How can term structure models be used by central banks?

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Abstract

This article provides an overview of recent developments in term structure modeling and its uses by central banks. The topic is important to central banks and policymakers, who are often interested in extracting economic information from long-term interest rates, and elaborating policies to influence them. I review some of the term structure models that allow for time-varying risk premia and that have served as the workhorse models in the analysis of the term structure of interest rates by central banks. These models have been used to measure policy rate expectations, to study the interest rate transmission mechanisms of unconventional monetary policies, to estimate inflation and liquidity risk premia in real government bond markets and to obtain useful policy indicators in an interest rate lower bound environment, such as the shadow rate.

1 Introduction

The term structure of interest rates is the relationship between the interest rates, or yields, on bonds of different maturities that are traded at each point in time. As it describes investors' choices on bonds and interest rates across maturities, the term structure thus carries information about market participants' expectations of future short-term interest rates and future economic conditions, as well as their willingness to bear interest rate risk.

Policymakers are often interested in term structure analysis as they wish to extract economic information from long-term interest rates, and elaborate policies to influence them (see Woodford 1999). The aim of this article is thus to provide an overview of recent developments in term structure modeling and its uses by central banks.

The simplest approach for term structure modeling is the one designed for its estimation. Because available data provide us with an incomplete set of points relating interest rates to maturities, the estimation of term structure curves is often desirable, providing central banks with a continuous set of interest rates that can be used for various purposes.

One important aspect of the standard approaches of term structure modeling, however, is that they are consistent with the expectations hypothesis, which asserts that long-term interest rates are formed from investors' expectations of future short-term interest rates. However, economic theory predicts that investors are typically risk-averse, implying that long-term interest rates may also be driven by the interest rate compensation that investors demand for buying and holding an *n*-year bond until maturity rather than rolling over a short-term interest rate (see Friedman and Savage 1948, Cochrane 2001), a measure that is often called the term premium. I discuss this phenomenon using term structure models that allow for time-varying term premia and discuss why these models are better at capturing many aspects of interest rates that are puzzling from the perspective of the expectations hypothesis.

* I would like to thank Jan Alsterlind, Ulf Söderström, Claes Berg, Ola Melander, David Vestin and David Kjellberg for comments and suggestions. All remaining errors are my own. The opinions expressed in this article are the sole responsibility of the author(s) and should not be interpreted as reflecting the official views of Sveriges Riksbank.

Although the quest for more robust estimates of time-varying term premia is still ongoing, several studies have used term structure models to investigate the transmission mechanisms of bond purchases to interest rates.[1] By analyzing the recent experience of unconventional monetary policy in Sweden, I also discuss how government bond purchases have affected interest rates, by measuring their impacts on short-rate expectations and term premia.

Policymakers are also often interested in measuring market participants' inflation expectations. As markets for inflation-linked securities have grown in recent years, the interest rates on these instruments, in combination with those on nominal government bonds, have become an important source of information on investors' inflation expectations.[2] However, these rates also include inflation and liquidity risk premia that compensate investors for the risk of facing higher inflation rates than they previously expected and for the risk of holding an instrument with low market liquidity. I also review some of the term structure models that have been used to estimate time-varying inflation and liquidity risk premia, in an attempt to obtain a "cleaner" measure of inflation expectations embedded in government bond interest rates.

Finally, in a world where policy interest rates have reached record lows, I also discuss term structure models that have been recently proposed to deal with a situation where the policy interest rate reaches its lower bound (see Wu and Xia, 2016, Bauer and Rudebusch, 2016, among others). Besides allowing for more reasonable estimates of short-rate expectations, these term structure models also allow for the estimation of other informative indicators such as the time to the expected interest rate liftoff, the expected pace of monetary policy tightening and the policy rate that would prevail if the interest rate lower bound did not exist.

The remainder of this article is organized as follows. The next section describes the formation of interest rates in a market economy and the transmission mechanisms of monetary policy to these interest rates. The third section introduces some existing term structure models and describes some of their uses by central banks. The fourth section concludes.

2 The formation of interest rates and the transmission of monetary policy

2.1 Interest rates: basic concepts

The most basic interest rate in fixed income analysis is the interest rate on the default-risk-free zero coupon bond. This security gives the holder SEK 1 at maturity and is priced at discount at time *t*, with no risk of default. More specifically, letting P_t^n denote the price of an *n*-maturity zero-coupon bond at time *t*, bond prices are obtained according to the following,

$$P_t^n = exp\,(-\,n y_t^n\,), \quad (1)$$

where y_t^n is the annualized continuously compounded nominal yield on this bond, i.e. the return the investor will receive at maturity. Similarly, one can solve (1) for y_t^n to obtain

$$y_t^n = -\frac{1}{n}\, ln\,(\,P_t^n\,). \quad (2)$$

1 A number of term structure models have been used for this purpose (see Vayanos and Vila 2009, Christensen and Rudebusch 2012, Greenwood and Vila 2014, Bauer and Rudebusch 2014, among others). This article is focused on the use of no-arbitrage affine term structure models (see Duffie 2001, Singleton 2006 and Piazzesi 2010 for a comprehensive review).

2 As explained later, this is often called the "break-even inflation", i.e. the rate of inflation that would give an investor the same return at maturity on a nominal and a real bond.

The term structure of interest rates, or yield curve, is then a function that maps interest rates and bond prices into maturities at a given point in time. Although the average yield curve is often found to be positively sloped and slightly concave, its shape varies over time, carrying useful information about investors' expectations of the future state of the economy.

Alternatively, one can characterize the term structure of interest rates in terms of forward rates, which is the interest rate the investor would require today to invest in a bond over a period in the future.[3] In that case, the return the investor would receive on that investment is the *n*- to *m*-maturity forward rate, which is given by

(3) $$f_t^{n,m} = \frac{1}{m-n}\,(m \times y_t^m - n \times y_t^n).$$

As the limit of the maturity difference $m - n$ goes to zero, $lim_{m \to n} f_t^{n,m}$, one can then obtain the *n*-maturity instantaneous forward rate, f_t^n, which is the interest rate required today to invest in a bond with the shortest possible maturity at a future point in time, *n*.

One can then construct the relationship between bond yields and forward rates as the following,

(4) $$y_t^n = \frac{1}{n} \int_0^n f_t^i \, di,$$

which simply states that a zero-coupon bond yield is equal to the average of instantaneous forward rates over the lifetime of the bond.

As will be explained later, because short-term interest rates tend to follow very closely the interest rate set by the central bank, from a central bank perspective, forward rates are useful because they allow for a better understanding of the movements in longer-term interest rates caused by factors other than the current policy rate, such as policy rate expectations.

2.2 The expectations hypothesis and the transmission of monetary policy to interest rates

In its strong form, the expectations hypothesis is a proposition that states that investors price bonds as if they were risk-neutral, meaning that they do not care about the level of uncertainty in a long-term investment. This means that long-term bond interest rates are determined by current and future expected short-term interest rates, in such a way that the return on the investment in a long-term bond is the same as the expected return obtained from rolling a short-term interest rate over the lifetime of the same bond.

This hypothesis assumes that the various maturities are perfect substitutes, and suggests that the expectations of future short-term interest rates is the only factor needed to construct a complete term structure, determining its shape at each point in time.[4] However, economic theory predicts that investors have some degree of risk-aversion and are typically concerned about the risk that short-term interest rates do not evolve as expected over the lifetime of the bond. This implies the existence of a gap between long-term interest rates and the average of expected short-term rates. This gap is often called the term premium and serves as a measure of the compensation that investors demand for buying and holding a long-term zero-coupon bond until maturity rather than rolling over a short-term interest rate.

3 The forward rate is the interest rate that makes a risk-neutral investor indifferent to buying and holding a longer-maturity bond until maturity or buying and rolling over a shorter-maturity bond. For instance, an investor can buy a two-year bond and hold it for two years, or he can buy a one-year bond, and then at the end of the first year, buy another one-year bond. Under these two scenarios, the investor knows the interest rates for both the two-year bond and the first one-year bond, but he does not know the actual interest rate for the second one-year bond, because it is an interest rate in the future. In this case, the forward rate is the predicted interest rate on the second one-year bond, which would give the investor the same return under either investment strategy.

4 As is customary in the literature, I am disregarding here the Jensen's inequality term, which is modest at maturities up to ten years when volatility is low.

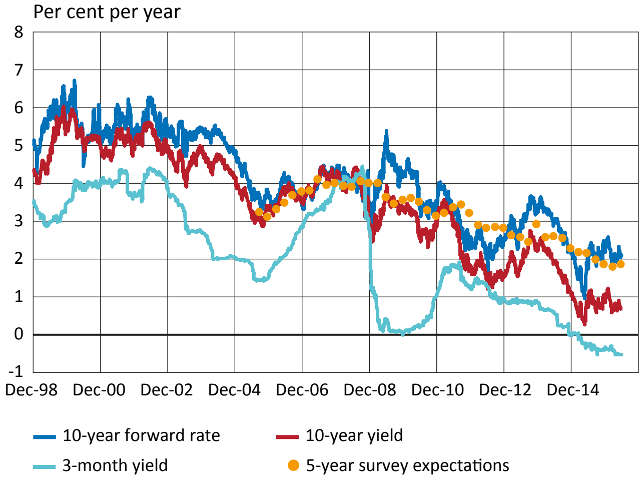
Empirically, the expectations hypothesis has failed to fully explain the behavior of interest rates. Several seminal studies including Fama (1984), Fama and Bliss (1987), Campbell and Shiller (1991), Stambaugh (1988), Cochrane and Piazzesi (2005), among others, have uncovered evidence of non-zero and time-varying risk premia in bond markets, thus violating the expectations hypothesis. Indeed, if the expectations hypothesis was sufficient to explain the term structure, then long-horizon short-rate expectations would typically converge to its steady state.[5] However, the fact that long-term yields and forward rates are highly time-varying is at odds with the expectations hypothesis implying that these may also be driven by time-varying term premia (see Figure 1 for a comparison between long-term yields, forward rates and survey expectations). This has led financial economists to reformulate the determination of interest rates, with equation (4) being rewritten as,

$$(5) \qquad y_t^n = \frac{1}{n} E_t \left(\int_0^n r_{t+1}\, di \right) + tp_t^n,$$

where r_t is the short-term interest rate, $E_t(\times)$ is an expectation operator and tp_t^n is the corresponding term premium. It is interesting to note that if the expectations hypothesis is valid, we then have that $f_t^n = E_t(r_{t+n})$, that is, the n-maturity instantaneous forward rate is the expectation of the short-term rate at time $t + n$, measured at time t.

Notice from (5) that bond yields are directly affected by movements in the short-term interest rate and its expectations. This implies that conventional monetary policy has a direct impact on the term structure of interest rates. In Sweden, the Riksbank implements conventional monetary policy by setting the repo rate and by steering the overnight rate towards this rate through short-term market operations, such as daily fine-tuning transactions and weekly issues of Riksbank certificates.

Figure 1. Bond yields, forward rates and survey expectations
Per cent per year



Note. The yields and forward rates shown were estimated using the Svensson (1994) method. The survey expectations are the average of money market participants' expectations obtained from TNS Sifo Prospera.
Sources: The Riksbank and own calculations

Although changes in the repo rate primarily affect interest rates in the interbank market, government bonds of different maturities are also directly impacted. A cut in the repo rate by the Riksbank commonly leads to a fall in repo rate expectations, which in turn tends to move longer-maturity market rates in the same direction. The Riksbank can also influence repo rate expectations directly by communicating its future monetary policy intentions or by providing forward guidance more directly through its repo rate path, i.e. the Riksbank's own

5 The short-rate steady state may be constant or time-varying, depending on one's underlying (model) assumptions.

repo rate forecasts. Since February 2015, the Riksbank has also purchased nominal and real government bonds of different maturities in the secondary market as a means of lowering longer-maturity interest rates in the economy and providing further monetary stimulus. This unconventional monetary policy is expected to operate by lowering expectations of future repo rates as well as by lowering term premia across maturities, which arises from the reduction in the available supply of the assets purchased.

Changes in the interbank and government bond interest rates for different maturities then tend to impact other borrowing rates for banks, such as interest rates on deposit bank accounts and bonds of mortgage institutions. Changes in banks' borrowing rates in turn affect their lending rates to households and firms, as well as interest rates on corporate debt securities such as commercial paper and corporate bonds.

One can then augment (5) to describe the different interest rates in the economy through the following,

(6) $$\tilde{y}_t^n = \frac{1}{n} E_t \left(\int_0^n r_{t+i} \cdot di \right) + tp_t^n + x_t^n,$$

where x_t^n is anything beyond short-rate expectations and term premia that may affect $\tilde{y}_t^n$ such as credit risk, liquidity risk, banks' profit margins or banks' funding costs. Swedish government bond interest rates are typically free of default and credit risk and are then determined by repo rate expectations and term premium only.[6] The other interest rates in the economy typically embed some liquidity and credit risks.

3 The uses of term structure models by central banks

Term structure models are important tools that central banks use to describe and better understand the behavior of interest rates. In this section, I describe the various uses of term structure models by central banks. These range from simple curve fitting techniques to models that deal with more complex issues such as the decomposition of interest rates into short-rate expectations and their various premiums.

3.1 Term structure estimation

Term structure estimation is a benchmark in the analyses of the interest rate behavior. The issue is that available data commonly provide us with an incomplete set of points relating interest rates to maturities. However, obtaining continuous, interpolated term structure curves is often desirable, and this is what constitutes term structure estimation, or yield curve fitting.

The literature on term structure estimation can be divided into parametric and nonparametric methods. Parametric methods, which have the Nelson and Siegel (1987) and the Svensson (1994) models as their flagship, have at least two reasons for their popularity. First, they are relatively easy to estimate. In fact, if some of their parameters are assumed to be fixed over time, they can be estimated by simple linear regression techniques.[7] If not, one has to resort to non-linear regression methods. Second, their functional forms impose more smoothness on the shapes of the estimated curves, as desirable by macroeconomists and many central banks (see Gürkaynak et al. 2007).

6 For practical purposes, I assume in this article that government bonds are free of credit risk. However, it is important to note that sovereign credit risk is not negligible in some countries, being an important source of determination of interest rates on government bonds.

7 Typically, one can estimate the Nelson and Siegel (1987) and the Svensson (1994) models using linear regressions by simply assuming that the decay parameters in their exponential terms are constant over time.

However, parametric methods are not immune to problems. For instance, they do not impose the presumably desirable theoretical restriction of absence of arbitrage across maturities (Filipovic 1999 and Diebold et al. 2005) and face some problems in fitting more flexible curves and curves with long maturity spectrums.

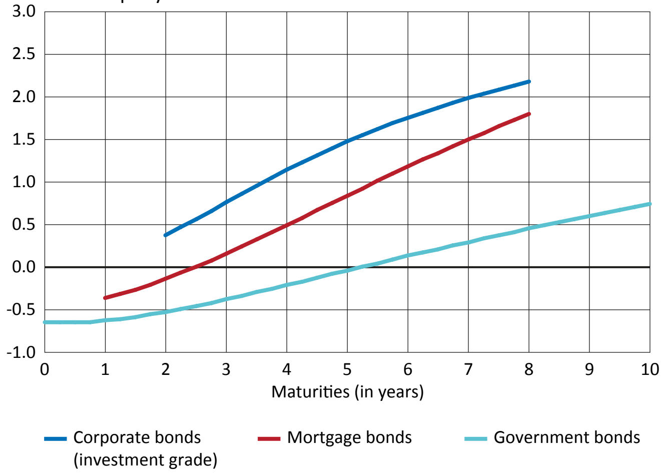
On the other hand, nonparametric methods, which have the spline methods of McCulloch (1971, 1975), Vasicek and Fong (1982) and Fisher et al. (1995) as their flagship, do not assume a particular functional form, being more robust to misspecification and exhibiting greater flexibility by fitting all kinds of term structure curves with very small fitting errors. The greater flexibility, however, comes at a cost. These methods tend to exhibit greater instability in fitting the shorter and longer-term maturities of the term structure, and their estimation typically involves a large number of parameters. Another problem is that the location and the number of interpolation points in the maturity space must be typically chosen before estimation.

Hence, when one must decide what estimation method to use, one is basically confronted by the issue of how much flexibility to allow in the term structure estimation. If a nonparametric method is chosen, a very flexible curve could be estimated, but it would be done with considerable variability in yields and forward rates. On the other hand, through parametric methods, more smoothness could be imposed on the shapes of the term structure, while some of the fit would be sacrificed. The choice in this dimension depends on the purpose that the curves are intended to serve. A trader looking for small pricing anomalies may be very concerned with how a specific security is priced relative to those securities immediately around it and would, probably, choose the more flexible method to estimate the term structure curve. By contrast, a macroeconomist may be more interested in measuring monetary policy expectations through the forward curve or in understanding the fundamental determinants of the yield curve, preferring a greater degree of smoothness. The BIS (2005) states that out of the thirteen main central banks of the world, at least nine use the parametric methods of Nelson and Siegel (1987) and Svensson (1994) with the Svensson (1994) method being the most popular one. The other typical methods used are the smoothing spline method proposed by Fisher et al. (1995) and the variable roughness penalty method that is used by the Bank of England.

The Riksbank uses the Svensson (1994) method to estimate daily term structure curves for a number of debt securities, including government bonds, mortgage bonds and corporate bonds. Figure 2 shows estimated term structure curves for these assets. Notice that the government bond curve has the lowest interest rates, followed by mortgage bonds and corporate bonds. This has to do with the fact that government bonds have typically lower credit risk and are more liquid than the other securities.

Figure 2. Term structure of interest rates for different asset classes (May 26, 2016)

Per cent per year

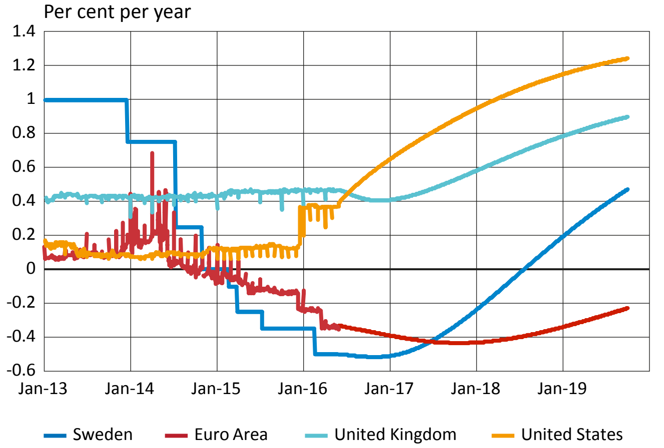


Note. The yield curves shown were estimated using the Svensson (1994) method.
Source: The Riksbank

In order to obtain a measure of market participants' expectations of the repo rate in the future, the Riksbank also estimates smoothed forward curves on FRA (Forward Rate Agreements) and RIBA (Riksbank Futures) contracts' interest rates. These types of instruments have been popular among central banks in the last years mainly due to their availability in high frequencies as well as their good predictive power regarding future central bank actions in the near term (see Gürkaynak et al. 2007). Besides estimating forward curves for Sweden, the Riksbank also estimates daily forward curves for the US, the UK and the Euro Area (see Figure 3) in order to track market participants' expectations of future policy rates in these economies.

Figure 3. Forward curves on interest rate forwards and futures (May 26, 2016)

Per cent per year



Note. The forward curves shown were estimated using the Svensson (1994) method.
Source: The Riksbank

3.2 Decomposing government bond interest rates into short-rate expectations and term premia

Although the ordinary term structure estimation methods described above have the advantage of being relatively simple to handle and estimate, they do not allow for the decomposition of interest rates into short-rate expectations and term premia, and are, therefore, consistent with the expectations hypothesis.[8] For instance, it is not uncommon to assume that the forward rates calculated from these methods are a pure measure of short-rate expectations, as term premia are thought to be constant and/or equal to zero. However, as explained above, empirical research has shown that the expectations hypothesis has failed to explain the behavior of interest rates in several bond markets, which has led researchers to develop more theoretically founded methods to deal with this issue.

Affine term structure models (ATSM henceforth) provide an alternative to the common term structure estimation methods and have become enormously popular among central banks in the last ten years. This class of models (ATSM) encompasses the pure expectations hypothesis but also allows for a tractable and structured way of modeling constant as well as time varying term premia. By imposing the desirable theoretical restriction of absence of arbitrage across maturities, ATSMs allow for a convenient decomposition of government bond interest rates into the average of short-term interest rate expectations and a corresponding time-varying term premium. Through this decomposition, central banks are able to better understand the behavior of interest rates over time as well as to study the transmission of monetary policy to interest rates more directly. Furthermore, obtaining more sensible measures of short-rate expectations is crucial, as interest rate expectations are an important input for central banks' macroeconomic models in which private agents' decisions about consumption, investment, labor supply and price-setting are driven by the current policy rate as well as its expectations.

The literature on ATSMs is vast and covers a large range of models. I discuss here some of the models that have been used by central banks more recently. They differ mainly according to the estimation method and the number and type of variables, or factors, included in the model specification.

The first model is the one proposed by Kim and Wright (2005), which is one of the ATSMs estimated by the Federal Reserve Board staff. Its main distinct feature is the assumption that the behavior of any *n*-maturity yield and the corresponding short-rate expectations and term premium components are driven by three latent factors that are filtered from yields within the model estimation. This model has been quite popular among central banks and has been used by the Federal Reserve Board staff for many years, serving as a benchmark for several other studies.[9]

The second model is proposed by Joslin et al. (2011). Its main innovation is the inclusion of factors that are observables, and that can be linear combinations of yields, such as its three or four first principal components, or even the yields themselves. Moreover, part of the parameters of the model can be estimated by ordinary least squares (OLS), which facilitates the model estimation enormously, helping to solve one of the most serious problems with ATSMs (see Ang and Piazzesi 2003).

Interest rates tend, however, to be very persistent, meaning that typical data samples used in dynamic term structure estimation may be too short to capture a sufficient number of interest rate cycles. This induces the appearance of the problem of small-sample bias that may arise in the estimation of ATSMs and that affects the decomposition of yields into short-rate expectations and term premia (see Kim and Orphanides 2012 and Bauer et al. 2012, 2014).

Several studies have then proposed ways to get around this problem. For instance, Kim and Orphanides (2012) propose a way of providing additional relevant information to the Kim and Wright (2005) model by incorporating information from surveys of financial market participants about short-term interest rate forecasts. The basic idea is that the additional information on short-rate expectations can help in the estimation of more precise

8 Moreover, they have no clear foundation on economic and financial theory. For instance, they allow for arbitrage opportunities across interest rates of different maturities (see Christensen et al. 2009 and Christensen et al. 2012).

9 The Federal Reserve Board makes available daily estimates from the model. The estimates can be downloaded from https://www.federalreserve.gov/pubs/feds/2005/200533/200533abs.html.

parameters, delivering more realistic estimates of the short-rate expectations and term premia components.

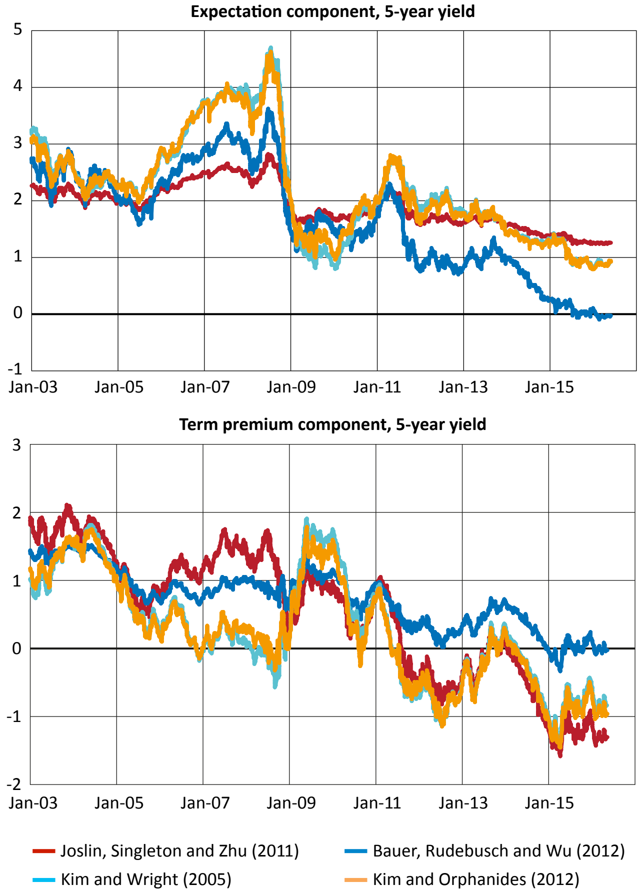
Another attempt to solve the small-sample bias problem is provided by Bauer et al. (2012), who propose a number of simulation-based methods that can be applied to the Joslin et al. (2011) for example. The idea behind their approach is to correct for the bias that tends to underestimate the interest rate persistence in ATSMs so that short-rate expectations converge more slowly to their sample mean than in non-bias-corrected models. This may deliver estimates of term premia and short-rate expectations that are more consistent with economic theory (see Bauer et al. 2012, 2014).

Figure 4 shows estimates of the short-rate expectations and term premium components for the five-year Swedish government bond yield. These are obtained from the four ATSMs discussed above. The Kim and Orphanides (2012) model is enriched with monthly repo rate expectations of money market participants obtained from surveys.[10] Notice that the Kim and Wright (2005) and the Kim and Orphanides (2012) model deliver similar estimates of the five-year yield decomposition, suggesting that the survey expectations do not provide much information to the Kim and Wright (2005) model. As noted by Bauer et al. (2012) results also suggest that more variation is attributed to the expectation component of the five-year yield after applying the bias-correction method to the Joslin et al. (2011) model. Interestingly, in this case, the five-year expectation component is much lower than for the other models at the end of the sample. This can be explained by the higher interest rate persistence captured by the Bauer et al. (2012) model, which induce short-rate forecasts to revert to its sample mean at a much slower speed.

From Figure 4 we also observe that most models deliver estimates of the short-rate expectations and term premium components that both contribute to the decline in the five-year yield, with the declines in term premium being, in general, more pronounced. Notice also that the five-year term premium has been low and even negative in more recent periods, according to most models.

10 These are measured by TNS Sifo Prospera.

Figure 4. Decompositions of the five-year Swedish government bond yield into the average of short-rate expectations and the associated term premium
Per cent per year



Note. The estimates of short-rate expectations and term premium components were obtained using the affine term structure models of Kim and Wright (2005), Kim and Orphanides (2012), Joslin et al. (2011) and Bauer et al. (2012).
Sources: The Riksbank and own calculations

There are at least four possible explanations for why long-term term premia have been compressed in Sweden. The first is the low inflation environment in Sweden, Europe and the United States observed since late 2013, which has led bondholders to be willing to accept less compensation for bearing inflation risk.[11] Another important factor is the low uncertainty about the near-term outlook for policy rates in Sweden and major economies. The low inflation environment increases the likelihood that policy rates around the world will remain low for some time, lowering uncertainty about future policy rates and helping to compress term premia in long-term yields. It is likely that the zero-lower bound in the US policy rate also contributed to lowering uncertainty about future policy rates in the US, as investors were quite sure that the Fed would keep the fed funds rate at zero for some time. Another possible explanation for the observed decline in Swedish government bond term premia is the bond purchases by the Riksbank (see De Rezende 2016), in Europe, Japan and elsewhere. It is likely that bond purchases in foreign economies have possibly caused a “spillover” effect into the demand for Swedish bonds, pushing down their term premia. And lastly, it is important to note that government bonds typically work as a hedge against different types of risk that may hurt returns on other riskier assets, and may be especially demanded by certain institutional investors due to liquidity and regulatory reasons. Investors may then be willing

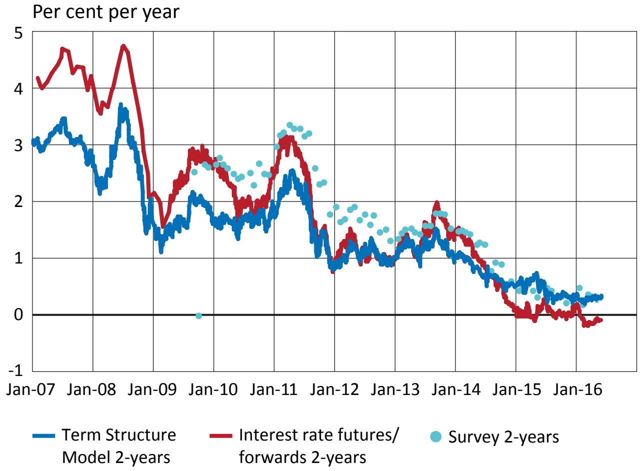
11 Historically, the most important risk for long-term bondholders has been the risk of unexpected inflation increases, as they deteriorate the returns associated with a nominal bond.

to accept low or even negative compensation for holding long-term government bonds, which helps to explain why term premia have been negative more recently.

Although term structure models have been quite popular in the last ten years, central banks have also used two other methods for measuring policy rate expectations. One first common method is the use of interest rate futures and forwards. Besides being considered good predictors of future central bank actions, its main distinctive feature is its availability in high frequencies, providing central banks with information about investors' expectations at any point in time. Its main drawback, however, is that interest rate futures and forwards are not free of risk premia, tending to overestimate – or underestimate in some cases – the right policy rate expectations (see Piazzesi and Swanson 2008). Another common method is the use of surveys, which have been especially popular for being clean from the risk premia that plague financial market instruments. The main drawback of surveys, however, is their availability in low frequencies. In addition, they may be subject to measurement error due to the typical availability of different respondents at each time they are conducted, which may bias the estimates of policy rate expectations such as the consensus forecast.[12]

Figure 5 shows measures of repo rate expectations for the two-year horizon. They were obtained from surveys, interest rate futures and forwards, and from affine term structure models. Notice that although the three measures are similar in terms of dynamics, they seem to differ in terms of levels. For instance, the term structure model predicts the repo rate to be lower than the estimates of interest rate futures for the period before mid-2014 and higher from 2015. This is expected since forward premia were mostly positive before 2014, turning negative afterwards (see Figure 4). Notice also that surveys deliver the highest estimates of repo rate expectations before 2015, but gets quite close to the affine models afterwards.

Figure 5. Measures of repo rate expectations, 2-year horizon
Per cent per year



Note. The survey expectations are the average of money market participants' expectations obtained from TNS Sifo Prospera. The interest rate futures/forwards were obtained using the Svensson (1994) model and data of RIBA and FRA contracts. The term structure expectations were obtained from the average of estimates from two term structure models: Joslin et al. (2011) and Bauer et al. (2012).
Sources: The Riksbank and own calculations

3.3 Studying the interest rate transmission mechanisms of unconventional monetary policies

In the aftermath of the global financial crisis of 2008, and in the face of deteriorating economic conditions and deflationary pressures, a number of central banks reduced their

12 The consensus forecast is typically the mean or the median of individual forecasts.

policy interest rates to their effective lower bounds. With limited room for further rate cuts, central banks have then taken actions to lower longer-term interest rates mainly by purchasing large amounts of government debt and other types of assets, and by providing forward guidance.

The Riksbank has been implementing unconventional monetary policy through the purchase of nominal and real government bonds. With the slower than expected recovery in foreign economies and the considerable downward pressure on Swedish consumer prices, in February 2015, the Executive Board of the Riksbank announced that the Riksbank would start buying nominal government bonds with maturities of up to five years on the secondary market to the amount of SEK 10 billion. The purchases took place by means of auctions in which the Riksbank's monetary policy counterparties and the Swedish National Debt Office's primary dealers were able to participate. Later on, further monetary policy easing continued to be desirable, in particular because of concerns about the strengthening of the Swedish krona (SEK), and the Riksbank announced further extensions of its bond purchase program. At the same time, the repo rate was gradually lowered, reaching the level of –0.50 per cent in February 2016. The Riksbank has also published its projected repo rate path since 2007 as a way to inform the public about its future monetary policy intentions. Table 1 shows a description of the Riksbank's monetary policy announcements in the period ranging from February 2015 to April 2016.

Table 1. Riksbank's monetary policy announcements from February 2015 to April 2016

Date	Announcement description
Feb 12, 2015	Riksbank cuts repo rate to −0.10 percent, buys government bonds for SEK 10 billion and is prepared to do more at short notice
Mar 18, 2015	Riksbank cuts repo rate to −0.25 percent and buys government bonds for SEK 30 billion
Apr 29, 2015	Riksbank buys government bonds for SEK 40-50 billion and lowers the repo-rate path significantly
Jul 2, 2015	Repo rate cut to −0.35 percent and purchases of government bonds extended by SEK 45 billion
Sep 3, 2015	Repo rate unchanged at −0.35 per cent
Oct 28, 2015	The Riksbank purchases government bonds for a further SEK 65 billion and keep the repo rate at −0.35 per cent for a longer time
Dec 15, 2015	Repo rate unchanged at –0.35 per cent – still highly prepared to act
Feb 11, 2016	Repo rate cut to –0.50 per cent
Apr 21, 2016	Riksbank to purchase government bonds for a further SEK 45 billion and repo rate held unchanged at –0.50 per cent

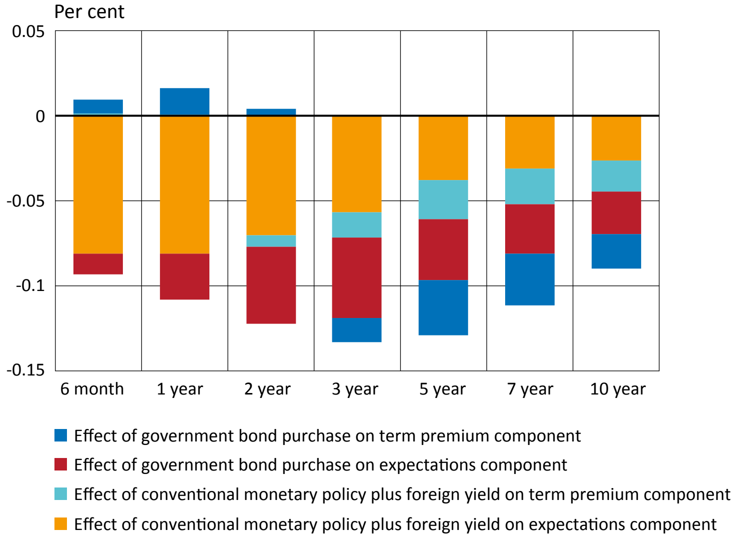
The reasoning behind these policies lies in their transmission to interest rates. For instance, by announcing asset purchases, central banks may send a signal to market participants that they intend to keep policy rates low for longer than otherwise, lowering the expected path of future policy rates and, consequently, long-term interest rates. This is the signaling channel of government bond purchases, which works through changing expectations of future policy rates. The other is the portfolio balance channel, which arises from the reduction in the available supply of the assets purchased. In this channel, under the assumption that bonds of different maturities are not perfect substitutes and that maturity-specific bond demands by certain investors exist (see Vayanos and Vila 2009), central banks may be able to affect bond yields by changing the risk premia that investors require for holding the securities purchased. Central banks may also influence market expectations by communicating their future monetary policy intentions and by providing forward guidance about their future policy rate path.

While it is widely accepted that asset purchases have helped to reduce long-term interest rates, the understanding of their interest rate transmission channels is still partial and has become an important topic in this literature. For instance, using data for the US, Gagnon et al. (2011) argue that the Federal Reserve's Large Scale Asset Purchases primarily lowered long-term government bond rates through the portfolio balance channel. This is also emphasized by D'amico and King (2013). On the other hand, Krishnamurthy and VissingJorgensen (2011), Christensen and Rudebusch (2012) and Bauer and Rudebusch (2014) discuss that the signaling channel was the main driver of the observed fall in the US long-term interest rates. Using ATSMs together with event study regressions De Rezende (2016) shows that government bond purchases have had important portfolio balance and signaling effects in Sweden, which seem to operate by mainly lowering intermediate maturity short-rate expectations and longer-maturity term premia. In addition, De Rezende (2016) discusses that the Riksbank was effective in lowering government bond yields across the full yield maturity spectrum when implementing conventional and unconventional policies together.

The monetary policy announcement made by the Riksbank on July 2, 2015 is a good example of how conventional and unconventional policies seem to work and interact. On that day, the decisions to cut the repo rate by 10 basis point and to purchase government bonds for a further SEK 45 billion were largely unexpected by market participants. The surprise regarding the interest rate cut affected short-rate expectations strongly, driving the fall observed in short-term government bond yields. At the same time, bond purchases contributed, to a large extent, to lowering the short-rate expectations and term premia components in the two-year to five-year and in the five-year to ten-year segments of the yield curve, respectively, suggesting that both the signaling and the portfolio balance channels seemed to have contributed to the fall in mid- and long-term yields (see Figure 6 and De Rezende 2016 for more details).

Figure 6. Effects of the monetary policy announcement of July 2, 2015 on government bond interest rates and its components

Per cent



Notes. the effects of bond purchase announcements on term premia and the average of short-rate expectations are computed using an event study regression approach (please see De Rezende 2016 for more details).
Sources: The Riksbank and own calculations

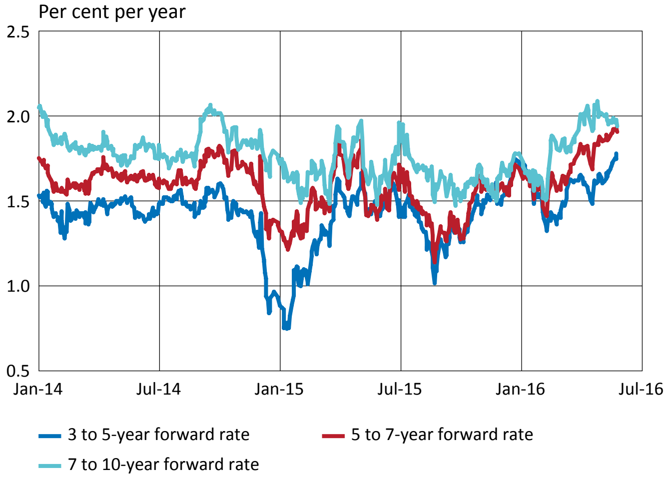
3.4 Measuring inflation expectations

Markets for inflation-protected debt securities have grown dramatically in recent years. The idea behind their issuance is to provide investors with the possibility of eliminating inflation risks in fixed-income investments while providing a real rate of return guaranteed by governments. Interestingly, the interest rates on these securities, when used in combination with those of nominal bonds, have allowed central banks to compute measures of investors' expectations of future inflation. This is often called the "break-even inflation", i.e. the rate of inflation that would give an investor the same return at maturity on a nominal and a real bond. However, as for nominal bonds, real bond issues only happen for particular maturities and coupon rates, meaning that it is not possible to get measures of inflation expectations directly from these issues. As for nominal bonds, central banks have then used term structure models to obtain interpolated real term structure curves that can be used to obtain measures of inflation expectations for any horizon.

The Riksbank estimates real term structure curves daily using inflation-linked securities issued by the Swedish National Debt Office and computes different measures of break-even inflation. Figure 7 shows forward break-even inflation rates for the period from January 2014 to July 2016.

Figure 7. Forward break-even inflation rates for Sweden

Per cent per year



Note. The forward breaks-even inflation rates shown were computed using the difference between nominal and real forward rates. These were estimated using the Svensson (1994) method.
Sources: The Riksbank and own calculations

3.5 Other uses

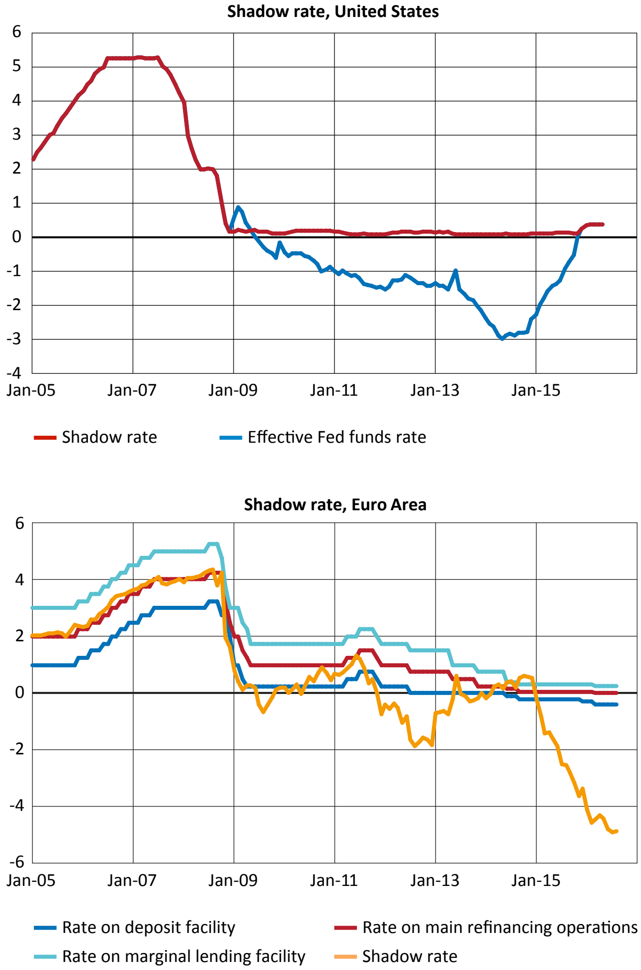
As discussed above, as markets for inflation-protected securities have grown in recent years, the interest rates on these instruments have been used by central banks as an important source of information about investors' expectations of future inflation. Unfortunately, these rates also include risk premia that compensate investors for inflation risk, which may add noise in break-even inflation rates. In an attempt to obtain a "cleaner" measure of the inflation expectations embedded in nominal and real government bond interest rates, some studies have then used term structure models to estimate time-varying inflation risk premia present in break-even inflation rates. Typical models in this literature were developed by Christensen et al. (2010), Joyce et al. (2009), García and Werner (2010), Abrahams et al. (2015), among others.

Another typical problem with inflation-linked bonds is the lack of liquidity in certain markets and in specific periods of time. As discussed by Sack and Elsasser (2004), Shen (2006), Pflueger and Viceira (2011), among others, this induces the appearance of liquidity risk premia on inflation-linked bonds' interest rates, which may distort the measures of break-even inflation commonly used by central banks. Some articles have then proposed term structure models to get around this problem by estimating the liquidity risk premia in these markets and using them together with estimates of inflation risk premia to obtain more reasonable measures of investors' inflation expectations. For instance, D'Amico et al. (2010) show that ignoring the liquidity premia in the US index-linked bond market produces large pricing errors for these securities. Abrahams et al. (2015) shows that adjusting break-even rates for inflation and liquidity risks substantially improves forecasts of US inflation. Haubrich et al. (2012) suggests that the US index-linked bonds were significantly underpriced prior to 2004 and again during the 2008-2009 financial crisis, with the lack of liquidity being one of the possible explanations for this phenomenon.

As the policy rate approaches its lower bound, standard ATSMs may lose their ability to fit short-term interest rates, generate point and distributional short-rate forecasts, and extract accurate policy rate expectations. A modified version of the more common ATSMs has then been proposed to deal with these situations. These are the so-called shadow-rate term structure models, which have been popularized by Wu and Xia (2016), Bauer and Rudebusch (2016), Krippner (2012), among others. Besides allowing for the estimation of more reasonable short-rate expectations, these models also allow for the estimation of useful indicators for central banks such as the time to the expected interest rate liftoff, the expected pace of monetary policy tightening, as well as the shadow rate, which is commonly understood as a measure of the policy rate that would prevail in case the lower bound was not present.

Figure 8 shows estimates of the shadow rate for the US and the Euro Area obtained from the Wu and Xia (2016) model. Notice that as policy rates approach their respective lower bounds in both economies, the estimated shadow rates start decoupling from the actual policy rates. The divergence between the shadow and the actual policy rate becomes larger when the interest rate lower bound is binding and increases as longer-maturity interest rates become particularly compressed and assumedly constrained by the lower bound. As some of the unconventional monetary policies put in practice in these economies are expected to affect longer-term interest rates primarily, the shadow rate has then been used as a measure of the current stance of monetary policy. Some studies, however, have criticized this idea. For instance, Bauer and Rudebusch (2016) argue that common shadow rate estimates are highly sensitive to model specification, the choice of the lower bound value and the data choice at the short end of the yield curve. Similarly, Krippner (2014) argues that shadow rates are subject to variation with modelling choices. He then proposes the use of economic stimulus measures, which are based on the area between the expected shadow rate path and the long-term nominal interest rate level, as an alternative measure of the stance of monetary policy.

Figure 8. Shadow rate for the US and the Euro Area
Per cent per year



Note. The estimates of the shadow rates were obtained from Jing Cynthia Wu's website: http://faculty.chicagobooth.edu/jing.wu.
Source: http://faculty.chicagobooth.edu/jing.wu

4 Concluding remarks

This article provides an overview of the recent developments on term structure modeling and its uses by central banks. The topic is important for central banks and policymakers who wish to extract economic information from long-term interest rates, and elaborate policies to influence them. The simplest proposition of the determination of the term structure of interest rates is the expectations hypothesis. I describe some of the term structure models that are consistent with the expectations hypothesis and discuss why they are insufficient for explaining the behavior of interest rates. I then review term structure models that allow for time-varying risk premia and discuss why they are more consistent with economic theory and data. These models have been especially useful for studying the interest rate transmission mechanisms of unconventional monetary policy such as government bond purchases and forward guidance, which are expected to affect long-term interest rates through short-rate expectations and term premia. In addition, I describe how central banks have used term structure models to estimate inflation and liquidity risk premia in real government bond markets, in order to obtain "cleaner" measures of market participants' inflation expectations. Finally, as policy rates have approached their lower bounds in many economies, some term structure models have been developed to deal with this situation.

Besides allowing for the estimation of more reasonable short-rate expectations, these models also allow for the estimation of useful policy indicators such as the shadow rate, which is commonly understood as a measure of the policy rate that would prevail in case the lower bound was not present.

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Appendix A – the generalized affine term structure model

The generalized discrete-time Gaussian dynamic ATSM assumes that zero-coupon bond yields are functions of p pricing factors. More specifically, the $p \times 1$ vector of pricing factors X_t follows a VAR(1) process under the objective probability measure $\mathbb{P}$,

(7) $$X_{t+1} = \mu + \phi X_t + \Sigma \varepsilon_{t+1},$$

where $\varepsilon_{t+1} \sim iid\ N(0, I_i)$ and Σ is a $p \times p$ lower triangular matrix. The stochastic discount factor (SDF) that prices all assets under the absence of arbitrage is assumed to be conditionally lognormal

(8) $$M_{t+1} = exp\,(-r_t - \frac{1}{2} \lambda_t' \lambda_t - \lambda_t' \varepsilon_{t+1}),$$

where $\lambda_t = \lambda_0 + \lambda_1' X_t$ is a $p \times 1$ vector of risk prices. The short rate is allowed to vary freely, without imposing any restrictions or asymmetries in the conditional distributions of short-rate expectations. The short-term interest rate is then affine in the pricing factors, $r_t = \delta_0 + \delta_1' X_t$. Under the risk-neutral measure $\mathbb{Q}$, the vector of pricing factors follows the dynamics,

(9) $$X_{t+1} = \mu^Q + \phi^Q X_t + \Sigma \varepsilon_{t+1},$$

where $\mu^Q = \mu - \Sigma \lambda_0$ and $\phi^Q = \phi - \Sigma \lambda_1$.

Under no-arbitrage bond prices are then exponential affine functions of the state variables, $P_t^n = exp\,(A_n + B_n' X_t)$, where A_n is a scalar and B_n is a $p \times 1$ vector that satisfy the recursions

(10) $$A_{n+1} = A_n + \mu^{Q\prime} B_n + \frac{1}{2} B_n' \Sigma \Sigma' B_n - \delta_0$$

(11) $$B_{n+1} = \phi^{Q\prime} B_n - \delta_1,$$

which start from $A_1 = -\delta_0$ and $B_1 = -\delta_1$. Model implied yields are computed as $y_t^n = -n^{-1} \log P_t^n = -n^{-1}\,(A_n + B_n' X_t)$.

It is interesting to note that the functions A_n and B_n are computed under the risk-neutral measure $\mathbb{Q}$ and not under the objective probability measure $\mathbb{P}$. The difference is determined by the risk premium demanded by investors to invest in an n-year bond and that is embodied in X_t. Following this argument, the term premium is then defined as the return difference between buying and holding an n-year bond until maturity and rolling over the short-term interest rate,

(12) $$TP_t^n = y_t^n - \frac{1}{n} \sum_{i=0}^{n-1} E_t^P\,(r_{t+1})\,.$$

Appendix B – the generalized shadow rate term structure model

Because the model described above is linear in Gaussian factors, it potentially allows nominal interest rates to go below its lower bound, facing difficulties in fitting the yield curve in a lower bound environment. One way of getting around this problem is to use shadow rate term structure models, an approach that has proven to be helpful for describing yields and the stance of monetary policy in a lower bound environment. This class of models posits the existence of a shadow interest rate that is linear in Gaussian factors, with the actual short-term interest rate being the maximum of the shadow rate and the effective lower bound. More specifically, the model assumes that the short-term interest rate is the maximum of the shadow rate s_t and a lower bound $\underline{r}$,

(13) $$r_t = max(\underline{r}, s_t) \qquad s_t = \delta_0 + \delta_1' X_t.$$