Staff memo

The neutral rate of interest – theory and evidence for Sweden

Henrik Lundvall, Ingvar Strid and David Vestin

April 2025

Contents

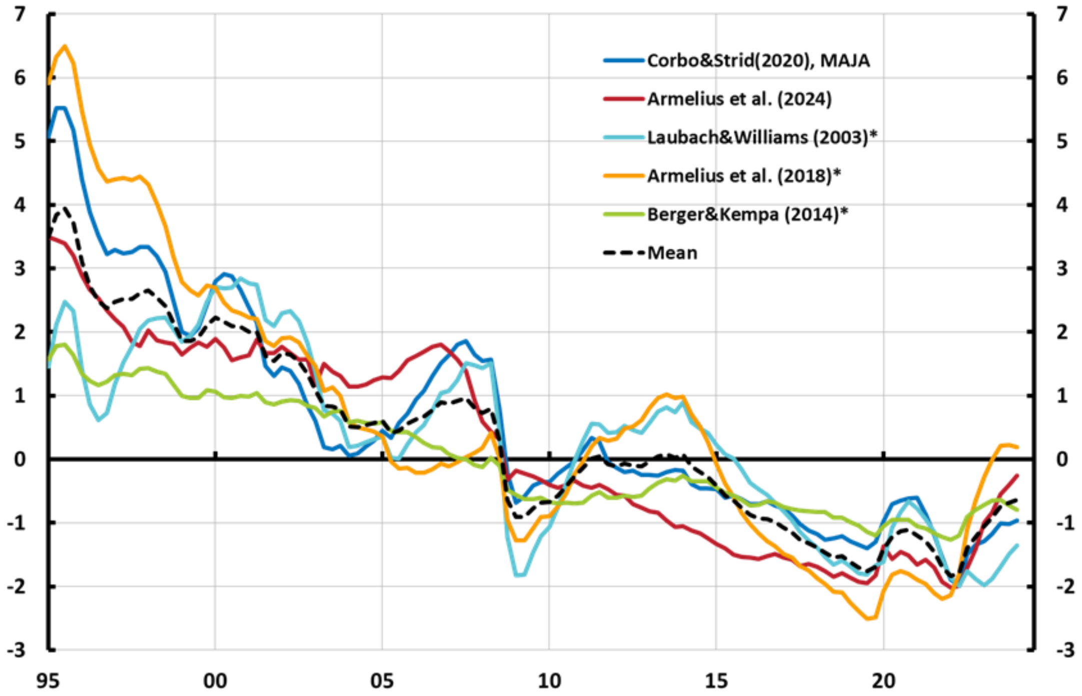
Summary

This paper estimates the trend component of the neutral rate of interest for Sweden. We first discuss the concept of the neutral rate of interest through the lens of a New-Keynesian model and connect the theoretical discussion to the empirical literature. We separate the neutral rate into a business cycle component and a trend component and argue that the empirical literature focuses almost exclusively on estimating the trend component. We follow this approach and provide several estimates of the trend component, using a range of empirical macroeconomic models including the Riksbank's structural model of the Swedish economy, MAJA.

Figure 1 below summarises our main empirical findings. We show estimates of the trend component of the real neutral rate obtained with five macroeconomic models. First, it appears highly likely that the real neutral rate in Sweden has exhibited a downward trend over the past several decades. Second, a reasonable mean estimate of the real neutral rate trend in 2024, based on this class of models, appears to lie roughly within the interval between -1.5 and 0.5 percent. Although our results indicate that the trend component may have increased in recent years, the estimated level is still negative, according to most models.

It is well known that estimates of the neutral rate and its trend are uncertain, for several reasons. The sensitivity of our estimates to prior assumptions on the variability of the trend component suggests a considerable degree of uncertainty regarding the current and future level of the neutral rate. For this reason, we urge caution when using such estimates and conclude that assessments of the long-run level of the neutral rate should be based on a broad range of information.

Figure 1. Estimates of the real neutral rate trend in Sweden using five macroeconomic models 1995q2-2024q2.



Note: the estimates which are marked with a star (*) are produced and discussed in this paper while the estimates based on the models of Armelius et al (2024) and Corbo and Strid (2020) have been obtained from the authors.

Authors:Henrik Lundvall, Ingvar Strid and David Vestin, who work at the monetary policy department.[1]

---

[1] The authors would like to thank Mikael Apel, Hanna Armelius, Vesna Corbo, Mattias Erlandsson, Jens Iversen, Stefan Laséen, Anna Seim and Anders Vredin for valuable comments and suggestions. In particular we would like to thank Hanna Armelius and her co-authors for providing an updated estimate of the real neutral rate trend based on the model in Armelius et al (2024).

1 Introduction

What is the current level of the neutral interest rate and how will it evolve going forward? This is currently the topic of an intensive debate that engages central bankers and academics alike. The backdrop is the substantial and swift monetary policy tightening that occurred in 2022 and 2023, after a period of more than 10 years when real policy interest rates were kept at unusually low levels.

The neutral rate of interest is often defined as the real interest rate that is neither contractionary nor expansionary. However, discussions about the natural/neutral rate are often obstructed by confusion regarding the precise definition of the concept. Woodford (2003) uses the term *natural* or *Wicksellian* real interest rate to denote the real interest rate that would prevail in an equilibrium with flexible prices and wages, in which output is equal to its potential level – we will henceforth refer to this as the neutral rate.[2] In an influential empirical study Laubach and Williams (2003), henceforth LW, estimate the trend component of the neutral rate associated with the trend level of output growth. In a footnote, the authors point out that this object is different from " … the higher-frequency component of the natural rate …" defined by Woodford (2003).

A large body of subsequent empirical work follows in the footsteps of LW, focusing on estimating the long-run trend or slow-moving component of the neutral rate. This analysis is useful for central banks since their modelling and forecasting framework needs an anchor for where interest rates are heading in the longer term, beyond the conventional 3-year forecasting horizon.[3] In its monetary policy report from December last year, the Riksbank communicated its latest assessment of the long-term (nominal) neutral interest rate in Sweden.[4] That assessment is based on international studies, financial-market data and estimations of different models.

This memo has two key objectives. The first is to explore the link between the theoretical definitions of the neutral rate of interest and the object typically estimated in empirical work. The second is to describe the semi-structural models, e.g. LW, and the associated estimates of the trend component of the neutral rate− including a robustness analysis − that is part of the analytical toolkit the Riksbank uses when forming its assessment.

We base our conceptual discussion on a simple, theoretical New-Keynesian model and refer to the 'neutral' rate as the real interest rate that would prevail in an equilibrium with flexible prices and wages. According to New-Keynesian theory, this neutral rate is the appropriate gauge against which to measure the stance of monetary policy.[5] We

---

[2] With our definition, Woodfords natural rate and our neutral rate are hence the same thing. We use the term neutral as it is in line with the Riksbank's communication, as well as with how the empirical literature use the term – though we will highlight that the latter estimates the trend component.

[3] Many central banks provide estimates of the neutral rate and discuss the methodologies used to obtain such estimates, see Adjalala et.al. (2024), Meyer et. al. (2022) and Brandt et.al. (2024) for useful examples.

[4] See Riksbank (2024) and Seim (2024).

[5] Our definition of neutral thus coincides with Woodfords definition of natural. Woodford (2003) uses the term natural or 'Wicksellian' interest rate for the rate that would prevail in the equilibrium with flexible

then interpret the object estimated using semi-structural models such as LW as the trend component of the neutral rate.

The rest of this memo is organised as follows. In Section 2, the neutral rate of interest, as defined by Woodford, is decomposed into a trend component and a cyclical component. This decomposition is then linked to the semi-structural models such as LW designed to capture the long-run component. In Section 3 we present several estimates of the long-term component of the Swedish neutral rate and shed some light on their sensitivity to different assumptions. Section 4 concludes by providing a discussion of the usefulness of these measures.

---

prices and wages, with a clear reference to Wicksell's early work on the topic. Because this natural rate of interest is the theoretically appropriate gauge of the stance of monetary policy, we believe clarity is best served by treating the two terms – 'natural' and 'neutral' – as synonymous. Platzer et al. (2022) also use the term 'neutral' rate to designate the real rate of interest compatible with a neutral stance of monetary policy. They emphasize that the neutral rate has both a short-run and a long-run component, suggesting the long-run component be labelled the 'natural rate'. While we agree with Platzer et al. (2022) that the neutral rate has both a high-frequency (or 'cyclical') component and a long-run ('trend') component, we believe the safest way to avoid confusion is to explicitly refer to them as the different components of the neutral rate, when relevant.

2 Decomposing the neutral rate

In this section, we start from the New-Keynesian definition of the neutral rate and decompose it into a trend component and a cyclical component. This is helpful for clarifying the link between the theoretical definition of the neutral rate and the empirical literature estimating the trend component.

2.1 Definition of the neutral rate of interest

We define the neutral rate of interest as the real interest rate that would prevail if all prices and wages were fully flexible.[6] That is, given the state of the current economic environment, including all current shocks, it is the real rate of interest that would equate the demand for savings with that for investment in the equilibrium with no nominal rigidities. This rate, we may note, is independent of monetary policy; according to standard monetary policy, if all nominal prices (including wages) are flexible, monetary policy does not influence real variables. The natural rate of output is defined analogously as the level of output that would prevail in the flex-price equilibrium.

With this definition, all shocks that affect the flex-price equilibrium may, in principle, affect both the neutral rate of interest and the natural rate of output. Thus, if we want a comprehensive analysis of the neutral rate of interest, including both its current level and its likely value some years from now, our model should ideally capture all factors important to the analysis. Such factors includes both short-run drivers, e.g., temporary productivity disturbances and cyclical fiscal policy, and long run factors, e.g., demographic change.

We will now illustrate these concepts in a tractable NK-model with three disturbances – a transitory technology shock, a preference shock capturing slow-moving changes in the discounting behaviour of agents, and a short-run cost-push shock. The latter captures a time-varying ability of firms to pass on costs to prices and/or time variation in the degree of competitiveness.

The model, from Chapter 3 of Galí (2017), can be written in compact form as

$$y_t - y_t^n = E_t(y_{t+1} - y_{t+1}^n) - (r_t - r_t^n), \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - y_t^n) + u_t, \tag{2}$$

$$L_t = \pi_t^2 + \lambda(y_t - y_t^n)^2 \, , \tag{3}$$

---

[6] As mentioned above, we borrow this definition from Woodford (2003) who prefers to use the term 'natural', in part as a tribute to the work of Wicksell on the topic.

$$r_t^n = \rho - \alpha a_t + (1-\rho_w)w_t, \tag{4}$$

$$y_t^n = \upsilon a_t + c, \tag{5}$$

$$a_t = \rho a_{t-1} + \varepsilon_t, \tag{6}$$

$$w_t = \rho_w w_{t-1} + \varepsilon_t^w \tag{7}$$

Equation (1) is the Euler equation[7], derived from the intertemporal choice of consumers. It relates the deviation of output from the natural rate of output, $y_t - y_t^n$, to expectations about future such deviations, $E_t(y_{t+1} - y_{t+1}^n)$, and to the deviation of the actual real interest rate from its neutral rate, $r_t - r_t^n$. Next, equation (2) is the Phillips curve resulting from the dynamic pricing decisions of firms. The current rate of inflation, π_t, is a function of future expected inflation, $E_t\pi_{t+1}$, the output gap, $y_t - y_t^n$, and the exogenous, short-run cost-push shock, u_t. The loss-function of the central bank, equation (3), reflects its flexible inflation targeting strategy, with concerns for both inflation (in deviation from target) and for resource utilisation (in deviation from the natural level of output, as measured by the output gap). The microeconomic theory implies that there are welfare costs associated with deviations of inflation from the inflation target, and with deviations of output from the natural level of output.[8]

Equations (4) and (5) are the model solution for the neutral rate of interest, r_t^n, and the natural rate of output, y_t^n. In our model, a negative transitory technology shock leads to an increase in the neutral real interest rate in the short run. The reason is that output would tend to fall, and thus consumption would be low relative to its future value. In such a scenario, all agents want to borrow in order to transfer income from the future to the present. In the model, this cannot happen in equilibrium, and the real rate therefore has to rise to the point where no one wants to borrow. In the case of a positive preference shock (w), households value the present more than the future, with the similar implication for the real interest rate: it has to rise in order to eliminate the incentive to borrow.

Finally, equations (6) and (7) describe the evolution of the two exogenous shocks that exhibit some degree of persistence: the transitory technology shock, a_t, and w_t, the slow-moving shock to agents' discounting behaviour, which are both assumed to follow AR(1) processes.

Despite its simplicity, this model is rich enough to illustrate some key points. First, there are shocks that affect the neutral rate of interest, without affecting the natural rate of output – in our case the w_t shock. This is a feature that will be present also in more general models.

Second, some shocks will not create any trade-off for monetary policy. This can be seen by rewriting the Euler equation and the Phillips curve by forward recursive substitution to get

---

[7] We have imposed that Y=C in the simple model without investment and trade.

[8] In this simple model, the inflation target, $\bar{\pi}$, is assumed to be zero. This is why the deviation from target, $\pi_t - \bar{\pi}_t$, may be written: $\pi_t - \bar{\pi} = \pi_t$.

$$y_t - y_t^n = -E_t \sum_{j=0}^{\infty} \left(r_{t+j} - r_{t+j}^n\right) \tag{8}$$

$$\pi_t = u_t + \kappa E_t \sum_{j=0}^{\infty} \beta^j \left(y_{t+j} - y_{t+j}^n\right), \tag{9}$$

where we have used the assumption that the cost-push shock, u_t, takes the expected value of zero in future periods.

Equation (8) reveals why the neutral rate of interest is a relevant benchmark when assessing the stance of monetary policy. If the central bank conducts monetary policy so that the current real interest is always equal to the neutral rate of interest – such that market expectations of future interest rate gaps are zero – then the current output gap will be zero. That is, such a policy will stabilise the real economy at the desirable potential (natural) level of output. However, if $E_t \sum_{j=0}^{\infty} \left(r_{t+j} - r_{t+j}^n\right) > 0$, so that the real interest rate is on average higher than the neutral rate, the output gap will contract below zero and monetary policy will, in this sense, be “contractionary”. If instead $E_t \sum_{j=0}^{\infty} \left(r_{t+j} - r_{t+j}^n\right) < 0$, output will expand above its potential level, such that the stance of monetary policy may be labelled as “expansionary”. Only when the real rate is always equal to its neutral rate is monetary policy neither contractionary nor expansionary; we will refer to such a stance as “neutral”.[9]

At first, this definition may give the impression that monetary policy should always strive to be “neutral”, in the above sense of replicating the neutral rate of interest and the natural level of output. However, a closer inspection of the Phillips curve (equation 2) reveals that such a policy does not always stabilize inflation at target. In our simple model, this is due to the presence of the mark-up shock, u_t. When a positive mark-up shock hits the economy, a central bank with the loss function in equation 3 needs to balance the inflationary pressure with a contractionary monetary policy that opens up a negative output gap. Since it is impossible to have both the inflation gap and the output gap be zero simultaneously, the central bank must choose the least costly attainable combination. In this situation, a neutral monetary policy stance is not appropriate unless the weight given to real stability is infinitely high relative to that of inflation stability. Woodford uses these terms this way, thus reserving the use of the words for the balancing of objectives of monetary policy against each other.

Notice that an alternative definition of stance, based on the difference between the actual real interest rate and the trend component of the neutral rate, would of course yield a different result. In this case, any interest rate increase resulting in an actual real interest rate above the trend component would classify as contractionary, and vice-versa. With such a definition, however, we could easily have a case where inflation is running hot and output gaps are positive, and monetary policy is still classified as contractionary. Since this alternative definition abstracts from the cyclical driving forces, it offers no link to the objectives of monetary policy, which is to stabilise inflation and resource utilisation.

---

[9] $E_t \sum_{j=0}^{\infty} \left(r_{t+j} - r_{t+j}^n\right) = 0$ may of course hold even if $E_t(r_{t+j} \neq r_{t+j}^n)$ for 2 or more periods $(t+j)$. Such a policy will not be consistent over time, however; the only policy consistent with closed output gaps in both the current and future expected periods is one where, for all $j = 0,1, \ldots$, $E_t(r_{t+j} = r_{t+j}^n)$.

Finally, we note that there is an issue of horizon. A larger, more realistic model will include many shocks and the neutral rate is therefore likely to be quite volatile, as it will be affected by several different, short-run disturbances. However, forecasts of future values of the neutral rate will be more stable; the longer the horizon of the forecast, the more dominant will the persistent shocks be. In our simple model, we may assume that the technology shock has transitory effects and that the z-shock has more persistent effects, representing slow-moving features, such that $\rho_w \gg \rho$. Thus, in this simple model, the technology shock stands in for any type of shock that, in a more realistic model, drives the business cycle component of the neutral rate. Likewise, the preference shock may be thought of as representing any shock that, in a richer model, drives the trend.

2.2 Decomposing the neutral rate into trend and cyclical components

We now proceed to explicitly decompose the neutral rate into a trend component, r_t^*, and a cyclical component, r_t^b. This is useful as it clarifies the relationship between the neutral rate and its trend component, where the latter is the focus of most empirical studies. Standard business cycle models do not include the trend component; it therefore needs to be estimated in a more reduced-form.

In the context of our simple model, where $r_t^n = \rho - \alpha a_t + (1 - \rho_w) w_t$, we obtain

$$r_t^n = r_t^b + r_t^* \tag{10}$$

$$r_t^* = \rho + (1 - \rho_w) w_t \tag{11}$$

$$r_t^b = -\alpha a_t \tag{12}$$

Here, we see that r_t^* captures both the long-run mean of the natural rate, as well as the slow-moving component, the preference shock w_t. Business cycle variation in r_t^n will instead turn up in r_t^b, which is determined by the transitory technology shock a_t. We may therefore interpret r_t^* as the trend component of the natural rate and r_t^b as its cyclical component. To draw a precise line between cyclical and trend components is obviously impossible and practical applications therefore require some degree of judgment.[10]

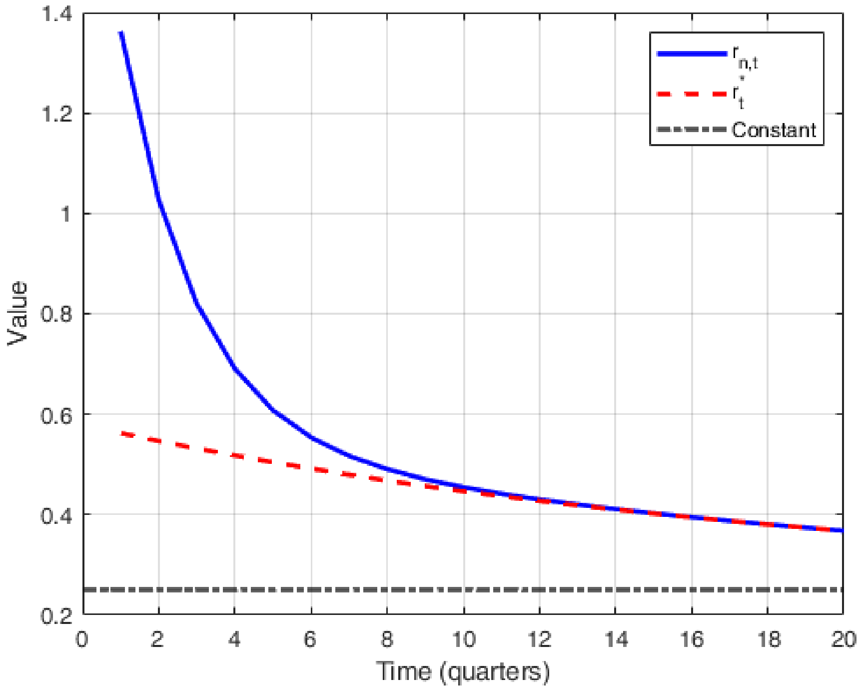
Figure 2 illustrates the case when the economy has been hit by both a negative productivity shock and a positive preference shock. As discussed above, both these shock realisations will tend to increase the neutral rate of interest, illustrated by the blue line in Figure 2. The red line plots the response of the long-run neutral rate (the trend component, or r_t^*) – in our model driven by the preference shock w_t. The cyclical component of the neutral rate, r_t^b, is thus given by the difference between the blue line and the red line. We see that there is a substantial discrepancy between the

---

[10] The same is true in the empirical literature. For example, when using the band-pass filter, the analyst needs to arbitrarily choose which frequencies that should define the trend.

neutral rate of interest and the long-run neutral rate in the short run, but that the two converge when looking sufficiently far ahead.

Figure 2. The response of the trend- and cyclical components of the neutral rate of interest to transitory and persistent shocks.



Note that if we define business cycle shocks as relatively short-lived disturbances (such as the one that would be generated by our cost-push shock u, not present in Figure 2), there is typically no trade-off in the long-run between closing the output gap and stabilizing inflation at target. The reason is that the kind of shocks that do generate such a trade-off, discussed above, will typically be relevant only at the business cycle frequency, such that a reasonably calibrated monetary policy will make sure they do not carry lasting effects on the economy. This explains why some authors, e.g. Laubach and Williams (2003), include price stability as part of the definition of the neutral rate of interest.

2.3 Adding more features to the model

Of course, our simple model is not empirically realistic. To capture the full aspects of the neutral rate of interest, we would need to add to the model more business cycle features as well as additional elements that could explain movements in the trend component.

Beginning with features related to the trend component, in recent years, a vast literature has emerged that aims to explain the trend decline in global real interest rates that started around 1990 and that, in 2020, had resulted in an accumulated decrease of several percentage points over three decades. This literature has put forth a number of possible explanations for the trend decline, including a slowdown in productivity growth, demographic change, a shortage of safe assets and increased income inequality.[11] These features are typically modelled one at a time; an example is the paper by Auclert et. al. (2024) who provides an in-depth analysis of how an aging population affects several macroeconomic variables, including the real interest rate. For a small open economy such as Sweden, trend developments in the neutral interest rate are almost entirely decided by structural changes abroad.[12]

Moving to the business cycle literature, with a particular focus on models intended for monetary policy analysis like Smets and Wouters (2007) and Corbo and Strid (2020), adding features like habit persistence and inflation indexation will add endogenous state variables, such as lagged output and lagged inflation, to our simple model. Solving a model of this type will yield (reduced form) equations for output and inflation that more closely resembles the specifications in semi-structural models, e.g. the Laubach and Williams (2003) model (see section 3 below for details).

2.4 Semi-structural models for estimation of the trend component of the neutral rate

Many applications that concern monetary policy are focused on the business cycle dimension. Such models will typically feature a tight link between the equilibrium real interest rate and productivity growth. Since the real interest rate trend differs from the productivity growth trend (because of the above mentioned, additional driving forces), something needs to be done to reconcile the two. Since it is too costly, in such medium-scale business cycle models, to include proxies for all the structural features that drive the neutral rate, it is common to introduce a single exogenous variable intended to capture these factors. For example, we may assume that the neutral rate features a highly persistent exogenous shock that can lead to long-lasting deviations from steady-state. This is the approach taken both in relatively simple reduced form business cycle models such as Laubach and Williams (2003) as well as in larger DSGE models such as the one used by the Riksbank, Corbo and Strid (2020, 2025). Thus, in this empirical literature, it is common to focus exclusively on estimating the long-run neutral rate (the trend component of the neutral rate), and model this using simplified assumptions about driving forces. In such specifications, trend productivity growth may even be non-stationary,[13] such that a change in the growth rate is expected to last indefinitely, moving the natural real interest in the same direction. However, due to the inability to model driving forces explicitly and assess them using actual data, such approaches run the risk of incorrectly classifying cyclical fluctuations in real interest rates as trend movements. If an estimate of r^* displays a relatively high degree of variability, the interpretation that it captures slow moving components

---

[11] An overview of these trend developments, a discussion of the main driving forces and references may be found in IMF (2023). See also Flodberg (2024) and Lundvall (2023).

[12] See e.g. the discussion in Lundvall (2023).

[13] Note that in our theoretical analysis, the stationary technology shock was interpreted as business cycle shock. Here we are talking about trend productivity – which would naturally be classified as a trend shock.

becomes questionable. In section 3 we illustrate how assumptions on the variability of the factors driving the trend component of the neutral rate affect the trend estimates.

Consistent with our interpretation, Holston, Laubach and Williams (2017) interpret r^* as capturing the "highly persistent" or "low frequency determinants" of the neutral rate. In a similar vein Kiley (2020) notes that while the natural rate contains transitory components and therefore is expected to fluctuate considerably, r^* "consists solely of the permanent component and is likely to fluctuate less, if at all, over the business cycle and instead should evolve slowly over time". Our interpretation, which is in line with these authors, thus hinges on these factors indeed being slow-moving factors and we should anticipate limited variability in the resulting r^* estimate.[14]

3 Estimation of the neutral rate in Sweden using a range of macroeconomic models

In this section we estimate the trend component of the real neutral rate using a range of semi-structural models. We highlight the sensitivity of the estimates to assumptions about the variability of the trend. While the estimates are uncertain they suggest that the trend component of the real neutral rate has declined in the past decades and is now at a low level.

3.1 Empirical framework

In this section we present three small-scale semi-structural (or multivariate unobserved component, UC) models to estimate the trend component of the real neutral rate, r^*, in Sweden. These models decompose key macroeconomic variables into trend (represented as "star" variables) and cyclical components (denoted by "g" for gap). Unlike univariate de-trending methods like the HP-filter the components are identified by combining a small multivariate economic model with filtering techniques. We note that since the star variables in these models correspond closely to the trend component of the natural rates (of output and interest) in the New Keynesian model discussed above in section 2, it follows that the associated gaps are defined differently, i.e. in the semi-structural models they are the deviation from a long run trend component. We maintain close adherence to the original model formulations.

---

[14] Pescatori and Turunen (2016) include several additional variables in the specification of the neutral rate in their semi-structural model: the current account surplus of developing countries, the equity premium, and a measure of economic uncertainty. This introduces high-frequency variation in the estimate of r* and makes it difficult to interpret their estimate as the trend component the neutral rate. For example, the contribution of movements in the equity premium to r* is quite volatile.

Holston, Laubach and Williams (HLW)

Our version of the HLW model closely follows that of Holston, Laubach and Williams (2017), which builds on Laubach and Williams (2003).[15] We may note that while this is a model for a closed economy it has been applied also to estimate r* in small open economies.[16]

The output gap ($y_{g,t}$) is related to the real interest rate gap ($r_{g,t}$) through the investment-saving (IS) equation

$$y_{g,t} = a_{1,y} y_{g,t-1} + a_{2,y} y_{g,t-2} + (a_r/2)\big(r_{g,t-1} + r_{g,t-2}\big) + \sigma_{\varepsilon_{y_g}} \varepsilon_{y_g,t} \qquad (13)$$

where $\varepsilon_{y_g,t}$ is a (demand) shock with a standard normal distribution, $N(0,1)$ which is scaled by the standard deviation $\sigma_{\varepsilon_{y_g}}$. The output gap is the log deviation of output, y_t, from potential (or trend) output, y_t^*.[17] The real interest rate gap is given by

$$r_{g,t} = r_t - r_t^* \qquad (14)$$

where r_t^* is the neutral rate trend component and where the real interest rate is defined as

$$r_t = R_t - E_t \pi_{t+1} \qquad (15)$$

where R_t is the nominal interest rate and where $E_t\pi_{t+1}$ is the model-consistent one quarter ahead expected inflation.[18] Inflation is related to the output gap through the following Phillips curve relationship

$$\pi_t = b_\pi \pi_{t-1} + I_{TR}(1 - b_\pi) \sum_{k=2}^{4} \pi_{t-k} + b_y y_{g,t-1} + \sigma_{\varepsilon_\pi} \varepsilon_{\pi,t} \quad (16)$$

Here π_t is defined as the log deviation of inflation from a constant inflation target and $\varepsilon_{\pi,t}$ is a supply shock with a standard normal distribution, $N(0,1)$. The indicator variable I_{TR} is introduced to allow for the possibility of non-stationary inflation. We note that when $I_{TR} = 1$ inflation is restricted to follow an integrated AR(4) process, i.e. the sum of the coefficients on the inflation terms equals 1 and we have an accelerationist Phillips curve as in HLW. When we estimate the model with a Taylor rule we instead assume $I_{TR} = 0$ and $b_\pi < 1$ ensuring a stationary inflation process.

---

[15] For an overview of and references to the literature following Laubach and Williams (2003), see, e.g.,. Brand et al. (2018).

[16] For example, the Federal reserve bank of New York apply the HLW model regularly to compute estimates of r* for Canada. A similar model is also used by Meyer et al. (2022) to estimate r* for Norway.

[17] In many papers in the semi-structural literature, e.g. Holston, Laubach and Williams (2017), the output gap as well as the other variables are assumed to be expressed in percent. The output gap is obtained as the percentage deviation from trend by multiplying $y_{g,t}$ with 100. Our setup is more influenced by the literature on the estimation of DSGE models and we instead make the appropriate scalings of variables in the observation equations (see below).

[18] Holston, Laubach and Williams (2017) refer to r* as the highly persistent, or low frequency, component of the natural rate. This is in line with our definition of r* in chapter 2.

The neutral rate trend (r_t^*) is assumed to depend on potential GDP growth (g_t) and a variable capturing all other influences on the trend (z_t). This latter factor will sometimes be referred to as the 'other factor' for brevity.

$$r_t^* = cg_t + z_t \tag{17}$$

where

$$g_t - \mu_g = \rho_g\left(g_{t-1} - \mu_g\right) + \sigma_{\varepsilon_g}\varepsilon_{g,t} \tag{18}$$

and

$$z_t = \rho_z z_{t-1} + \sigma_{\varepsilon_z}\varepsilon_{z,t} \tag{19}$$

where $\varepsilon_{g,t}$ and $\varepsilon_{z,t}$ are standard normally distributed shocks. We introduce autoregressive coefficients (ρ_g, ρ_z) in these equations to allow for other processes than random walks.[19] With $\rho_z = 1$ z_t thus follows a driftless random walk. We may note that r_t^* as it is defined here is consistent with the definition provided in chapter 2, i.e. it is intended to capture the trend component of the real neutral rate.[20]

Potential GDP is modelled as a random walk with stochastic drift (g_{t-1})

$$y_t^* = y_{t-1}^* + g_{t-1} + \sigma_{\varepsilon_{y^*}}\varepsilon_{y^*,t} \tag{20}$$

where $\varepsilon_{y^*,t}$ is a shock to the log of the level of potential GDP.[21] The log of the level of GDP is provided by

$$y_t = y_t^* + y_{g,t} \tag{21}$$

We may note that this is a different concept of the output gap than the one used in the New Keynesian model described in chapter 2 where the gap was defined as a deviation from the natural rate of output, y_t^n. In correspondence with the difference between the neutral rate (r^n) and its trend component (r^*), potential GDP (y^*) should be thought of as the trend component of the natural rate of output (y^n), see the discussion in the Appendix for details on the relation between the two gap measures.[22]

---

[19] There is no theoretical reason why both drivers of the neutral rate need to be non-stationary. Re-estimating the LW model on US data and comparing different specifications using the marginal likelihood, Lewis and Vazquez-Grande (2018) find that the data supports a specification where z_t is subject to transitory shocks. This is in contrast to the results in Laubach and Williams (2003).

[20] The New Keynesian model in chapter 2 does not allow for secular growth. If we let $g_t = 0$ and z_t=$\rho + (1-\rho_w)w_t$ in the HLW model we obtain the definition of r_t^* in chapter 2.

[21] The model for the trend component of GDP is identical to the one in Clark (1987) with $\rho_g = 1$. The trend component is modelled as in a univariate dynamic linear trend model while the cyclical component is now determined by a multivariate economic model (rather than as noise).

[22] The New Keynesian model in chapter 2 does not contain a secular growth trend.

We now have 9 equations for 10 endogenous variables. To close the model we assume that monetary policy is governed by a Taylor rule

$$R_t - r_t^* = \rho_R(R_{t-1} - r_{t-1}^*) + (1-\rho_R)\big(\rho_\pi \pi_{t-1} + \rho_y y_{g,t-1}\big) + \sigma_{\varepsilon_R}\varepsilon_{R,t} \tag{22}$$

where we thus assume that the central bank takes into account the trend level of the neutral rate when it sets the policy rate. [23]

Alternatively we may close the model by assuming that the real interest rate follows a random walk

$$r_t = r_{t-1} + \sigma_{\varepsilon_r}\varepsilon_{r,t} \tag{23}$$

This specification is closer to the original HLW model where the real interest rate is treated as an exogenous variable.

We call the two versions of the HLW model the Taylor rule (TR) and real rate random walk (RRW) versions, respectively. The model contains a total of six innovations ($\varepsilon_{y_g,t}$, $\varepsilon_{\pi,t}$, $\varepsilon_{g,t}$, $\varepsilon_{z,t}$, $\varepsilon_{y^*,t}$, $\varepsilon_{R,t}$) which are assumed to be independent and normally distributed, i.e. $cov\big(\varepsilon_{i,t}, \varepsilon_{j,t-k}\big) = 0$ for all shock pairs (i,j) and leads or lags k.[24]

Our specification differs from Holston, Laubach and Williams (2017) in two key aspects. They use a four-quarter moving average of past inflation as a proxy for inflation expectations and treat the ex ante real interest rate as an exogenous variable.[25] We, instead, assume model-consistent inflation expectations and close the model with either a Taylor rule or an exogenous process for r_t.This allows our nominal interest rate (R_t) to respond endogenously to the economy, unlike the exogenously determined rate in HLW. HLW also assume $c = 1$, $\rho_g = 1$ and $\rho_z = 1$ which implies that GDP is integrated of order 2 (I(2)) and that the neutral rate trend is I(1).[26] Based on our analysis (see below) we find that these restrictions are reasonable also when estimating the model on Swedish data and we therefore impose them (see further discussion below).

We solve the set of equations 13-21 and 22 (Taylor rule) or 23 (random walk for the real rate) and the solution is represented as the state equation in the linear and Gaussian (LGSS) state space model, where the state vector is given by[27]

---

[23] We follow Brand and Mazelis (2019) in incorporating a Taylor rule in the HLW model. Since they estimate the model on data from the 1960s they also need to incorporate a time-varying inflation target in the model. Since we estimate the model on Swedish data for the inflation targeting period we assume that the inflation target is constant and equal to 2 percent.

[24] For an example of a model which allows for non-zero correlations between some shocks, see Brand and Mazelis (2019).

[25] Laubach and Williams (2003) proxy inflations expectations with the forecast of the four-quarter change in the price index for personal consumption expenditures excluding food and energy (“core PCE”) generated from a univariate AR(3) of inflation estimated over the prior 40 quarters. Preliminary estimation of the model suggests that the modelling of inflation expectations does not have crucial effects on our results.

[26] A variable is integrated of order k if it becomes stationary, I(0), after being differenced k times.

[27] In the literature on estimation of the neutral rate using semi-structural models it is common to present the IS and Phillips curve equations directly as observation equations. Our approach is to let all endogenous variables in the model enter the vector of (econometric) state variables, an approach which is common in the estimation of DSGE models, see e.g. An and Schorfheide (2007) or Herbst and Schorfheide (2015). We use the software Dynare to solve the model instead of obtaining the state space representation by hand.

$$X_t = \left(y_{g,t}\ r_{g,t}\ r_t\ R_t\ \pi_t\ r_t^*\ g_t\ z_t\ y_t^*\ y_t\ \right)^T$$

Armelius et al (2018) and Berger and Kempa (2014)

We use slightly modified versions of the open-economy semi-structural models of Berger and Kempa (2014) and Armelius et al (2018). These models, which incorporate the real exchange rate, should in principle be better suited than HLW to estimate the neutral rate trend in small open economies like Sweden. These models also allow for somewhat more flexible modelling of the cyclical components through a VAR framework.

We first describe the Armelius et al (2018) model. The equations describing the trend model for output and the real interest rate are identical to those of the HLW model presented above: the neutral rate trend (r_t^* , eq. 17), potential GDP growth (g_t, eq. 18), the other factors affecting the neutral rate trend (z_t, eq.19) and potential GDP (, eq. 20). Also the definitions of the real interest rate gap ($r_{g,t}$, eq. 14), the real interest rate (r_t, eq. 15) and output (y_t, .eq. 21) are identical.

The output gap ($y_{g,t}$), the real interest rate gap ($r_{g,t}$) and the real exchange rate gap ($q_{g,t}$) are modelled using a VAR model

$$X_{g,t} = \sum_{k=1}^{K} \Psi_k\, X_{g,t-k} + R^{1/2}\varepsilon_t \tag{24}$$

where the vector of cyclical components is given by

$$X_{g,t} = \left(y_{g,t}\ , r_{g,t}\ , q_{g,t}\ \right)^T$$

and the notation for the elements of the coefficient matrices is

$$\Psi_k = \left(\Psi_{k,ij}\right)$$

and the vector of innovations is

$$\varepsilon_t = \left(\varepsilon_{y_g,t}\ \ \varepsilon_{r_g,t}\ \varepsilon_{q_g,t}\right)^T$$

where

$$R^{1/2} = diag\left(\sigma_{y_g}\ \sigma_{r_g}\ \sigma_{q_g}\right)$$

Here ε_t is a vector of shocks with a $N(0, I)$ distribution where I is the identity matrix. The equations in the VAR model can be interpreted as an IS equation ($y_{g,t}$), a monetary policy reaction function ($r_{g,t}$) and a generalised real interest rate parity condition ($q_{g,t}$). The real exchange rate gap is defined as

$$q_{g,t} = q_t - q_t^* \tag{25}$$

where the real exchange rate trend component (q_t^*) is defined as a driftless random walk

$$q_t^* = q_{t-1}^* + \sigma_{\varepsilon_{q^*}} \varepsilon_{q^*,t} \tag{26}$$

The evolution of inflation is captured by the Phillips curve

$$\pi_t = b_\pi \pi_{t-1} + b_{\Delta q} \Delta q_{t-1} + b_y y_{g,t} + \sigma_{\varepsilon_\pi} \varepsilon_{\pi,t} \tag{27}$$

where

$$\Delta q_{t-1} = q_t - q_{t-1} \tag{28}$$

is the quarterly change in the real exchange rate. An increase in q_t implies a real depreciation of the Krona and we therefore expect $b_{\Delta q} > 0$. Our specification follows Berger and Kempa (2014) but differs slightly from Armelius et al (2018) who use the change in the *nominal* exchange rate.[28] Otherwise the model outlined above is identical to the one presented by Armelius et al (2018) assuming the lag $K = 1$ in the cyclical model. In total, our version of the model has 13 equations for 13 endogenous variables.

The Armelius et al (2018) and Berger and Kempa (2014) models are very similar, with the former largely building on the latter. Berger and Kempa (2014) use a restricted version of the cyclical model (eq. 24) and the lag K=2. In particular they model the real exchange rate as a univariate process, and they allow the real interest rate gap to depend only on the real exchange rate gap. Our implementation of their model generalises the cyclical model toward an unrestricted BVAR model with K=4 lags. The main difference between our implementations of the Armelius et al (2018) and Berger and Kempa (2014) models therefore concerns the number of lags in the cyclical model.

We note that the three models outlined above are very similar in the specification of the trend model, and in particular the neutral rate trend, while there are some differences in the specification of the cyclical model.

Observation equations and data

The semi-structural models outlined above are represented as linear and Gaussian state-space (LGSS) models.[29] The solution to the systems of equations defining each of the models constitute the state equations of the LGSS model. We next describe the relationship between the observed variables and the state variables.

The models are estimated using quarterly data from 1995Q1 to 2024Q2.[30] For the HLW model we use four data series: annualised quarterly GDP growth, annualised

---

[28] Since most of the variation in the real exchange rate is due to variation in the nominal exchange rate, rather than variation in the relative price level of Sweden and the foreign economy, we expect this assumption to be less important.

[29] We use Dynare to solve the model. Since the models are backward-looking an alternative, however, would be to solve the model by hand. The state-space representations of the models considered here do not contain any exogenous observed variables.

[30] A starting point of the sample in the mid-1990s is suitable since Sweden introduced a flexible exchange rate regime in 1993 and an inflation target of 2 percent in 1995.

quarterly CPIF inflation (consumer price index with a fixed interest rate), the policy interest rate and the Riksbank's measure of the GDP gap.[31] The data series are displayed in figure 7 in appendix B. The observation equations for these variables are presented below.

$$\Delta y_t^{obs} = \Delta y^{obs,ss} + 400(y_t - y_{t-1}) + \sigma_{\Delta y^{obs}} \varepsilon_{\Delta y^{obs},t} \tag{29}$$

$$\pi_t^{obs} = \pi^{obs,ss} + 400\pi_t + \sigma_{\pi^{obs}} \varepsilon_{\pi^{obs},t} \tag{30}$$

$$R_t^{obs} = R^{obs,ss} + 400R_t + \sigma_{R^{obs}} \varepsilon_{R^{obs},t} \tag{31}$$

$$y_{g,t}^{obs} = y_g^{obs,ss} + 100y_{g,t} + \sigma_{y_g^{obs}} \varepsilon_{y_g^{obs},t} \tag{32}$$

where we allow for observation errors. Here we let the intercepts (or steady state values) be[32]

$$\Delta y^{obs,ss} = 400\mu_g = 2.3\%$$

$$\pi^{obs,ss} = 2\%$$

$$R^{obs,ss} = 2.5\%$$

$$y_g^{obs,ss} = 0$$

When estimating the Armelius et al. (2018) and Berger and Kempa (2014) models we add two additional data series: the annualised quarterly change in the real exchange rate and the Riksbank's measure of the real exchange rate gap, i.e. the deviation of the real exchange rate from its long-run trend.[33] Here we use the trade-weighted index called KIX20 which includes the euro area countries and the United States. The observation equations for these two variables are provided below.

$$\Delta q_t^{obs} = \Delta q^{obs,ss} + 400(q_t - q_{t-1}) + \sigma_{\Delta q^{obs}} \varepsilon_{\Delta q^{obs},t} \tag{33}$$

$$q_{g,t}^{obs} = q_g^{obs,ss} + 100q_{g,t} + \sigma_{q_g^{obs}} \varepsilon_{q_g^{obs},t} \tag{34}$$

where

$$\Delta q^{obs,ss} = 0$$

$$q_g^{obs,ss} = 0$$

---

[31] An alternative which is not pursued here would be to use a so called shadow rate in place of the policy rate to account for the possible effects of Riksbank asset purchases in the period since 2015. The analysis in Pescatori and Turunen (2016) for the United States shows that the use of a shadow rate yields a lower estimate of the neutral rate in the period following the financial crisis in 2007-09 when the Federal Reserve implemented quantitative easing.

[32] When the variables are assumed to be non-stationary they do not have a steady state. However, technically, we may still allow for the intercept in the equation, i.e. it does not affect the estimation.

[33] The estimation of the long run real exchange rate is discussed in Belfrage (2021) and Belfrage et al (2020).

Using the GDP and real exchange rate gaps as data when estimating the model implies that potential GDP (y_t^*) and the long-run real exchange rate (q_t^*) are perfectly identified.[34] After preliminary estimations we choose to typically estimate the models without observation errors, i.e. we calibrate the standard deviations of all these errors to zero.

3.2 Estimation method

As is common in the estimation of dynamic stochastic general equilibrium (DSGE) models we employ a Bayesian approach to estimate the parameters (collected in the vector θ) of each model (indexed by M). This involves combining prior assumptions on the model parameters, $p(\theta|M)$, with the likelihood function for the data, $p(D|\theta, M)$, to obtain the posterior distribution of the parameters.

$$p(\theta|D, M) \propto \; p(D|\theta, M)\, p(\theta|M) \tag{35}$$

This approach has also become more common in estimating the parameters in the type of semi-structural models applied for estimation of the trend neutral rate.[35] We calibrate a subset of the parameters in the respective models and assign independent prior distributions to the remaining parameters which are estimated. The likelihood function is evaluated using the Kalman filter where we use a diffuse prior on the (potentially non-stationary) initial state vector. We sample from the posterior distribution of the parameters using the random walk Metropolis-Hastings algorithm targeting a 20-35% acceptance rate.[36] We compute smoothed (and filtered) estimates of the state variables and innovations for a subset of the posterior parameter draws using the state and innovation smoothers.[37] We primarily focus on the smoothed estimates of the neutral rate trend and its parts, the potential growth rate (g) and the other factors affecting the rate (z), averaged across the posterior parameter distribution.[38] All computations are performed using the Dynare software.[39] We obtain $m = 100{,}000$ draws from the posterior distribution when estimating each model and use a subset (i.e. a thinned chain) of those for the computation of posterior statistics.

---

[34] Pescatori and Turunen (2016) also use a measure of the output gap when they estimate a HLW type model on US data. A difference from our approach is that they assume it is a less precise signal.

[35] Berger and Kempa (2014) and Armelius et al. (2018) are two examples of papers in which the parameters are estimated using Bayesian methods. Holston, Laubach and Williams (2017) estimate their model using maximum likelihood and the median unbiased estimator of Stock and Watson (1998) to handle the so called pile-up problem. Maximum likelihood estimation of time series models that contains a small permanent component and a large transitory component tend to yield an estimate of the variance of the permanent component which piles up at zero. Buncic (2024) notes that the likelihood of encountering pile-up problems in the HLW model is small and, furthermore, corrects the estimation of this model.

[36] Berger and Kempa (2014) and Armelius et al. (2018) apply importance sampling to sample from the posterior distribution.

[37] See e.g. Durbin and Koopman (2012) for a review of filtering and smoothing in state-space models.

[38] The smoothed estimate of the neutral rate trend is given by $r_{t|T}^* = E(r_t^*|D_{1:T})$ where D denotes the data.

[39] Dynare is available at www.dynare.org.

3.3 Estimation results: Holston, Laubach and Williams (2017)

Prior and posterior distributions of model parameters

Table 1 presents prior distributions and posterior mean estimates for the HLW model parameters. Following Holston, Laubach and Williams (2017), and in line with preliminary estimation results on Swedish data, we calibrate $c = 1$, $\rho_g = 1$ and $\rho_z = 1$, while estimating the remaining parameters.[40] These calibrations imply I(1) processes for the neutral rate trend and its components. We employ normal prior distributions for most of the parameters, a beta prior distribution for the interest rate smoothing parameter in the Taylor rule and inverse gamma prior distributions for the innovation standard deviations.[41] We scale the innovation standard deviations in order to avoid an excessive influence of the prior on the posterior estimates. The priors for the parameters in the IS and Phillips curve equations are largely uninformative, while the priors for the monetary policy rule parameters have standard values.[42] In the case of the HLW model it turns out that we do not need to impose additional restrictions on the parameters to obtain reasonable signs of the estimated parameters (i.e. signs consistent with standard economic theory). Preliminary estimations of the two versions of the HLW model versions reveal a relatively flat slope of the Philips curve, i.e. a low value for b_y, and we choose to calibrate this parameter to the value 0.05, a choice which does not significantly affect the model fit (as measured by the marginal likelihood).[43] Conversely, the IS curve slope, a_r, appears rather steep. [44]

---

[40] When estimating the parameter c we obtain a posterior mode estimate close to 1 and the posterior density is very similar to the prior density (normal distribution centered on 1 with standard deviation 0.5, N(1,0.5)), which suggests that the parameter is not well identified. Estimating the persistence parameters ρ_g and ρ_z individually or jointly with N(1,0.5) priors we find that also these parameters are estimated close to 1 and the respective marginal posterior distributions are tightly estimated. Thus we do not find support for the result of Lewis and Vazquez-Grande (2018) of a transitory z-component using Swedish data. Armelius et al. (2018) conclude that the assumptions on the processes for g_t and z_t are not of major importance for their estimate of r*.

[41] The general approach is to use normal prior distributions for unbounded parameters, gamma or inverse gamma priors for innovation standard deviations (i.e. parameters which are bounded from below) and beta prior distributions for parameters that are bounded both from below and above.

[42] See, e.g., Corbo and Strid (2020).

[43] This is in line with Corbo and Strid (2020) who estimate rather flat Phillips curves in a DSGE model for Sweden.

[44] The filtering uncertainty in estimating r* depends crucially on the steepness of these two parameters, as demonstrated by Fiorentini et al (2018). Low (i.e. flat) estimates imply that the precision of the estimate of r* deteriorates. It is common that both parameters are estimated to be flat, see Brand et al. (2018) who collect estimates from a large number of studies.

Table 1. Prior parameter distributions and posterior mean estimates for alternatives of the HLW model estimated on Swedish data for the period 1995Q2-2024Q2.

	Prior distribution				**Posterior mean**					
					Taylor rule			**Real rate RW**		
	Dist.	Mode	Std.	Scale	Baseline	Low σ_{ε_z}	TN prior	Baseline	Low σ_{ε_z}	TN prior
$a_{1,y}$	N	0.5	1		0.86	0.84	0.84	0.85	0.87	0.86
$a_{2,y}$	N	0	1		-0.028	-0.018	-0.018	-0.030	-0.022	-0.030
a_r	N	0	1		-0.51	-0.55	-0.60	-0.38	-0.27	-0.36
b_π	N	0.5	1		0.56	0.56	0.56	0.20	0.20	0.20
ρ_R	B	0.8	0.1		0.91	0.91	0.91			
ρ_π	N	1.75	0.2		1.57	1.57	1.61			
ρ_y	N	0.1	0.05		0.14	0.15	0.15			
$\sigma_{\varepsilon_{y_g}}$	IG, TN	0.2	∞	1/100	1.20	1.20	1.21	1.19	1.20	1.21
σ_{ε_π}	IG, TN	0.2	∞	1/100	0.40	0.40	0.40	0.45	0.45	0.45
σ_{ε_z}	IG, TN	0.2	∞	1/100	0.083	0.069	0.062	0.12	0.16	0.075
$\sigma_{\varepsilon_{y^*}}$	IG, TN	0.2	∞	1/100	0.028	0.028		0.078	0.078	
σ_{ε_g}	IG, TN	0.2	∞	1/100	0.053	0.053	0.046	0.10	0.10	0.10
σ_{ε_R}	IG, TN	0.2	∞	1/100	0.126	0.31	0.49	1.93	1.93	1.93

Note: The prior distributions are the normal (N), beta (B) and inverse gamma (IG) distributions. The scaling parameters of the IG distribution scale the prior mode. The truncated normal distribution (TN) is applied with standard deviation equal to 2 and it is truncated at 0. The low prior for the standard deviation of the shock to z is obtained by scaling the prior mode with 1/1000 (instead of 1/100).

Our primary focus is to assess the robustness of the estimated neutral rate trend to alternative assumptions regarding a subset of the model's parameters. The priors assigned to the standard deviations of the unobserved shock processes (σ) are of particular importance in this context. The use of an inverse gamma prior effectively prevents the so called pile-up problem since it rules out values of the parameters too close to zero. The other side of the coin is that this prior may effectively rule out relevant parameter values close to zero, thereby possibly exaggerating the variability of some shock processes. We therefore estimate these parameters under alternative assumptions for the prior mode in the inverse gamma distributions or with truncated normal priors that allow the estimates to reach zero. Our main focus is the parameter governing the variability of the “other factor” (z_t), σ_{ε_z}, since it has a major impact on the estimated neutral rate trend. Since it is challenging to elicit a prior on this parameter we are particularly interested in assessing the sensitivity of both its estimate and the associated estimate of the neutral-rate trend to the choice of prior distribution.

We begin by examining the **Taylor rule (TR) version of the model**. To evaluate the influence of prior assumptions, we conduct sensitivity analysis. For each parameter individually, we shift the mode of the inverse gamma prior density by a factor of 10 and re-estimate the model.[45] The posterior mode estimates of the parameters $\sigma_{\varepsilon_{y_g}}$ (output gap IS equation), σ_{ε_π} (Phillips curve) and σ_{ε_g} (potential GDP growth) are robust to the change in the centering of the prior, i.e. they change by little and therefore appear well identified by the data. We note that the result for σ_{ε_g} presumably hinges on

---

[45] Kiley (2020) performs a similar experiment but changes the prior mean by a factor 2.

us using data on the Riksbank measure of the GDP gap in the estimation. On the other hand, the posterior estimates of $\sigma_{\varepsilon_{y^*}}$ (innovation to potential GDP) and σ_{ε_R} (monetary policy shock) are sensitive to the prior mode which suggests that these parameters are not well-identified by the data. The posterior estimate of σ_{ε_z} (other factors in the neutral rate trend) does not appear excessively sensitive to the change in the centering of the inverse gamma prior in the TR version of the model.

Next, we explore the effects of alternative prior assumptions on the innovation standard deviations by estimating these parameters using truncated normal priors. This approach, which allows the standard deviations to take values arbitrarily close to zero contrasts with the use of inverse gamma priors.[46] We first obtain the posterior mode estimate $\sigma_{\varepsilon_{y^*}} = 0$, which could potentially be seen as an illustration of the so called pile-up problem. Calibrating $\sigma_{\varepsilon_{y^*}}$ to zero and estimating the remaining five innovation standard deviations (along with the other model parameters) we find that the estimates of $\sigma_{\varepsilon_{yg}}$, σ_{ε_π} and σ_{ε_g} are invariant to the choice of prior (i.e. inverse gamma or truncated normal) while the estimate σ_{ε_r} is significantly altered. The estimate of σ_{ε_z} is 0.06 compared to 0.08 when the IG prior is used, a relatively small difference.

Table 1 reports the parameter estimates obtained under three different sets of assumptions, with a focus on the parameter σ_{ε_z} which governs the variability of the process for the other factors affecting the neutral rate trend (z_t). These assumptions include a baseline prior, an alternative with a lower prior mode for σ_{ε_z} and the alternative with a truncated normal prior for all the innovation standard deviations. The posterior estimates of σ_{ε_z} show relatively small differences across these three cases. The log marginal likelihoods of these three models are −235.8, −238.5 and −241.0, respectively, suggesting that the differences in model fit are modest.[47]

The results of a similar analysis of prior robustness for the **real rate random walk (RRW) version of the model** are in many ways similar, while crucially different for the key parameter of interest σ_{ε_z}. The posterior mode estimates of $\sigma_{\varepsilon_{yg}}$, σ_{ε_π}, σ_{ε_g} and σ_{ε_r} (real interest rate) are robust to alterations of the inverse gamma prior distribution hyperparameters, whereas the estimates of σ_{ε_z} and $\sigma_{\varepsilon_{y^*}}$ are sensitive to alterations. Estimating the model with truncated normal priors for the standard deviations we again find $\sigma_{\varepsilon_{y^*}} = 0$ and the estimates of σ_{ε_z} and σ_{ε_r} are significantly altered compared to those obtained with the inverse gamma prior. Also, lowering the prior mode of σ_{ε_z} by a factor 10 yields a much lower posterior estimate, suggesting that the parameter is not well identified by the data.

To summarise, our analysis indicates that the estimates of several parameters governing the neutral rate trend process are somewhat sensitive to the choice of prior distribution and hyperparameters. The sensitivity of the parameter σ_{ε_z}, which governs the

---

[46] Ferroni et al. (2016) use exponential or truncated normal priors to allow the standard deviations of structural shocks in DSGE models to obtain the value zero. They also use observation errors to avoid stochastic singularity. Since our model is short, i.e. we have six innovations and four observables, we do not need to use observation errors.

[47] The marginal likelihood is calculated using the modified harmonic mean (MHM) estimator.

variability of the "other factors" component, to its prior distribution is notably larger for the real rate random walk (RRW) version of the model compared to the Taylor rule (TR) version. In the section below, we illustrate how these differences in prior assumptions affect the estimated neutral rate trend.

Neutral rate trend estimates

We present estimates of the neutral rate trend from both versions of the model (TR and RRW) under three assumptions on the prior for the innovation standard deviations: a baseline assumption on the prior mode of σ_{ε_z}, a prior mode for σ_{ε_z} which is a tenth of the baseline mode, and truncated normal priors for all innovation standard deviations.

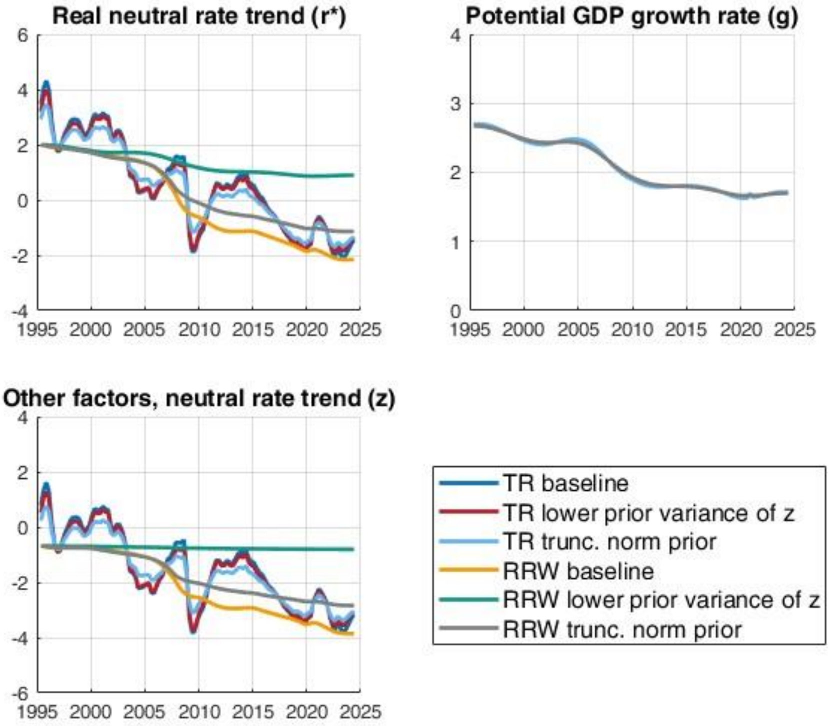
Figure 3 displays the neutral rate trend estimates. Since we use data on both GDP growth and the GDP gap the potential GDP growth rate is well identified and the estimate is virtually identical across all variants of the model.[48] The Riksbank's estimate of potential GDP growth suggests a decline of approximately one percentage point since 1995, which translates directly into a similar effect on the real neutral rate trend. Consequently, the differences between the real neutral rate estimates primarily reflect differences in the estimated "other factor" (z_t). Given our calibration $c = 1$ the estimated real neutral rate is simply the sum of these two components. In the Taylor rule (TR) version of the model, the baseline estimate of the neutral rate (blue line, visually overlapping the red line) exhibits considerable variability, which suggests that the estimate reflects cyclical factors to some extent. In contrast, the estimate obtained using the real rate random walk (RRW) version of the model is smoother (yellow line). Lowering the prior mode for σ_{ε_z} does not appreciably change the TR model estimate (red line, visually overlapping the blue line), while it leads to a significant change in the RRW model estimate (green line). Finally, using the truncated normal prior for the innovation standard deviations yield a low estimate of σ_{ε_z} in the RRW version of the model and a smoother estimate of the neutral rate (grey line). The corresponding estimate for the TR version of the model suggests that this model yields a reasonably robust estimate of the trend neutral rate (light blue line).[49]

---

[48] A similar result is obtained by Pescatori and Turunen (2016) who also use the GDP gap as an observable.

[49] In figure 9 in appendix B we show that the estimate of r* for the Taylor rule version of the model is reasonably robust to ending the sample in 2019Q4 (instead of 2024Q4) and not using the GDP gap as data in estimation.

Figure 3. Estimated real neutral rate trend, HLW (2017) model.



In summary all our estimates of the real neutral rate trend in Sweden using the HLW model suggest that it has declined in the past three decades and, furthermore, the estimates of the level in 2024 are reasonably unanimous (excluding the highest and lowest estimates), roughly in the region -2 to -1 percent. With a 2 percent inflation target this yields a nominal trend level in the interval 0 to 1 percent. However, we note that the estimates are sensitive to the prior assumptions on the process for the other factors affecting the neutral rate, z_t. This sensitivity is especially pronounced in the real rate random walk (RRW) version of the model.

3.4 Estimation results: Armelius (2018)

Table 2 presents the prior distributions and posterior mean estimates for the parameters in the Armelius et al (2018) model. We calibrate $c = 1$, $\rho_g = 1$ and $\rho_z = 1$, and based on preliminary estimation also $\Psi_{1,13} = 0$.[50] Sensitivity analysis similar to that for the HLW model above suggests that the parameter estimates are sensitive to alteration of the prior mode for σ_{ε_z} and $\sigma_{\varepsilon_{y^*}}$ (and to some extent also σ_{r_g}) but robust to such alterations for the remaining standard deviations of innovations. We report parameter posterior mean estimates, and corresponding neutral rate trend estimates, for three alternative assumptions: a baseline with inverse gamma priors for the innovation standard deviations, an alternative with a lower prior mode for σ_{ε_z} and an al-

---

[50] In preliminary estimation using a normal prior with mean 1 and standard deviation 0.5 we obtain a posterior estimate of c equal to 0.88 with a 90% posterior probability interval ranging between 0.15 and 1.62.

ternative with truncated normal priors for the innovation standard deviations. We observe a significantly lower posterior mean estimate of σ_{ε_z} when the prior mode is reduced by a factor 10. Using truncated normal priors yields parameter estimates that are broadly similar to those obtained in the baseline case.

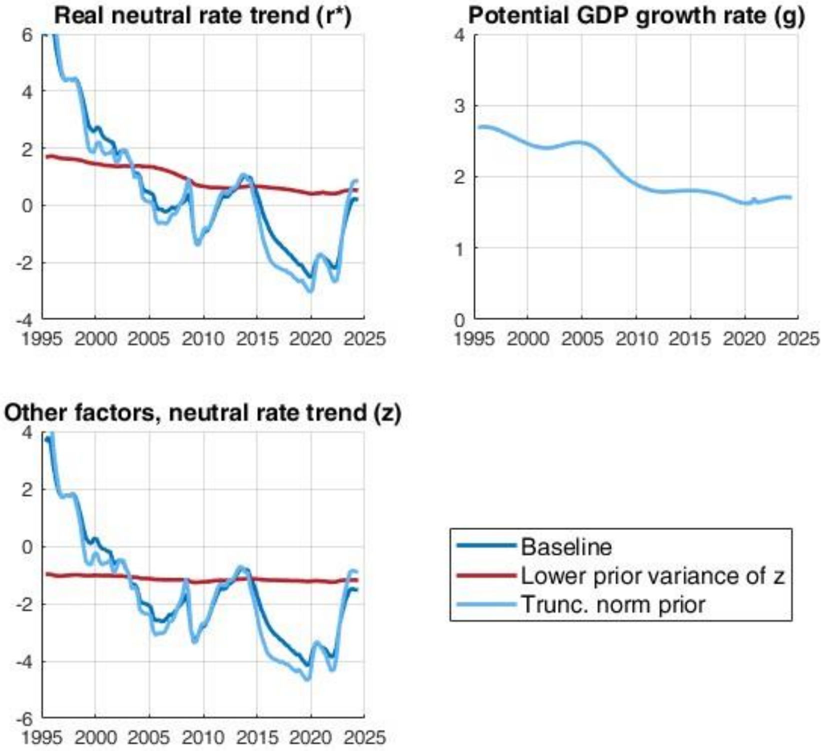
Table 2. Prior parameter distributions and posterior mean estimates for alternatives of the Armelius (2018) model.

	Prior distribution				**Posterior mean**		
	Dist.	Mode	Std.	Scale	Baseline	Low σ_{ε_z}	TN prior
$\Psi_{1,11}$	N	0.5	0.1		0.80	0.81	0.81
$\Psi_{1,12}$	N	0	0.1		-0.038	-0.095	-0.015
$\Psi_{1,21}$	N	0	0.1		0.024	0.020	0.026
$\Psi_{1,22}$	N	0.5	0.1		0.85	0.98	0.75
$\Psi_{1,23}$	N	0	0.1		0.0061	0.0047	0.0061
$\Psi_{1,31}$	N	0	0.1		0.11	0.12	0.11
$\Psi_{1,32}$	N	0	0.1		0.0065	0.011	0.014
$\Psi_{1,33}$	N	0.5	0.1		0.88	0.89	0.87
b_π	N	0.5	0.2		0.0048	-0.0029	-0.001
$b_{\Delta q}$	N	0.1	0.1		0.0049	0.0041	0.0043
b_y	N	0.1	0.1		0.023	0.026	0.021
σ_{ε_z}	IG, TN	0.2	∞	1/100	0.077	0.013*	0.090
$\sigma_{\varepsilon_{q^*}}$	IG, TN	0.2	∞	1/100	0.63	0.62	0.63
$\sigma_{\varepsilon_{y^*}}$	IG, TN	0.2	∞	1/1000	0.028	0.028	
σ_{ε_g}	IG, TN	0.2	∞	1/1000	0.053	0.053	0.046
$\sigma_{\varepsilon_{y_g}}$	IG, TN	0.2	∞	1/100	1.22	1.20	1.25
σ_{ε_R}	IG, TN	0.2	∞	1/1000	0.057	0.086	0.017
σ_{q_g}	IG, TN	0.2	∞	1/10	0.23	0.23	0.24
σ_{ε_π}	IG, TN	0.2	∞	1/100	0.47	0.47	0.47

Note: The prior distributions are the normal (N), beta (B) and inverse gamma (IG) distributions. The scaling parameters of the IG distribution scale the prior mode. The truncated normal distribution (TN) is applied with standard deviation equal to 2 and it is truncated at 0. The low prior for the standard deviation of the shock to z is obtained by scaling the mode with 1/1000 (instead of 1/100).

Figure 4 shows the estimated real neutral rate under the three prior assumptions. In the case of the low prior mode and consequently a low posterior mean estimate of σ_{ε_z} we obtain a rather smooth estimate of the neutral rate (red line). This estimate is largely influenced by the potential growth rate (g) with minimal variability in the other factors (z). In contrast, when using either the baseline prior or the truncated normal prior for σ_{ε_z}, the posterior mean estimate of σ_{ε_z} is larger, and we observe greater variability in both the estimates of z and the estimated neutral rate. It appears that these estimates capture cyclical factors to some extent. Our baseline estimate of the neutral rate is broadly similar in level to the estimate presented in Armelius et al. (2018), although our estimate exhibits greater volatility.[51]

Figure 4. Estimated real neutral rate trend, Armelius (2018) model.



3.5 Estimation results: Berger and Kempa (2014)

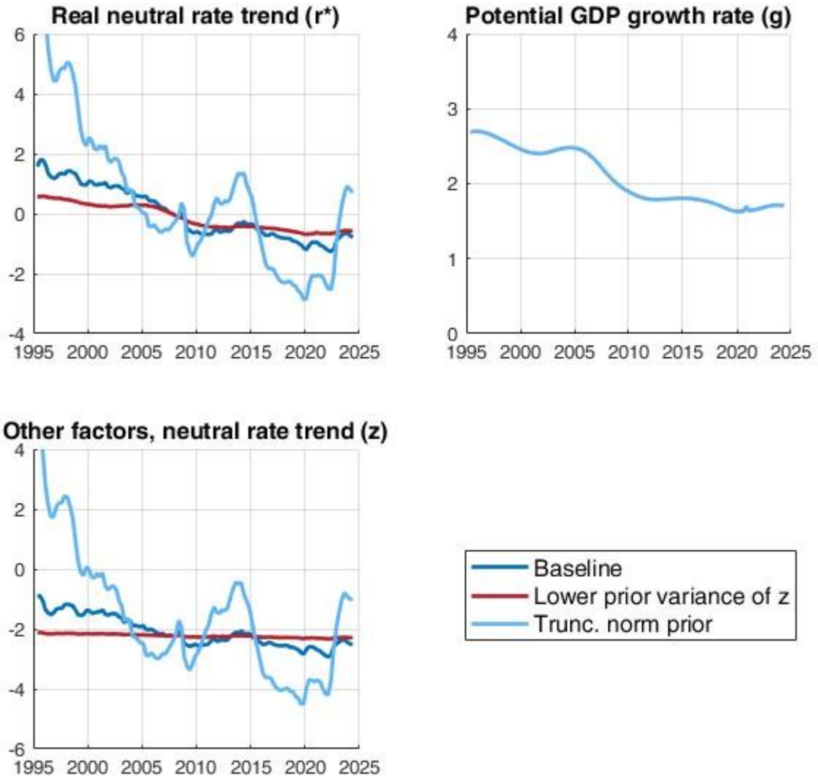
We estimate three variants of the Berger and Kempa (2014) model: a baseline with inverse gamma priors for the innovation standard deviations, an alternative with a lower prior mode for σ_{ε_z} and an alternative with truncated normal priors for the innovation standard deviations. As for the other two models we calibrate $c = 1$, $\rho_g = 1$ and $\rho_z = 1$ while estimating the remaining parameters. The estimates of the neutral rate are displayed in figure 5. Again, the differences in the estimates primarily reflect the different posterior estimates of the standard deviation of the innovation to z which reflect the different priors assumptions.[52]

---

[51] In figure 10 in appendix B we show estimates of r* with the sample ending in 2019Q4 (instead of 2024Q4) and not using the GDP gap as data in estimation.

[52] The posterior mean of σ_{ε_z} is 0.05 (baseline), 0.01 (low prior mode) and 0.09 (truncated normal prior). The prior distributions and posterior estimates of the remaining parameters are available upon request from the authors.

Figure 5. Estimated real neutral rate trend, Berger and Kempa (2014) model.



3.6 Recoverability

Recent research by Buncic et al (2024) has demonstrated that the ability of different models to recover unobservable variables (shocks and "stars") varies across models. We investigate the recoverability of these latent variables, defined as the ability to accurately recover them when the model is assumed to be correct and its parameters are assumed to be known. Our approach involves two steps. First we simulate a long artificial dataset, including time series for the model's unobservables, using the model and the posterior mode estimates of the model parameters.[53] Second, the smoother is applied to estimate the unobserved variables using the simulated data on the observed variables. As a final step, we compare the true (simulated) and estimated series for the unobserved variables to evaluate the model's ability to recover these variables under ideal conditions. We summarise this comparison using a scalar measure

---

[53] We simulate a series of 10,000 observations.

of recovery: the correlation between the first differenced simulated and smoothed series, respectively. A higher value indicates better recoverability of the true series.

Table 3 reports the recoverability measures for three different models, each evaluated under three different sets of observed variables used in the Kalman smoothing procedure. As expected, we observe that the ability to recover the unobservable variables generally increases as more data is used for state estimation. Conversely, when too few data series are used in the estimation process, the ability to recover the unobservables deteriorates substantially, with the correlation coefficients approaching values close to zero. Finally, our analysis suggests that both the HLW TR model and the Armelius et al (2018) model perform reasonably well in terms of recoverability when realistic datasets are used for state estimation. The performance of the RRW version of the HLW model, however, is notably worse.

Table 3. Recovery measures for the neutral rate, potential GDP growth and other factors in three semi-structural models

		Recoverability statistic		
Model	Observables	Neutral rate	Pot. GDP growth	Other factors
LW TR	Δy_t^{obs} π_t^{obs} R_t^{obs} $y_{g,t}^{obs}$	1.00	0.84	0.99
LW TR	Δy_t^{obs} π_t^{obs} R_t^{obs}	0.99	0.13	0.99
LW TR	Δy_t^{obs} π_t^{obs}	0.01	0.12	0.00
LW RRW	Δy_t^{obs} π_t^{obs} R_t^{obs} $y_{g,t}^{obs}$	0.12	0.83	0.10
LW RRW	Δy_t^{obs} π_t^{obs} R_t^{obs}	0.10	0.11	0.10
LW RRW	Δy_t^{obs} π_t^{obs}	0.07	0.11	0.07
Arm	Δy_t^{obs} π_t^{obs} R_t^{obs} $y_{g,t}^{obs}$ Δq_t^{obs} $q_{g,t}^{obs}$	0.83	0.83	0.83
Arm	Δy_t^{obs} π_t^{obs} R_t^{obs} Δq_t^{obs}	0.83	0.11	0.83
Arm	Δy_t^{obs} π_t^{obs} Δq_t^{obs}	0.02	0.09	0.00

Note: The recovery measure shown in the table is the correlation between the first differenced simulated series and the first differenced smoothed estimate of the series. The simulated series is of length 10,000.

3.7 Summary: estimates of the neutral rate trend using a range of models

In the preceding sections, we presented results from estimating several variants of three small-scale semi-structural models using Swedish data to investigate the uncertainty associated with estimates of the neutral rate. Here, we report estimates of $r*$state from two further models: the Riksbank's dynamic stochastic general equilibrium (DSGE) model MAJA, as described in Corbo and Strid (2020) and updated in Corbo and Strid (2025), and the semi-structural model developed by Armelius et al. (2024).[54] A notable difference between these two models and the three small-scale models we have considered thus far is that they incorporate substantially more data in the estimation of r^* which should improve identifcation.

---

[54] Estimation of the neutral rate trend (or trend real interest rate) in MAJA is discussed in a recent Riksbank staff memo, see Corbo and Strid (2025). For Armelius et al. (2024) the authors have kindly supplied us with an updated estimate of r* for Sweden.

MAJA is a two-region DSGE model for Sweden and its main trading partners, the euro area and the United States. The real neutral rate trend in Sweden, r*, is modelled similarly to equation 17 and it is assumed to be primarily driven by global factors, aligning closely with the real neutral rate trend in the foreign economy. The unobservables are estimated using data on 25 data series and the ability to recover r* is good. Armelius et al. (2024) observe that existing studies based on the Laubach and Williams (2003) framework often use limited information to identify r* and other unobservables. They address this by augmenting the framework with a dynamic factor model linked to several economic indicators to improve identification.[55]

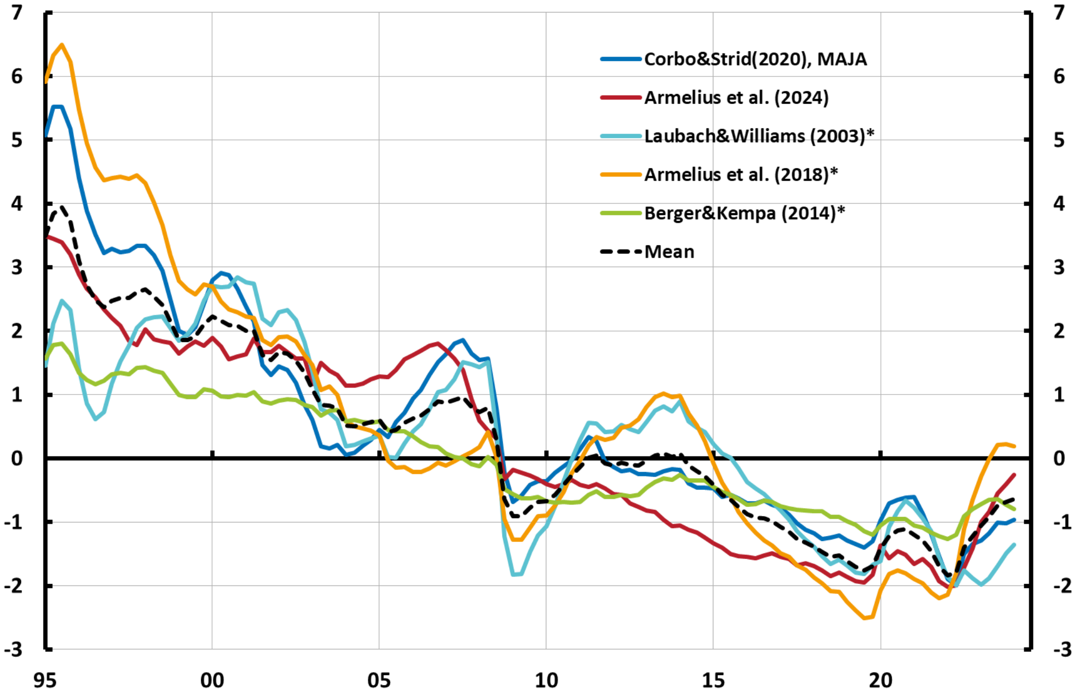
Figure 6 displays the baseline estimates of the real neutral rate obtained from each of the models considered in our analysis. Although these estimates are subject to considerable uncertainty and depend on specific assumptions made within each model, we believe they allow us to draw three broad conclusions regarding the evolution of the neutral rate trend in Sweden. First, it appears highly likely that the real neutral rate in Sweden has exhibited a downward trend over the past several decades. This observation is consistent with evidence presented in other research studies focusing on both Sweden and other advanced economies. Second, the downward trend shows signs of stabilising or even reversing in the post-pandemic period. Third, taking into account the inherent model uncertainty, a reasonable mean estimate of the real neutral rate trend in 2024, based on this class of models, appears to lie roughly within the interval between -1.5 and 0.5 percent. This yields an interval for the corresponding nominal neutral rate trend equal to 0.5 to 2.5 percent.[56] This range suggests that even though policy rates were raised significantly in many countries around the world in response to the surge in global inflation observed in 2021 and 2022, structural factors are likely to be exerting a continued downward pressure on interest rates.

---

[55] In addition to the GDP, inflation and an interest rate data typically used for estimation of these models they use data on 7 additional series: consumption, gross fixed capital formation, exports, imports, unemployment rate, capacity utilization and a consumer confidence indicator.

[56] Brand et al (2025) has recently published a range of semi-structural model estimates of r* for the euro area. Their estimates in 2024 are roughly in the interval -0.5 to 1, i.e. somewhat higher than our estimates for Sweden. Benigno et al (2024) reports estimates for the euro area using a set of macroeconomic models. In 2023 the range of estimates for r*is roughly between -1 and 1.5 percent.

Figure 6. Estimates of the real neutral rate trend in Sweden using five macroeconomic models 1995q2-2024q2.



Note: the estimates which are marked with a star (*) are produced and discussed in this paper while the estimates based on the models of Armelius et al (2024) and Corbo and Strid (2020) have been obtained from the authors.

4 Discussion and concluding remarks

How useful are measures of the neutral rate of interest? As we have discussed, it is important for a central bank to have a quantitative assessment of the trend in real interest rates, which is driven by factors outside the typical business cycle oriented frame of analysis. The estimates presented in this paper can be used for that purpose and, taken together, they provide some information on the sensitivity of these measures to variations in key assumptions. Our results indicate that, while useful, these measures have to be interpreted with caution, as they are quite sensitive to key prior assumptions within models and in addition there are differences between the estimates obtained with different models. In particular, measures from empirical models that rely on relatively few data series, together with a rich number of shocks, display quite some variation when altering key assumptions, as previously emphasized by Buncic (2024).

Short-run measures of the neutral rate

But what about the more short-run, i.e. business cycle, fluctuations in the neutral rate? We will argue that, under some specific assumptions, these measures have less practical relevance for an inflation-targeting central bank. To make this point, let us describe – in a highly simplified and stylized way – how staff at such a central bank may prepare monetary policy decisions. The objective is to make consistent forecasts of inflation and resource utilization, for a number of alternative monetary policy assumptions, so that policy makers may choose the most preferred combination of these three variables (i.e. the most preferred combination of forecasts for inflation, resource utilisation and the policy interest rate). These paths thus summarize (together with risk assessments and other additional material) the information that is relevant for the monetary policy decision. Since many central banks do not use the flex-price output gap as their preferred measure of resource utilisation, they have no compelling reason to estimate and assess the short-run component of the neutral rate in order to complete the above exercise.[57] This, probably, is a key reason why most such central banks do not publish any estimates or assessments of the neutral rate, but rather only its trend component.

Defining monetary policy stance

A final issue for discussion is the concept of monetary policy stance. We note in passing that there is strictly speaking no need to define this concept from the viewpoint of the monetary policy decision making process, as described above. But nevertheless, central bankers may want to use such terms in external communication. There are at least two competing, possible usages of these terms.

A first is based on Woodfords discussion, where we can somewhat loosely say a neutral stance is a real interest rate path that closes the flex-price output-gap as soon as possible (reflecting possible transmission lags and so forth).[58] If monetary policy is conducted in such a way that actual real interests changes in lock-step with the neutral rate, then the flex-price output gap will close, and in this precise sense monetary policy will be neutral. In other words, monetary policy will ensure that actual output equals the flex-price level of output, just like it would have been if prices and wages were fully flexible. If the shock poses no trade-off for monetary policy, this will also result in inflation stabilising at target. A shock that does impose a trade-off will instead require the central bank to balance its objectives against each other. For example, to balance an inflationary shock – the cost-push shock in equation (2) - , the central bank may choose to induce a negative output gap, in which case Woodford would say that monetary policy is contractionary. With this definition, there will be a correspondence between a contractionary monetary policy and a negative flex-price output gap.

A second alternative definition of monetary policy stance instead focuses on the difference between the actual real interest rate and the trend component of the neutral rate. If the actual real interest rate is above the trend component of the neutral rate, with this definition, monetary policy would be classified as contractionary.

---

[57] Rather than using a flex-price output gap, many central banks focus on the deviation of output or employment from a slow moving-trend.

[58] This usage is broadly consistent with the discussion in Section 2. Within the small model outlined in Section 2, there are no lags in the transmission of monetary policy. By setting the policy instrument appropriately, it is thus possible for the central bank to keep the output gap closed at all times (if it wishes to do so). In the context of that model, one may therefore define as neutral a monetary policy stance that ensures the gap is always closed. However, more realistic models do include transmission lags; after a shock has hit the economy, it is often simply not reasonable for policy makers to try to close the output gap immediately. In the context of such a model, a more pragmatic definition of a neutral stance is needed.

Suppose, for example, that a business cycle shock raises the neutral rate and, if unchecked, therefore tends to open up a positive flex-price gap and lead to inflation running above target. First, in the case that we use the flex-price gap as our measure of resource utilization, we would say that a monetary policy that raises the real interest rate in lock-step with the neutral rate is contractionary (in contrast to the Woodfordian definition above which would label it as neutral). Indeed, any policy that raises the real rate above the trend-component is classified as contractionary. With this definition, there is no correspondence between the stance of monetary policy and the sign of the flex-price output gap. Second, if a more conventional output gap measure is used, based on deviations of output from a slow moving trend, the discrepancy between theory and practice may be even larger. For example, a positive technology shock that drives up output will, in the simple model discussed in Section 2, lead to a fall in the neutral rate of interest. If monetary policy does not accommodate this (i.e. if the policy real interest rate is not lowered enough to fully compensate for the fall in the neutral rate), the sticky price equilibrium will feature rising output but a *negative* flex-price output gap that will tend to excerpt a downward pressure on inflation. A central bank examining a more conventional output gap will most likely conclude that the output gap is positive, and use language describing monetary policy as expansionary – since the real interest is below the long run trend component.

There is no right or wrong here, we simply end by noting that there is ample scope for confusion about terms like the neutral rate of interest and the stance of monetary policy. Therefore, it is essential that central bankers are as clear as possible with what they mean when they use these concepts. We suggest communication that focus on the monetary policy objectives and what the central bank strives to achieve, in terms of these objectives.

5 References

Adjalala, Frida, Alves, Felipe , Desgagnés, Hélène, Dong, Wei, Dmitry Matveev and Laure Simon (2024), “Assessing the US and Canadian neutral rates”, Staff Analytical Note 2024-9.

An, Sungbae and Frank Schorfheide (2007), “Bayesian Analysis of DSGE Models,’’ *Econometric Reviews*, 26(2-4), 113–172.

Armelius, Hanna, Martin Solberger, Erik Spånberg and Pär Österholm (2024), ”The evolution of the natural rate of interest: evidence from the Scandinavian countries”, *Empirical Economics* (2024) 66, s. 1633-1659.

Armelius, Hanna, Martin Solberger and Erik Spånberg (2018), ”Domestic policy or global influences? The Swedish natural rate of interest and international spillovers”, Research report, No 2018:1, Stockholm University, Department of Statistics.

Auclert, Adrien, Hannes Malmberg, Frédéric Martenet and Matthew Rognlie (2024), ”Demographics, wealth, and global imbalances in the twenty-first century”, Working Paper.

Belfrage Carl-Johan (2021), “The development of the Swedish real exchange rate over a longer perspective”, *Sveriges Riksbank Economic Review* 2021:2.

Belfrage Carl-Johan, Paolo Bonomolo and Pär Stockhammar (2020), ‘A time-varying equilibrium VAR model of the long-run real exchange rate’, Staff Memo, Sveriges Riksbank.

Benigno, Gianluca, Boris Hoffman, Galo Nuno, and Damiano Sandri (2024), “Quo vadis, r*? The natural rate of interest after the pandemic”, BIS Quarterly Review, March 2024.

Berger, Tino och Bernd Kempa (2014), “Time-varying equilibrium rates in small open economies: Evidence for Canada”, *Journal of Macroeconomics*, 39 (2014), s. 203-214.

Brand, Claus, Noëmie Lisack and Falk Mazelis (2024), “Estimates of the natural interest rate for the euro area”, ECB Economic Bulletin, Issue 1.

Brand, Claus, Marcin Bielecki and Adrian Penalver (editors) (2018), “The natural rate for intetest: estimates, drivers and challenges to monetary policy”, ECB Occasional Paper, No 217, December 2018.

Brand, Claus and Falk Mazelis (2019), “Taylor-rule consistent estimates of the natural rate fo interest”, ECB working paper, No 2257, mars 2019.

Brand, Claus, Noemi Lisack and Falk Mazelis (2025), “Natural rate estimates for the euro area: insights, uncertainties and shortcomings”, ECB Economic Bulletin, Issue 1 2025.

Buncic, Daniel (2024), “Econometric issues in the estimation of the neutral interest rate”, Economic Modelling, 132.

Buncic, Daniel, Adrian Pagan and Tim Robinson (2024), ”Recovering stars in macroeconomics”, manuscript.

Clark, Peter K. (1987), “The cyclical component of U.S. economic activity”, The Quarterly Journal of Economics, Vol. 102, No. 4, pp. 797-814.

Corbo, Vesna and Ingvar Strid (2020), ”MAJA: A two-region DSGE model for Sweden and its main trading partners”, Riksbank Wrking Paper No. 391.

Corbo, Vesna and Ingvar Strid (2025), ”Forecasts and narratives in a macroeconomic model with a real interest rate trend”, Staff memo, Sveriges riksbank, forthcoming.

Durbin, James and Siem Jan Koopman (2012), *Time series by state space methods,* Oxford University Press.

Ferroni, Filippo, Stefano Grassi and Miguel A. León-Ledesma (2019), “Selecting structural innovations in DSGE models”, *Journal of Applied Econometrics*, 34, pp. 205-220.

Fiorentini, Gabriele, Alessandro Galesi, Gabriel Pérez-Quirós and Enrique Sentana (2018), “The rise and fall of the natural interest rate”, Documentos de Trabajo, no 1822, Banco de Espana.

Flodberg, Caroline (2024), “Structural forces determine interest rates in the longer run”, Sveriges Riksbank Economic Commentary No. 4 2024.

Herbst, Edward and Frank Schorfheide (2015), *Bayesian Estimation of DSGE models*, Princeton University Press.

Holston, Kathryn, Thomas Laubach and John C. Williams (2017), “Measuring the natural rate of interest: International trends and determinants”, *Journal of International Economics*, Volume 108, Supplement 1, pages 59-75.

IMF (2023), “The natural rate of interest: drivers and implications for policy”, World Economic Outlook, April 2023, Chpt. 2.

Laubach, Thomas, and John C. Williams (2003) “Measuring the Natural Rate of Interest.” *The Review of Economics and Statistics*, vol. 85, no. 4, pp. 1063–70.

Lundvall, Henrik (2023), “Drivkrafter bakom globala trender i den neutrala räntan”, Bilaga 2 till Långtidsutredningen 2023.

Meyer, Sara, Paul Ulvedal and Erik Wasberg (2022), “The neutral real interest rate”, Staff Memo No. 7, Norges Bank.

Seim, Anna (2024), “Neutral interest rate - meaning, limitations and assessment”, Remarks given at Sveriges Riksbank on November 26.

Smets, Frank and Rafael Wouters (2007), "Shocks and Frictions in US Business Cycles", *American Economic Review,* Vol. 97, No. 3, pages 586-606.

Woodford, Michael (2003), "Interest and prices", Princeton University Press.

6 Appendix A – The definition of the output gap

Defining the output gap as the deviation from the efficient flex-price level is nowadays standard in the literature.. The reason this concept is relevant is that this deviation captures, theoretically, the welfare of the agents in the economy.

Thus, it is this gap that policy should optimally strive to close, provided that a shock that hits the economy does not imply that the central bank has to trade off an inflation gap against an output gap.

In practice, these output gaps are unobservable, and model dependent which creates an empirical challenge. It is common in practice to define some other output-gap using, e.g., an HP-filter or the deviation from trend. To illustrate the effect of this in our simple theoretical model in which $y_t^n = \upsilon a_t + c$, suppose we exclude the cyclical technology shock from our definition and instead define $y_t^* = c$, such that potential output is just a constant. Of course, in a more complicated model we may think of this as representing the fact that *some* cyclical shocks have been excluded from the trend output and the natural rate. With these definitions, we note that

$$y_t^n = y_t^* + \upsilon a_t$$

Using this in the small New Keynesian model presented in section 2 of the paper we get[59]

$$y_t - y_t^* = E_t(y_{t+1} - y_{t+1}^*) - (r_t - r_t^*)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa(y_t - y_t^*) + u_t - \upsilon a_t$$

$$L_t = \pi_t^2 + \lambda\left(y_t \ \ - y_t^* - \upsilon a_t\right)^2$$

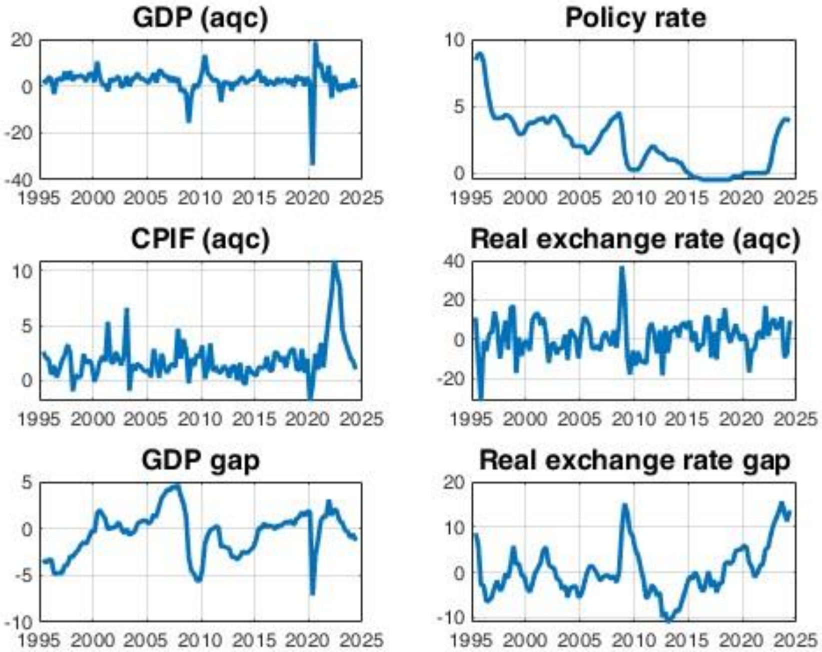
If this new definition of the real interest rate gap is closed, such that the new definition of the gap is zero, it does not mean that the output gap relevant for capturing cost pressures is closed, since short-run productivity shocks can lead to discrepancies. Thus, closing this gap at all times might still lead to non-zero inflation, even when there are no mark-up shocks. Furthermore, even if $y_t - y_t^*$ is zero, the loss function will still carry an output-related component equal to $\lambda(\upsilon a_t)^2$, indicating that this output gap is not the one monetary policy should care about from a theoretical viewpoint. That is, from a normative point of view, it is not this gap that should be closed.

---

[59] It can be shown, using the microfoundations of the model, that $\upsilon(1-\rho) - \alpha = 0$, such that the technology shock drops from the first equation (see Chapter 3 in Galís (2015).

7 Appendix B – Estimates of the real neutral rate trend

Figure 7. Data series used in estimation of the semi-structural models.



Note: GDP, CPIF and the real exchange rate are in annualised quarterly change (aqc) in percent. The policy rate is in percent. The GDP and real exchange rate gaps are percent deviations from the potential (or long run) level.

Figure 8. Median smoothed estimate of the real neutral rate trend and 80 percent probability intervals in the baseline versions of the models.

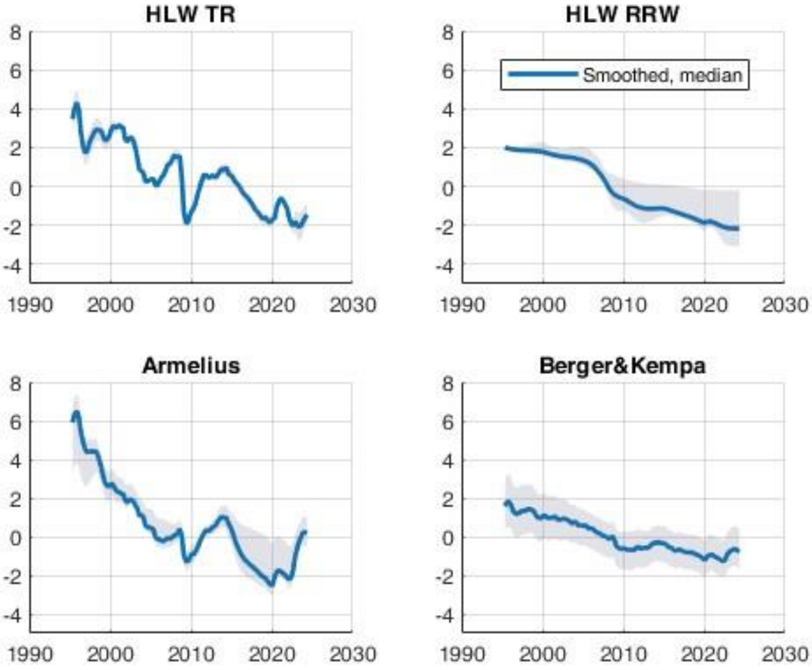


Figure 9. Sensitivity analysis for the baseline HLW-TR model: sample ends in 2019Q4 (red) and the model is estimated without the GDP gap as data (light blue)

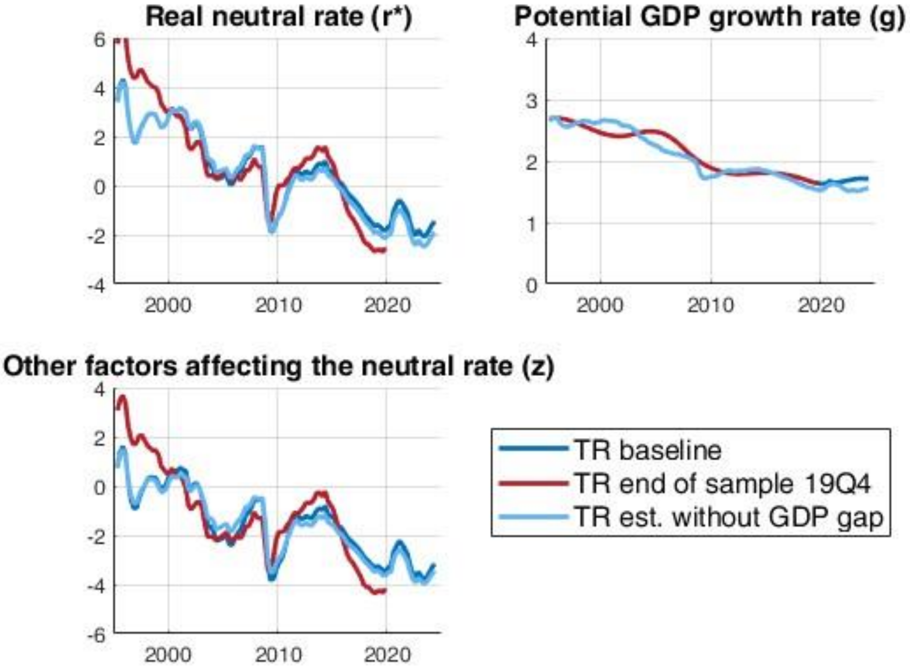
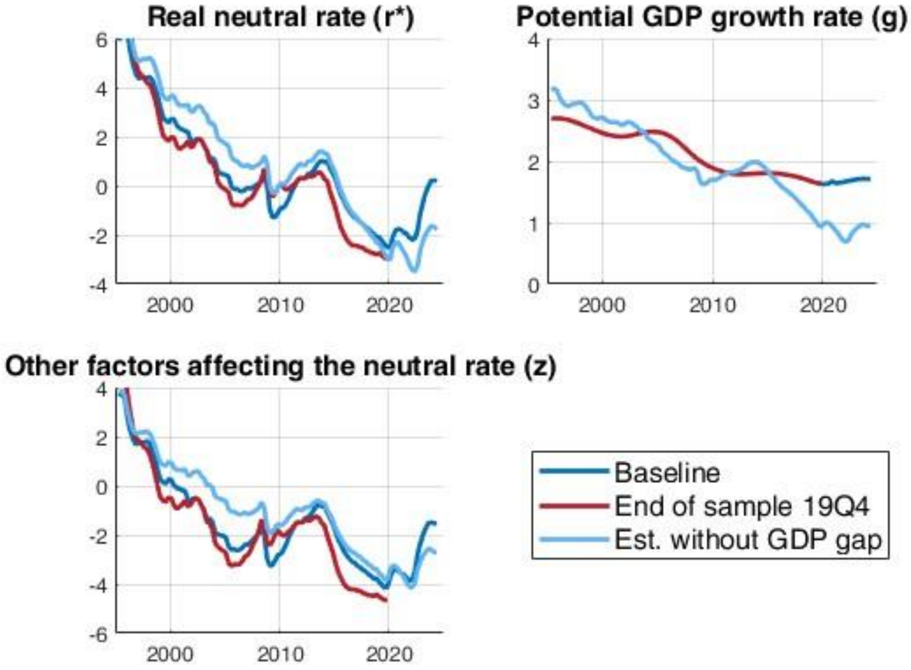


Figure 10. Sensitivity analysis for the baseline Armelius et al. (2018) model: sample ends in 2019Q4 (red) and the model is estimated without the GDP gap as data (light blue)



SVERIGES RIKSBANK
Tel +46 8 - 787 00 00
registratorn@riksbank.se
www.riksbank.se

PRODUCTION SVERIGES RIKSBANK