Fire Sale Bank Recapitalizations

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Abstract

We develop a general equilibrium model of banks' capital structure, featuring heterogeneous portfolio risk and an imperfectly elastic supply of bank equity stemming from financial market segmentation. In our model, equity is costly and serves as a buffer against insolvency. Banks are ex-ante identical, but may need to recapitalize by selling equity claims after their portfolio risk becomes public knowledge. When the need to issue outside equity arises simultaneously in a large number of banks, the market for equity becomes crowded. Reminiscent of asset fire sales, banks do not fully internalize the effect of their individual equity issuance on the endogenous cost of equity and their future ability to recapitalize. As a result, they are inefficiently under-capitalized in equilibrium, and the incidence of insolvency is inefficiently high. This constrained inefficiency provides a new rationale for macroprudential capital regulation that arises despite the absence of deposit insurance, moral hazard, and asymmetric information; it also has implications for the regulation of payout policies and the design of bank stress testing.

Keywords: Macroprudential policy, capital regulation, capital structure, financial market segmentation, incomplete markets, constrained inefficiency.

JEL Classification: D5, D6, G21, G28.

1 Introduction

While there is a rich theoretical and empirical literature on asset fire sales (e.g. Shleifer and Vishny 2010), comparable effects on the liability side of the balance sheet have not received the same degree of attention. In this paper, we analyze what we call "fire sale bank recapitalizations", i.e. situations in which a large number of banks simultaneously access a crowded equity market to issue shares.[1]

We find that banks do not fully incorporate the general equilibrium effect of individual equity issuance on their (and their competitors') future ability to recapitalize. Banks are therefore under-capitalized in equilibrium and the incidence of insolvency is inefficiently high. We derive our results in a general equilibrium model of banks' capital structure in which the supply of equity financing is imperfectly elastic. In our model the optimal leverage ratio is determinate because markets are segmented and incomplete, and bankruptcies arise in equilibrium. A simultaneous need for recapitalizations by a large number of banks leads to an elevated cost for equity issuance due to crowded markets, when the aggregate supply of equity funding is imperfectly elastic.[2] This feature is reminiscent of the literature on (long-term) asset fire sales and is associated with a higher incidence of insolvency. As a consequence, our findings have implications for the macroprudential regulation of bank capital and payout policies. As we will argue below, they are also relevant for the design of bank stress tests.

Key for the mechanism presented in this paper is the fact that some banks' portfolios will exceed a certain threshold level of risk and, hence, be too risky to recapitalize by selling equity claims. This threshold level of risk is a function of the individual equity buffer and the cost of equity. If the cost of equity issuance is determined endogenously in general equilibrium, a wedge emerges between the marginal cost of equity from the perspective of the individual bank and the marginal social cost. In our model this is the case, because of an imperfectly elastic supply of investments in bank equity stemming from financial market segmentation.

Motivation Some commentators have argued that there is "no scarcity problem with respect to bank equity" (Stein, 2013; p. 13), and we are sympathetic to this view in

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[1]While banks can -and do- respond to a capital short-fall in various ways, evidence suggests that equity growth plays an important role. See De Jonghe and Öztekin (2015) for comprehensive international evidence on how banks (de)leverage, and Kok and Schepens (2013) for evidence specifically on European banks' recapitalization efforts during 2004-2011.

[2]Notice that this is different from traditional reasons for elevated issuing costs arising due to adverse selection problems à la Myers and Majluf (1984).

the long-run. At the same time, however, there is a literature concerned with scarcity and "slow moving capital" at very short horizons (Mitchell et al. 2007; Duffie 2010), and evidence suggestive of crowded equity markets in the medium-term. During the first US bank stress test (the Supervisory Capital Assessment Program) in May 2009, for example, the Federal Reserve assessed the 19 largest bank holding companies and identified 10 banks with a significant equity short-fall. These 10 "excessively risky" banks were mandated to raise sufficient equity from the market within six month, or to face a permanent recapitalization from the government.[3] As a result of the limited time horizon, banks were unable to address the short-fall solely by operations on the asset side of their portfolios; instead, they were forced to complement these measures by selling additional equity claims. This resulted in the highest ever monthly U.S. equity issuance volume in May 2009. Within a few weeks banks raised $60bn in new common equity and over $125 by the end of the year.[4] Most of the large bank holdings managed to raise the required equity, albeit at a substantial dilution cost.[5]

Similarly, when the U.S. housing market turned unexpectedly in 2007, those banks with exposure to defaulting mortgages needed to recapitalize in order to comply with risk-based capital regulation and to reassure creditors. Without a well-functioning market for mortgage-backed securities at the time, and generally heightened uncertainty, this meant that a significant number of banks had to compete for a limited number of willing and/or able equity investors. In our view, our model applies most naturally to short or medium term scenarios like these.[6]

Notice that the mortgage example also indicates, that our mechanism is more likely to be relevant among firms that hold more correlated portfolios. Because there are reasons to believe that this is more common among financial firms (e.g. Farhi and Tirole 2012) than among non-financial firms, we consider our mechanism to be particularly applicable to the banking sector. Conditional on correlated portfolios, our mechanisms is also more relevant when risk is directly linked to a required level of capital; to this extent, the mechanism bears potential relevance for the design of risk-weighted capital regulation.

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[3]The results of the Supervisory Capital Assessment Program (SCAP), as well as the details on its design and implementation are published online: http://www.federalreserve.gov/newsevents/press/bcreg/20090507a.htm.

[4]See Hanson et al. (2011) and an U.S. equity market issuance summary by Reuters published online: http://www.lse.co.uk/ukIpoNews.asp?ArticleCode=4a39ycmc7drz9zm.

[5]As mentioned earlier, elevated issuing costs may arise due to an adverse selection problem (Myers and Majluf 1984). Hanson et al. (2011), however, argue that the strong regulatory involvement in the SCAP likely muted the adverse selection problem associated with equity issuance in this case.

[6]Over a longer horizon, it would also be more difficult to maintain that the supply of equity funding is imperfectly elastic; which is central to our analysis.

Model and results We build a two period general equilibrium model of bank equity where banks choose their optimal level of outside equity ex-ante, knowing that they may need to sell additional equity once their portfolio risk becomes publicly known. A key building block of our model is the imperfectly elastic supply of equity in the short- and medium-term, which allows us to link bank solvencies to conditions in the market for equity. Our micro-foundation for this inelastic supply is based on a stylized characterisation of financial market segmentation, in which the marginal equity investor requires ever higher compensation for providing capital. This, in turn, implies a cost of equity that is increasing in aggregate demand.

Banks are identical ex-ante and have exclusive access to risky investment projects. At the initial date, they issue demandable debt and equity in order to invest, while the actual risk of each bank's portfolio is revealed at the intermediate date. Undercapitalized banks then have to recapitalize in order to prevent a run from debt holders and insolvency. The ability of a given bank to recapitalize by selling equity, however, depends on its individual portfolio risk *and* on the market conditions in (i.e. the crowdedness of) the equity market.

We find that the market equilibrium is *constrained efficient*, as long as the level of ex-ante capitalization only affects the magnitude of future recapitalization needs at the bank level *(intensive margin)*, leaving solvency unaffected. Instead, the market equilibrium is *constrained inefficient* due to a *pecuniary externality*, if ex-ante equity issuance also affects the threshold level of portfolio risk for which recapitalizations are still feasible *(extensive margin)* and -consequently- bank solvencies in future contingencies. If this is the case, our model predicts an inefficiently low ex-ante capitalization whenever higher equity buffers today are associated with lower aggregate recapitalization needs in the future (which can be assured under plausible conditions). Because this overleveraging at the bank level is associated with an inefficiently high incidence of insolvency in equilibrium, the externality in our model has systemic implications that are relevant for the design of macroprudential policies. In other words, the effects associated with "fire sale bank recapitalizations" give rise to a new rationale for the regulation of bank capital and payout policies. Notably, our results do not hinge on features that are usually associated with models of bank capital regulation, such as deposit insurance, moral hazard or asymmetric information.

In addition, our results also hold absent aggregate risk. A model in which aggregate risk pushes the cost of equity upwards, precisely when a large number of banks need to recapitalize simultaneously (e.g. in a recession), however, is likely to amplify the role of an imperfectly elastic supply of equity funding. This, and a richer microfoundation for the imperfectly elastic supply of equity, will be particularly relevant when it comes to exploring the quantitative implications of fire sale bank recapitalizations and is left for

future work.

Frictions and features   The *constrained inefficiency* arises due to a *pecuniary externality* in combination with incomplete deposit contracts, and incomplete markets for ex-ante risk-sharing. Incomplete deposit contracts and incomplete markets do not generate an inefficiency (Allen and Gale 2004), except if the ability to recapitalize in the future depends on the future cost of bank equity *(extensive recapitalization margin)*. Bank managers can successfully conduct a market-based recapitalization only if they can promise sufficiently high expected returns to new equity investors. While the promised return is bounded by the probability and the magnitude of high portfolio returns, the required risk-adjusted compensation for new equity investors depends on market conditions due to the imperfectly elastic supply of equity. In other words, the ability of a given bank to issue shares in order to recapitalize is governed by a *"recapitalization constraint"* that is a function of the endogenous cost of bank equity. It is this cost in the constraint, which gives rise to the inefficiency. This result is reminiscent of the findings on pecuniary externalities and incomplete ex-ante risk markets in combination with borrowing constraints (Lorenzoni 2008).

Related literature   Our paper is related to the literature on fire sales (Shleifer and Vishny 1992), although we do not consider fire sales of assets. Instead, we focus on "fire sales" of equity claims and highlight the role of a potentially insufficient precautionary motive for holding equity buffers ex-ante. This is reminiscent of the studies by Allen and Gale (1994, 2004, 2007) and others on the role of the precautionary and the speculative motive, which are a characteristic of papers with fire sales of assets.

The paper also relates to the extensive literature on optimal capital structures. In contrast to the classical model of Modigliani and Miller (1958), markets in our model are incomplete, implying that banks with a level of portfolio risk that is too high are subject to creditor runs and insolvency; as a result, the optimal capital structure is determinate. While we analyze the capital structure of banks and the implications on macroprudential regulation, a related paper by Gale and Gottardi (2015) studies a dynamic general equilibrium model where firms choose their capital structure and investments, trading off tax advantages of debt with the risk of costly default. In their paper with fire sales of assets, the equilibrium exhibits inefficient under-investment, because firms do not internalize that an increased use of debt by all firms can lower their tax burden. Conversely, we find in our model that banks are inefficiently over-leveraged.

The *constrained inefficiency* results in Gale and Gottardi (2015) and in our paper are related to the work of Lorenzoni (2008). Whilst Lorenzoni's *borrowing constraint*

depends on asset prices and affect leverage directly, however, our paper features a *recapitalization constraint* that is essentially a solvency constraint depending on leverage and on equity market conditions. Furthermore, our paper is also related to an earlier literature on price externalities and incomplete markets in combination with informational or other frictions (e.g. Greenwald and Stiglitz 1986; Geanakoplos and Polemarchakis 1986).

Recent related papers on the capital structure of banks and on bank regulation include Admati et al. (2011), Admati et al. (2013), DeAngelo and Stulz (2015) and Allen et al. (2014). Furthermore, there is a large literature on the role of agency problems in shaping the capital structure of banks (Kashyap et al. 2008). Philippon and Schnabl (2013) discuss efficient recapitalizations of banks by a government in the presence of a debt overhang problem. Agency problems also play a prominent role in the macroprudential literature on capital regulation. Our paper is, however, more closely related to the macroprudential literature that motivates the need for regulation based on externalities (see Nicolò et al. (2012) for a review paper).

The remainder of the paper is organized as follows. Section 2 introduces the setup and section 3 discusses the decentralized equilibrium. Thereafter, section 4 solves a benchmark model, analyses the constrained planner problem and presents the main results. In section 5 we provide a numerical example to illustrate the key insight. Section 6 concludes.

2   Setup

We consider a stylized model of financial intermediation. Time is discrete, there are three dates: $t = 0, 1, 2$ and we abstract from discounting. The economy comprises a continuum of islands with unit mass that are indexed by i. There is one homogeneous perishable good that can be consumed or invested.

Agents and endowments   There are three types of agents: bank managers, households, and investors. All agents are risk-neutral. At $t = 0$ each island is inhabited by one bank manager and a continuum of mass one of households, who both maximize their expected total consumption. Each household is endowed with one unit of the perishable good at $t = 0$. Managers have no endowment (w.l.o.g.), but the talent to run a bank; i.e. exclusive access to a production technology. At $t = 0$ there is also a continuum of mass one of "global" investors who do not reside on a specific island. They live for two periods and are endowed with one unit of the perishable good at $t = 0$ and $t = 1$.

Imperfectly elastic supply of equity: financial market segmentation   A key model feature is an imperfectly elastic supply of equity. It is microfounded with the help of a stylized model of financial market segmentation but could, in principle, also stem from other sources.[7] To ease the exposition we consider a setup where households and global investors are separated. While the former are the natural suppliers of deposits, the latter are the natural suppliers of bank equity. Specifically, households and investors are modeled as follows.

Households' utility function is given by $U^{HH} = C_1^{HH} + C_2^{HH}$; they can either consume their endowments right away, or deposit it with the local bank on their island. They can neither place deposits with banks on other islands (e.g. because of prohibitively high transport costs) nor participate in equity markets (e.g. because they are financially illiterate and face prohibitively high financial market participation costs).

Global investors' utility is given by $U^I = C_1^I + C_2^I$; they cannot store their endowments at dates $t = 0, 1$, but they are financially sophisticated. At $t = 0$ and $t = 1$ they decide whether to access the *global* equity market and purchase bank equity. Alternatively, they can realize their outside option that yields a private benefit of $R > 1$ in each period. Each time investors decide to enter the equity market, however, they need to acquire expertise and study market conditions. More specifically, entering the market requires investors to incur an idiosyncratic utility cost. At the beginning of dates $t = 0, 1$ each investor j draws her individual utility cost for financial market participation $\theta c_{jt} \geq 0$. We assume $\theta > 0$ and $c_{jt} \sim U[0, 1]$.[8]

Financial intermediation   Each island is populated by one bank; this bank has monopoly power in the local deposit market.[9] Bank managers collect uninsured deposits at $t = 0$. In addition, they raise outside equity on competitive global equity markets at $t = 0, 1$, by offering a share of their franchise to those investors that incur the participation cost. Deposits are assumed to be demandable at $t = 1$. Furthermore, we impose a sequential service constraint as in Diamond and Dybvig (1983). Both assumptions are supported by empirical evidence and can be endogenized by introducing a disciplining role for liquid deposits.[10]

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[7]A deeper analysis of issues related to aggregate risk and a richer microfoundation for the imperfectly elastic supply of equity are left for future work. Other sources that may contribute to an inelastic supply of equity such as asymmetric information (Myers and Majluf 1984) may potentially interact with the key mechanism of this model and, hence, affect the qualitative results.

[8]The results would not change if the participation cost of an individual investor were the same at both dates.

[9]An extension to competitive deposit markets, as well as the endogenous segmentation into "international" equity investors and "domestic" depositors is left for future research. We expect the key insights of the paper to be qualitatively unaffected.

[10]See, for example, Calomiris and Kahn (1991) and Grossman and Hart (1982).

Technology Banks invest all resources at $t = 0$ into a long-term technology with a stochastic return. At the beginning of $t = 1$ the portfolio quality of each bank becomes publicly known. With probability $0 < p < 1$ a bank turns out to be *safe* and can achieve a high portfolio return of $R^H > 1$ at $t = 2$ with certainty. With probability $1 - p$, instead, a bank turns out to be *risky*. Each risky bank i achieves a high portfolio return of $R^H > 1$ at $t = 2$ with probability $q_i \in [0, 1)$ and a low portfolio return of $0 < R^L \leq 1$ with probability $1 - q_i$. To meet deposit withdrawals at $t = 1$, banks can use a private liquidation technology to physically liquidate some of their investments at $t = 1$ after the portfolio quality is revealed. Liquidation is assumed to be costly and yields only a small fraction $0 < \gamma \ll 1$ of the expected return, i.e. γR^H for safe banks and $\gamma \left[q_i R^H + (1 - q_i) R^L \right]$ for risky banks. For simplicity we consider $\gamma \to 0$.

Equity markets Equity markets at $t = 0, 1$ are assumed to be competitive spot markets that operate on an economy-wide ("global") level. Only global investors invest in bank equity.[11]

Information, bank runs & bankruptcy At the beginning of $t = 1$ the portfolio risk of each bank (q_i) becomes public information.[12] A risky bank faces a run on its demandable deposits and goes bankrupt whenever it is unable to honor its initial promise to depositors. This happens, when the bank is too risky to recapitalize at $t = 1$ (i.e. if q_i is so low, that potential equity investors are better off realizing their outside option).

Timing of events

- $t = 0$

    1. Investors at $t = 0$ draw their individual participation cost parameter $c_{j,0}$ and decide whether to invest in bank equity or to realize their outside option.

    2. Banks issue equity and offer deposit contracts.

    3. Depositors decide whether to invest in bank deposits or to consume.

    4. Banks collect deposits and invest all resources in the long-term technology.

- $t = 1$

    1. Banks' portfolio risk becomes public knowledge.

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[11]An interesting extension left for future research is to allow for interbank equity markets.

[12]For example because the regulator conducts a stress test and publishes the results.

2. Investors at $t = 1$ draw their individual participation cost parameter $c_{j,1}$ and decide whether to invest in bank equity or to realize their outside option.

3. Risky banks attempt to raise additional equity. Risky banks that are too risky to recapitalize by selling additional shares to equity investors face a bank run and go bankrupt. The remaining resources after liquidation go to depositors.

- $t = 2$

    1. Payoffs are realized.

    2. Solvent banks repay depositors (in full); equity investors are paid the contingent returns they were promised for their investments at $t = 0$ and $t = 1$, respectively.

Before proceeding with the equilibrium analysis, we introduce the following assumption.

Assumption 1. $R^H < R$

Assumption 1 guarantees that equity funding is more costly than debt/deposit funding, even in the absence of financial market participation costs. This is more restrictive than necessary, but significantly facilitates the exposition of the key mechanism.

3 Equilibrium

Denote as $\bar{r}_0^D$ the expected return demanded by local households by and as $\bar{r}_0^E$ and $\bar{r}_1^E$, respectively, the expected return demanded by global equity investors. In addition, let d_i^S denote the amount of deposit funding supplied by depositors on island i, whilst d_i^D is the demand for deposit funding by bank i. Similarly, let $e_{0,i}^D$ and $e_{1,i}^D$ be the demand for equity financing by bank i at dates $t = 0$ and $t = 1$; the supply of equity funding by a global equity investor j at date t, instead, is $e_{t,j}^S$. Before proceeding with the characterization and solution of the individual agents' problems, we first define the decentralized equilibrium.

Definition. *The allocation (d_i^S, $e_{0,j}^S$, $e_{1,j}^S$; d_i^D, $e_{0,i}^D$, $e_{1,i}^D$), $\forall i, j$ and the price vector ($\bar{r}_0^D$, $\bar{r}_0^E$, $\bar{r}_1^E$) constitute an equilibrium if the following conditions are met:*
(i) given ($\bar{r}_0^E$, $\bar{r}_1^E$), ($\bar{r}_0^D$; d_i^D, $e_{0,i}^D$, $e_{1,i}^D$) solves the optimization problem for each bank i;
(ii) given ($\bar{r}_0^E$, $\bar{r}_1^E$), ($e_{0,j}^S$, $e_{1,j}^S$) solves the optimization problem for all equity investors j;
(iii) given $\bar{r}_0^D$, d_i^S solves the optimization problem of local depositors on all islands i;
(iv) all local deposit markets clear at $t = 0$: $d_i^S = d_i^D$ for all i;
(v) the global equity markets clear at the two dates: $\int_j e_{t,j}^S \, dj = \int_i e_{t,i}^D \, di$ for $t = 0, 1$.

Depositor problem   At date $t = 0$ the representative depositor on island i maximizes expected utility $E\left[U^{HH}\right] = \bar{r}_0^D\, d_i^S + \left(1 - d_i^S\right)$ for $d_i^S \in [0, 1]$. Solving this problem, and aggregating over all depositors on island i (mass 1), we obtain the supply function for local deposits:

$$d_i^S = \begin{cases} 1, & if\, \bar{r}_0^D \geq 1 \\ 0, & if\, \bar{r}_0^D < 1. \end{cases}$$

At $t = 1$ depositors find it optimal to withdraw their funds whenever their local bank turns out to be risky and does not have a sufficient equity buffer to ensure the repayment of r^D in all states of the world. This is because the first depositors who withdraw can expect a return of r^D due to the sequential service constraint and the strictly positive liquidation value of risky banks, i.e. $\gamma > 0$.

Investor problem   Next, we consider the problem of a representative equity investor, indexed by j. At $t = 0$ and $t = 1$, atomistic investors receive an endowment of 1 and draw a uniformly distributed capital market participation cost $c_{j,t} \in [0, 1]$ defining their type. At both dates, they can either invest their endowment in a private investment opportunity yielding a utility of $R > 1$, or they can pay the participation cost $\theta c_{j,t} \geq 0$, invest in equity and consume the return to equity at $t = 2$. Bank equity pays an expected return of $\bar{r}_t^E$, where expectations at $t = 1$ are formed conditional on knowledge of the issuing bank's types. The problem of investor j is then given by:

$$\max_{e_{j,0}^S, e_{j,1}^S} \quad E\left[U^I\right] = \quad E\left[C_1^I + C_2^I\right]$$

$$s.t.: \quad E\left[C_1^I\right] = \quad \left[\bar{r}_0^E - \theta c_{j,0}\right] e_{j,0}^S + R\left(1 - e_{j,0}^S\right)$$

$$E\left[C_2^I\right] = \quad \left[\bar{r}_1^E - \theta c_{j,1}\right] e_{j,1}^S + R\left(1 - e_{j,1}^S\right)$$

$$e_{j,t}^S \in [0, 1], \forall t$$

For all j and t, the optimal supply of equity by each atomistic investor is therefore given by:

$$e_{j,t}^S = \begin{cases} 1, & if\, \bar{r}_t^E - \theta c_{j,t} \geq R \\ 0, & if\, \bar{r}_t^E - \theta c_{j,t} < R. \end{cases}$$

This implies, that all investors who draw $c_{j,t} > \hat{c}_t$, where $\bar{r}_t^E - \theta \hat{c}_t = R$, invest their entire endowment into the private outside option whilst all investors with $c_{j,t} \leq \hat{c}_t$ invest in bank

equity. As a result, aggregate investment in equity at date t is given by:

$$E_t^S \left(\bar{r}_t^E \right) = \int_j \left(e_{t,j}^S \right) dj = \int_0^{\hat{c}_t} [1] \, dc_{j,t} = \hat{c}_t = \frac{\bar{r}_t^E - R}{\theta}.$$

Notice that supply for investments in bank equity is strictly increasing in $\bar{r}_t^E$.

Recapitalization at $t = 1$ An individual risky bank of type q_i is able to recapitalize if it can promise a sufficiently high upside to equity investors at $t = 1$, i.e. if:

$$q_i \frac{\left(1 + e_{0,i}^D \right) R^H - r^D + \overbrace{r^D - \left(1 + e_{0,i}^D \right) R^L}^{\equiv e_{1,i}^D \left(r^D, e_{0,i}^D \right)}}{e_{1,i}^D \left(r^D, e_{0,i}^D \right)} + (1 - q_i) \, 0 \geq \bar{r}_t^E, \tag{1}$$

where we use the fact that banks do not find it optimal to choose higher recapitalization volumes than necessary, i.e. $e_{1,i}^D \geq r^D - \left(1 + e_{0,i}^D \right) R^L$ holds with equality. Inequality (1) reveals that a recapitalization is feasible at $t = 1$ provided that R^H and/or q_i are sufficiently high relative to $\bar{r}_t^E$.

The aggregate demand for bank equity depends on $\bar{r}_1^E$, the distribution of the ex-ante equity issuance volumes, say $\Gamma_{e_{0,i}^D}$, and the corresponding distribution of the q_i's:

$$E_1^D \left(\bar{r}_1^E, \Gamma \left(e_{0,i}^D \right) \right) = \int_0^1 \mathbb{1}_{q_i \geq \hat{q}(e_{0,i}^D; \bar{r}_1^E)} \, (1 - p) \overbrace{\left(r^D - \left(1 + e_{0,i}^D \right) R^L \right)}^{= e_{1,i}^D \left(r^D, e_{0,i}^D \right)} f(q_i) \, dq_i \,,$$

where:

$$\hat{q} \left(e_{0,i}^D; \bar{r}_1^E \right) \equiv \max \left\{ 0, \min \left\{ 1, \bar{r}_1^E \frac{r^D - \left(1 + e_{0,i}^D \right) R^L}{\left(1 + e_{0,i}^D \right) \left(R^H - R^L \right)} \right\} \right\}.$$

All risky banks with a $q_i \geq \hat{q} \left(e_{0,i}^D; \bar{r}_1^E \right)$ are able to raise capital by selling claims to equity, whereas all other banks are unable to recapitalize; the inability to raise additional capital, however, leads to insolvencies with a zero payoff for banks. Notice that a higher cost of equity at $t = 1$ results in fewer banks being able to recapitalize, while a higher equity buffer (a higher $e_{0,i}^D$) increases the chances for a given bank to be able to recapitalize.

The key insights of this paper hinge on the observation that the recapitalization constraint is governed by $\hat{q} \left(e_{0,i}^D; \bar{r}_1^E \right)$ and that it depends on market conditions at $t = 1$. Hence, the frequency of insolvencies is affected by the individual choice of $e_{0,i}^D$ and by the endogenous cost of issuing equity claims, $\bar{r}_1^E$, at $t = 1$.

Bank problem at $t = 0$ — Next we turn to the problem of banks. A given bank is safe with probability $0 < p < 1$ and risky with probability $1 - p$. Conditional on being identified as risky, a given bank i's probability of a high return is q_i. The bank chooses the amount of deposits and initial equity that it would like to raise, as well as the deposit rate that it offers on the local market and the equity that it raises in addition at $t = 1$; taking $\bar{r}_0^E$ and $\bar{r}_1^E$ as given, the corresponding problem of bank i on island i then writes:

$$
\max_{\substack{\bar{r}^D, d_i^D, \\ e_{0,i}^D, e_{1,i}^D}} \left\{ p \left[\overbrace{R^H \left(d_i^D + e_{0,i}^D \right)}^{\substack{\text{gross return} \\ \text{if bank is safe}}} - \underbrace{d_i^D\, r^D}_{\text{payment to depositors}} \right] - \overbrace{e_{0,i}^D\, \bar{r}_0^E}^{\substack{\text{expected return} \\ \text{to investors at t=0}}} + \right.
$$
$$
\left. (1-p) \int_{\widehat{q}(e_{0,i}^D)}^1 \left(\begin{array}{c} q_i \underbrace{\left[R^H \left(d_i^D + e_{0,i}^D \right) - d_i^D\, r^D + e_{1,i}^D \right]}_{\substack{\text{gross return if bank is risky} \\ \text{but has a high return}}} + \\[2pt] (1-q_i) \underbrace{\left[R^L \left(d_i^D + e_{0,i}^D \right) - d_i^D\, r^D + e_{1,i}^D \right]}_{\substack{\text{gross return if bank is risky} \\ \text{but has a low return}}} \\[2pt] - \underbrace{e_{1,i}^D\, \bar{r}_1^E}_{\substack{\text{expected return} \\ \text{to investors at t=1}}} \end{array} \right) f(q_i)\, dq_i \right\}
$$

$$(2)$$

Subject to solvency in the low portfolio return state and to depositor participation:

$$
e_{1,i}^D \geq \max \left\{ 0, d_i^D\, r^D - R^L \left(d_i^D + e_{0,i}^D \right) \right\}
$$

$$
\bar{r}^D = \left(p + (1-p) \int_{\widehat{q}(e_{0,i}^D)}^1 [1]\, dq_i \right) r^D \geq 1,
$$

where $f(q_i)$ is the probability density function describing the distribution of $q_i's$ and $\widehat{q}\left(e_{0,i}^D\right)$ is the portfolio risk for which a recapitalization is just feasible, i.e. for which (1) holds with equality. All banks that are more risky than the threshold type $\widehat{q}$, i.e. that are of a type $q_i < \widehat{q}$, are subjected to a depositor run that renders them insolvent, as they are unable to recapitalize. Because banks are local monopolists the bank offers the smallest possible deposit rate that still ensures depositor participation; i.e. $\bar{r}^D = 1$. At this

price, the bank would demand $d_i^D = \infty$. Anticipating market clearing on all local deposit markets, we can therefore set $d_i^D = d_i^S = 1$. Similarly, because equity at $t = 1$ is costly, the bank raises as little additional equity as possible, implying that the solvency constraint will be satisfied with equality. Finally, notice that banks will not find it optimal to issue more equity than necessary to be solvent in all states of the world, i.e. $e_{0,i}^D \leq \overline{e_{0,i}^D} \equiv \frac{r^D - R^L}{R^L}$.

Market clearing   Using the previous results and assumptions, we have:

- Clearing of the deposit market: $d_i^D = d_i^S \equiv D = 1, \forall i$.

- Clearing of the equity market at dates $t = 1, 2$: $E_t^S = E_t^D$, where $E_t^D = \int_0^1 e_{t,i}^D \, di$.

This concludes the description of the decentralized equilibrium.

4   Results

Next, consider a stylized version of the previous model where we introduce different types of risky banks. Conditional on being risky, q_i, takes:

- a value of $q_i = \overline{q}$ with probability $0 \leq s \leq 1$

- a value q_i that is drawn from a uniform distribution, $q_i \sim U[0, \overline{q}]$, with probability $1 - s$.

This stylized model allows us to parameterize the relative intensity of the *extensive recapitalization margin* vis-a-vis the *intensive recapitalization margin* and to derive implications for the efficiency of ex-ante equity issuance. When s is closer to 1, the distribution becomes more skewed and a larger mass of risky banks have a high probability of a high portfolio return and, hence, better chances to recapitalize at $t = 1$. At the same time, the mass of banks that are affected by changes in the recapitalization constraint. i.e. in $\widehat{q}$, is reduced whenever $\widehat{q} < \overline{q}$. Hence an increase in s mutes the extensive recapitalization margin.

We first analyze the decentralized economy in section 4.1 and then efficiency in section 4.2. Throughout our analysis we are interested in scenarios where at least some of the risky banks are able to recapitalize, i.e. where $\widehat{q} < \overline{q}$. Hence, we first solve the model under this conjecture and then analyze under which conditions it can be verified.

4.1   Decentralized economy

Suppose that $\widehat{q} < \overline{q}$ and that all risky banks with $q_i = \overline{q}$ are able to recapitalize at $t = 1$, meaning that the depositors of the risky banks of type $q_i = \overline{q}$ are always repaid.

4.1.1 Bank's problem

The ex-ante probability of a depositor to be repaid is:

$$\overline{p}(\widehat{q}) \equiv p + (1-p)s + (1-p)(1-s)\frac{\overline{q}-\widehat{q}}{\overline{q}} < 1,$$

where $\widehat{q}$ denotes the threshold probability that solves inequality (1). All risky banks with a probability q_i of achieving a high return that exceeds the threshold, i.e. $q_i \geq \widehat{q}$, are able to raise equity by issuing additional shares. Conversely, all risky banks with a probability $q_i < \widehat{q}$ are unable to recapitalize and are liquidated.

Hence, the bank problem at $t = 0$ reads:

$$\max_{\overline{e}^D_{0,i} \geq e^D_{0,i} \geq 0} \left\{ \begin{array}{l} p\left[\left(1+e^D_{0,i}\right)R^H - \overline{p}(\widehat{q})^{-1}\right] - e^D_{0,i}\,\overline{r}^E_0 \\[2mm] +(1-p)s\left[\overline{q}\left(1+e^D_{0,i}\right)\left(R^H-R^L\right) - \left[\overline{p}(\widehat{q})^{-1} - \left(1+e^D_{0,i}\right)R^L\right]\overline{r}^E_1\right] \\[2mm] +(1-p)(1-s)\left[\begin{array}{l}\frac{1}{\overline{q}}\int_{\widehat{q}}^{\overline{q}} q\left(1+e^D_{0,i}\right)\left(R^H-R^L\right)dq \\[2mm] -\frac{\overline{q}-\widehat{q}}{\overline{q}}\,\overline{r}^E_1\left[\overline{p}(\widehat{q})^{-1} - \left(1+e^D_{0,i}\right)R^L\right]\end{array}\right] \end{array} \right\} \tag{3}$$

s.t.

$$\widehat{q} = \overline{r}^E_1\,\frac{\left(p+(1-p)s+(1-p)(1-s)\frac{\overline{q}-\widehat{q}}{\overline{q}}\right)^{-1} - \left(1+e^D_{0,i}\right)R^L}{\left(R^H-R^L\right)\left(1+e^D_{0,i}\right)},$$

where we used:

$$e^D_{1,i}\left(e^D_{0,i};\overline{r}^E_1\right) = r^D - R^L\left(1+e^D_{0,i}\right)$$

$$r^D\left(e^D_{0,i};\overline{r}^E_1\right) = \overline{p}\left(\widehat{q}\left(e^D_{0,i};\overline{r}^E_1\right)\right)^{-1}$$

from optimality at $t = 1$ and deposit market clearing.

Furthermore, the costs $\overline{r}^E_0$ and $\overline{r}^E_1$ are taken as given. From equity market-clearing, we have:

$$\overline{r}^E_0 = R + \theta\int_0^1 e^D_{0,i}\,di.$$

$$\overline{r}^E_1 = R + \theta\int_0^1\left[(1-p)s + (1-p)(1-s)\frac{\overline{q}-\widehat{q}\left(e^D_{0,i};\overline{r}^E_1\right)}{\overline{q}}\right]e_1\left(e^D_{0,i};\overline{r}^E_1\right)di.$$

14

The first-order necessary condition of the bank's problem is given by:

$$
\begin{aligned}
e_{0,i}^D: \quad & p\left[R^H - \frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_{0,i}^D}\right] - \overline{r}_0^E \\
& + (1-p)\,s\left[\bar{q}R^H + \left(\overline{r}_1^E - \bar{q}\right)R^L - \overline{r}_1^E\frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_{0,i}^D}\right] \\
& + \frac{(1-p)(1-s)}{\bar{q}}\left(\int_{\widehat{q}}^{\bar{q}} q\left(R^H - R^L\right)dq - \widehat{q}(1+e_0)\left(R^H - R^L\right)\frac{d\widehat{q}}{de_{0,i}^D}\right) \\
& + \frac{(1-p)(1-s)}{\bar{q}}\left[\overline{r}_1^E e_1 \frac{d\widehat{q}}{de_{0,i}^D} - (\bar{q}-\widehat{q})\,\overline{r}_1^E\left(\frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_{0,i}^D} - R^L\right)\right] \\
& = 0, \; if\; \overline{e_{0,i}^D} > e_{0,i}^D > 0.
\end{aligned}
$$

For the special case when $s \to 1$, the first-order necessary condition simplifies to:

$$
e_{0,i}^D: \quad p\,R^H - \overline{r}_0^E + (1-p)\left[\bar{q}R^H + \left(\overline{r}_1^E - \bar{q}\right)R^L\right] = 0, \; if\; \overline{e_{0,i}^D} > e_{0,i}^D > 0. \tag{4}
$$

In what follows, section 4.1.2 examines the role of the extensive recapitalization margin. Then section 4.1.3 presents the system of equations before the equilibrium is analyzed in section 4.1.4.

4.1.2 Extensive margin

Observe that the recapitalization constraint, governed by the threshold $\widehat{q}$, is a function of the equity issuance of an individual bank at $t = 0$ and of the cost of equity at $t = 1$, which itself is determined by aggregate choices through competitive equity markets. In problem (3) banks only internalize the direct effect of selecting higher equity ex-ante via $\frac{d\widehat{q}}{de_{0,i}^D}$, but not the indirect general equilibrium effect via $\frac{d\widehat{q}}{d\overline{r}_1^E}\frac{d\overline{r}_1^E}{de_0}$.

Suppose, $\widehat{q}$ takes on an interior solution, i.e. $\widehat{q} \in (0,\bar{q})$. By application of the Implicit Function Theorem, it follows that:

$$
\frac{d\widehat{q}}{de_{0,i}^D} = -\frac{\overline{r}_1^E\,\overline{p}(\widehat{q})^{-1}}{(R^H - R^L)\left(1 + e_{0,i}^D\right)^2}\left(1 - \frac{\overline{r}_1^E\,(1-p)\,(1-s)}{(R^H - R^L)\left(1 + e_{0,i}^D\right)\bar{q}\,\overline{p}(\widehat{q})^2}\right)^{-1}
$$

$$
\frac{d\widehat{q}}{d\overline{r}_1^E} = \frac{e_{1,i}^D\left(e_{0,i}^D;\overline{r}_1^E\right)}{(R^H - R^L)\left(1 + e_{0,i}^D\right)}\left(1 - \frac{\overline{r}_1^E\,(1-p)\,(1-s)}{(R^H - R^L)\left(1 + e_{0,i}^D\right)\bar{q}\,\overline{p}(\widehat{q})^2}\right)^{-1}.
$$

Furthermore, supposing the date $t = 0$ choices of banks are symmetric (i.e. $e_{0,i}^D = e_0^D \; \forall\, i$),

we have from the demand and supply for investments in bank equity that:

$$\frac{d\bar{r}_1^E}{dE_0^D} = \theta \left[\begin{array}{c} -\frac{(1-p)(1-s)}{\bar{q}} \frac{d\hat{q}}{de_{0,i}^D}\Big|_{e_{0,i}^D=e_0} e_1\left(e_0,\bar{r}_1^E\right) \\ + \left((1-p)s + (1-p)(1-s)\frac{\bar{q}-\hat{q}}{\bar{q}}\right)\left(\frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\hat{q})^2} \frac{d\hat{q}}{de_{0,i}^D}\Big|_{e_{0,i}^D=e_0} - R^L\right) \end{array} \right].$$

From this, we obtain our first set of results:

Lemma 2. (Recapitalization constraint) *By continuity, $\exists\ \underline{s}_0 \in [0,1)$, such that for all $s \geq \underline{s}_0$:*

1. *the likelihood of being able to issue shares at $t = 1$ increases in the individual level of equity issued at $t = 0$, i.e. $\frac{d\hat{q}}{de_{0,i}^D} < 0$,*

2. *the likelihood of being able to issue shares at $t = 1$ decreases in the cost of equity at $t = 1$, i.e. $\frac{d\hat{q}}{dr_1^E} > 0$,*

3. *and the cost of equity at $t = 1$ decreases in the aggregate level of equity issued at $t = 0$, i.e. $\frac{d\bar{r}_1^E}{dE_0^D} < 0$, provided that the $t = 0$ choices of banks are symmetric.*

Notice that the scenario where $\frac{d\bar{r}_1^E}{dE_0^D} < 0$ is also the plausible scenario. Here a higher aggregate equity issuance ex-ante is associated with lower aggregate recapitalization needs in the future. In other words, higher equity buffers reduce the magnitude and, hence, the cost of future recapitalizations. Intuitively, the result of Lemma 2 prevails if the intensive recapitalization margin is sufficiently important relative to the extensive recapitalization margin. This is guaranteed for sufficiently high values of *s*.

4.1.3 System of equations

Suppose an interior solution exists and suppose that the equilibrium is symmetric,[13] i.e. the choices at date $t = 0$ are symmetric. Then the system of equations is given by six equations in six unknowns, where the first equation constitutes the first-order necessary

---

[13]Later we rule out asymmetric equilibria.

condition of the problem in (3):

$$
\begin{aligned}
\bar{r}_0^E = \; & p\left[R^H - \frac{(1-p)(1-s)}{\bar{q}\,\bar{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_0}\right] \\
& + (1-p)s\left[\bar{q}\cdot R^H + \left(\bar{r}_1^E - \bar{q}\right)R^L - \bar{r}_1^E\frac{(1-p)(1-s)}{\bar{q}\,\bar{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_0}\right] \\
& + \frac{(1-p)(1-s)}{\bar{q}}\left(\int_{\widehat{q}}^{\bar{q}}q\left(R^H - R^L\right)dq - \widehat{q}(1+e_0)\left(R^H - R^L\right)\frac{d\widehat{q}}{de_0}\right) \\
& + \frac{(1-p)(1-s)}{\bar{q}}\left[\bar{r}_1^E e_1\frac{d\widehat{q}}{de_0} - (\bar{q}-\widehat{q})\,\bar{r}_1^E\left(\frac{(1-p)(1-s)}{\bar{q}\,\bar{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_0} - R^L\right)\right] \\
e_1 = \; & \bar{p}\left(\widehat{q}\right)^{-1} - (1+e_0)\,R^L \\
\bar{p}\left(\widehat{q}\right) = \; & p + (1-p)s + (1-p)(1-s)\,\frac{\bar{q}-\widehat{q}}{\bar{q}} \\
\widehat{q} = \; & \frac{\bar{r}_1^E e_1}{(R^H - R^L)(1+e_0)} \\
\bar{r}_0^E = \; & R + \theta e_0 \\
\bar{r}_1^E = \; & R + \theta\left((1-p)s + (1-p)(1-s)\,\frac{\bar{q}-\widehat{q}}{\bar{q}}\right)e_1\left(e_0, \bar{r}_1^E\right).
\end{aligned}
\tag{5}
$$

Notice that the system in (5) can be reduced to two equations in two unknowns $(e_0, \widehat{q})$.

4.1.4 Equilibrium

Equilibrium existence and uniqueness can be established for a relevant parameter range by first analyzing the special case $s = 1$ and then generalizing the results. Proposition 3 presents sufficient conditions for existence and uniqueness.

Proposition 3. (Existence and uniqueness)

(a) *If $s = 1$, there exists a unique equilibrium where all risky banks with $q_i = \bar{q}$ can recapitalize, provided that:*

$$
\bar{q} \geq \frac{\left[R + \theta\left(1 - p\right)\left(1 - R^L\right)\right]\left(1 - R^L\right)}{R^H - R^L}.
\tag{6}
$$

In equilibrium $E_0^* = \max\left\{0, \min\left\{E_0', \overline{E_0}\right\}\right\}$, where E_0' solves (4) and $\overline{E_0} = \frac{1-R^L}{R^L}$.

(b) *By continuity, $\exists\,\underline{s} \in [0,1)$, such that for all $s \geq \underline{s}$ there exists a unique equilibrium with $\widehat{q}^* \in [0, \bar{q}]$ provided that $\bar{r}_1^E \leq p\left(\frac{1}{2} - \frac{1-p}{\bar{q}}\right)^{-1}$. The equilibrium is symmetric. For $s \to 1$ it is characterized by $e_0^* = E_0^*$.*

Proof. See Appendix A.1. □

Uniqueness of a symmetric equilibrium can be established provided that inequality (6) holds and that the value of s is high, meaning that the intensive recapitalization margin is relatively important. This conditions are sufficient but not necessary for the result of

Proposition 3 to hold. Intuitively, inequality (6) demands that the "best" risky banks have a sufficiently high probability of a good return to assure that at least they are able to recapitalize at the intermediate date. It follows that there must exist a $\widehat{q} \in [0, \overline{q})$ above which risky banks can recapitalize. Whenever s is sufficiently high, it can be shown that there exists a unique equilibrium and that it is symmetric. Notice that $p(\frac{1}{2} - \frac{1-p}{\overline{q}})^{-1} > 2$, so that the sufficient condition $\overline{r}_1^E \leq p(\frac{1}{2} - \frac{1-p}{\overline{q}})^{-1}$ is not restrictive, as it is satisfied for all reasonably high costs of equity.

4.2 Efficiency: a second-best benchmark

Next, we analyze efficiency. We use the benchmark of a constrained planner who selects $e_{0,i}^D$ for each bank and cannot do anything more than that. Different to the individual banks, the planner takes into account how the equity issuance at $t = 0$ affects the cost of equity at both dates and how this affects the recapitalization need $e_{1,i}^D(e_{0,i}^D; \overline{r}_1^E)$ at $t = 1$, and thereby the incidence of insolvency in equilibrium. We assume that the planner maximizes total surplus in the economy, i.e. she maximizes the sum of bank profits across all islands and the net surplus[14] of equity investors at $t = 0$, $S_0(E_0)$, and at $t = 1$, $S_0(E_0, \overline{r}_1^E)$.

4.2.1 Constrained planner problem

The constrained planner problem reads:

$$
\max_{\overline{e}_0^D \geq e_0^D \geq 0}
\left\{
\begin{array}{c}
p \left[\left(1 + e_0^D\right) R^H - \overline{p} \left(\widehat{q}\right)^{-1} \right] - e_0^D \, \overline{r}_0^E \\[2ex]
+ (1-p)s \left[\overline{q} \left(1 + e_0^D\right) \left(R^H - R^L\right) - \left[\overline{p}\left(\widehat{q}\right)^{-1} - \left(1 + e_0^D\right) R^L\right] \overline{r}_1^E \right] \\[2ex]
+ (1-p)(1-s)
\left[
\begin{array}{c}
\frac{1}{\overline{q}} \int_{\widehat{q}}^{\overline{q}} q \left(1 + e_0^D\right) \left(R^H - R^L\right) dq \\[2ex]
- \frac{\overline{q} - \widehat{q}}{\overline{q}} \, \overline{r}_1^E \left[\overline{p}\left(\widehat{q}\right)^{-1} - \left(1 + e_0^D\right) R^L \right]
\end{array}
\right] \\[2ex]
{\color{blue} + S_0 \left(e_0^D\right) + S_1 \left(e_0^D, \overline{r}_1^E\right)}
\end{array}
\right\}
\tag{7}
$$

---

[14] The surplus in excess of the outside option R.

s.t.

$$\widehat{q}\left(e_0^D, \bar{r}_1^E\right) = \frac{\bar{r}_1^E \, e_1\left(e_0, \bar{r}_1^E\right)}{(R^H - R^L)(1 + e_0)}$$

$$e_1\left(e_0^D, \bar{r}_1^E\right) = \overline{p}\left(\widehat{q}\right)^{-1} - \left(1 + e_0^D\right) R^L$$

$$\bar{r}_0^E = R + \theta e_0^D$$

$$\bar{r}_1^E = R + \theta \left[(1-p)s + (1-p)(1-s)\frac{\bar{q}-\widehat{q}}{\bar{q}}\right] e_1\left(e_0^D, \bar{r}_1^E\right)$$

$$S_0\left(e_0^D\right) = \frac{\theta}{2}\left(e_0^D\right)^2$$

$$S_1\left(e_0^D, \bar{r}_1^E\right) = \frac{\theta}{2}\left(\left[(1-p)s + (1-p)(1-s)\frac{\bar{q}-\widehat{q}}{\bar{q}}\right] e_1\left(e_0^D, \bar{r}_1^E\right)\right)^2$$

The two surplus terms in the objective function and the four last constraints in blue color differ from the bank's problem. Supposing an interior solution exists, the constrained efficient solution is characterised by almost the same system of equations as in the decentralized equilibrium. The only difference is that the first equation in 5 has to be replaced by the optimality condition derived from the planner problem in (7).

4.2.2   Envelope argument

To analyze efficiency, we use an envelope argument. In particular, we compare the optimality condition of the banks' problem with the optimality condition of the constrained planner problem. Let $E_0^* = (e_{0,i}^D)^* \; \forall i$ be the equilibrium level of outside equity at date $t = 0$ and denote the left-hand side of the planner's optimality condition by G. Evaluating G at E_0^* leads to:

$$G\big|_{e_0^D = E_0^*} = -\frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2}\left[p + (1-p)\,s\,\bar{r}_1^E + \overline{p}\left(\widehat{q}\right)^2\left(\bar{q} - \widehat{q}\right)\right]\frac{d\widehat{q}}{d\bar{r}_1^E}\frac{d\bar{r}_1^E}{de_0^D}$$

Proposition 4. (Constrained inefficiency) *Given the result in Lemma 2, the equilibrium is characterized by an inefficient under-capitalization, i.e. $\frac{d}{de_0^D}\big|_{e_0^D = E_0^*} > 0$, for all $1 > s \geq \underline{s}_0$ and by an efficient capitalization if $s = 1$.*

The result in Proposition 4 holds for sufficiently high values of s that assure we are in the plausible scenario when $\frac{d\bar{r}_1^E}{de_0^D} < 0$ (Lemma 2). Notably, the equilibrium is constrained efficient for the special case where $s = 1$ provided that inequality (6) holds. This is because the ex-ante capitalization here only affects the magnitude of future recapitalization needs, leaving solvency unaffected.

Instead, if $s < 1$ then the extensive margin is added, which creates a wedge between the marginal private cost of equity and the marginal social cost of equity. Furthermore, the frequency of insolvencies is affected by market conditions at $t = 1$ if $s < 1$. Bank

managers do not fully internalize how their individual choice at $t = 0$ is linked to the incidence of insolvency. Formally, the pecuniary externality materializes in the recapitalization constraint governed by $\widehat{q}\left(e_0^D, \bar{r}_1^E\right)$.[15]

5 A numerical example

Let us consider a numerical example to illustrate the results using the parameters in Table 1. The model parameters imply a probability that a given bank has a recapitalization need of 5% and a probability that a given bank faces a bankruptcy of $< 1\%$. For the model

Variable	R^H	R^L	p	$\bar{q}$	$\underline{q}$	R	θ
Value	1.3	0.7	0.95	0.8	0.3	R^H	0.2

Table 1: Model parameters

with the intensive margin only (i.e. $s = 1$), the equilibrium is constrained efficient with $E_0^* = E_0^{SP} = 0.023$.[16] Instead, for the model with the intensive and extensive margin (i.e. $s = 0.8$), banks are inefficiently under-capitalized. Here $E_0^* = 0.074 < E_0^{SP} = 0.178$, as can be seen in Table 2.[17]

Variable	E_0	$\bar{r}_0^E$	$\bar{r}_1^E$	E_1	$\widehat{q}$
Market equilibrium if $s = 1$	0.023	1.305	1.303	0.284	
Planner solution if $s = 1$	0.023	1.305	1.303	0.284	
Market equilibrium if $s = 0.8$	0.074	1.315	1.302	0.252	0.510
Planner solution if $s = 0.8$	0.178	1.336	1.301	0.176	0.325

Table 2: Results

Notably, the key welfare effect comes through the impact on the incidence of bank insolvency. While the critical threshold is $\widehat{q} = 0.325$ for the planner solution, it is considerably higher in the market equilibrium where banks are under-capitalized ($\widehat{q} = 0.510$). The implied ex-ante probability of a bankruptcy is $< 1\%$ for the planner solution and $> 2\%$ for the market equilibrium.

---

[15]This feature is in the spirit of the literature on collateral constraints that depend on market prices (Lorenzoni 2008, Korinek 2012).

[16]The index SP indicates the solution to the planner problem.

[17]With debt being normalized to unity, $E_0^{SP} = 0.178$ corresponds to a capital ratio of about 15%. Notice that in this example $\bar{r}_0^E > \bar{r}_1^E$. This result hinges on the model parameters and the opposite relation is possible. In a variation of the model with aggregate risk, e.g. with a random p or R_L, the cost for equity issuance will be highest in the crisis state at $t = 1$.

6 Conclusion

In this paper, we demonstrate that banks tend to be inefficiently under-capitalized ex-ante when their ability to recapitalize by issuing equity claims depends on future market conditions. In the presence of an imperfectly elastic aggregate supply of equity in the short- and medium-term, inefficient under-capitalization arises when higher aggregate equity buffers today are associated with a lower aggregate recapitalization need tomorrow. Under this plausible scenario, banks do not internalize that their equity buffers not only improve their individual chances to be solvent in the future, but are also positively associated with the chances of their peers to be solvent in the future. This second effect arises due to the endogeneity of the cost of bank equity and leads to an inefficiently high incidence of insolvency. Importantly, the inefficient under-capitalization of banks does not rely on the usual suspects: deposit insurance, moral hazard or asymmetric information.

Based on the efficiency analysis, we can draw conclusions for regulatory policy. The *pecuniary externality* in the market for bank equity has systemic implications that provide a new rationale for macroprudential capital regulation. A regulator can achieve the *second-best* outcome by setting the appropriate ex-ante capital charges or, in a re-interpretation of the model, by regulating banks' payout policies. At the same time, our model also contributes to the literature on the design of public bank stress tests, as it may be interpreted as an argument favouring staggered stress tests over extensive simultaneous testing exercises.

Future research aims at generalizing the results in a richer model of financial market segmentation with competitive deposit markets, that also allows for interbank equity markets at the intermediate date. Furthermore, the ambiguous implications of higher capital requirements on the endogenous social cost of bank equity, as well as the role of risk-weighted versus risk-insensitive capital regulation in our model, deserve an in-depth analysis. A more ambitious extension is to develop a richer model of the asset side of the bank balance sheet and to allow for asset fire sales alongside fire sale bank recapitalizations. Finally, both short-term debt contracts and the optimality of public bank stress test by the financial regulator can be rationalized by formally introducing a discipling role for liquid deposits.

References

Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. Pfleiderer (2013). The leverage ratchet effect. *MPI Collective Goods Bonn 2013/13*.

Admati, A. R., M. F. Hellwig, P. M. DeMarzo, and P. Pfleiderer (2011). Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive. *MPI Collective Goods Bonn 2010/42*.

Allen, F., E. Carletti, and R. Marquez (2014). Deposits and Bank Capital Structure. *mimeo*.

Allen, F. and D. Gale (1994). Limited Market Participation and Volatility of Asset Prices. *The American Economic Review 84*(4), 933–955.

Allen, F. and D. Gale (2004). Financial Intermediaries and Markets. *Econometrica 72*(4), 1023–1061.

Allen, F. and D. Gale (2007). *Understanding Financial Crises*. Oxford University Press, New York.

Calomiris, C. W. and C. M. Kahn (1991). The Role of Demandable Debt in Structuring Optimal Banking Arrangements. *American Economic Review 81*(3), 497–513.

De Jonghe, O. and O. Öztekin (2015). Bank capital management: International evidence. *Journal of Financial Intermediation 24*(2), 154–177.

DeAngelo, H. and R. M. Stulz (2015). Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks. *Journal of Financial Economics 116*(2), 219–236.

Diamond, D. W. and P. H. Dybvig (1983). Bank runs, deposit insurance, and liquidity. *The Journal of Political Economy 91*(3), 401–419.

Duffie, D. (2010). Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance 65*(4), 1237–1267.

Farhi, E. and J. Tirole (2012). Collective moral hazard, maturity mismatch, and systemic bailouts.

Gale, D. and P. Gottardi (2015). Capital Structure, Investment, and Fire Sales. *The Review of Financial Studies 28*(9), 2502–2533.

Geanakoplos, J. D. and H. M. Polemarchakis (1986). Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete. In W. Heller, R. M. Starr, and D. A. Starrett (Eds.), *Essays in Honor of Kenneth J. Arrow, volume 3: Uncertainty, Information, and Communication* (Essays in ed.)., pp. 65–95. Cambridge University Press.

Greenwald, B. C. and J. E. Stiglitz (1986). Externalities in Economies with Imperfect Information and Incomplete Markets. *The Quarterly Journal of Economics 101*(2), 229–264.

Grossman, S. J. and O. Hart (1982). Corporate Financial Structure and Managerial Incentives. In J. J. McCall (Ed.), *Corporate Financial Structure and Managerial Incentives*, pp. 107–140. University of Chicago Press.

Hanson, S. G., A. K. Kashyap, and J. C. Stein (2011, February). A Macroprudential Approach to Financial Regulation. *Journal of Economic Perspectives 25*(1), 3–28.

Kashyap, A., R. Rajan, and J. Stein (2008). Rethinking Capital Regulation. *Paper prepared for the 2008 Jackson Hole Symposium, August 21-23, 2008 on Maintaining Stability in a Changing Financial System.*

Kok, C. and G. Schepens (2013). Bank reactions after capital shortfalls. *ECP Working Paper* (1611).

Korinek, A. (2012). Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses. *mimeo*.

Lorenzoni, G. (2008). Inefficient Credit Booms. *The Review of Economic Studies 75*(2008), 809–833.

Mitchell, M., L. H. Pedersen, and T. Pulvino (2007). Slow moving capital. In *American Economic Review*, Volume 97, pp. 215–220.

Modigliani, F. and M. Miller (1958). The cost of capital, corporation finance and the theory of investment. *The American Economic Review 48*(3), 261–297.

Myers, S. C. and N. S. Majluf (1984). Corporate financing and investment decisions when firms have information that investors do not have. *Journal of Financial Economics 13*, 187–221.

Nicolò, G. D., G. Favara, and L. Ratnovski (2012). Externalities and Macroprudential Policy. *IMF Staff Discussion Note SDN/12/05*.

Philippon, T. and P. Schnabl (2013). Efficient Recapitalization. *Journal of Finance 68*(1), 1–42.

Shleifer, A. and R. Vishny (2010). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives 25*(1), 29–48.

Shleifer, A. and R. W. Vishny (1992). Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance 47*(4), 1343–66.

Stein, J. C. (2013). Liquidity Regulation and Central Banking. *Remarks at "Finding the Right Balance," 2013 Credit Markets Symposium, Sponsored by the Federal Reserve Bank of Richmond, Charlotte, North Carolina, April 19.*

A Proofs

A.1 Proof of Proposition 3

We prove results (a) and (b) in turn. **Result *(a)*:** Reformulating inequality (1), we can derive the lowest possible level of initial equity that allows for a market-based recapitalization in the intermediate period as:

$$\underline{e_0^D}\left(q_i, \bar{r}_1^E\right) = \max\left\{0, \frac{\bar{r}_1^E \, r_0^D}{q_i \, (R^H - R^L) + \bar{r}_1^E \, R^L} - 1\right\}.$$

Provided that inequality (6) holds, we have that $\underline{e_0^D}(\bar{q}, \bar{r}_1^E) = 0$ when evaluated at $e_{0,i}^D = 0 \, \forall i$. As a result, for $s = 1$ all risky banks can recapitalize. In equilibrium, the aggregate E_0^* solves the first-order necessary condition whenever the solution is interior. In this case the individual level of $e_{0,i}^D$ is indeterminate. If the right-hand side of the first-order condition is negative (positive) when evaluated at $E_0 = 0$ ($E_0 = \overline{E_0}$), then $E_0^* = 0$ ($E_0^* = \overline{E_0}$). This proves **Result *(a)***.

   Result *(b)*: The proof consists of two steps. First, we show that there can only exist equilibria characterized by a symmetric choice at $t = 0$. Second, we proof existence of an equilibrium that is unique in the class of symmetric equilibria.

   Step 1: In this part of the proof we take $\bar{r}_0^E$ and $\bar{r}_1^E$ as given an analyze the system of equations that solves the problem of an individual bank, which can be expressed as:

$$
\begin{aligned}
F\left(e_{0,i}^D, \widehat{q}\right) &\equiv \bar{r}_0^E - p\left[R^H - \frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2} \frac{d\widehat{q}}{de_{0,i}^D}\right] \\
&\quad - (1-p)\,s\left[\bar{q}\left(R^H - R^L\right) - \bar{r}_1^E\left(\frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2} \frac{d\widehat{q}}{de_{0,i}^D} + R^L\right)\right] \\
&\quad - \frac{(1-p)(1-s)}{\bar{q}}\left(\int_{\widehat{q}}^{\bar{q}} q\left(R^H - R^L\right) dq - \widehat{q}\left(1 + e_{0,i}^D\right)\left(R^H - R^L\right) \frac{d\widehat{q}}{de_{0,i}^D}\right) \\
&\quad - \frac{(1-p)(1-s)}{\bar{q}}\bar{r}_1^E\left[\left(\overline{p}\left(\widehat{q}\right)^{-1} - \left(1 + e_{0,i}^D\right)R^L\right)\frac{d\widehat{q}}{de_{0,i}^D} - (\bar{q} - \widehat{q})\left(\frac{(1-p)(1-s)}{\bar{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_{0,i}^D} - R^L\right)\right] \\
&= 0 \\
G\left(e_{0,i}^D, \widehat{q}\right) &\equiv \widehat{q}\left(R^H - R^L\right)\left(1 + e_{0,i}^D\right) - \bar{r}_1^E\left(\overline{p}\left(\widehat{q}\right)^{-1} - \left(1 + e_{0,i}^D\right)R^L\right) = 0 \\
\overline{p}\left(\widehat{q}\right) &= p + (1-p)\,s + (1-p)\,(1-s)\frac{\bar{q} - \widehat{q}}{\bar{q}}
\end{aligned}
$$

We prove that, for given market prices, there exists at most one solution that solves this system of equations provided the sufficient condition that s is large. As a result, if an equilibrium exists then it must be characterized by a symmetric choice at $t = 0$. To show this, we derive the comparative statics of $F\left(e_0, \widehat{q}\right)$ and $G\left(e_0, \widehat{q}\right)$. The following results

will be useful:

$$\frac{d\bar{p}(\widehat{q})}{d\widehat{q}} = -\frac{(1-p)(1-s)}{\bar{q}} < 0$$

$$\left.\frac{d\bar{p}(\widehat{q})}{d\widehat{q}}\right|_{s\to 1} = 0$$

$$\left.\frac{d^2\widehat{q}}{d\left(e_{0,i}^D\right)^2}\right|_{s\geq \underline{s_0}} > 0$$

$$\frac{d^2\widehat{q}}{de_{0,i}^D d\widehat{q}} < 0.$$

First, consider F:

$$\left.\frac{\bar{q}}{(1-p)(1-s)}\frac{dF\left(e_{0,i}^D,\widehat{q}\right)}{de_{0,i}^D}\right|_{s\to 1} > 0 \text{ if } \bar{r}_1^E \leq \frac{p}{\frac{1}{2}-\frac{1-p}{\bar{q}}}.$$

and:

$$\frac{\bar{q}}{(1-p)(1-s)}\frac{dF\left(e_{0,i}^D,\widehat{q}\right)}{d\widehat{q}} > (<) \, 0 \text{ if } s\to 1 \text{ and } \frac{R^H-R^L}{1-R^L} > (<) \, \bar{r}_1^E.$$

We have that $\frac{\bar{q}}{(1-p)(1-s)}\frac{dF}{de_{0,i}^D} > 0 \text{ if } s\to 1 \text{ and } \bar{r}_1^E \leq \frac{p}{\frac{1}{2}-\frac{1-p}{\bar{q}}}$ and $\frac{\bar{q}}{(1-p)(1-s)}\frac{dF}{d\widehat{q}} > (<)\,0 \text{ if } s\to 1 \text{ and } \frac{R^H-R^L}{1-R^L} > (<0) \, \bar{r}_1^E$. Hence $F\left(e_{0,i}^D,\widehat{q}\right)$ gives us the following relation: $\frac{d\widehat{q}}{de_0} \neq 0 \text{ if } s\to 1 \text{ and } \bar{r}_1^E \leq 2$.

Next, consider G:

$$\frac{dG\left(e_{0,i}^D,\widehat{q}\right)}{de_{0,i}^D} = 0 \text{ if } s\to 1$$

and:

$$\frac{dG\left(e_{0,i}^D,\widehat{q}\right)}{d\widehat{q}} > 0 \text{ if } s\to 1.$$

We have that $\frac{dG}{de_{0,i}^D} = 0$ and $\frac{dG}{d\widehat{q}} > 0$ if $s\to 1$. Hence $G\left(e_{0,i}^D,\widehat{q}\right)$ gives us the following relation: $\frac{d\widehat{q}}{de_0} \to 0$ if $s\to 1$. As a result, there is at most one crossing of F and G in the $\left(e_{0,i}^D,\widehat{q}\right)$ space. Hence, an equilibrium must be symmetric in the $t=0$ choice.

Step 2: Suppose a symmetric equilibrium exists and recall that the system of equa-

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tions describing the equilibrium can be reduced to two equations in two unknowns $(e_0, \widehat{q})$:

$$
\begin{aligned}
H(e_0, \widehat{q}) \quad &\equiv \overline{r}_0^E - p\left[R^H - \frac{(1-p)(1-s)}{\overline{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_0}\right] \\
&\quad - (1-p)s\left[\overline{q}\left(R^H - R^L\right) - \overline{r}_1^E(e_0, \widehat{q})\left(\frac{(1-p)(1-s)}{\overline{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_0} + R^L\right)\right] \\
&\quad - \frac{(1-p)(1-s)}{\overline{q}}\left(\int_{\widehat{q}}^{\overline{q}} q\left(R^H - R^L\right)dq - \widehat{q}(1+e_0)\left(R^H - R^L\right)\frac{d\widehat{q}}{de_0}\right) \\
&\quad - \frac{(1-p)(1-s)}{\overline{q}}\left[\overline{r}_1^E(e_0, \widehat{q})\left(\left(\overline{p}(\widehat{q})^{-1} - (1+e_0)\,R^L\right)\frac{d\widehat{q}}{de_0} - (\overline{q} - \widehat{q})\left(\frac{(1-p)(1-s)}{\overline{q}\,\overline{p}(\widehat{q})^2}\frac{d\widehat{q}}{de_0} - R^L\right)\right)\right] \\
&= 0 \\
I(e_0, \widehat{q}) \quad &\equiv \widehat{q}\left(R^H - R^L\right)(1+e_0) - \overline{r}_1^E\left(\overline{p}(\widehat{q})^{-1} - (1+e_0)\,R^L\right) = 0 \\
\overline{p}(\widehat{q}) \quad &= p + (1-p)s + (1-p)(1-s)\frac{\overline{q} - \widehat{q}}{\overline{q}} \\
\overline{r}_0^E \quad &= R + \theta e_0 \\
\overline{r}_1^E \quad &= R + \theta\left((1-p)s + (1-p)(1-s)\frac{\overline{q} - \widehat{q}}{\overline{q}}\right)\left(\overline{p}(\widehat{q})^{-1} - (1+e_0)\,R^L\right).
\end{aligned}
$$

We prove that there exists at most one solution that solves this system of equations provided the sufficient condition that s is large. As a result, if a symmetric equilibrium exists, it will be unique. To show this, we derive the comparative statics of $H(e_0, \widehat{q})$ and $I(e_0, \widehat{q})$, taking into account the general equilibrium effects. The following results will be useful:

$$
\left.\frac{d^2\widehat{q}}{de_0^D\,d\overline{r}_1^E}\right|_{s > \underline{s}_0} = -\frac{1}{\left(R^H - R^L\right)\left(1 + e_0^D\right)^2} < 0
$$

$$
\frac{d\overline{p}(\widehat{q})}{d\overline{r}_1^E} = -(1-p)(1-s)\frac{\frac{d\widehat{q}}{d\overline{r}_1^E}}{\overline{q}} < 0.
$$

First, consider H:

$$
\frac{dH\left(e_0^D, \widehat{q}\right)}{de_0^D} > 0 \; if \; s \to 1.
$$

To see this, notice that $\left.\frac{d\overline{r}_1^E}{de_0}\right|_{s \to 1} = -\theta(1-p)R^L$. Furthermore:

$$
\frac{dH\left(e_0^D, \widehat{q}\right)}{d\widehat{q}} = 0 \; if \; s \to 1.
$$

Next, consider I:

$$
\left.\frac{dI\left(e_0^D, \widehat{q}\right)}{de_0^D}\right|_{s \to 1} = 0
$$

and:

$$
\left.\frac{dI\left(e_{0,i}^D, \widehat{q}\right)}{d\widehat{q}}\right|_{s \to 1} > 0.
$$

We have that $\frac{dH}{de_0} > 0\ if\ s \to 1$ and $\frac{dH}{d\widehat{q}} = 0\ if\ s \to 1$. Hence $H(e_0, \widehat{q})$ gives us the following relation: $\frac{d\widehat{q}}{de_0} \to -\infty\ if\ s \to 1$. Further, $\frac{dI}{de_o} = 0\ if\ s \to 1$ and $\frac{dI}{d\widehat{q}} > 0\ if\ s \to 1$. Hence $I(e_0, \widehat{q})$ gives us the following relation: $\frac{d\widehat{q}}{de_0} = 0\ if\ s \to 1$. Taken together, if it exists, the equilibrium is unique in the class of symmetric equilibria provided that $s \to 1$, because $H(e_0, \widehat{q})$ and $I(e_0, \widehat{q})$ have a single crossing. By continuity, $\exists\, \underline{s} \in [0, 1)$, such that the previous results hold for all $s \geq \underline{s}$, where $\underline{s} \geq \underline{s}_0$. Provided that inequality (6) holds, the existence of an interior solution for $\widehat{q}$ follows and, hence, the above described symmetric equilibrium exists. This proves **Result (b)**. *(q.e.d.)*.

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