Staff memo

A time-varying equilibrium VAR model of the long-run real exchange rate

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Summary

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Many studies show that there is a tendency for the real exchange rate to revert to an equilibrium level, or the long-run real exchange rate (LRRER). This implies that a good assessment of the LRRER can be helpful when making exchange rate forecasts.

At the Riksbank, the assessment of the LRRER is supported by a time-varying equilibrium vector autoregressive (TVE-VAR) model. In this model, it is assumed that each observed variable is the sum of two unobserved components: the first one describes the evolution of the variable in the long run while the second one describes the deviation of the variable from that long-run trend. Both components are estimated simultaneously using Bayesian methods.

In this memo, we describe this model and the method used to estimate it. We also show some of its properties and estimated LRRERs.

[1] We would like to thank Vesna Corbo, Paola Di Casola, Mattias Erlandsson, Jesper Hansson and Stefan Laséen for their valuable comments. The opinions expressed are those of the authors and should not be interpreted as reflecting views of the Riksbank.

1. Introduction

Financial institutions, international organizations such as the IMF, and central banks devote a lot of attention to the assessment, and determinants, of equilibrium exchange rates (see e.g. IMF (2019)). At the Riksbank, forecasts for the krona exchange rate rely on the common finding in the literature that there is a tendency for the real exchange rate to revert to an equilibrium level (see e.g. Ca'Zorzi and Rubaszek (2018)).[2] A good assessment of this equilibrium level is hence quite helpful. In what follows, we describe the main method for calculating the equilibrium krona exchange rate used at the Riksbank, interchangeably referred to here as the trend or the long-run real exchange rate (LRRER). Much of the literature relies on the assumption that this equilibrium level of the real exchange rate can be captured by a historical mean of the real exchange rate series; see e.g. Rogoff (1996) and Taylor (2006). That assumption amounts to relying on relative purchasing power parity as the sole long-run determinant of the equilibrium real exchange rate.

However, that approach is a poor representation if the real exchange rate displays a trend. There are signs of a trend in the Swedish real effective exchange rate (although it varies depending on measure, see Sveriges Riksbank (2019)) and there is indeed support in economic theory for the presence of such a trend, if for instance there is a trend in relative productivity or the terms of trade. At the same time, there are large short-term variations in the real exchange rate and there is reason to expect them to be at least partly related to a set of variables, such as interest rate differentials and the current account balance.

A key characteristic of the long-run real exchange rate is thus that it is an unobservable trend variable that is influenced by trends in other variables. Ideally, this should be captured by any model or method that is designed to support the assessment of the long-run real exchange rate. At the same time, it is of interest to know not just that trend level of the real exchange rate (and what explains it), but also what explains the gap between the actual real exchange rate and the trend – in other words, it is desirable to simultaneously account for the important structural and cyclical contributions to the real exchange over time. Furthermore, in an empirical analysis, the trend and gap factors should ideally be estimated together, since information about the trend level (and thus the size of the gap between the actual and the trend real exchange rate) is useful when estimating how different variables contribute to the gaps. At the same time, information about the variables that influence the size of the gap is useful when estimating the trend.

This staff memo describes the properties of a time-varying equilibrium vector autoregressive (TVE-VAR) model. It has all the desirable properties mentioned above and is therefore used to support the Riksbank's assessments of the long-run real exchange rate.

This methodology is introduced and described in detail in Akkaya *et al.* (2019), and it is similar to the one in Del Negro *et al*. (2018), and Johannsen and Mertens (2020).[3]

The rest of this staff memo is organized as follows: Section 2 briefly describes the TVE-VAR model for the LRRER and the data. Section 3 presents results from the proposed baseline specification and shows some sensitivity analysis. Finally, Section 4 concludes.

2. The TVE-VAR model and data

The TVE-VAR approach involves the assumption that only permanent changes in the values of the explanatory variables reflect changes in fundamentals that affect the LRRER. The methodology is

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[2] Askestad *et al*. (2019) describe the krona forecasting process at the Riksbank and use out-of-sample forecast evaluations to contrast it with alternative methods for forecasting the krona. They find a relatively good forecasting performance for medium-term forecasts that rely on a slow convergence of the real krona exchange rate towards the Riksbank's assessed equilibrium level.

[3] How the current methodology differs from these studies is described in Section 2.

designed to simultaneously estimate the short-run and long-run components of the economic variables. More formally, it is assumed that each observed variable is the sum of two unobserved components: the first one describes the evolution of the variable in the long run, while the second one describes the deviation of the variable from its long-run trend. The second component is by nature temporary and is therefore expected to converge to zero in the medium term. The long-run components of the variables are described by linear and Gaussian models (the time-varying equilibrium parts of the model) while the short-run components follow a stationary VAR with unconditional expected value equal to zero. The technique is essentially a trend-cycle decomposition, which has been extensively used in the literature for a long time. Here we combine this technique with VAR modelling. Moreover, the methodology allows for drifting parameters in VARs, and it introduces the possibility of including variables in levels without requiring the nonstationary variables to be cointegrated.

As explained in Akkaya *et al.* (2019), the TVE-VAR is an extension of the steady-state Bayesian VAR model proposed by Villani (2009) in which the equilibria are assumed to be constants and the user is allowed to set priors on those steady states. In the TVE-VAR model, the equilibria are described by a model, instead of by simple priors on the values of the steady states as in Villani (2009). This allows us to make inferences on long-run equilibria using economic structures that can be inspired by theoretical models, or by statistical properties of the time series at hand.

The TVE-VAR model is estimated using data for 1995Q1-2019Q2 on the following six variables:[4]

- The *real exchange rate (RER).* A trade-weighted real effective krona exchange rate deflated by consumer price indices[5], calculated by the Riksbank and referred to as real KIX (where KIX simply stands for krona index). The estimation uses the log of this series.
- *Relative GDP per capita (Yrel)*. Measured as GDP per person aged 15-64 for Sweden relative to its trading partners (using their weights in KIX), constructed using GDP data from national sources and the OECD as well as population series from the World Bank. The estimation uses the log of this ratio in deviation from the sample mean.
- The *terms of trade (ToT)*. Measured as the ratio of Sweden's export deflator to its import deflator as reported in the national accounts. The estimation uses the log of this ratio in deviation from the sample mean.
- The *current account (CA)*. Measured as Sweden's current account balance in percent of GDP.
- The *policy rate differential (RepoDiff)*. Measured as the difference between Sweden's repo rate and a KIX-weighted average of the EONIA rate and the policy rates of Norway, the United Kingdom and the United States.
- The *VIX* index, i.e. the CBOE volatility index.[6] Data provided by Macrobond.

The usual VAR parameterization is given by:

$$\mathbf{\Pi}(L)\boldsymbol{x}_t = \boldsymbol{e}_t, \qquad (1)$$

where $\mathbf{\Pi}$ is the matrix describing the dynamics of the system (possibly with contemporaneous relations), $\mathbf{\Pi}(L) = \boldsymbol{I} - \mathbf{\Pi}_1 L - \mathbf{\Pi}_2 L^2 - \cdots - \mathbf{\Pi}_m L^m$ is a lag polynominal of order *m* with the property $Lx_t = x_{t-1}$. $\boldsymbol{x}_t$ is an *nx1* vector of observed variables and $\boldsymbol{e}_t$ is an *nx1* vector of i.i.d. Gaussian error terms fulfilling $E(\boldsymbol{e}_t) = 0$ and $E(\boldsymbol{e}_t \boldsymbol{e}'_t) = \boldsymbol{\Sigma}$.

An alternative parameterization proposed by Villani (2009) is given by:

[4] Unless otherwise indicated, the data is from Statistics Sweden or national sources. The data is exhibited in Figure 2.

[5] HICP for the euro area, CPIF (CPI with fixed mortgage interest rate) for Sweden, and CPI for the remaining countries. The index is constructed using data on Sweden's 32 largest trading partners.

[6] Other measures of uncertainty (e.g. vstoxx, different Economic Policy Uncertainty measures and aggregates thereof) have also been tested but were found to exert less influence on the real exchange rate.

$$\mathbf{\Pi}(L)(\boldsymbol{x}_t - \mathbf{\Psi}) = \boldsymbol{e}_t, \qquad (2)$$

where $\mathbf{\Psi}$ is an *nx1* vector describing the steady-state values of the variables in the system. The model in (2) will be referred to as the steady-state VAR.

As in Akkaya *et al.* (2019), the steady-state VAR in (2) is here further generalized by allowing time-varying parameters in the $\mathbf{\Psi}$ vector, that is

$$\mathbf{\Pi}(L)(\boldsymbol{x}_t - \mathbf{\Psi}_t) = \boldsymbol{e}_t \qquad (3)$$

Thus, the deviations from the time-varying steady states are modelled using a VAR model and the model in (3) will be referred to as the *TVE-VAR.*[7]

Following Lane and Milesi-Ferretti (2004) and Sellin (2007) we assume that the long-run real exchange rate is a function of the long-run relative GDP per capita and the long-run terms of trade, all expressed in logs:[8]

$$\overline{RER}_t = \alpha + \beta_1 \overline{Yrel}_t + \beta_2 \overline{ToT}_t + \varepsilon_t, \qquad (4)$$

where the bar denotes the long-run component and ε_t is an i.i.d. normally distributed error term. The long-run relative GDP per capita, terms of trade and current account are assumed to follow the simple statistical models:[9]

$$\begin{aligned} \overline{Yrel}_t &= \overline{Yrel}_{t-1} + g_t^Y \qquad (5) \\ g_t^Y &= \rho^Y g_{t-1}^Y + \varepsilon_t^Y \end{aligned}$$

$$\begin{aligned} \overline{ToT}_t &= \overline{ToT}_{t-1} + g_t^P \qquad (6) \\ g_t^P &= \rho^P g_{t-1}^P + \varepsilon_t^P \end{aligned}$$

$$\begin{aligned} \overline{CA}_t &= \overline{CA}_{t-1} + g_t^C \qquad (7) \\ g_t^C &= \rho^C g_{t-1}^C + \varepsilon_t^C \end{aligned}$$

Thus, in total there are six parameters to estimate for the long run: $\alpha, \beta_1, \beta_2, \rho^Y, \rho^P$ and ρ^C, in addition to the parameters in the variance-covariance matrix. The long-run components of the policy rate differentials and the VIX index are all assumed to be constant and do not directly enter in the long-run relationship of the real exchange rate (their effects are captured by the constant term). Neither does the current account, see footnote 9. These variables do however enter the VAR in

[7] The Hannan-Quinn information criterion was minimized for *m=2* lags, so that is what is used throughout this staff memo. Details about how this model is estimated are given in Appendix A.

[8] Lane and Milesi-Ferretti (2004) and Sellin (2007) also included the net foreign asset position as do some other studies that seek to explain real exchange rate developments, but we have excluded that variable as it is afflicted by measurement issues and arguably of questionable importance to industrial countries (insignificant for that group of countries in the study by Lane and Milesi-Ferretti (2004), perhaps because their borrowing on international capital markets is unconstrained (see Christopoulos *et al*. (2012)). Unlike what we do in this staff memo, they do not consider time-varying steady states. Sellin calibrates the parameter coefficients of equation (4) and models the short-run dynamics using a vector error correction model. We have found no evidence for cointegration between the variables in equation (4) during our sample period.

[9] It can also be argued that it would make sense to include the current account in equation (4) but that does not change the estimate of the LRRER in a quantitative meaningful way (results are available from the authors upon request). Hence, for reasons of parsimony, the current account is not included in equation (4) in the baseline specification of the TVE-VAR model. However, due to the non-stationarity of the current account series, see Figure 2, we have let the deviations from the *time-varying* steady state of this variable enter the VAR specification in equation (3).

equation (3) that describes the short-run component (that is the gap between the observed variable and its long-run level).

The parameters of the model are estimated using the Bayesian methods described in Appendix A. Generally, the estimation looks good, especially when using many draws from the posterior distribution (see the discussion and results in Section 3 and Appendices B and C). The prior means for the coefficients of relative GDP and terms of trade in Table 1 below follow the ones used in Sellin (2007).[10]

Table 1: Prior and posterior means and standard deviations of the long-term parameters.

	Prior means	Prior standard deviations	Posterior means	Posterior standard deviations
Constant, α	4.848 (127.5)	0.01	4.840 (126.4)	0.008
Coefficient of $\overline{Yrel}$, β_1	-0.7	0.35	-0.743	0.375
Coefficient of $\overline{ToT}$, β_2	-0.7	0.35	-1.933	0.261
AR(1) coefficient of $\overline{Yrel}$, ρ^Y	0.8	0.1	0.830	0.069
AR(1) coefficient of $\overline{ToT}$, ρ^P	0.8	0.1	0.813	0.085
AR(1) coefficient of $\overline{CA}$, ρ^C	0.8	0.1	0.930	0.049

Note: These estimates are based on 200 000 draws from the posterior distribution.

The bivariate correlations, i.e. the correlation between every pair of parameters, are generally small, see Table A1 in Appendix B.

The estimated impulse response functions of the long-run relationship in equation (4) are in line with what we would expect based on macroeconomic theory, see Figures A2 and A3 in Appendix B. A one standard deviation higher relative GDP per capita trend or terms of trade trend significantly decreases the LRRER estimate at all horizons (and vice versa).

3. Results

The results from the baseline specification, as described in equations (3) to (7) in the previous section, are shown in Section 3.1 and a sensitivity analysis with discussions in Section 3.2.

3.1 Results from the baseline specification

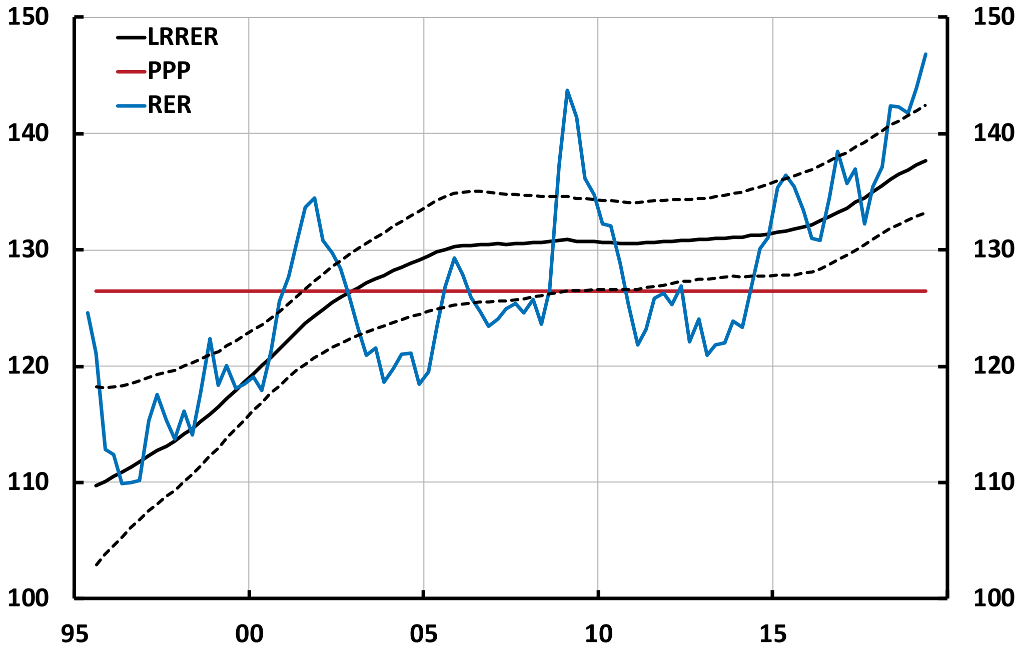
Figure 1 below shows the estimated LRRER from the baseline specification together with its 95-percent posterior coverage interval (dashed lines), the real exchange rate and the PPP (the constant in equation (4)).[11] Deviations between LRRER and PPP are due to trends in relative productivity and terms of trade, see equation (4), and deviations between RER and LRRER are due to gaps between actual and trend levels (interpreted as the short-run components) of all the variables as captured by the VAR model.

[10] See Appendix A for a reasoning behind the other prior means and variances and a description of the prior distributions of the parameters. Figure A1 in Appendix A shows the prior and posterior distributions.

[11] Here the prior on the PPP (Purchasing Power Parity) is the midpoint of the most recent Riksbank assessment of the interval of the LRRER, i.e. [120, 135].

Figure 1. The estimated LRRER

Real KIX, index, 18 November 1992 = 100

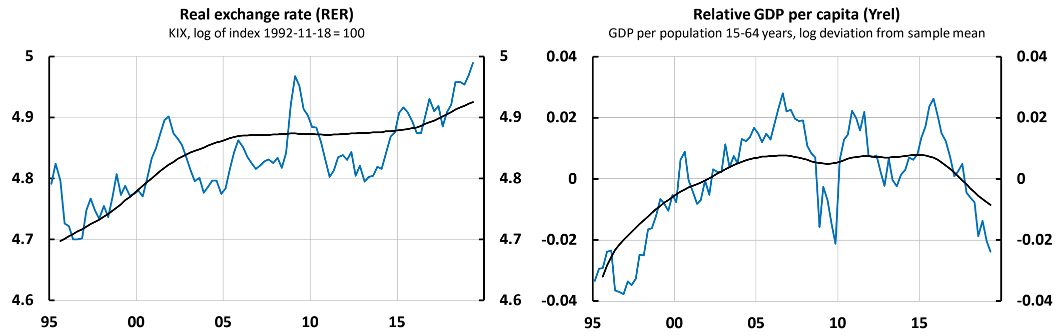


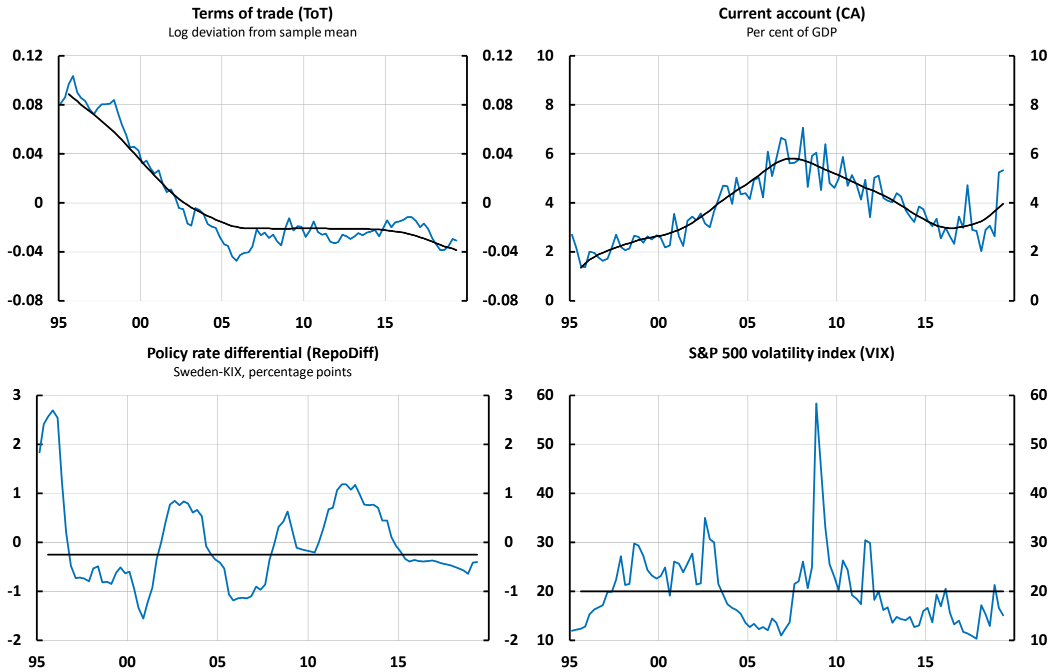
Note: The dashed lines represent the 95-percent posterior coverage interval. PPP refers to the constant in equation (4). The estimates are based on 200 000 draws from the posterior distribution.

Sources: National sources, Statistics Sweden and the Riksbank

The actual levels and estimated long-run components of each variable are shown in Figure 2. The model picks up a weakening of the LRRER between 1995 and 2005, driven by the weakening trend in the terms of trade, and another weakening of the LRRER after 2015 driven by a further weakening of the terms of trade trend as well as a weakening trend in relative GDP per capita. According to the model, the krona real exchange rate was significantly weaker than the LRRER at the end of the sample period and this coincided with relative per capita GDP being below its trend level.

Figure 2. The data and the estimated long-term components





Note: Outcomes are shown in blue, the estimated long-term components in black. The short-run components are calculated as the differences between the two. The estimates are based on 200 000 draws from the posterior distribution.

Sources: National sources, Macrobond, Statistics Sweden, OECD, Thomson Reuters, the World Bank and the Riksbank.

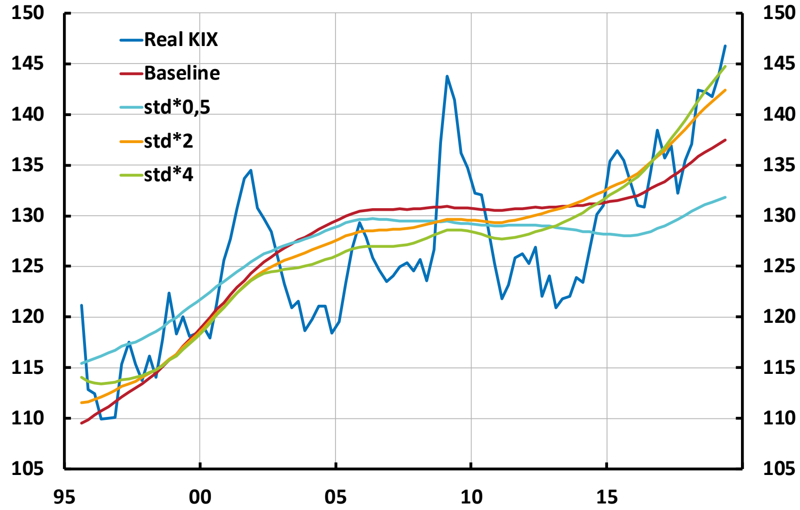
The results so far have been based on 200 000 draws from the posterior distributions. So called cumulative moving averages of the parameter estimates, see Figure A4 in Appendix B, indicate that they converge already after around 50 000 draws (also supported by Figures A5-A7 in Appendix C).[12]

3.2 Sensitivity analysis

In this section, we present sensitivity analysis of the baseline specification. For example, as always using Bayesian estimation, results are subject to the choices of the prior means, distributions, standard deviations and hyperparameters. To a very large extent, this staff memo follows previous studies, e.g. using the same prior means as in Sellin (2007), as well as using very standard probability distributions and hyperparameters. There is, however, no real consensus about what standard deviations to use for the parameters, and thus how much weight to give to data (tighter priors means lesser weight to data and vice versa). In Figure 3, the LRRER from the baseline specification (in dark blue) is compared to the ones where the prior standard deviations of the parameters in Table 1 have been multiplied by 0.5, 2 and 4.

[12] The cumulative moving average is constructed so that the first observation of the running mean is the mean of the first observation of the parameter estimates series, the second observation is the mean of the first two observations of the parameter estimates series and so on.

Figure 3. LRRER, the effects of different standard deviations of the parameters
Real KIX, index, 18 November 1992 = 100



Note: The lines “std*0.5”, “std*2” and “std*4” show the LRRER when estimated as in the baseline specification but with prior standard deviations on the parameters in the long-run relationships (see equations (4)-(7) and Table 1) multiplied by 0.5, 2 and 4, respectively. Due to estimation time restrictions, these estimates are based on 50 000 draws from the posterior distribution.

Sources: National sources, Statistics Sweden and the Riksbank

It is clear that giving more weight to data (as by e.g. multiplying the standard deviations by four) yields a more volatile LRRER throughout the entire period and a much weaker LRRER at the end of the sample (closer to the actual real exchange rate).[13]

Estimating the LRRER using the Bayesian techniques described in this staff memo is computationally time-consuming. It is therefore informative to know approximately how many draws from the posterior distribution that are needed in order to get satisfactorily robust estimates. Figures A5-A7 in Appendix C show how the estimated LRRER depends on the number of draws. It is clear that around 50 000 draws from the posterior distributions are required for the estimated LRRER to be satisfactorily robust to different random seeds. Table A2 in Appendix C shows that also the parameter estimates, especially the coefficients for relative GDP per capita and terms of trade, change considerably using a small number of draws or changing the prior standard deviations for the parameters. It is clear that increasing the prior standard deviations (thus giving more weight to data) yields larger parameter estimates (in absolute numbers) in the long-run relationship for the LRRER (i.e. the coefficients for relative GDP per capita and terms of trade in equation (4)).

So far, the results are based on data for the period 1995Q1-2019Q2. Shortening of the sample, from the beginning or from the end, affects the estimates of the LRRER, see Figures 4 and 5.

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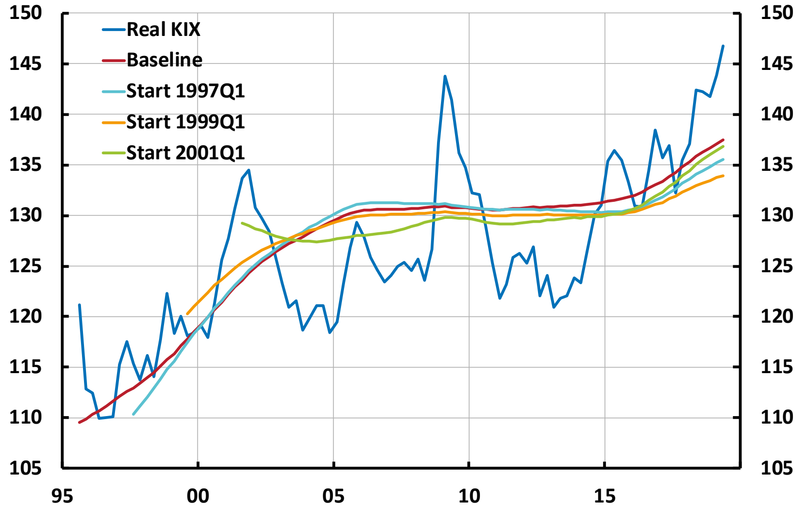
[13] How the prior standard deviations in Table 1 have been chosen is briefly explained in Appendix A.

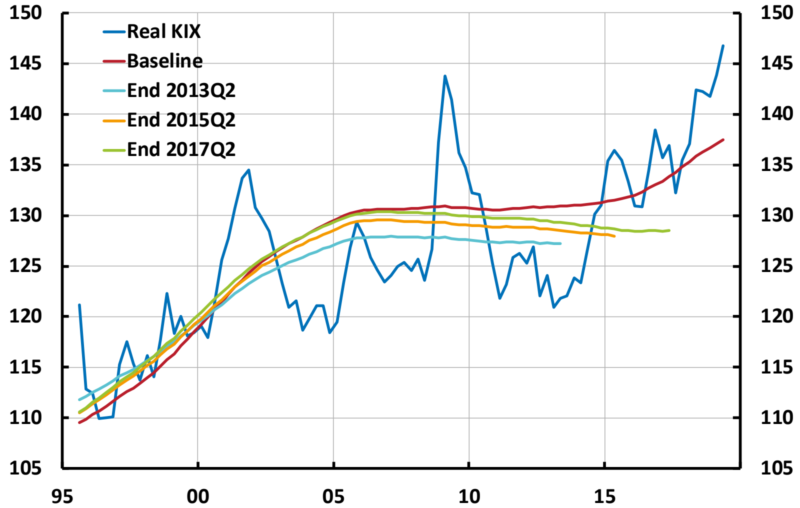
Figure 4. LRRER, different starting points
Real KIX, index, 18 November 1992 = 100



Note: The lines "Start 1997Q1", etc. show the LRRER when estimated as in the baseline specification but with different starting points for the sample period. Due to estimation time restrictions, these estimates are based on 50 000 draws from the posterior distribution.

Sources: National sources, Statistics Sweden and the Riksbank

Figure 5. LRRER, different endpoints
Real KIX, index, 18 November 1992 = 100



Note: The lines "End 2013Q2", etc. show the LRRER when estimated as in the baseline specification but with different endpoints for the sample period. Due to estimation time restrictions, these estimates are based on 50 000 draws from the posterior distribution.

Sources: National sources, Statistics Sweden and the Riksbank

This section has shown that the LRRER estimates depend a lot on the TVE-VAR settings, i.e. the prior standard deviations, the number of draws from the posterior distribution and the starting points and endpoints. Knowing this, it makes sense that the baseline specification is based on the entire data sample (1995-2019), as many as 200 000 draws from the posterior distribution (where around 50 000 draws seem to yield relatively stable results) and on reasonably loose priors (so that data are allowed to affect the results).

4. Conclusions and ideas for future work

The forecasting method for the Swedish krona in the medium term currently used at the Riksbank relies on the common finding in the literature that there is a tendency for the real exchange rate to revert to the long-run real exchange rate (LRRER). A good assessment of both the LRRER and the (temporary) deviations from the LRRER is thus of importance not only for the krona forecasts but also for the understanding of economic developments in general.

In this staff memo, we have described the properties of a model that very neatly meets those needs: the time-varying equilibrium vector autoregressive (TVE-VAR) model.[14] In this model, the long-run components of the variables are described by a linear and Gaussian model (the time-varying equilibrium part of the model). The short-run components follow a stationary VAR with unconditional expected value equal to zero. The parameters of the model are estimated using Bayesian methods.

We have shown that around 50 000 draws from the posterior distribution are needed in order to get robust estimates of the LRRER. However, no matter the number of draws, the estimates are sensitive to different choices of prior tightness as well as different choices of starting points and endpoints of the estimation samples. In the standard specification of the TVE-VAR, the priors are generally loose, i.e. data is given the possibility to affect the parameter estimates.

At the Riksbank, there are on-going efforts to improve and extend the TVE-VAR model. Forecasting has not been the prime focus of this staff memo but in future work the forecasting performance of the TVE-VAR could be compared with the forecasting performance of e.g. naïve forecasts, VECM's, the Villani (2009) steady-state BVAR model and the Riksbank's published forecasts. As for alternative model specifications, a few options to consider are: adding a linear trend in the equation for the LRRER (or in some of the equations for the long-run relative GDP, terms of trade and current account); identifying shocks in the short-run VAR model (in order to enable relevant decompositions and conditional forecasts); and making use of other measurement variables. In order to understand the properties of the TVE-VAR model better, one might also consider estimating the model using simulated data.

[14] Currently, however, the TVE-VAR model is not used directly for forecasting the krona at the Riksbank, only indirectly through its contribution to the assessment of the LRRER.

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[15] This paper is work in progress: a draft is available upon request from the authors, and a presentation can be found at media.ed.ac.uk/media/Paolo+Bonomolo.mp4/1_riiz5rfk/101834671.

Appendix A. Estimating the TVE-VAR model

The TVE-VAR model is in state-space form so that:

$$\boldsymbol{\Psi_t} = \boldsymbol{H\theta_t}\,, \qquad \text{(A1)}$$

where the latent process $\boldsymbol{\theta_t}$ is modelled as:

$$\boldsymbol{\theta_t} = \boldsymbol{G\theta_{t-1}} + \boldsymbol{P\xi_t} \qquad \text{(A2)}$$

and $\boldsymbol{H}$, $\boldsymbol{G}$ and $\boldsymbol{P}$ are parameter vectors of which only $\boldsymbol{G}$ is estimated (the others calibrated), and $\boldsymbol{\xi_t}$ is a vector of uncorrelated and normally distributed shocks with $E(\boldsymbol{\xi_t}) = \boldsymbol{0}$ and $E(\boldsymbol{\xi_t \xi'_t}) = \boldsymbol{\Sigma}$.

The TVE-VAR model of the LRRER consists of the following 13 equations describing the long-run part.[16]

$$
\begin{aligned}
\overline{RER}_t &= \alpha + \beta_1 \overline{Yrel}_t + \beta_2 \overline{ToT}_t + \varepsilon_t \\
\overline{Yrel}_t &= \overline{Yrel}_t \\
\overline{ToT}_t &= \overline{ToT}_t \\
\overline{CA}_t &= \overline{CA}_t \\
\overline{RepoDiff}_t &= \overline{RepoDiff}_{t-1} \\
\overline{VIX}_t &= \overline{VIX}_{t-1} \\
\overline{PPP}_t &= \overline{PPP}_{t-1} \\
\overline{Yrel}_{t+1} &= \overline{Yrel}_t + g_t^Y \\
g_t^Y &= \rho_Y g_{t-1}^Y + \varepsilon_t^Y \\
\overline{ToT}_{t+1} &= \overline{ToT}_t + g_t^P \\
g_t^P &= \rho_P g_{t-1}^P + \varepsilon_t^P \\
\overline{CA}_{t+1} &= \overline{CA}_t + g_t^C \\
g_t^C &= \rho_C g_{t-1}^C + \varepsilon_t^C
\end{aligned}
$$

Thus, equation A1 becomes:

$$
\begin{bmatrix} \overline{RER}_t \\ \overline{Yrel}_t \\ \overline{ToT}_t \\ \overline{CA}_t \\ \overline{RepoDiff}_t \\ \overline{VIX}_t \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} \overline{RER}_t \\ \overline{Yrel}_t \\ \overline{ToT}_t \\ \overline{CA}_t \\ \overline{RepoDiff}_t \\ \overline{VIX}_t \\ \overline{PPP}_t \\ \overline{Yrel}_{t+1} \\ g_{t+1}^Y \\ \overline{ToT}_{t+1} \\ g_{t+1}^P \\ \overline{CA}_{t+1} \\ g_{t+1}^C \end{bmatrix}
$$

[16] $\overline{PPP}_t$ is a constant that equals one for all t.

And equation A2:

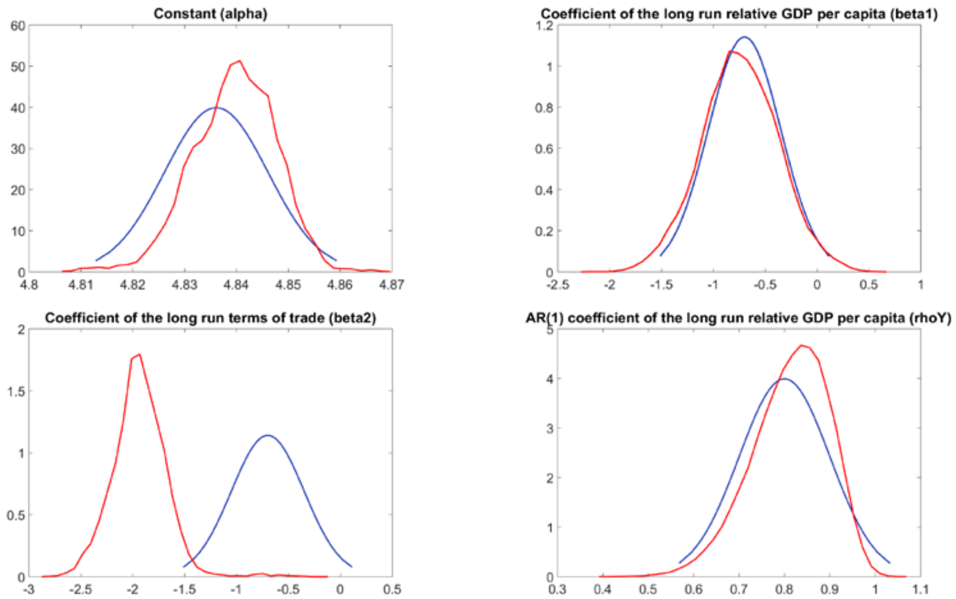
$$
\begin{bmatrix} \overline{RER}_t \\ \overline{Yrel}_t \\ \overline{ToT}_t \\ \overline{CA}_t \\ \overline{RepoDiff}_t \\ \overline{VIX}_t \\ \overline{PPP}_t \\ \overline{Yrel}_{t+1} \\ g^Y_{t+1} \\ \overline{ToT}_{t+1} \\ g^P_{t+1} \\ \overline{CA}_{t+1} \\ g^C_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \alpha & \beta_1 & 0 & \beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_Y & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_P & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_C \end{bmatrix} \begin{bmatrix} \overline{RER}_{t-1} \\ \overline{Yrel}_{t-1} \\ \overline{ToT}_{t-1} \\ \overline{CA}_{t-1} \\ \overline{RepoDiff}_{t-1} \\ \overline{VIX}_{t-1} \\ \overline{PPP}_{t-1} \\ \overline{Yrel}_t \\ g^Y_t \\ \overline{ToT}_t \\ g^P_t \\ \overline{CA}_t \\ g^C_t \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \varepsilon^Y_t \\ \varepsilon^P_t \\ \varepsilon^C_t \end{bmatrix}
$$

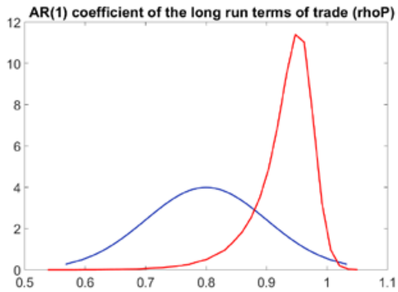
We are interested in approximating the joint posterior distribution:

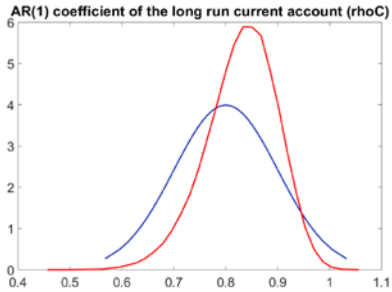
$$\boldsymbol{f}(\boldsymbol{\Pi}, \boldsymbol{\theta}, \boldsymbol{G}, \boldsymbol{\Sigma} | data)$$

Following Villani (2009), the estimation is done using a Gibbs sampler. Also, the choice of prior distributions of the parameters follows the one made in Villani (2009). Specifically, the priors of the variances follow an Inverse Gamma distribution and the priors on all other parameters are assumed (multinomial) Gaussian. The prior means for the coefficients of relative GDP and terms of trade in $\boldsymbol{G}$ follow Sellin (2007). The prior variances are set so that two standard deviations confidence bands around the prior means are still within a region of acceptable parameter estimates. E.g. we have set the prior means and standard deviations of the AR(1) parameters to 0.8 and 0.1 respectively (as an estimated AR(1) parameter larger than one would indicate non-stationarity), see Table 1. We have set a tighter prior on the constant as we do not want the LRRER level to react too much to new RER outcomes. Prior and posterior distributions of the six parameters guiding the long-term levels of the trending variables (the real exchange rate, relative GDP per capita, terms of trade and the current account) are shown in Figure A1.

Figure A1. Prior (blue) and posterior distributions (red) of the long term parameters







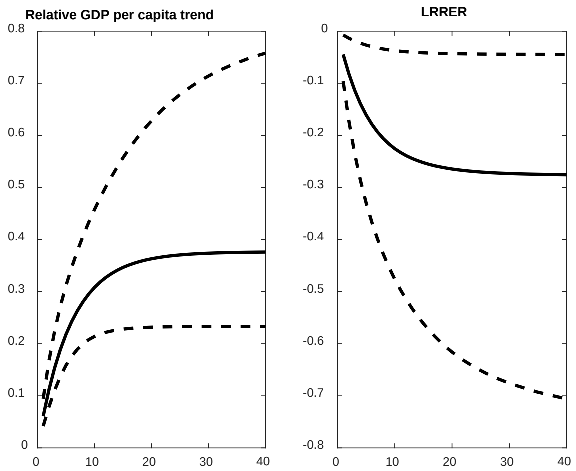
Note: The estimates are based on 200 000 draws from the posterior distribution

As can be seen in Figure A1, the data is informative, especially about the coefficient of the long-run terms of trade and the AR(1) coefficients of the long-run relative GDP per capita and terms of trade. For those parameters, the posterior means and variances differ substantially from their prior counterparts. The hyperparameters of the model are uncontroversial and follow the literature[17], and the lag length in the model is set to *m*=2.

[17] See, for example, Doan (1992) and Villani (2009) where e.g. the overall tightness is set to 0.2, the cross-variable tightness to 0.2 and the lag length decay parameter to 1.

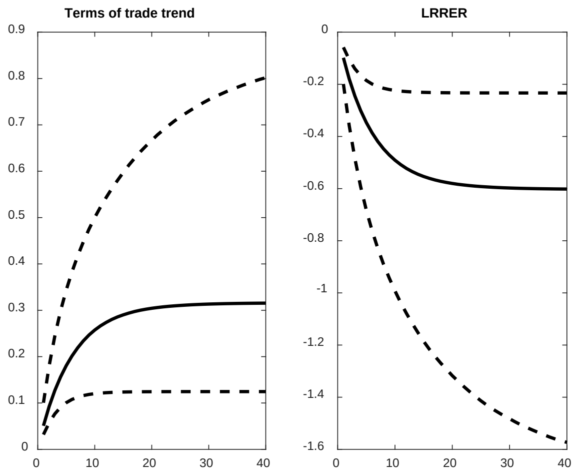
Appendix B. Results from the baseline specification

Figure A2. Impulse responses to a one standard deviation relative GDP per capita trend shock
Percent



Note: The solid line is the median, dashed lines are approximately 95 percent probability bands. The standard deviation of the relative GDP per capita trend shock is 0.061 percent with a contemporaneous LRRER response of -0.045 percent. In the long run, the response of the relative GDP per capita trend is 0.38 percent and the LRRER response is around -0.28 percent. For reference, the standard deviation of the shock to LRRER (in equation (1)) is 0.177 percent. The responses are significantly non-zero at all horizons. The impulse responses are based on 200 000 draws from the posterior distribution.

Figure A3. Impulse responses to a one standard deviation terms of trade trend shock
Percent



Note: The solid line is the median, dashed lines are approximately 95 percent probability bands. The standard deviation of the terms of trade trend shock is 0.051 percent with a contemporaneous LRRER response of -0.098 percent. In the long run, the response of the terms of trade trend is 0.32 percent and the LRRER response is around -0.60 percent. For reference, the standard deviation of the shock to LRRER (in equation (1)) is 0.177 percent. The responses are significantly non-zero at all horizons. The impulse responses are based on 200 000 draws from the posterior distribution.

Figure A4. Cumulative moving averages of the parameter estimates

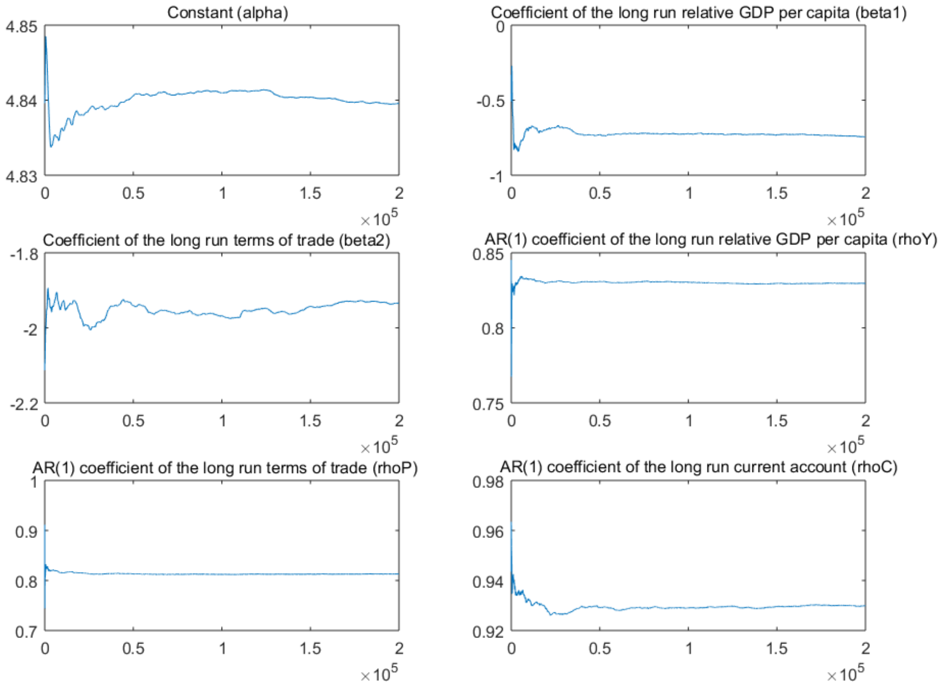


Table A1. Bivariate correlations between the parameters

	α	β_1	β_2	ρ^Y	ρ^P	ρ^C
α	1	0.03	0.21	0.01	-0.01	0.07
β_1	0.03	1	0.11	-0.02	-0.15	-0.08
β_2	0.21	0.11	1	0.02	0.03	0.18
ρ^Y	0.01	-0.02	0.02	1	0.01	0.02
ρ^P	-0.01	-0.15	0.03	0.01	1	0.01
ρ^C	0.07	-0.08	0.18	0.02	0.01	1

Note: The estimates are based on 200 000 draws from the posterior distribution.

Appendix C. Sensitivity analysis

Figure A5. LRRER, three different seeds. 1 000 draws

Real KIX, index, 18 November 1992 = 100

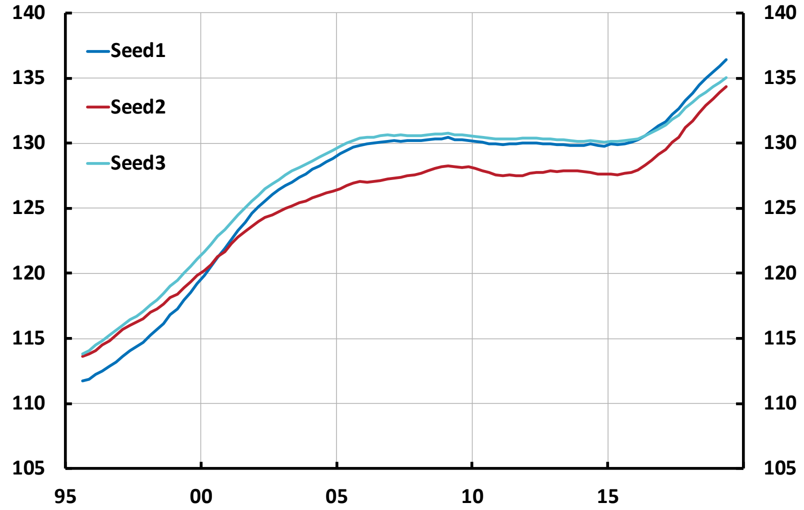


Figure A6. LRRER, three different seeds. 10 000 draws

Real KIX, index, 18 November 1992 = 100

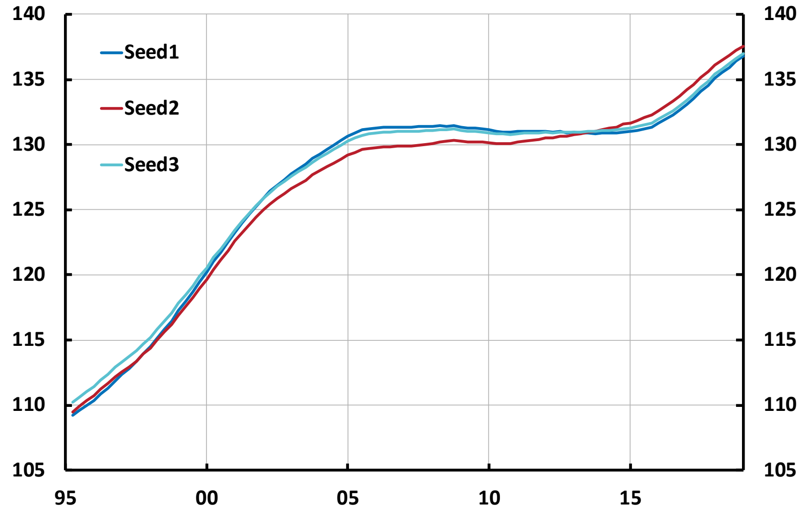


Figure A7. LRRER, three different seeds. 50 000 draws

Real KIX, index, 18 November 1992 = 100

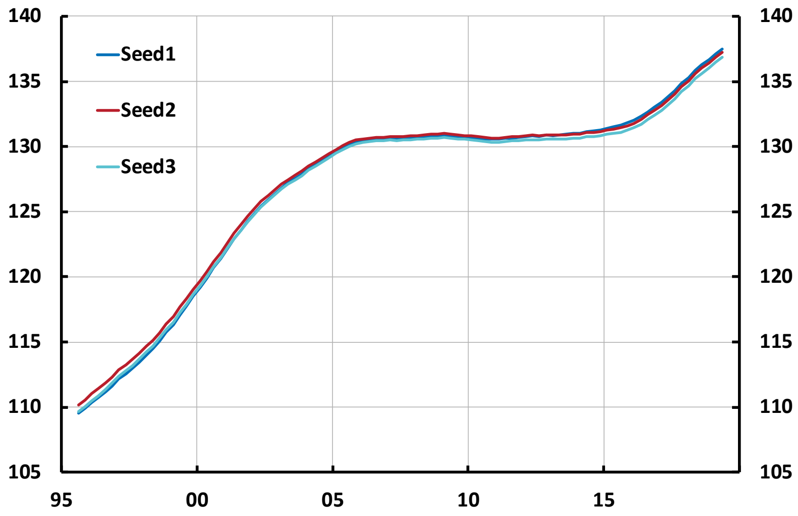


Table A2: Posterior means for the long-term parameters (seed 1)

		200k	1k	10k	50k	0,5*std	2*std	4*std
Constant, α	**Logs**	4.840	4.838	4.841	4.839	4.838	4.841	4.828
	Levels	(126.4)	(126.1)	(126.6)	(126.4)	(126.2)	(126.6)	(124.9)
Coefficient of $\overline{Yrel}$, β_1		-0.743	-1.254	-0.842	-0.731	-0.665	-1.456	-3.006
Coefficient of $\overline{ToT}$, β_2		-1.933	-1.576	-1.822	-1.929	-1.093	-2.566	-2.800
AR(1) coefficient of $\overline{Yrel}$, ρ^Y		0.830	0.826	0.832	0.830	0.815	0.828	0.825
AR(1) coefficient of $\overline{ToT}$, ρ^P		0.813	0.834	0.818	0.815	0.813	0.810	0.821
AR(1) coefficient of $\overline{CA}$, ρ^C		0.930	0.945	0.940	0.930	0.882	0.927	0.922

Note: The estimates in the 200k column is the baseline specification, also shown in Table 1. The three rightmost columns are based on 50 000 draws from the posterior distribution.

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