Is the Swedish neutral interest rate affected by international developments?

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In this study we use a small scale macroeconomic model to estimate a Swedish 'neutral' real interest rate. The word 'neutral' here refers to the interest rate that according to the model is consistent with a closed output gap, which means that one can compare the level of the actual interest rate with that of the neutral interest rate to see whether monetary policy is expansionary or contractionary. In line with other recent international studies, we find that the Swedish neutral real rate has fallen in recent decades and that it is currently negative. Another important conclusion is that the decline in Swedish interest rates over the past couple of decades can be largely explained by the decline in neutral interest rates abroad, where the greatest influence comes from interest rates in the United States.

1 Introduction

Global interest rates are currently very low, while inflation is below target in many countries. However, today's low interest rates can be viewed in the light of a decline in both nominal and real interest rates that has been going on for some decades now. In the United States, the Federal Reserve has recently begun to raise its policy rate, but the interest rate is still unusually low, given the low rate of unemployment. One possible explanation for today's low interest rates is that neutral real interest rates have fallen, a subject which has received considerable international attention in a number of studies (see, for instance, Rachel and Smith, 2015; Williams, 2016; Holston, Laubach and Williams, 2017, and Christensen and Rudebusch, 2017). According to the so-called New Keynesian theory, which is the most common model for monetary policy analysis in the academic literature, the neutral interest rate is usually defined as the interest rate that is neither expansionary nor contractionary when the economy is close to its potential. The concept of a neutral interest rates goes back to the prominent Swedish economist Knut Wicksell (1936), who in his most influential work defined the neutral real interest rate as the interest rate consistent with stable prices and balanced resource utilisation (that is, when production is at its potential level and unemployment is at its equilibrium level). In the popular textbook 'Interest and Prices', Michael Woodford (2003) shows how Wicksell's concept of a neutral real interest rate can be defined and used in modern models. It is therefore not surprising that the former Chair

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of the US Federal Reserve, Janet Yellen, has explicitly referred to the low US neutral interest rate as an explanation for the current unusually low interest rates (see, for instance, Yellen, 2015). In Sweden, the Riksbank has in a number of publications referred to the global decline in interest rates to explain the extraordinarily low Swedish interest rates (see, for instance, Sveriges Riksbank, 2017b).

According to the New Keynesian theory, it is thus necessary to have an estimate of the level of the neutral interest rate to be able to determine whether monetary policy is expansionary or contractionary. When the policy rate is above the neutral level, monetary policy is contractionary, and conversely, the policy rate must be below the neutral level for monetary policy to be expansionary (see, for instance, Giammarioli and Valla, 2004, for an overview). If the neutral interest rate has shown a falling trend, it may therefore imply that the repo rate needs to be set lower to attain the same stimulus to the economy. By estimating the neutral interest rate, we can obtain a quantitative measure of how large a part of the decline in Swedish interest rates is due to monetary policy and how large a part is due to structural factors. According to the estimate we present in this article, monetary policy is currently expansionary in Sweden, but a very low policy rate is needed to make this the case. We also analyse the international influence on the Swedish neutral real interest rate. Our results indicate that the Swedish neutral interest rate is strongly influenced by international neutral interest rates, particularly the US one.

The outline of the article is as follows: In section 2 we describe our model, in section 3 we report the estimation results, in section 4 we analyse the international influences and in section 5 we summarise our conclusions. Details regarding the method of estimation and some associated results have been put into an appendix at the end.

2 The neutral interest rate must be estimated using models

In accordance with the New Keynesian theory we mentioned in the previous section, we assume that there exists an interest level that is consistent with a balanced resource utilisation, but that this 'neutral' level for the interest rate can vary over time. The neutral interest rate cannot be observed directly, which means that it can be difficult to know its level at any given point in time. We therefore use statistical methods to estimate it, in a similar way as we would estimate the potential level of GDP to form an opinion of the size of the output gap.

Laubach and Williams' (2003) method has become something of a standard for estimating the neutral interest rate. Their approach is based on a long-term relationship, which can be derived from the consumers' utility function, between the real neutral interest rate (r^*) and potential growth (g) according to

$$r^* = \frac{1}{\sigma} g + \rho, \quad (1)$$

where σ is the intertemporal elasticity of substitution and ρ measures time preference (the subjective discount rate).[1] As neither the neutral interest rate nor the potential growth rate is observable, one has to make certain theoretical assumptions on how they affect inflation, GDP and the nominal policy rate. In this way, the neutral interest rate is linked to the observed economy.

Our model has the same theoretical starting point as Laubach and Williams' model. However, in line with Berger and Kempa (2014), we make two additions to the basic model. Firstly, we add an exchange rate channel to capture the fact that Sweden is a small, open

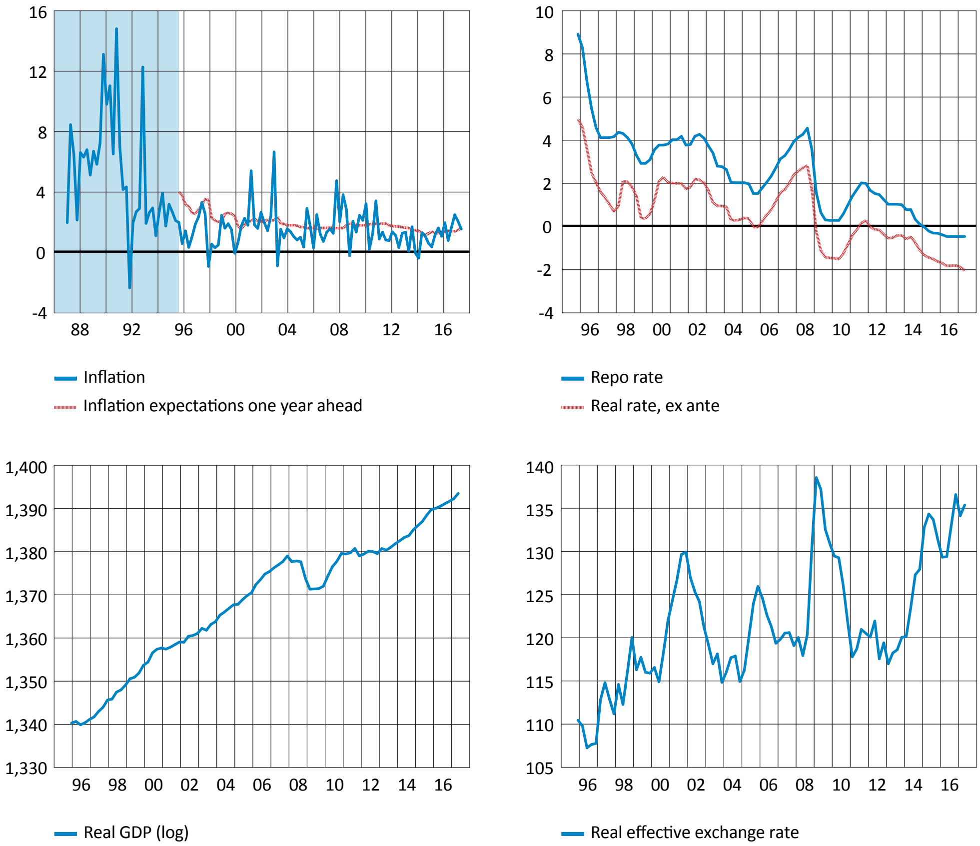
1 We disregard population growth here.

economy. Secondly, we use Bayesian methods of estimation, which can facilitate the identification of links between the model's potential, or unobserved, variables (which are estimated) and the variables that are observable (see, for instance, Pedersen, 2015).

2.1 Data

We use quarterly data for seasonally adjusted real GDP, CPIF inflation[2], the repo rate and the exchange rate index KIX[3]. We define the real interest rate as the nominal repo rate minus expected inflation one year ahead. As in Laubach and Williams (2003), we estimate inflation expectations at a given point in time by the forecast for inflation four quarters ahead from a simple regression on past inflation.[4] The estimation period begins in the last quarter of 1995 and extends to the second quarter of 2017. Data is shown in Figure 1, together with the estimated inflation expectations and the real interest rate.

Figure 1. GDP, exchange rate, repo rate, inflation and estimated inflation expectations



2.2 Model

We assume that real GDP (y_t), the real interest rate (r_t) and the real effective exchange rate (q_t) consist of two components: an equilibrium level (or a *potential* level) that is expressed with an asterisk, and a gap that is expressed with a tilde according to

(2) $$y_t = y_t^* + \tilde{y}_t,$$

(3) $$r_t = r_t^* + \tilde{r}_t,$$

2 The CPIF is the consumer price index with a fixed interest rate, which as of September 2017 is the Riksbank's official target variable, and was also the implicit target variable for some time prior to this (Sveriges Riksbank, 2017a).

3 For descriptions of KIX, see Erlandsson and Markowski (2006) and Alsterlind (2006). We create a real effective exchange rate by deflating KIX with a series which expresses the relative difference between international consumer prices and consumer prices in Sweden, where the international price is a weighted average (using the same weights as is used in constructing KIX) of consumer prices in different countries.

4 The regression is defined according to an AR(3) process with a rolling estimation window of 40 quarters.

(4) $q_t = q_t^* + \tilde{q}_t,$

where y_t^* is potential GDP (in logarithmic form), r_t^* is the neutral real interest rate and q_t^* is the equilibrium exchange rate. Based on equation (1), we assume that there is a relationship between the neutral real interest rate and potential growth g_t (defined in the model as the first difference of y_t^* plus a disturbance term, see equation (8)), but where deviations from this relationship are allowed. The deviations are modelled with another non-observable and time-varying series z_t. The component z_t could consist of factors that affect the interest rate but are not directly linked to domestic potential growth, such as increased global saving, an increase in global demand for safe assets, structural changes in fiscal policy, etc. (see Armelius et al., 2014, Rachel and Smith, 2015, and Bean et al., 2015, for more detailed discussions). In line with Laubach and Williams (2003), we assume the following relationship between the neutral real interest rate r_t^*, potential growth g_t and the component z_t:

(5) $r_t^* = c\, g_{t-1} + z_{t-1},$

(6) $z_t = z_{t-1} + \varepsilon_t^z,$

where c is a parameter and ε_t^z is a disturbance term. The component z_t is thus assumed to follow a process where the change from the previous period is determined by another, independent, random variable. Such a process is usually referred to as a random walk.

As we do not have any strong opinion regarding the equilibrium exchange rate, it is also modelled using a random walk,

(7) $q_t^* = q_{t-1}^* + \varepsilon_t^q,$

where ε_t^q is a disturbance term. Like Laubach and Williams (2003), we also assume that potential GDP follows a trend, but that there can be random disturbances ($\varepsilon_t^{y^*}$ and ε_t^g) both to the level and growth rate according to

(8) $y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*},$

(9) $g_t = (1 - \varphi_2)\varphi_1 + \varphi_2\, g_{t-1} + \varepsilon_t^g,$

where φ_1 and φ_2 are parameters.[5]

The different gaps in the model are assumed to affect each other dynamically. When the output gap $\tilde{y}_t$ is positive, for instance, it is expected to lead to a stronger exchange rate and a higher interest rate. The interaction between the gaps is estimated by a vector autoregressive model (hereinafter referred to as a VAR),

(10) $\tilde{x}_t = \Psi \tilde{x}_{t-1} + \tilde{\varepsilon}_t,$

where $\tilde{x}_t = (\tilde{y}_t, \tilde{r}_t, \tilde{q}_t)'$ is a time series vector of the gaps, Ψ is a 3×3-matrix of parameters that capture the way the variables are dynamically impacted by the previous period's gap and $\tilde{\varepsilon}_t = (\varepsilon_t^{\tilde{y}}, \varepsilon_t^{\tilde{r}}, \varepsilon_t^{\tilde{q}})'$ is a vector of error terms. The first equation in this system can be regarded as a form of dynamic demand curve, or IS curve,

$$\tilde{y}_t = \psi_{11}\tilde{y}_{t-1} + \psi_{12}\tilde{r}_{t-1} + \psi_{13}\tilde{q}_{t-1} + \varepsilon_t^{\tilde{y}},$$

5 This means that log-GDP in level follows a random walk with a stochastic drift g_t, but that the growth rate is stationary. This is also in line with what is usually expected in the euro area (see, for instance, Mésonnier and Renne, 2007). We therefore expect that φ_2 is less than 1 i absolute value, so that g_t is a stationary process with mean φ_1. In Armelius, Solberger and Spånberg (2018) a sensitivity analysis is performed for some different specifications of both g_t and z_t, which lead, for instance, to log-GDP being integrated of second order. The results of the sensitivity analysis suggest that the different specifications are of minor importance in estimating the neutral interest rate.

where the output gap is influenced by the exchange rate gap and the interest rate gap from the previous period.[6] When the policy rate is higher than the neutral interest rate, monetary policy is contractionary, which gradually reduces the output gap. We therefore expect that the output gap will have a negative correlation with lagged values of the interest rate gap. In the same way, a stronger exchange rate should gradually lead to lower exports and thereby also lower output. The neutral interest rate is thus the interest rate that is compatible with a balanced resource utilisation (that is, a closed output gap) where the exchange rate is neither over-valued nor under-valued, in the absence of other shocks.

Finally, we use a Philips curve that describes how inflation and resource utilisation are assumed to be linked,

$$(11) \qquad \pi_t = \delta_1 + \delta_2 \pi_{t-1} + \delta_3 \Delta q^n_{t-1} + \delta_4 \tilde{y}_t + \varepsilon^{\pi}_t,$$

where π_t is inflation at time t, which apart from being backward-looking (that is, it depends on the previous period's inflation) also depends on changes in the nominal exchange rate (q^n_t) and the output gap.[7] Here, the nominal exchange rate captures changes in international prices and thus the contribution from imports. Equation (11) contains several important details. To begin with, inflation should not have any trend if it is firmly anchored around the Riksbank's inflation target. We therefore expect that δ_2 is greater than 0 but less than 1 so that inflation is stationary around the target, but that it also takes some time to bring inflation back to the target when a deviation has occurred. Furthermore, a depreciation of the exchange rate (that is, $\Delta q^n_t > 0$) means that foreign goods become more expensive, which with some time lag will lead to increased inflation in Sweden. We therefore expect that the sign of δ_3 is positive. As economic activity should covary with inflationary pressures, we finally expect δ_4 to be positive, but not particularly large. Note that the exchange rate affects inflation directly in the Phillips curve (11) in nominal terms, and the GDP gap in the VAR system (10) in real terms as the output gap and the interest rate gap are expressed in real terms. Monetary policy, on the other hand, only affects inflation via the output gap in our model.

3 The results show a low neutral Swedish interest rate at present

We estimate the model which consists of equations (2) to (11) using Bayesian methods (see Appendix A). In Bayesian methodology, the user starts out from his or her own priors on what is to be estimated, and then updates these priors using data.[8] Figure 2 shows the means and 90 percent probability intervals for the estimated time series r^*_t, $\tilde{r}_t$, z_t and g_t (see section 2.2). In the upper left-hand panel of Figure 2 it is clear that the Swedish neutral real interest rate has had a declining trend since the middle of the 1990s and that it is currently very low. There are also no signs that the neutral interest rate would have bottomed out and be on the way up at the end of the estimation period. Thus, even though we only use Swedish data in our estimation, we find that the Swedish neutral real interest rate seems to follow the same pattern as a number of studies have shown for neutral interest rates in other countries (see, for instance, Rachel and Smith, 2015; Laubach and Williams, 2016; and Holston, Laubach and Williams, 2017).

If we study the estimated real interest rate gap in the upper right-hand panel of Figure 2, we see that monetary policy according to the model was clearly contractionary during the

6 Our modelling of the gaps differs somewhat from those in Laubach and Williams (2003) and Berger and Kempa (2014). They let the gaps interact more restrictively.

7 Here, Δ denotes the difference operator such that $\Delta x_t = x_t - x_{t-1}$, for a time series x_t.

8 The priors are expressed as probability distributions, which are then updated using Bayes theorem, given the data (see, for instance, Gelman et al., 2013).

years prior to the international financial crisis in 2008, and then turned clearly expansionary after the crisis. When the interest rate was then raised in 2010, the interest rate gap became slightly positive once again. We can also see that, according to our estimates, monetary policy has been expansionary in Sweden since 2014. However, it should be noted that the uncertainty surrounding these estimates is fairly large, which is common for this type of model.[9] This was, for example, one of the main conclusions by Laubach and Williams (2003).

In the lower panels of Figure 2 we can see that the component z_t has a clear downward sloping trend, while potential growth g_t is more stable. In total, the neutral interest rate has fallen from 3 per cent at the end of 1995 to −1.8 per cent at the beginning of 2017. By far the largest part of the decline (corresponding to around 4 percentage points) is explained in the model by the component z_t. The trend fall in the neutral interest rate is thus due to a decline in structural factors that are independent of both monetary policy and domestic potential growth (we will return to this in the next section). This indicates that the Riksbank has been right when claiming in its analysis of Swedish interest rates that the trending decline in the real interest rate is caused by structural global factors (see, for instance, Sveriges Riksbank, 2017b).

Figure 2. Estimated neutral real interest rate, its corresponding gap and components

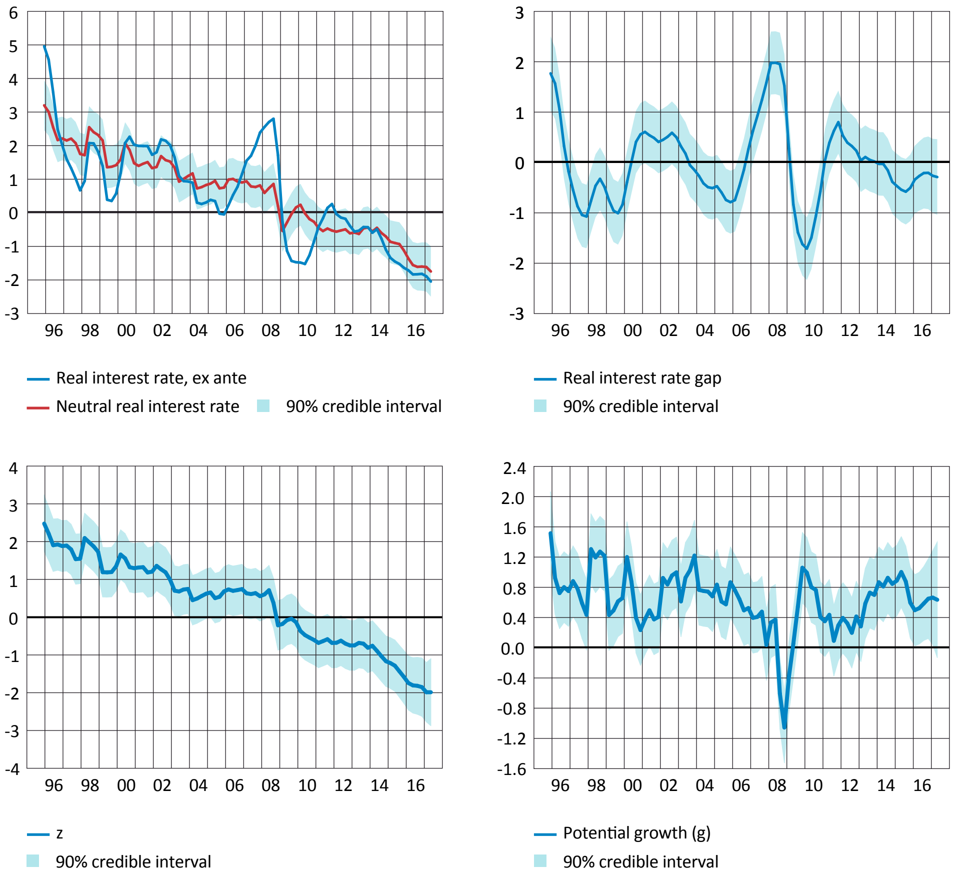


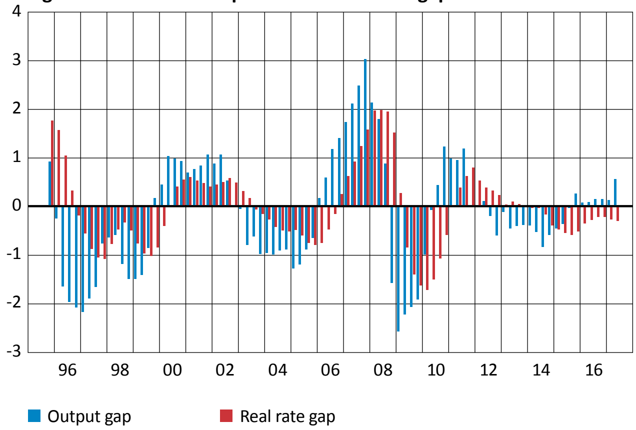
Figure 3 shows how the estimated output gap $\tilde{y}_t$ and the interest rate gap $\tilde{r}_t$ interact according to the dynamics given by the model.[10] When an output gap opens up, the Riksbank responds by changing the direction of monetary policy. This creates an interest rate gap,

9 It is worth pointing out that in the academic literature there are those who think that one should be cautious in using inflation as a good signal of the size of the output gap, which is what we do indirectly in our model. Juselius et al. (2016) and Borio (2017) consider, for instance, that financial imbalances can make the macroeconomic relationships more complicated. According to them, output cannot be considered to be balanced as long as financial imbalances are being built up.

10 Unlike Berger and Kempa (2014), we have not set our priors such that the interest rate gap is assumed to have an effect on the output gap (see Table A1 in Appendix A). We do, however, find such an effect in our estimates (see also the impulse response analysis in Armelius, Solberger and Spånberg, 2018).

which in turn will make production gradually return to its potential. For instance, a severely negative output gap was created during the financial crisis in 2008, which was immediately followed by the previously contractionary interest rate gap instead becoming expansionary. In recent years, both the output and the interest rate gaps have been relatively small according to these estimates. Moreover, as changes in the output gap often seem to occur prior to changes in the interest rate gap, our estimates indicate that monetary policy has not been a major driving force in creating output gaps in Sweden, despite keeping this possibility open in the model. Monetary policy appears rather to have reacted after the output gap was opened for other reasons.[11]

Figure 3. Estimated output and interest rate gaps

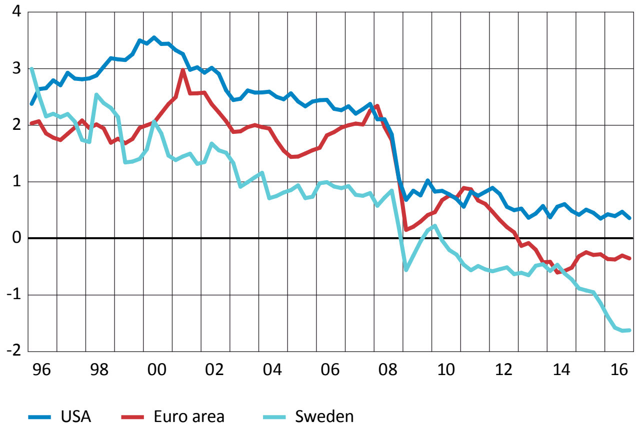


4 International influences are important for Sweden's neutral interest rate

In the previous section, we were able to observe that a large part of the decline in the Swedish neutral real interest rate seems to depend on structural factors that, in our model, are captured by the component z_t. For a small, open economy, it is possible that these factors come from abroad, particularly considering that real interest rates are low in many other advanced economies. Consequently, in this section, we analyse the international influence on the Swedish neutral interest rate. To do this, we use estimated neutral interest rates for the euro area and United States from a new study by Holston, Laubach and Williams (2017).[12] These interest rates are shown in Figure 4, together with our estimate of the Swedish neutral interest rate.

11 Lindé (2003), for example, shows that shocks from abroad can explain a substantial part of the Swedish business cycle.
12 The estimated neutral interest rates for the United States and euro area can be downloaded from John Williams' personal page on the San Francisco Fed's website: http://www.frbsf.org/economic-research/economists/john-williams. It also includes interest rates for Canada and the United Kingdom, but not for Sweden.

Figure 4. Neutral real interest rates in the United States, euro area and Sweden



The neutral interest rates show trends throughout the entire period. Therefore, we cannot rely on simple statistical methods such as correlation analysis or principal component analysis to capture any covariation between the interest rates, as those methods assume that the time series are stationary and move around stable mean values. Instead, the natural choice is to use error correction models, which allow the time series to be characterised by trends.[13] In this article, we use a simple approach based on Engle and Granger (1987). The method proceeds from the following time series regression:

(12) $$r^*_{SE,t} = \beta_0 + \beta_{US} r^*_{US,t} + \beta_{EA} r^*_{EA,t} + \varepsilon^J_t,$$

where β_j are parameters, $r^*_{SE,t}$ is our estimated neutral real interest rate for Sweden, $r^*_{US,t}$ and $r^*_{EA,t}$ are the estimated neutral real interest rates for the United States and euro area respectively and ε^J_t is a disturbance term. Equation (12) can be re-written so that the disturbance term is placed on the left hand side according to

(13) $$\varepsilon^J_t = r^*_{SE,t} - \beta_0 - \beta_{US} r^*_{US,t} - \beta_{EA} r^*_{EA,t},$$

which describes the deviation from a possible equilibrium standardised on the Swedish neutral interest rate. If the neutral interest rates in equation (12) are each non-stationary but the disturbance term ε^J_t is stationary, then the neutral interest rates are said to be *cointegrated*. This would involve at least one of the interest rates adjusting (making an error correction) to deviations from the equilibrium that arises when $\varepsilon^J_t = 0$. Table 1 shows a test to determine whether the neutral interest rates are cointegrated. We see that, using a unit root test, we cannot reject that the neutral interest rates are separately non-stationary, whereas we can reject that the disturbance term is non-stationary. This suggests that the neutral interest rates are cointegrated.[14]

Table 1. Unit root tests

	$r^*_{SE,t}$	$r^*_{US,t}$	$r^*_{EA,t}$	ε^J_t
P value	0.968	0.959	0.810	0.019

Note. The null hypothesis is that the time series has a unit root, which is to say it is non-stationary.

13 As the interest rates themselves are estimates, we should interpret further estimation with some caution. However, we expect that the statistical analysis we perform is valid in sufficiently large samples.

14 We use the Dickey-Fuller test, with critical values that are calculated based on the results in MacKinnon (1996).

Using the equation (13), we can now move on and estimate an error correction equation for the Swedish neutral interest rate according to

(14) $$\Delta r^*_{SE,t} = \mu + \gamma_{SE}\Delta r^*_{SE,t-1} + \gamma_{US}\Delta r^*_{US,t-1} + \gamma_{EA}\Delta r^*_{EA,t-1} + \alpha\varepsilon^J_{t-1} + v_t,$$

where $\Delta r^*_{SE,t}$ is the change in the Swedish neutral real interest rate, μ is a constant, γ_j are short-term parameters that relate to changes in the respective neutral interest rates in the previous period, α is an error correction coefficient that determines how a change in the Swedish neutral interest rate depends on deviations from the equilibrium in the previous period (that is, how the interest rate relates to deviations in ε^J_{t-1} from 0) and v_t is a disturbance term.

Error correction models based on equations (12)–(14) thus estimate a long-term equilibrium between the time series and deviations from the long-term equilibrium in a short-term relationship.[15] Using these models, we can measure how great an impact the time series have on the equilibrium, as well as how rapidly a time series reverts to this equilibrium when there is a deviation.[16] If our hypothesis is correct and the Swedish neutral interest rate has fallen due to factors originating abroad, we should be able to find statistical support for a relationship between the Swedish neutral interest rate and foreign neutral interest rates. Examples of such factors could include an increased global propensity to save or decreased demand for investment in the world economy. We test the relationship between the Swedish neutral interest rate and the neutral interest rates in the euro area – where the greatest share of Swedish exports go – and the United States, as the US dollar is an important currency for international financial flows.

The error correction estimation results are shown in Table 2. The upper part of the table shows a statistically significant relationship in the long run between changes in the US neutral interest rate and changes in the Swedish neutral interest rate. In addition, the lower part of the table shows a short-run significant relationship, in which about 40 per cent of changes in the US neutral interest rate spill over onto the Swedish interest rate in the short term (the estimated γ_{US} is 0.393). The Swedish interest rate also compensates for about one-fifth of deviations in level against the international interest rates each quarter (the estimated α is −0.181). Normally then, after just over one year, the Swedish neutral rate has returned to the level implied by the estimated equilibrium.

15 We only examine the short-term relationship for the Swedish interest rate. A more detailed analysis with short-term relationships also for the international interest rates can be found in Armelius, Solberger and Spånberg (2018), who also consider that there may be more than one equilibrium.

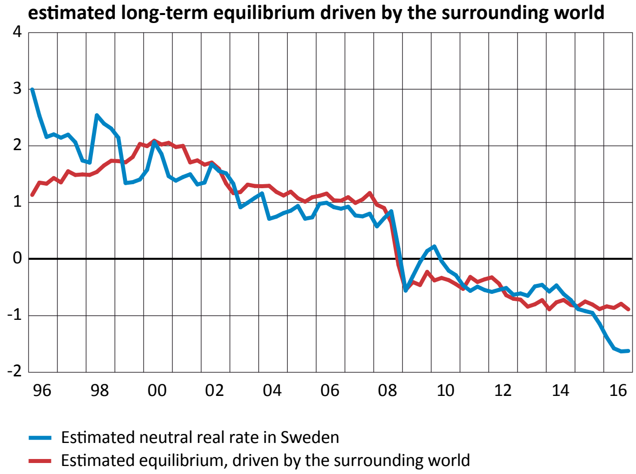
16 The interconnected concepts cointegration and error correction are described in more detail in, for example, Hatanaka (1996). A more accessible description (in Swedish) can be found in Englund, Persson and Teräsvirta (2005).

Table 2. Cointegration analysis of neutral real interest rates

	Parameter	Estimate	Standard error	P value
Cointegrating equation	β_0	−1.127	0.114	0.000
	β_{US}	0.815	0.124	0.000
	β_{EA}	0.158	0.135	0.244
Error correction equation	μ	−0.043	0.048	0.084
	γ_{SE}	0.083	0.109	0.449
	γ_{US}	0.393	0.172	0.025
	γ_{EA}	−0.116	0.142	0.418
	α	−0.181	0.054	0.001

Figure 5 shows the estimated equilibrium from equation (13) together with the estimated Swedish neutral interest rate.[17] We see that the Swedish neutral interest rate since 2015 is slightly lower than what can be explained by the international model. However, the largest part of the decline in the Swedish neutral interest rate is still due to the decline in neutral interest rates internationally.

Figure 5. Estimated Swedish neutral real interest rate against estimated long-term equilibrium driven by the surrounding world



Lastly, and to provide further perspective on how Swedish and international neutral interest rates are connected, we investigate whether there exist Granger causality (Granger, 1969) between the neutral interest rates, that is, whether changes in one of the interest rates precede (and thereby can be used to predict) changes in the other interest rates. For example, we say that the US neutral interest rate *Granger-causes* the Swedish neutral interest rate if at least one of the coefficients $\phi_1, \phi_2, \ldots, \phi_k$ are significantly different from zero in the regression

$$(15) \qquad r^*_{SE,t} = \lambda + \phi_1 r^*_{US,t-1} + \ldots + \phi_k r^*_{US,t-k} + \theta_1 r^*_{EA,t-1} + \ldots + \theta_k r^*_{EA,t-k} + \omega_1 r^*_{SE,t-1} + \ldots + \omega_k r^*_{SE,t-k} + u_t$$

17 The equation (13) describes deviations in the cointegrated system $r^*_{SE,t} - \beta_0 - \beta_{US} r^*_{US,t} - \beta_{EA} r^*_{EA,t}$ from its equilibrium 0. As the relationship is normalised on the Swedish neutral interest rate, we can express the deviations from equilibrium in terms of deviations in the Swedish neutral interest rate $r^*_{SE,t}$ from the time series $m_t = \beta_0 + \beta_{US} r^*_{US,t} + \beta_{EA} r^*_{EA,t}$. The latter time series is therefore shown together with the Swedish neutral interest rate in Figure 5.

where λ is a constant, $\theta_1, \theta_2, \ldots, \theta_k$ are coefficients for lagged values of the neutral interest rate in the euro area, $\omega_1, \omega_2, \ldots, \omega_k$ are coefficients for lagged values of the Swedish neutral interest rate and u_t is a disturbance term.

To take into account that the interest rates are potentially non-stationary, we use the robust methods of Toda and Yamamoto (1995).[18] Their method implies that for instance the null hypothesis that 'the US neutral interest rate does not Granger-cause the Swedish neutral interest rate', that is, that $\phi_1 = \phi_2 = \ldots = \phi_k = 0$ in equation (15), can be tested with a conventional Chi-square test.

The result of this test is shown on the first row of Table 3. As we can see, the US neutral interest rate has a statistically significant influence on the Swedish neutral interest rate. Table 3 also shows tests of a few other Granger causality hypotheses in which, in certain cases, we have exchanged the left-hand series in equation (15) for one of the international neutral interest rates. None of the other null hypotheses in Table 3 can be rejected, which means that we find no statistical support that the euro area's neutral interest rate Granger-causes the Swedish neutral interest rate, or that the Swedish interest rate Granger-causes either of the other two interest rates. It may seem odd that the influence of the euro area's neutral interest rate is not significant in these tests. However, if we study Figure 4, we can see that it has had large fluctuations that did not directly precede fluctuations in the Swedish neutral interest rate. For example, the euro area's neutral interest rate has had a tendency to rise more in the upturns that started around 2000, 2005 and 2009.

Table 3. Granger causality analysis of neutral real interest rates

Null hypothesis	Chi-square statistic	P value
$r^*_{US,t}$ *does not Granger-cause* $r^*_{SE,t}$	8.740	0.003
$r^*_{EA,t}$ *does not Granger-cause* $r^*_{SE,t}$	0.216	0.642
$r^*_{SE,t}$ *does not Granger-cause* $r^*_{US,t}$	0.221	0.638
$r^*_{SE,t}$ *does not Granger-cause* $r^*_{EA,t}$	0.013	0.911

All in all, the analysis in this section indicates that the Swedish neutral interest rate is influenced in both the short and long run by fluctuations in international neutral interest rates, in particular that of the US.

5 Conclusions

In this article we have estimated the Swedish neutral real interest rate in a small scale macroeconomic model using Swedish data. Our results indicate that the Swedish neutral interest rate has had a downward sloping trend in recent decades and that it currently is negative. According to our estimates, a very low repo rate is required at present for monetary policy to be expansionary, which it has been since 2014. The greater part of the decline in the neutral interest rate is explained in the model by components that are independent of both monetary policy and domestic potential growth. The Swedish neutral interest rate has thus followed roughly the same pattern as most studies have found for neutral interest rates in other developed economies.

18 The method involves fitting a VAR model of the interest rates in which an extra lag is used in the estimation, while the Granger causality hypotheses are tested under the ordinary number of lags. By using this method, conventional statistical inference can be performed regardless of whether the series are stationary or non-stationary. We select the number of lags according to the Schwarz information criterion.

We have also investigated whether the decline in the Swedish neutral interest rate can be explained by international relationships. Real interest rates have fallen around the world and, in its communication, the Riksbank usually claims that the decline in the level of Swedish interest rates can be traced to structural factors abroad. We have therefore analysed the international influence by estimating a long-term relationship between our estimated Swedish neutral interest rate and estimated neutral interest rates for the United States and the euro area. Our results indicate that structural factors can explain the greater part of the decline in the level of interest rates in Sweden. We find statistical support for a global influence on the Swedish neutral interest rate in both the long and short run. The Swedish neutral interest rate follows fluctuations in, above all, the US neutral interest rate, while the influence from the euro area is smaller.

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Appendix A

In this appendix, we report prior and posterior distributions of the model's parameters. The equations (2)–(11) in section 2.2 can be written as a state-space model and estimated using the Kalman filter.[19] Our estimation methods, which largely follow Berger and Kempa (2014), are described in detail in Armelius, Solberger and Spånberg (2018). Each disturbance term is assumed to have a variance designated by a sigma, σ_j, in which j takes the associated series notation. For these variance parameters, we use gamma distributions. For the other parameters in the model, we use normal distributions. The prior and posterior distributions are summarised in Table A1.

The prior expected value for the growth coefficient c in equation (5) is set to 4, which approximates a 1-to-1 relationship between the neutral real interest rate and the annual potential growth. In equation (9), the prior expected value for the parameter φ_1 is set to 0.57, which corresponds to an annual equilibrium growth rate of about 2.3 per cent (φ_1 is the mean of the process), and the prior expected value for φ_2 is set to 0.8, in line with a persistent potential growth rate. For all gaps in the VAR system (10), the prior expected values are set to 0.5 for own lags and 0 for remaining lags, so that the gaps are dynamically independent a priori. In the Phillips curve (11), the prior expected value for the slope δ_2 is set to 0.5 and, for the other coefficients, the prior expected values are set so that the 90 per cent intervals cover 0.

The posterior distributions are largely in line with our expectations. For example, the posterior mean values for the coefficients in the Phillips curve (11) have signs that, in advance, may be considered to be reasonable (see section 2.2). The 90 per cent probability intervals are fairly wide. At the same time, the posterior distributions for the disturbance terms' variance parameters are considerably more condensed than their prior distributions, which suggests that data provide valuable information to our model.

19 See, for example, Durbin and Koopman (2012) for a detailed review of state-space models and underlying methods of estimation such as the Kalman filter.

Table A1. Prior and posterior distributions

		Prior distribution		Posterior distribution	
Equation	Parameter	Expected value	90 per cent interval	Expected value	90 per cent interval
Potential production and growth	σ^2_{y*}	0.50	[0.06, 1.28]	0.146	[0.061, 0.304]
	φ_1	0.57	[0.41, 0.73]	0.569	[0.504, 0.634]
	φ_2	0.80	[0.64, 0.96]	0.687	[0.627, 0.746]
	σ^2_g	0.25	[0.11, 0.43]	0.147	[0.102, 0.241]
Neutral interest rate	c	4.00	[2.34,5.65]	0.333	[0.231, 0.441]
	σ^2_z	0.25	[0.11, 0.43]	0.063	[0.048, 0.082]
Equilibrium exchange rate	σ^2_q	0.25	[0.11, 0.43]	0.236	[0.160, 0.346]
Output gap	ψ_{11}	0.50	[0.09, 0.91]	1.011	[0.937, 1.086]
	ψ_{12}	0	[−0.41, 0.41]	−0.389	[−0.492, −0.290]
	ψ_{13}	0	[−0.41, 0.41]	0.011	[0.002, 0.020]
	$\sigma^2_{\tilde{y}}$	0.50	[0.06, 1.28]	0.245	[0.167, 0.354]
Interest rate gap	ψ_{21}	0	[−0.41, 0.41]	0.306	[0.243, 0.370]
	ψ_{22}	0.50	[0.09, 0.91]	0.627	[0.557, 0.697]
	ψ_{23}	0	[−0.41, 0.41]	-0.016	[−0.024, −0.009]
	$\sigma^2_{\tilde{r}}$	0.50	[0.06, 1.28]	0.006	[0.001, 0.029]
Exchange rate gap	ψ_{31}	0	[−0.41, 0.41]	−0.114	[−0.290, 0.073]
	ψ_{32}	0	[−0.41, 0.41]	0.257	[0.095, 0.419]
	ψ_{33}	0.50	[0.09, 0.91]	0.916	[0.882, 0.949]
	$\sigma^2_{\tilde{q}}$	1.00	[0.13, 2.57]	5.727	[5.165, 6.350]
Phillips curve	δ_1	1.00	[0.18, 1.82]	1.298	[1.154, 1.444]
	δ_2	0.50	[0.09, 0.91]	0.120	[0.043, 0.193]
	δ_3	0.25	[−0.16, 0.66]	0.050	[0.012, 0.086]
	δ_4	0.25	[−0.16, 0.66]	0.180	[0.094, 0.272]
	σ^2_π	2.00	[0.68, 3.88]	1.419	[1.277, 1.578]