the slopes, factoring the difference in effective curvatures delivers*

$$\eta_b^{eff} - \eta_a^{eff} = \left(v_a^2 - v_b^2\right) - (v_a - v_b) = (v_a - v_b)(v_a + v_b - 1)$$

Since $v_a > v_b$, the more distant sector-b has the steeper Phillips curve if and only if $v_a + v_b > 1$, that is $2\phi + \delta(1+\delta)(1-\phi) > 1$, or

$$\kappa_b > \kappa_a \quad \Longleftrightarrow \quad \phi > \hat{\phi}(\delta) \equiv \frac{1-\delta(1+\delta)}{2-\delta(1+\delta)} \tag{B.20}$$

where the denominator is positive for every $\delta \in (0,1)$. The threshold $\hat{\phi}(\delta)$ is decreasing in δ and becomes non-positive for $\delta \geq \left(\sqrt{5}-1\right)/2 \approx 0.618$, in which case sector-b has the steeper curve for every ϕ. With $\phi = 0.4$ and $\delta = 0.6$, $v_a = 0.76$, $v_b = 0.616$, $v_a + v_b = 1.376 > 1$ and $\hat{\phi}(0.6) \approx 0.038$, so that $\kappa_b > \kappa_a$.

B.9. Aggregation

Step B.21 (Network-adjusted consumption weights) *In equilibrium, the system for sectoral outputs is $\boldsymbol{y} = (\boldsymbol{I} - \boldsymbol{\mathcal{A}})^{-1}\boldsymbol{c} = \boldsymbol{\mathcal{L}}\boldsymbol{c}$, the network-participation representation of Theorem 1. Substituting the household demand system in vector form, $\boldsymbol{c} = \boldsymbol{\omega} C$,*

$$\boldsymbol{y} = \boldsymbol{\mathcal{L}}\boldsymbol{\omega} C = \boldsymbol{\lambda} C \qquad \text{with} \quad \boldsymbol{\lambda} = \boldsymbol{\mathcal{L}}\boldsymbol{\omega}$$

so that λ_i measures the total (direct plus indirect) contribution of sector-i to aggregate consumption. This is the influence vector of Corollary 5, that is the destination marginal ξ_{ij} of the joint distribution; the sectoral slopes κ_i use the other marginal, $\varpi_i(\cdot)$. Aggregate inflation is defined accordingly as $\pi_t = \boldsymbol{\lambda}'\boldsymbol{\pi}_t$, with $\boldsymbol{\pi}_t = [\pi_{1,t}, \ldots, \pi_{m,t}]'$.

Step B.22 (Aggregate NKPC in marginal costs) *Pre-multiplying eq. (B.16) by $\boldsymbol{\lambda}'$, then*

$$\pi_t = \beta\,\mathbb{E}_t[\pi_{t+1}] + \boldsymbol{\lambda}'(\boldsymbol{\kappa} \odot \boldsymbol{mc}_t) \qquad \Longleftrightarrow \qquad \pi_t = \beta\,\mathbb{E}_t[\pi_{t+1}] + \sum_{i\in\Phi} \lambda_i\,\kappa_i\,mc_{i,t}$$

Step B.23 (Aggregate NKPC in the output gap) *Combining the labour supply schedule eq. (B.2) with the input demands eq. (B.4) and the production function, sectoral marginal costs are proportional to sectoral slack, $mc_{i,t} = \zeta_i g_{i,t}$ with $\zeta_i > 0$. Substituting into eq. (B.16) gives $\pi_{i,t} = \beta\mathbb{E}_t[\pi_{i,t+1}] + \tilde{\kappa}_i g_{i,t}$ with $\tilde{\kappa}_i = \kappa_i\zeta_i$, and aggregating with the same weights,*

$$\pi_t = \beta\mathbb{E}_t[\pi_{t+1}] + \kappa^{agg} g_t^{agg} \qquad \text{with} \quad \kappa^{agg} = \sum_{i\in\Phi} \lambda_i\tilde{\kappa}_i \quad \text{and} \quad g_t^{agg} = \frac{\sum_{i\in\Phi}\lambda_i\tilde{\kappa}_i g_{i,t}}{\sum_{i\in\Phi}\lambda_i\tilde{\kappa}_i}$$

Both the aggregation weights and the notion of slack are endogenous to the production structure: $\boldsymbol{\lambda}$ through the Leontief inverse (vertical channel, normalising by the buyer) and $\tilde{\boldsymbol{\kappa}}$ through v_i (horizontal channel, normalising by the seller). The aggregate Phillips curve therefore uses both representations of Theorem 1 at once, one in the weights and one in the slopes.

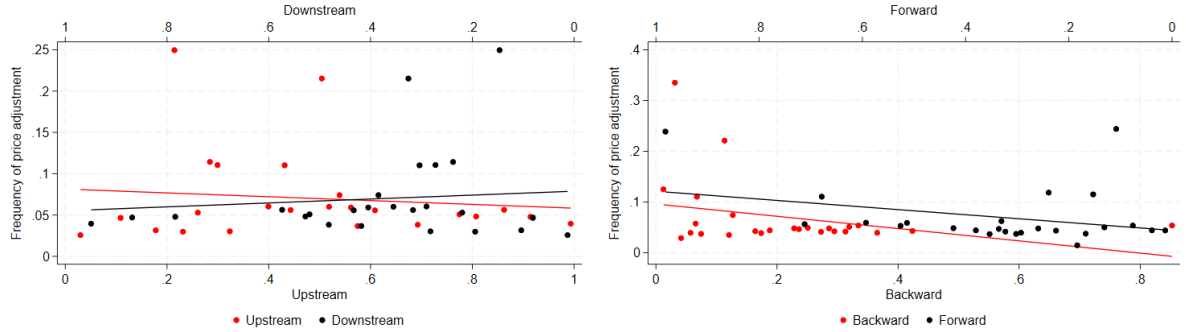
C. Robustness Checks

C.1. Frequency of price adjustments and network measures

In this section we corroborate the results of Section 5.1 by looking at the frequency of price adjustment as dependent variable. In fact we know that, in a standard new Keynesian model, the higher is the frequency of adjustment the steeper is the Phillips Curve (PC). Hence, the

degree of *Proximity* and *Participation* and their respective components should correlate with the frequency of price adjustment in the same way they do with the slope of the PC. To study this, we use the data from Gautier *et al.* (2024), which provides information on price adjustments at the product (COICOP) level for Austria, Belgium, Germany, Greece, Spain, France and Italy between 1995 and 2019. Once using the the COICOP-to-NACE cross-walk of Allayioti *et al.* (2025), we take the average frequency of adjustment within sector and country and relate it with the corresponding sectoral slope of the PC. Figure C.1a shows that sectors in which the level of *Upstreamness* and *Downstreamness* is high are characterized by less frequent price adjustments and a flatter PC. Figure C.1b shows that sectors in which the level of *Backwardness* and *Forwardness* are high exhibit respectively low and high frequency of price adjustments, along with flatter and steeper sectoral PCs. In light of this, we confirm the influence of network characteristics on sectoral supply curves.

FIGURE C.1: DECOMPOSITION OF SECTORAL PROXIMITY AND PARTICIPATION



(A) DOWNSTREAM/UPSTREAM (B) BACKWARD/FORWARD

Notes: Figure C.1a and C.1b (bin)scatter each component of *Proximity* and *Participation* measures – *i.e.*, *Downstreamness*, *Upstreamness*, *Forwardness* and *Backwardness* – on the average sectoral frequency of price adjustments (data from Gautier *et al.* 2024. The number of bins is 50. For variable description and sample definition see Section 4.

C.2. First-stage statistics

In Table C.1 we report the first stage statistics of eq. (23) of Section 5.2, when each endogenous variable is instrument with it own first lag. Along with the F-statistic for each instrumented regressor, we report also the Cregg-Donal F-statistics and Anderson-Rubin Wald F-statistics. Instruments result to be valid.

TABLE C.1: 1ST STAGE STATISTICS - TABLE 2

	(1)	(2)	(3)	(4)	(5)	(6)
	F-stat	F-stat	F-stat	F-stat	F-stat	F-stat
Δy	617.14	575.70	575.70	575.70	575.70	481.12
$\Delta y \times Downstream$			568.19		658.19	475.03
$\Delta y \times Upstream$			665.18		665.18	554.42
$\Delta y \times Forward$				652.04	652.04	544.76
$\Delta y \times Backward$				735.29	735.29	613.30
$\Delta y \times I(Post\ GFC)$						613.30
Cragg-Donald Wald F-stat	617.14	575.70	454.20	575.44	439.40	295.64
Anderson Rubin Wald F-stat	46.95	43.89	43.89	43.89	43.89	36.95

Notes: Table C.2 shows first stage statistics for the instrumentation of the endogenous variables of eq. (23) of Section 5.2. The instruments are the first lag of each endogenous variable.

C.3. Bartik instruments

In this appendix, we explore the validity of our results by using two alternative instruments for the endogenous variables of eq. (23).

Within sector (across countries) leave-one-out shock.– First, for each sector-i and country-j, we build the following shock:

$$shock^1_{i,j,t} = \frac{\sum_{j' \neq j} y_{i,j,t} - \sum_{j' \neq j} y_{i,j,t-1}}{\sum_{j' \neq j} y_{i,j,t-1}} \tag{C.1}$$

which captures the growth of the sector-i outside country-j. Following the procedure described in Section 5.2, we use the lag of $shock^1_{i,j,t}$ and its interaction to instrument the endogenous variables of eq. (23). Column 1 and 2 of Table C.2 shows results respectively when considering all margins and the structural break of the Phillips curve observed after the great financial crisis. The instruments are valid and the evidence of Section 5.2 remains valid.

Within country (across sectors) leave-one-out shock.– Second, for each sector-i and country-j, we build this alternative shock:

$$shock^2_{i,j,t} = \frac{\sum_{i' \neq i} y_{i,j,t} - \sum_{i' \neq i} y_{i,j,t-1}}{\sum_{i' \neq i} y_{i,j,t-1}} \tag{C.2}$$

which captures the growth of country-j at the net of the growth of sector-i (within country-j). Following the procedure described in Section 5.2, we use the lag of $shock^2_{i,j,t}$ and its interaction to instrument the endogenous variables of eq. (23). Column 3 and 4 of Table C.2 shows results respectively when considering all margins and the structural break of the Phillips curve observed after the great financial crisis. The instruments are valid. Despite the sign of each point estimates being consistence with the evidence of Section 5.2, only the $Downstream$, $Upstream$ and $Forward$ margins are significant.

C.4. Discounted output growth as explanatory variable

In this appendix, we follow more closely the literature on the estimation of the regional Phillips Curve. Despite we are using year-on-year output growth as a proxy of sectoral slackness, sectoral output remains very volatile. For this reason, here we consider the discounted sum of four future realizations of output rates as a measure of slack. Similarly to Hazell *et al.* (2022), we define economic slack as $\Delta y_{i,j,t} + \beta \Delta y_{i,j,t+1} + \beta^2 \Delta y_{i,j,t+2} + \beta^3 \Delta y_{i,j,t+3}$. All variables controls and instruments are the same as in Section 5.2. Column 4 and 5 of Table C.2 show results when not including or including the structural break in the Phillips curve in the controls. Also under this definition of slack, results hold.

C.5. The time variation of network characteristics

Here, we explore to which extent muting the time variation of sectoral network characteristics affects the results of Section 5.2. In practice, instead of considering the time-series of each variable *Downstreamness*, *Upstreamness*, *Forwardness* and *Backwardness*, we consider their mean at the country-sector level. Thus, the identifying model is:

$$\pi^{PPI}_{i,j,t} = \alpha_{i,j} + \theta_{(t)} + \theta_{(t \times j)} + \kappa_1 \phi^c y_{i,j,t} + \sum_{x \in \Omega} \kappa_x y_{i,j,t} \times x_{i,j} + W'_{i,j,t-1}\gamma + \varepsilon_{i,j,t} \qquad \text{(C.3)}$$

where all components of the indicator vector for network dimensions, $\Omega = \{Upstreamness; Downstreamness; Forwardness; Backwardness\}$, are time independent. All variables controls and instruments are the same as in Section 5.2. Column 7 and 8 of Table C.2 show results. The components of *Proximity* and *Participation* have similar effects on the slope of the PC as those already found in Section 5.2 even though the estimates of κ_1 are now smaller. This indicates that ignoring/considering the time variation of the explanatory variables *Proximity* and *Participation* reduces/increases the slope of the PC, *i.e.*, the time varying shape of the network is important for the dynamic of the Phillips curve over time.

TABLE C.2: THE SECTORAL PHILLIPS CURVE - ROBUSTNESS CHECKS

	(1) π^{ppi}	(2) π^{ppi}	(3) π^{ppi}	(4) π^{ppi}	(5) π^{ppi}	(6) π^{ppi}	(7) π^{ppi}	(8) π^{ppi}
ΔY	0.4378***	0.6165***	0.1451	0.0901	0.1826***	0.2522***	0.1722***	0.1478***
	(0.0931)	(0.1678)	(0.0952)	(0.1074)	(0.0340)	(0.0475)	(0.0415)	(0.0481)
$\Delta Y \times Downstream$	-0.0949*	-0.1073*	-0.0907**	-0.0878**	-0.0466***	-0.0451**	-0.0441**	-0.0437*
	(0.0560)	(0.0611)	(0.0391)	(0.0390)	(0.0170)	(0.0176)	(0.0225)	(0.0224)
$\Delta Y \times Upstream$	-0.0718**	-0.0712**	-0.0565**	-0.0617**	-0.0405***	-0.0478***	-0.0377***	-0.0369***
	(0.0299)	(0.0306)	(0.0246)	(0.0241)	(0.0118)	(0.0134)	(0.0110)	(0.0112)
$\Delta Y \times Forward$	0.0122***	0.0140***	0.0532***	0.0533***	0.0193***	0.0195***	0.0197***	0.0199***
	(0.0046)	(0.0051)	(0.0094)	(0.0095)	(0.0022)	(0.0023)	(0.0019)	(0.0019)
$\Delta Y \times Backward$	-0.0057***	-0.0054***	-0.0033	-0.0032	-0.0035***	-0.0031***	-0.0043***	-0.0044***
	(0.0015)	(0.0015)	(0.0022)	(0.0022)	(0.0008)	(0.0008)	(0.0009)	(0.0009)
Observations	9907	9907	9907	9907	9399	9399	9907	9907
R^2	0.5781	0.5625	0.4881	0.4871	0.3191	0.2773	0.5850	0.5850
Country-Industry Fe	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time-Fe	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country-Time Fe	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
IV	Bartik	Bartik	Bartik	Bartik	Lags	Lags	Lags	Lags
Other Controls	No	Yes	No	Yes	No	Yes	No	Yes
Cragg-Donald Wald F-stat	36.05	85.64	23.57	19.70	36.18	18.05	421.91	286.07
Anderson Rubin Wald F-stat	22.30	13.30	33.43	27.92	48.86	41.09	48.35	40.79

Notes: Table C.2 shows results of the estimation of eq. (C.3) in Appendix C.5. The dependent variable is PPI inflation, the explanatory variable is the growth rate of sectoral output. The latter is interacted with the sector-country mean components of *Proximity* and *Participation*. The controls are a country-sector fixed effect, a time fixed effect, a time-country fixed effect, the lag of PPI inflation and a structural break in the Phillips curve following the Great Financial Crisis. Endogenous variables are instrumented with their either with a Bartik-type shock or the first lag of endogenous variables. For variable description and sample definition see 4.

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