SVERIGES RIKSBANK
WORKING PAPER SERIES

331

Money, Credit and Banking and the Cost of Financial Activity

Paola Boel and Gabriele Camera

October 2016

WORKING PAPERS ARE OBTAINABLE FROM

www.riksbank.se/en/research
Sveriges Riksbank • SE-103 37 Stockholm
Fax international: +46 8 21 05 31
Telephone international: +46 8 787 00 00

Money, Credit and Banking and the Cost of Financial Activity†

Paola Boel
Sveriges Riksbank

Gabriele Camera
Chapman University
University of Basel

Sveriges Riksbank Working Paper Series No. 331

October 2016

Abstract

We extend the study of banking equilibrium in Berentsen, Camera and Waller (2007) by introducing an explicit production function for banks. Banks employ labor resources, hired on a competitive market, to run their operations. In equilibrium this generates a spread between interest rates on loans and on deposits, which naturally reflects the efficiency of financial intermediation and underlying monetary policy. In this augmented model, equilibrium deposits yield zero return in a deflation or very low inflation. Hence, if monetary policy is sufficiently tight then banks end up reducing aggregate efficiency, soaking up labor resources while offering deposits that do not outperform idle balances.

Keywords: banks; frictions; matching.
JEL: C70, D40, E30, J30

1 Introduction

The model in Berentsen, Camera and Waller (2007) (henceforth BCW) has demonstrated from first principles how banks improve social welfare by reallocating liquidity from those with idle balances to those who are liquidity constrained. The central result is that banks improve welfare for *any* positive nominal interest rate, even for policies arbitrarily close to the zero bound. This is because introducing banks always raises the return from holding idle balances. This result emerges under the assumption that the process of financial intermediation requires neither labor nor capital inputs, that is, banks have no operating

† The opinions expressed in this article are the sole responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank. Corresponding author: P. Boel, Sveriges Riksbank, Research Division, SE-103 37 Stockholm, Sweden; e-mail: paola.boel@riksbank.se

costs. Though this assumption has benefits in terms of theoretical simplicity, it abstracts away from an important empirical feature: banking activities are not cheap.

The average European country spends about one percent of GDP to conduct retail transactions, and half of these costs were incurred by banks and infrastructures (Schmiedel et al., 2012). For the U.S., in 2012 financial activities employed almost 8 million workers and generated 7.2% of GDP.[1] Two natural question thus present themselves: do banks retain their welfare-improving role when we enrich the model to account for financial intermediation costs? And what are the quantitative consequences in terms of monetary policy?

Here, we study equilibrium when intermediation costs are explicit: intermediating a loan requires labor resources. The model generates a well-defined bid-ask spread for interest rates which endogenously responds in natural ways to monetary policy and the efficiency of the intermediation technology. In turn, this implies that financial intermediation costs are quantitatively important. If the spread between rates on loans, deposits and bonds is taken to measure market liquidity, then market liquidity is affected by monetary policy—whereas it is always zero when financial intermediation is costless.

A main result is that banking equilibrium does not always exist close to the zero bound. It generally exists only for nominal interest rates bounded sufficiently away from zero. Intuitively, banks' revenues and deposit rates shrink as rates fall, and at some point paying interests on deposits is no longer profitable. Eventually, lending will also cease to be profitable if banks are sufficiently inefficient. If so, then a deflation that brings the economy close to the Friedman rule is incompatible with the existence of banks. Supporting banking equilibrium when interest rates are close to zero requires improvements in the efficiency in the way in which banks transform liquidity into loans. Otherwise, banks will collapse. This is a sharp departure from the prediction in BCW where interests on deposits are invariably positive away from the Friedman rule because banking soaks up no resources.

But should we care about the efficiency of financial intermediation? And is an economy without banks worse off than one where banks are active? Quantitatively, we find that intermediation efficiency and welfare cost of inflation exhibit a negative association.

[1] Data for employment is from the *Bureau of Labor Statistics*, as measured by Employment in Financial Activities. Data for the share of GDP is from the St. Louis *FRED Database*, as measured by Value Added by Private Industries: Finance and Insurance.

Intuitively, greater efficiency in banking lessens the adverse effect of inflation on market liquidity. However, banks absorb labor resources away from other productive activities, which—if the financial sector is large and inefficient—has an appreciable negative general equilibrium effect on wages, hence on prices. As a result, banks may reduce overall efficiency in a deflation, away from the Friedman rule, and even at small positive inflation rates—when deposit rates are zero. In this sense, the analysis shows that there can be "too much" finance for sufficiently low inflation. In this scenario paying interests on reserves—financing it through lump sum taxes—can correct the problem, as it allows banks to raise deposit rates.

Microfounded models of banking exist in which intermediation generates exogenously fixed costs. In Bencivenga and Camera (2011) there is a fixed resource cost to liquidate a bank deposit; in banking equilibrium, inflation and banks' utilization are positively associated, and paying interest on reserves may improve welfare because it increases the deposits' liquidity. In Chiu and Meh (2011) entrepreneurs suffer a fixed disutility cost from borrowing money from banks; financial intermediation is thus welfare-improving only at high inflation, and it shuts down at low inflation. There are also models of costly financial intermediation where banks intermediate real assets but there is no role for money (e.g., see Antunes et al., 2013, and references therein). By contrast, we work with a model where money has an explicit role, and banking costs are due to an explicit labor-based production technology for loans, so labor costs suffered by banks vary with the intermediation level.

We proceed as follows. Section 2 describes the model, Section 3 discusses existence of equilibrium, Section 4 presents the main theoretical results, Section 5 discusses the quantitative performance of the model and the last Section concludes.

2 The environment

The model builds on BCW (2007). Time is discrete, the horizon is infinite and there is a large population of infinitely-lived agents who produce and consume perishable goods and have time-separable preferences. In each period agents may visit two sequential anonymous markets, denoted by 1 and 2. Markets differ in terms of economic activities and preferences and in each market, workers can access a linear goods' production technology that transforms one unit of labor into one unit of consumption goods. In market 1, agents face an idiosyncratic trading risk. An agent can either consume or work, two mutually exclusive individual states. She can consume with probability α and can work with probability $1-\alpha$. Since consumers will want to buy consumption goods, we call them buyers. Buyers get utility $u(q)$ from $q>0$ consumption, with $u'>0$, $u''<0$, $u'(0)=\infty$, and $u'(\infty)=0$. Supplying $N\geq 0$ labor generates disutility $c(N)$, with $c'>0$, $c''\geq 0$ and $c'(0)=0$. In market 2 every agent can consume and produce using a backyard technology, supplying Y labor generates Y disutility, consuming X goods generates utility $U(X)$, with $U'>0$, $U''\leq 0$, $U'(0)=\infty$ and $U'(\infty)=0$. We let $U'(X^*)=1$. The discount factor across periods is $\beta\in(0,1)$. We retain the same informational structure as in BCW, which gives rise to a role for monetary exchange and banking activity.

A central bank exists that controls the supply of fiat money. Let M_t denote the per capita money stock at the start of a generic period t. The gross growth rate of the money stock across two periods is $\mu\geq\beta$. Agents receive nominal lump-sum transfers $T=(\mu-1)M_t$ in market 2 of period t. We let $V(m)$ denote the continuation payoff of an agent who has $m>0$ nominal balances at the start of a period.

Competitive banks exist that accept one-period monetary deposits and make one-period monetary loans at the start of market 1, before the goods market opens—one-period contracts are optimal due to quasi-linear preferences (see BCW, 2007). Banks are profit maximizing firms, owned by households. Consumers can borrow money from banks to supplement their transaction balances, and workers can deposit their idle balances in a bank. Borrowers are charged interest payments on their loans, in market 2; let r_H denote the nominal interest rate on consumer loans. Depositors are compensated with interest payments on their deposits, in market 2; let r denote the nominal interest rate on deposits. Following BCW, we assume that banks can force repayment of loans at no cost, and that banks can engage in intermediation because only they can operate a technology that keeps records of financial histories (but not trading histories). However, unlike BCW, we do not assume that intermediation is a costless process. Instead, we assume that to transform deposits into loans banks must operate a loans' production technology F that uses labor inputs. Hence, in market 1 banks hire workers on a competitive labor market and compensates them in market 2, when loans are returned and any bank's profit is distributed to households as dividends. For simplicity, we will develop the main results under the conjecture that no one makes a deposit unless they are compensated for it, i.e., deposits are made only if $r > 0$. This is equivalent to assuming some (infinitesimal) cost from holding a deposit account. Subsequently, we will relax this assumption and consider the case when deposits are made even if $r = 0$.

3 Equilibrium allocations

We focus on symmetric stationary monetary equilibria where all agents follow identical strategies and where real variables are constant over time. Consider a generic date t. Let ϕ denote the real price of money in market 2. Variables corresponding to the following date $t + 1$ are indexed by a prime. In stationary equilibrium real money balances are time invariant, so we have $\phi M = \phi' M'$, which implies that inflation is pinned down by the rate of money growth because

$$\frac{\phi}{\phi'} = \frac{M'}{M} = \mu.$$

Consider stationary equilibrium on a generic date t when banks are operating, and in which the price of money is ϕ. Let $W(m, l, d)$ denote the continuation payoff of an agent who, at the start of market 2, has m balances, l loans and d deposits. This portfolio depends the agent's history of trade in market 1. We have:

$$\begin{aligned} W(m, l, d) &= \max_{X,Y,m' \geq 0} U(X) - Y + \beta V(m') \\ \text{s.t. } Y &= X - \phi m - \phi p n_B - \phi(1 + r)d + \phi(1 + r_H)l + \phi m' - \phi T - \phi\zeta \end{aligned}$$

where m' is money taken into period $t + 1$, ϕ is the real price of money, T are lump-sum transfers, and ζ are banks' dividends. To understand the budget constraint note that only workers can supply labor to a bank and would want to make deposits, while only consumers would want to take out loans. A consumer who borrowed $l > 0$ money in market 1 must repay $(1 + r_H)l$ money in market 2 with r_H. A worker who deposited $d > 0$ money in market 1 receives $(1 + r)d$ money in market 2 with r. A worker who supplies n_B labor to banks in market 1 gets paid pn_B dollars in market 2 when the wage is p. Here, p corresponds to the price of goods in market 1 because in equilibrium workers must be indifferent between producing goods or working for a bank.

Substituting the constraint into W yields:

$$W(m,l,d) = \phi[m+pn_B+(1+r)d-(1+r_H)l+T+\zeta]+\max_{X,m'\geq 0}[U(X)-X-\phi m'+\beta V(m')]$$

The first order conditions in monetary equilibrium are:

$$(X) \quad : \quad U'(X)=1 \;\Rightarrow\; X=X^* \tag{1}$$

$$(m') \quad : \quad \phi = \beta V'(m') \tag{2}$$

The envelope conditions are

$$\frac{\partial W}{\partial m}=\phi, \qquad \frac{\partial W}{\partial d}=\phi(1+r), \text{ and } \qquad \frac{\partial W}{\partial l}=-\phi(1+r_H).$$

We also have $\dfrac{\partial W}{\partial n_B}=\phi p$.

Moving backwards in the sequence of markets, the expected value of holding m money balances at the start of market 1 when the price is p is

$$V(m)=\max_{q,l}\ \alpha[u(q)+W(m-pq+l,l)]+\max_{n,n_B,d}\ (1-\alpha)[-c(N)+W(m-d+pn,d)],$$

where $N=n+n_B$ is the total labor supply of a worker, and we note that W is a function of n_B. In equilibrium, when buyers spend all their cash and workers deposit all their cash, we have a simplified problem:

$$V(m)=\max_{q,\tau}\ \alpha\left[u(q)+W(0,m-\tau)\right]+\max_{n,n_B}\ (1-\alpha)\left[-c(N)+W(pn,m)\right]$$

where we use $\tau=pq$ to denote the amount of money spent by a buyer to buy q consumption at price p.

We now consider workers and consumers' decisions, separately. The worker's problem when banks are operating is:

$$\max_{n,n_B,d} \quad -c(n+n_B)+W(m-d+pn,d)$$
$$\text{s.t.} \quad d \leq m, \text{ and } n+n_B=N$$

The first-order conditions are:

$$c'(N)=p\phi \tag{3}$$
$$\phi r=\lambda_d \tag{4}$$

where λ_d is the multiplier on the deposit constraint. The labor supply N is determined by the real wage $p\phi$, and is independent of the worker's financial decisions. Moreover, (4) implies that if $r>0$, then the deposit constraint is binding and $d=m$. If $r=0$, then the agent would be indifferent between keeping the money or depositing in the bank only in the absence of costs for making deposits. We will work under the conjecture that the agent does not make a deposit if $r=0$ and consider the case in which he does, in a later section.

The problem for a consumer when banks are operating is:

$$\max_{q,l} \quad u(q)+W(m-pq+l,l)$$
$$\text{s.t.} \quad pq \leq m+l$$

The first-order conditions are:

$$u'(q)-\phi p-p\lambda=0$$
$$\phi r_H=\lambda$$

where λ is the multiplier on the budget constraint. These two equations jointly imply

$$u'(q)=\phi p(1+r_H).$$

Using the information from the worker's problem, we have

$$u'(q) = c'(N)(1 + r_H), \tag{5}$$

where $N = n + n_B$. The value q is also a function of n and n_B because the goods market clearing condition is

$$(1 - \alpha)n = \alpha q. \tag{6}$$

Using the envelope conditions, the marginal value of money satisfies

$$V'(m) = \phi \left[\frac{\alpha u'(q)}{p\phi} + (1 - \alpha)(1 + r) \right],$$

which, given $u'(q) = \phi p(1 + r_H)$, yields

$$V'(m) = \phi \left[1 + r + \alpha(r_H - r) \right]. \tag{7}$$

We now can move on to discuss the bank's problem. It is assumed that hiring $\ell^B > 0$ labor allows the bank to generate $F(\ell^B)$ real loans. Let $F(0) = 0$, $F' > 0$ and $F'' \leq 0$. The bank's profit maximization problem is

$$\begin{aligned} \max_{L,D,\ell_B} \quad & L r_H - D r - p\ell_B \\ \text{s.t.} \quad & L = pF(\ell_B), \text{ and } L \leq D. \end{aligned}$$

L and D are aggregate loans and deposits, respectively. It should be clear that banks will operate only if there is a positive spread between interest rate on loans and deposits, $r_H > r$. In that case, the second constraint holds with equality. Hence, conjecturing $r_H > r$, we substitute both constraints in the production function, and the bank's maximization problem becomes

$$\max_{\ell_B} pF(\ell_B)(r_H - r) - p\ell_B.$$

The FOC is $F'(\ell_B)(r_H - r) = 1$. Therefore

$$r_H - r = \frac{1}{F'(\ell_B)}. \tag{8}$$

Depending on F, banks may earn positive profits, which are distributed as dividends $\zeta = p[F(\ell_B)/F'(\ell_B) - \ell_B]$. Greater efficiency in financial intermediation results in lower spreads. Spreads, in turn, alter the marginal value of money in (7), which increases with the spread, i.e., money is more valuable when financial intermediation services are *less* efficient because loans are more expensive, so consumers must rely more on cash. Finally, note that the spread is non-negative for all levels of banking. The spread can be zero only in the limit as banks make no loan, i.e., if $F'(0) = \infty$.

Now consider market-clearing. The loan-production technology implies

$$L = \alpha(pq - m) = pF(\ell_B). \tag{9}$$

The lending constraint is $L = \alpha(pq - m) = D = (1 - \alpha)m$. This implies that per capita money holdings are equal to per capita expenditure in market 1, so that

$$\alpha pq = m. \tag{10}$$

Therefore, we get

$$\alpha q = \frac{F(\ell_B)}{1 - \alpha}, \tag{11}$$

where from labor market clearing we have

$$\ell_B = n_B(1 - \alpha).$$

Therefore, using (11) and the goods market clearing condition in (6), in equilibrium we have

$$(1 - \alpha)^2 n = F(n_B(1 - \alpha)). \tag{12}$$

4 Results

In this section we list the main findings for economies where—as in the typical model of this class—the Central Bank does pay any interest on commercial banks' reserves. Subsequently we will also consider the case where this can be done. Finally, we discuss the quantitative implications of the model.

4.1 Zero interest on reserves

This is the standard assumption in this class of models. Start by noting that the nominal interest rate on an illiquid bond is $i = \dfrac{\mu - \beta}{\beta}$. We use this definition to determine the equilibrium interest rate on bank deposits.

Result 1 (**Endogenous spread**). *In equilibrium there is an endogenous spread between loan and deposit rates,* $0 \le r = r^* < i < r_H$ *with*

$$r^* := \max\left(i - \frac{\alpha}{F'(\ell_B)}, 0\right), \quad \textit{and} \quad r_H = \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha}.$$

Hence, if $\mu \le \beta + \dfrac{\beta\alpha}{F'(0)}$*, then* $r = 0$.

To derive this result, note that the marginal value of money must satisfy (2) and (7). The basic Euler equation is thus

$$\frac{\mu}{\beta} = 1 + r + \alpha(r_H - r) \qquad \Rightarrow \qquad i - r = \alpha(r_H - r). \tag{13}$$

To determine the interest rates use equation (8), which identifies the spread, yielding the candidate deposit rate

$$r = i - \frac{\alpha}{F'(\ell_B)}. \tag{14}$$

Since agents can trade with cash and are free to hold cash outside of banks, the deposit rate must be positive in banking equilibrium. Hence, we have r^*.[2] If $r = 0$, then $\ell_B = 0$

[2]This is what happens in BCW at the Friedman rule, where rates on loans and deposits are zero. There, it is irrelevant if banks do or not operate. Here, it is consequential because banks employ workers, which

since banks are inactive and so the aggregate labor effort is $N = \dfrac{\alpha q}{1-\alpha}$ (no one works in a bank).

From (13) with $r = r^*$ we obtain

$$r_H = \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha}.$$

Note that $r_H > i$ so borrowers always pay a premium above the yield i on an illiquid bond. When $r^* > 0$, the loan rate is

$$r_H = i + \frac{1-\alpha}{F'(\ell_B)}. \tag{15}$$

The interest rate on deposit and on loans are governed by the productivity of the banking sector, F'. In particular, one can think of the model in BCW as a special case of our model, where banks are infinitely productive at any level of activity. In that case, $r^* = i$ and $r_H = r = i$, as in the BCW model. The link between banks' productivity and interest rates on deposits has another important implication.

Corollary 1 (**Banking and deflations**). *Banking equilibrium breaks down for deflations sufficiently close to the Friedman rule.*

When banking absorbs labor resources, banking breaks down for inflation rates sufficiently close to β. The reason is that $r < i$. Hence, for $i > 0$ we have that $r = r^* = 0$ for $\mu \leq \beta + \dfrac{\beta\alpha}{F'(0)}$, and sellers no longer make deposits. Thus, economies with less productive banking sectors must maintain higher inflation rates to support banking equilibrium. This result stands in contrast with the analysis in BCW. There, banks are active and improve the allocation for any inflation rate above the Friedman rule, while for $\mu \to \beta$ agents are indifferent between cash and deposits and the allocation with banking is equivalent to the allocation attained in a pure-currency economy.

has general equilibrium effects. For this reason, in a subsequent section we also study the case where sellers deposit their money when $r = 0$.

Result 2. *Consider $r^* > 0$. In banking equilibrium, market 1 allocations satisfy*

$$u'(q) = c'(N)\left[1 + \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha}\right]. \tag{16}$$

Economic efficiency increases in the deposit rate r^*.

Using r_H from Result 1 in (5) we obtain (16). To prove the second part of the result, expression (16) implies that q increases with r^* because $N > \frac{\alpha q}{1-\alpha}$, while u is concave and c is convex. Since r^* increases with banks' productivity, then economic efficiency also increases with banks' productivity. This in turn implies that the efficiency level achieved when banking services require labor inputs is always below the efficiency level attained in BCW, where

$$u'(q) = c'(N)(1+i),$$

and the aggregate labor effort is $N = \dfrac{\alpha q}{1-\alpha}$, which is less than labor effort in our model where banks also use labor inputs.

Introducing a more realistic model where intermediation services are costly has also a deep implication for the desirability of banking. A fundamental result in BCW is that financial intermediation *always* improves the allocation and welfare when $\mu > \beta$ (BCW, Corollary 1). Simply put, away from the Friedman rule a pure currency economy would always deliver an inferior allocation. This is no longer true when intermediation absorbs labor resources. There is a threshold inflation level below which active intermediation lowers macroeconomic efficiency. This threshold level depends on the productivity of the banking sector.

Result 3 (Banking may reduce welfare). *If $r > 0$ and inflation is sufficiently low, then the allocation in banking equilibrium can be Pareto-inferior to the allocation in an equilibrium without banks.*

To prove it, calculate the efficiency achieved when banks are and are not operating. In monetary equilibrium *without banks*, as in Lagos and Wright (2005) for example, we have

$$u'(q) = c'(N)\left[1 + \frac{i}{\alpha}\right],$$

with $N = \frac{\alpha q}{1-\alpha}$ because $\ell_B = 0$. Instead, in an equilibrium with active banks, substituting for $r^* > 0$, we have

$$u'(q) = c'(N)\left[1 + i + \frac{1-\alpha}{F'(\ell_B)}\right].$$

Note that

$$i < \frac{\alpha}{F'(\ell_B)} \qquad \Rightarrow \qquad 1 + \frac{i}{\alpha} \leq 1 + i + \frac{1-\alpha}{F'(\ell_B)}.$$

Note also that $r^* > 0$ requires $i > \frac{\alpha}{F'(0)}$. Hence, if $F'' < 0$, then we can find some $\frac{\alpha}{F'(0)} < i < \frac{\alpha}{F'(\ell_B)}$, for any given $\ell_B > 0$. Now suppose, by means of contradiction, that q in the banking economy is at least as large as q in the cash economy, i.e., $q_{cash} \leq q_{banks}$. Since $c'' \geq 0$, we have $c'\left(\frac{\alpha q}{1-\alpha}\right) \leq c'(N)$ because some workers must supply labor to banks, hence $N > \frac{\alpha q}{1-\alpha}$. It follows that since $i < \frac{\alpha}{F'(\ell_B)}$, then we have

$$u'(q_{cash}) = c'\left(\frac{\alpha q}{1-\alpha}\right)\left[1 + \frac{i}{\alpha}\right] \leq c'(N)\left[1 + i + \frac{1-\alpha}{F'(\ell_B)}\right] = u'(q_{banks}),$$

which contradicts $q_{cash} \leq q_{banks}$ since $u'' < 0$. It should be clear that banks always improve the allocation relative to a cash-only economy, when $F'' = 0$. Otherwise, if $F'' < 0$ and interest rates are sufficiently small, then the quantity q consumed on market 1 is lower than in an economy without banks. Simply put, banks decrease efficiency even if they pay interests on deposits. Banks, in this case, simply soak up too many labor resources—distorting prices—relative to the benefits provided to depositors.

This result stands in contrast to a fundamental result developed in BCW (Proposition 2), stating that banks always improve the allocation away from the Friedman rule. The intuition is that banks improve welfare because they compensate sellers for their idle balances and not because they slacken the cash constraint of buyers (who are indifferent to borrowing). In this manner everyone will bring more liquidity to the market because money will never sit idle, and market 1 production increases. As soon as banks stop compensating depositors, then intermediation becomes irrelevant. In BCW, banks are irrelevant only at the Friedman rule because they have not operating costs so that $0 < r = i = r_H$ for $\mu > \beta$. That is: deposits yield as much as an illiquid bond, and loans command that same rate. Instead, when intermediation is costly, we have an endogenous spread between loans and deposit rates, $0 < r < i < r_H$. In this case, deposits yield less than an illiquid bond, while loans command a premium over it. For inflation rates sufficiently small, banks end up compensating depositors too little and would charge too high a premium on borrowers, hence the allocation will be worse than in an economy without banks.

An example: Consider a technology that transforms labor into loans at constant marginal rate, $F(x) = Ax$ for $A > 0$. The equilibrium interest rate is

$$r^* := \min\left(i - \frac{\alpha}{A}, 0\right), \quad \text{and} \quad r_H = \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha}.$$

Hence, $r > 0$ only away from deflations, for inflation rates satisfying $\mu \geq \mu_L := \beta + \dfrac{\beta\alpha}{A}$.

Equation (12) implies

$$An_B = (1-\alpha)n.$$

The Euler equation (16) becomes:

$$u'\left(\frac{(1-\alpha)n}{\alpha}\right) = c'(n + n(1-\alpha)/A)\left[1 + \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha}\right]$$

This will give us n as a function of the parameters.

It is immediate that the economy is better off without banks, when $\mu \leq \mu_L$ because in this case $r = 0$. Now suppose $\mu > \mu_L$ so $r^* = i - \frac{\alpha}{A} > 0$. Let i_L denote the interest rate when $\mu = \mu_L$.

To determine when the economy is better off with or without banks we must use specific functional forms. Let $u = \ln q$ and $q = n$. Then, with banks:

$$\frac{\alpha}{(1-\alpha)n_1} = c'(n_1 + n_1(1-\alpha)/A)\left[1 + i + \frac{1-\alpha}{A}\right].$$

Note that as $A = \infty$ we get back the model in BCW. Without banks, as in Lagos and Wright (2005), we have

$$\frac{\alpha}{(1-\alpha)n_2} = 1 + \frac{i}{\alpha}.$$

The left hand side of each equation is $1/q$. The quantity consumed q is greater with banks when the right hand side of the first equation is less than the right hand side of the second equation when $n_1 = n_2$, i.e.,

$$c'(n + n(1-\alpha)/A)\left[1 + i + \frac{1-\alpha}{A}\right] \leq 1 + i/\alpha.$$

Substituting for $n_1 = n_2 = \frac{\alpha^2}{(1-\alpha)(\alpha+i)}$ we obtain

$$c'\left(\frac{(A+1-\alpha)\alpha^2}{A(1-\alpha)(\alpha+i)}\right)\left[1 + i + \frac{1-\alpha}{A}\right] \leq 1 + i/\alpha,$$

which can be solved for i. Note that the inequality above implies that the larger is the employment in the banking sector ($(1-\alpha)n_B$), the greater is the spread between borrowers and depositors rates and therefore the less likely it is that the banking sector is desirable. In other words, low inflation rates are not consistent with large banking sectors. Note also that if $c'' = 0$ then banking is always optimal, since we showed before that $1 + \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha} < 1 + i/\alpha$. The Euler equation under banking becomes:

$$1 + i + \frac{1-\alpha}{A} - \frac{1}{q} = 0$$

and therefore:

$$\frac{dq}{dA} > 0$$

This implies the more efficient is the bank, the more the agent consumes.

If instead $c'' > 0$, then generally we need an interest rate $i > i_L$ for banking to be optimal. To show it suppose $\mu = \mu_L$ so $i = i_L$ and $1 + \frac{i}{\alpha} - \frac{r^*(1-\alpha)}{\alpha} = 1 + i/\alpha$. Hence, banks improve upon the equilibrium only if $c' < 1$. This implies a value i sufficiently large when $c'' > 0$.

4.2 Interest on reserves

In this section we study how allocations change when the central bank pays interests on banks' reserves. Let η denote the interest on reserves paid by the central bank, which adjusts T to maintain the inflation rate at a desired level. In line with BCW, ours is a model of narrow "banking" in that banks cannot create more loans than the deposits they have, so the required reserve ratio is 100 percent of deposits. Hence, the bank's profit condition changes as follows

$$\begin{aligned} \max_{L,D,\ell_B} \quad & Lr_H - D(r-\eta) - p\ell_B \\ \text{s.t.} \quad & L = pF(\ell_B) \text{ and } L \leq D. \end{aligned}$$

The second constraint holds with equality if $r_H > r - \eta$, so we can think of interests on reserves as a subsidy on deposits payments. If the second constraint holds with equality and we substitute both constraints in the production function, the bank's maximization problem becomes:

$$\max_{\ell_B} \; F(\ell_B)(r_H - r + \eta) - p\ell_B$$

The FOC for the problem above is:

$$F'(\ell_B)(r_H - r + \eta) = 1$$

Therefore:

$$r_H - r = \frac{1}{F'(\ell_B)} - \eta \tag{17}$$

Note that paying interest on reserves reduces the spread.

Result 4. *Paying interest on reserves is optimal away from the Friedman rule. In equilibrium* $0 \leq r^{**} = r < i < r_H$ *with*

$$r^{**} := \min\left(i - \frac{\alpha}{F'(\ell_B)} + \alpha\eta, 0\right), \quad \textit{and} \quad r_H = \frac{i}{\alpha} - \frac{r^{**}(1-\alpha)}{\alpha}.$$

Hence, $r^{**} \geq r^*$ *and banking generally improves welfare.*

To derive this result, note that the basic Euler equation is still

$$\frac{\mu}{\beta} = 1 + r + \alpha(r_H - r) \qquad \Rightarrow \qquad i - r = \alpha(r_H - r),$$

so substituting $(r_H - r)$ from equation (17), which identifies the spread when the interest rate η is paid on reserves, we obtain the deposit rate

$$r = i - \frac{\alpha}{F'(\ell_B)} + \alpha\eta.$$

The deposit rate must be non-negative, in equilibrium. Hence we have r^{**}. From (13) with $r = r^{**}$ we obtain

$$r_H = \frac{i}{\alpha} - \frac{r^{**}(1-\alpha)}{\alpha}.$$

The Euler equation remains

$$u'(q) = c'(N)\left[1 + \frac{i}{\alpha} - r^{**}\frac{1-\alpha}{\alpha}\right].$$

Since generally $r^{**} \geq r^*$ then q is higher when the central bank pays interests on reserves.

5 Quantitative analysis

To calibrate the relevant parameters, we focus on a yearly model of the United States for the sample period 1965-2010. All data except for money supply are from the St. Louis Fed *FRED Database*. Interest rates are annualized. The nominal interest rate i is the yield on 3-month treasury bills, the nominal price level P is CPI for all items, aggregate nominal output PY is nominal GDP, the deposit rate r is the yield on 3-month certificates of deposits and the lending rate r_H is the bank prime loan rate.[3] The nominal money supply M is sweep-adjusted M1.[4]

To facilitates comparison with BCW and other studies based on Lagos and Wright (2005), we adopt the following functional forms: $u(q) = ((q+b)^{1-\delta} - b^{1-\delta})/(1-\delta)$ with $\delta = 0.99999$ and $b = 0.00001$ so we have approximately unit-elastic preferences in both markets; $U(x) = Bln(X)$ so that $X^* = B$; and $c(N) = N^\psi/\psi$ with $\psi = 1.1$.[5] For the

[3]Specifically, we use the following *FRED* series: TB3MS for 3-month t bills; CPALTT01USQ661S for CPI; GDP for nominal GDP; IR3TCD01USQ156N for CD rates; MPRIME for the prime rate.

[4]We use the series M1S from Cynamon, Dutkowsky and Jones, "Sweep-Adjusted Monetary Aggregates for the United States." Online document available at http://www.sweepmeasures.com, December 27, 2007. The methods used to create the sweep-adjusted data are described in Cynamon et al. (2006).

[5]Studies based on Lagos and Wright (2005) usually assume linear disutility. Setting $\psi = 1$ would have

bank's problem, we consider a constant return to scale production function $F(\ell_B) = A\ell_B$ (we discuss a robustness check for a DRS production function, below). We set $\beta = 0.96$, and for the remaining parameters α, A and B, we simultaneously match the theoretical expression with the empirical counterpart of money velocity, interest elasticity of money demand and interest rate spread. Derivations are in the Appendix.

The calibration yields the following results: $A = 53.0565$, $\alpha = 0.4579$ and $B = 1.8722$. In what follows, we use these parameter values to quantify the welfare implications of costly banking as well as for the welfare cost of inflation. With these calibration parameters the share of labor employed in the banking sector, relative to total labor, is constant at about 1% for empirically reasonable inflation levels, between 0 and 10 percent.

5.1 Do banks improve social welfare?

The payoff function V can be written as

$$V(m) = \alpha\left[u(q) + W(m - pq + l, l)\right] + (1-\alpha)\left[-c\left(N\right) + W(m - d + pn, d)\right].$$

Fixing μ, let q_μ and N_μ denote equilibrium quantities; use (1), (6), $T = m' - m$ and the expression for ζ to define ex-ante equilibrium welfare by V_μ, where

$$(1-\beta)V_\mu = \alpha u(q_\mu) - (1-\alpha)c(N_\mu) + U(X^*) - X^*. \tag{18}$$

Inflation μ affects ex-ante welfare because it distorts market one consumption and labor, and thus the expected trade surplus $\alpha u(q) - (1-\alpha)c(N)$.

To discuss the effect of banking on social welfare, when banking activities are costly and not, we offer Figures 1-2. The graphs plot the difference in ex-ante welfare with and without active banks as a function of the inflation rate $\mu - 1$ in an economy. The solid line reports data for the model with costly financial intermediation. The dashed line reports data for the model without costly financial intermediation, i.e., in the costless banking case of BCW. Figure 1 also reports the equilibrium deposit rate (right vertical

effectively no impact on the calibration results but it would prevent the study of cases where Result 3 does not hold, i.e., where banks do not necessarily improve welfare.

axis). There are two important observations regarding banks, when their activity absorbs labor resources.

Observation 1: *If the inflation rate is sufficiently small, then banks reduce welfare.*

Consider the solid line in Figure 1. If deflations are significant—μ is slightly above the Friedman rule—then banks offer zero deposit rates. In this scenario, agents simply hold cash instead of depositing money in the bank. Here, welfare with or without banks is identical so the solid line coincides with the x axis. As inflation increases banks become active because they offer a deposit rate that is small, but positive. This generates a drop in welfare because banks do provide small benefits to depositors—which is the main reason for the welfare-enhancing effects of banking in this class of models—but *also* soak labor resources away from productive activities. Overall, this harms macroeconomic efficiency. As inflation rates move into positive territory—slightly above 2% for the calibrated model—banks improve welfare compared to a pure-currency economy.

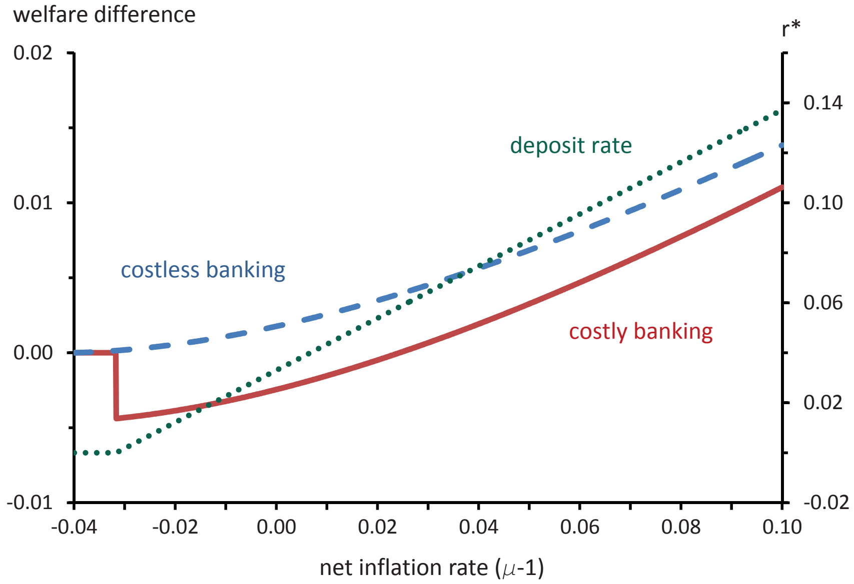

Figure 1: How banks affect welfare at low inflation

Notes for Figures 1-2: The figures report the difference in ex-ante welfare with and without banks, against inflation $\mu - 1$. The dashed line is the model in BCW, where banking is costless. The dotted line in Figure 1 is the equilibrium deposit rate (scale on right). The figures are drawn under the assumption that agents do not make deposits if $r = 0$, and for the calibrated values $A = 53.0565, \alpha = 0.4579, B = 1.8722$.

By means of comparison, Figure 1 also reports welfare differences in the calibrated model where banking activity uses no resources at all (as in BCW, that is). In that case deposit rates are *always* strictly positive (not reported in the figure) so banking activities can only raise welfare, compared to an economy where banks are inactive. The dashed line is monotonically increasing.

The lesson is that incorporating the realistic feature that intermediation is a costly activity reverses the result that banks are welfare improving even in deflations.

Observation 2: *If the inflation rate is sufficiently high, then banks increase welfare.*

Consider the solid line in Figure 2. As we move away from low inflation scenarios, the welfare gain associated with banking activity grows as depositing money in the bank is a way to partially deflect the inflation tax. But there is more. As we pass a high inflation threshold—that in the calibrated economy corresponds to 25%—banks are even more beneficial as compared to the model where banking is costless (dashed line). Since banks employ workers, this extra benefit of banking is due to general-equilibrium effects on wages and therefore prices of consumption goods. The magnitude of this effect depends on the banks' efficiency parameter.

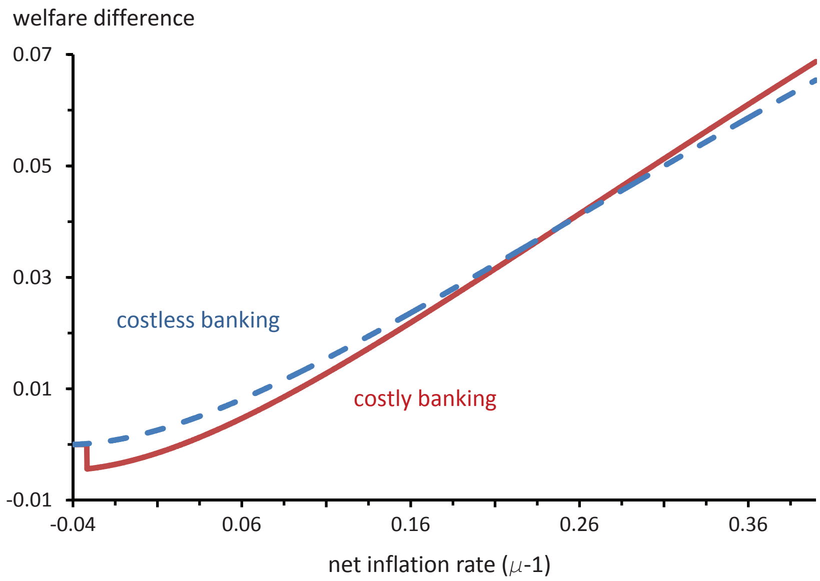

Figure 2: How banks affect welfare at high inflation

5.2 The welfare cost of inflation

We use a standard measure for the welfare cost of inflation for a representative agent, which is the compensating variation approach proposed in Lucas (2000). The welfare cost of inflation is the percentage adjustment in consumption (in both markets) that leaves the representative agent indifferent between (fully anticipated) inflation $\mu > \beta$ and a lower rate $z \geq \beta$. We use (18) to define ex-ante adjusted welfare $\bar{V}_z$ as

$$(1-\beta)\bar{V}_z = \alpha u(\bar{\Delta}_z q_z) - (1-\alpha)c(N_z) + U(\bar{\Delta}_z X^*) - X^*. \tag{19}$$

The welfare cost of μ instead of z inflation is the value $\Delta_z = 1 - \bar{\Delta}_z$ that satisfies $V_\mu = \bar{V}_z$.

To discuss the effect of banking on the welfare cost of inflation, when banking activities are costly and not, we offer Table 1 and Figures 3-4.

Observation 3: *Accounting for banking costs raises the welfare cost of small inflation significantly.*

Table 1 reports the welfare cost of inflation as opposed to the Friedman rule, comparing it with the results for (i) costless banking as in BCW and (ii) no banking as in Lagos and Wright (2005) using the same calibrated parameters.

Model	1%	5%	10%	20%
No banking	0.15	0.41	0.82	1.85
Costless banking BCW	0.04	0.11	0.23	0.56
Costly banking	0.22	0.27	0.35	0.60

Table 1: Welfare costs of inflation relative to the Friedman rule

Notes: Welfare cost is in percentage of consumption. Costly banking refers to the economy with costly financial intermediation, costless banking is identical to the model in BCW, while no banking refers to the economy in Lagos and Wright (2005). It is assumed that no deposits are made unless the deposit rate $r > 0$.

Figures 3-4 plot the welfare cost of inflation for a wider inflation range.

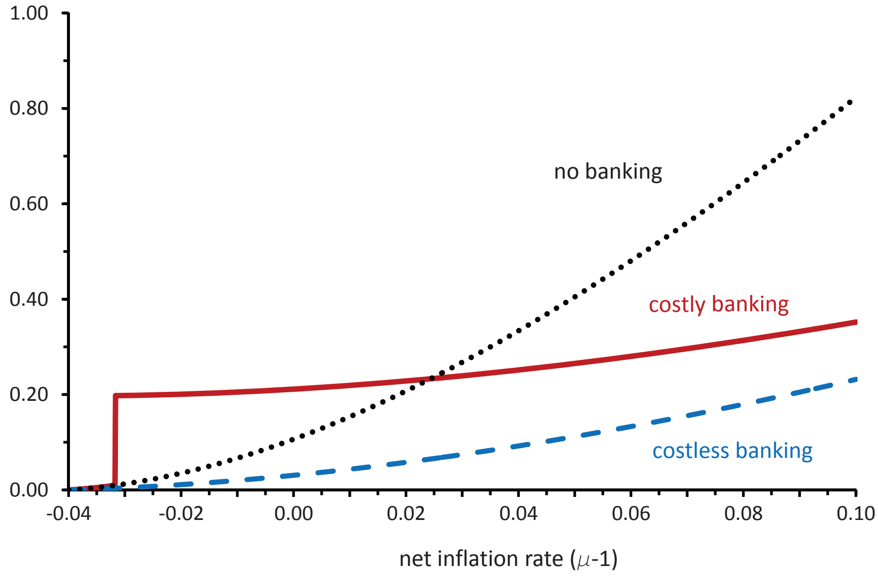

Figure 3: Welfare cost of inflation, for inflation up to 10%

Notes for figures 4-3: The figures report the percentage welfare cost of inflation, relative to the Friedman rule, against inflation $\mu - 1$. The dashed line is the model in BCW, where banking is costless. The figures are drawn under the assumption that agents make no deposits if $r = 0$, for the calibrated values $A = 53.0565$, $\alpha = 0.4579$ and $B = 1.8722$.

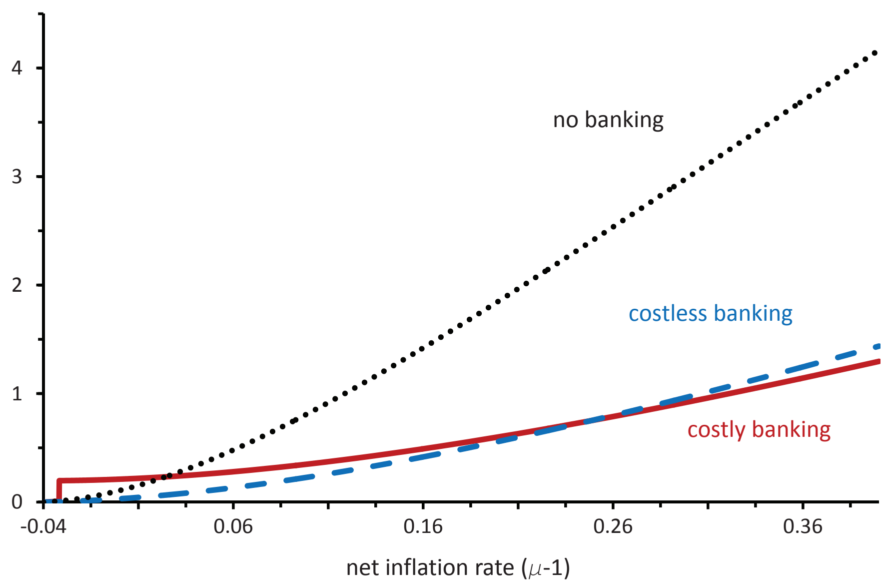

Figure 4: Welfare cost of high inflation

Table 1 and Figures 3-4 demonstrate that accounting for banking costs has significant quantitative ramifications for the welfare cost of inflation. Although this cost remains in the order of a fraction of one percent, as in most models of representative agents, the welfare costs of 1% inflation increases five-fold compared to the model where banking is costless, and more than doubles at 5% inflation. That is because in the model where banks are costless, infation distorts *only* saving decisions—hence consumption decisions. Instead, when the model accounts for banking costs, inflation also affects the return that depositors receive from their funds—lower than in the model where banks are costless. These two negative effects combine to push up the inefficiency associated with inflation, compared to the original, costless banking model.

This effect is reversed at high inflation rates, which is when the welfare costs of inflation are similar in the two models (costly and costless banking) because real savings are low in both models and the return to depositors is similar.

5.3 Extensions

In the previous analysis we have assumed that no deposits are made unless depositors are compensated. However, depositors are in fact indifferent between holding on to their money or leaving it in the bank. So, they might as well deposit it all into banks, even if $r = 0$ (say, bank deposits are safer than cash).

Under this scenario the model does not change very much. We still have

$$u'(q) = c'(N)\left(1 + \frac{i}{\alpha}\right),$$

although one has to be careful in comparing it with the no-banks expression because—though this expression resembles the one derived for the model without banks—the allocations are not identical in the two cases. This is due to a market clearing condition. The labor supply is $N > \frac{\alpha q}{1-\alpha}$ when banks are active, while it would have an $=$ sign with inactive banks. The main implication is that now banks induce general equilibrium effects even if deposit rates are zero.

To understand why this happens, take two separate steps. Consider first the economy where banks have no operating costs, as in BCW. There, banks influence allocations *only* by providing benefits to depositors, i.e., offering a return on their idle balances. In that economy, allocations with or without banks are identical when deposit rates are zero—which occurs only at the Friedman rule. Now, let us account for banking costs. Here, banks' activity soaks up labor resources and there is an additional effect—through labor markets—even if depositors receive no compensation for their deposits. All other expressions remain the same as the ones developed earlier.

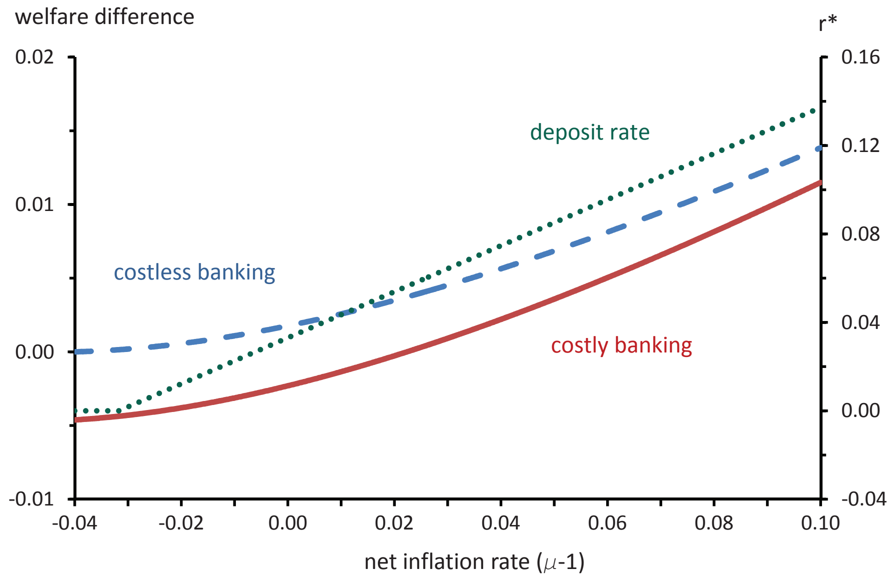

Figure 5: How banks affect welfare when agents always make deposits

Notes: The figure reports the difference in ex-ante welfare with and without banks, against inflation $\mu - 1$. The dashed line is the model in BCW, where banking is costless. The dotted line is the equilibrium deposit rate (scale on right). The figure is drawn under the assumption that agents make deposits even if $r = 0$, and for the calibrated values $A = 53.0565, \alpha = 0.4579, B = 1.8722$.

The impact of this variation in assumptions is seen at those deflation/inflation levels where $r = 0$. Banking now will induce a welfare loss all the way to the Friedman rule because banks hire workers away from production but do not compensate depositors. This is seen in Figure 5. It should be also clear that in this case there is no effect on the welfare cost of inflation, at positive inflation levels, since zero deposit rates in the calibrated model only occur for deflations close to the Friedman rule.

Finally, we note that all of the previous results go through if we consider a decreasing returns to scale technology for banks. In a separate calibration where $F(\ell_B) = Aln(1+\ell_B)$ we obtain $A = 53.4288$, $\alpha = 0.4580$ and $B = 1.8723$ and figures qualitatively identical to Figures 1-5; the welfare costs of inflation are similar to the ones obtained with CRS technology. A table comparing the welfare cost of inflation values for the CRS and the DRS case is in the Appendix.

6 Final comments

Over the last ten years, U.S. households have experienced very low inflation as well as zero yields on demand deposits. We have extended the study of banking equilibrium in Berentsen, Camera and Waller (2007) to show why yields on deposits may hit a zero lower bound as inflation shrinks to zero, and what are the welfare consequences of such policy. To do so, we have introduced an explicit production function for financial intermediation. Banks employ labor resources to run their operations, which they hire on a competitive market where they compete with firms. In equilibrium this gives rise to a spread between interest rates on loans and on deposits, which naturally reflects both the efficiency of financial intermediation and the underlying monetary policy. The main result is that a sufficiently tight monetary policy will induce banks to push down to zero the yield on demand deposits, thus removing the primary (and only) beneficial effect of banking in the BCW model. Hence, banks do not necessarily raise welfare under all possible monetary policies. However, when we calibrate the model to U.S. data, we find that these welfare costs are small in terms of how much households would give up to avoid very low inflation (and the need to resort to banks) in favor of the Friedman Rule where banks have no role to play. A natural question is whether or not this result would still hold if banks had an explicit role in intermediating also capital, for example as it happens in Bencivenga and Camera (2011). We leave this question for future research.

References

Antunes, A., Cavalcanti, T., and A. Villamil (2013). Costly Intermediation and Consumption Smoothing. *Economic Inquiry* 51 (1), 459-472.

Bencivenga, V., and G. Camera (2011). Banking in a Matching Model of Money and Capital. *Journal of Money, Credit, and Banking* 43 (7), 449-476.

Berentsen, A., Camera, G., and C. Waller (2007). Money, Credit and Banking. *Journal of Economic Theory* 135 (1), 171-195.

Chiu, J., and C.A. Meh (2011). Financial Intermediation, Liquidity, and Inflation. *Macroeconomic Dynamics* 15 (1), 83-118.

Cynamon, B.Z., Dutkowsky, D.H., and B.E. Jones (2006). Redefining the Monetary Aggregates: A Clean Sweep. *Eastern Economic Journal* 32 (4), 661-673.

Goldfeld, S., and D.E. Sichel (1990). The Demand for Money. In: Friedman, B.M., Hahn, F.H. (Eds.), Handbook of Monetary Economics. Elsevier, Amsterdam, 299-356.

Lagos, R., and R. Wright (2005). A Unified Framework for Monetary Theory and Policy Analysis. *Journal of Political Economy* 113 (3), 463-484.

Lucas, R., (2000). Inflation and Welfare. *Econometrica* 68 (2), 247-274.

Schmiedel, H., G. Kostova and W. Ruttenberg. 2012. The Social and Private Costs of Retail Payment Instruments—A European Perspective. European Central Bank Occasional Paper Series # 137.

Appendix

7 Derivations for CRS technology

Here we consider $F(\ell_B) = A\ell_B$. The assumption of constant returns to scale is convenient for two reasons. First, it matches the assumption on the good's production technology, which is also CRS. Second, it is consistent with the notion that we work with a representative bank in competitive markets, since the bank earns zero profits in equilibrium.

From (12) and $F(\ell_B) = A\ell_B$, we can find an expression for n_B in terms of n, which is:

$$n_B = (1-\alpha)n/A \tag{20}$$

Moreover, the Euler equation in (16) for $r > 0$ can be written as:

$$u'(q) = c'(N)\left[1 + i + \frac{1-\alpha}{F'(\ell_B)}\right]$$

Using the functional forms we chose for $u(q)$, $c(N)$ and $F(\ell_B)$, the Euler equation becomes:

$$(q+b)^{-\delta} = (n+n_B)^{\psi-1}\left[1 + i + \frac{1-\alpha}{A}\right]$$

This, given (20) and $\alpha q = (1-\alpha)n$ from goods market clearing, gives us an expression for n only as a function of the parameters:

$$\left[\frac{(1-\alpha)n}{\alpha} + b\right]^{-\delta} = \left[\frac{(A+1-\alpha)n}{A}\right]^{\psi-1}\left[1 + i + \frac{1-\alpha}{A}\right] \tag{21}$$

Now we need to pin down α, A and B by matching interest elasticity of money demand, money velocity and the interest rate spread with their empirical counterpart. The general formula for interest elasticity of money demand is:

$$\varepsilon_m = \frac{\partial(m\phi)}{\partial i} \times \frac{i}{m\phi}$$

We know $m = \alpha pq$ which, together with (3), implies $m\phi = \alpha c'(N)q$. Differentiating we get:

$$\varepsilon_m = \alpha c'(N)\frac{dq}{di} \times \frac{i}{\alpha q} = c'(N)\frac{dq}{di} \times \frac{i}{q}$$

From (2) and (7) the Euler equation can be written as $i = \alpha\left[u'(q)/c'(N) - 1\right] + (1-\alpha)r$. Using the implicit function theorem, we have:

$$\frac{dq}{di} = -\frac{-1}{\alpha\dfrac{u''(q)}{c'(N)}} = \frac{c'(N)}{\alpha u''(q)}$$

Therefore, the expression for the elasticity of money demand can be written as:

$$\varepsilon_m = \frac{\partial(m\phi)}{\partial i} \times \frac{i}{m\phi} = \frac{ic'(N)}{\alpha q u''(q)} \tag{22}$$

Given the functional forms for $u(q)$ and $c(N)$, the expression for ε_m becomes:

$$\varepsilon_m = -\frac{in^{\psi-2}\left(\dfrac{A+1-\alpha}{A}\right)^{\psi-1}}{\delta(1-\alpha)\left[(1-\alpha)n/\alpha+b\right]^{-(\delta+1)}} \tag{23}$$

where n is defined in (21). The empirical counterpart is estimated using the approach outlined in Goldfeld and Sichel (1990)[6] for which we obtain -0.1123.

Money velocity is $v = \frac{PY}{M}$ where PY is nominal GDP and M denotes nominal money holdings. In the model, nominal GDP has three components: market 1 goods output $p\alpha q$, market 1 banking output $pF(\ell_B)$ and market 2 nominal output p_2X. Therefore, the theoretical expression for money velocity in the model can be derived as follows:

$$v \;=\; \frac{PY}{M} = \frac{p\phi\alpha q + p\phi F(\ell_B) + X}{m\phi}$$

We know the following is true in equilibrium: $U'(X) = 1/B$ from (1), which implies $X = B$ given the functional form $U(X) = ln(X)$; $p\phi = c'(n+n_B)$ from (3); $\alpha q = (1-\alpha)n$ from (6); $m = \alpha pq$ from (10); $n_B = (1-\alpha)n/A$ from (20). Hence, velocity in equilibrium becomes:

$$v = 2 - \alpha + \frac{B}{n^{\psi}(1-\alpha)\left[\dfrac{A+1-\alpha}{A}\right]^{\psi-1}} \tag{24}$$

where n is given by (21). For the empirical counterpart of v, we find $v = 5.8956$.

Last, the parameter A is chosen to fit the interest rate spread which, given (8), is:

$$r_H - r = 1/A \tag{25}$$

The empirical counterpart for $r_H - r$ is equal to 1.8848%, which pins down a value $A = 53.0565$. Given the the values we set for β, δ, b and ψ we find $\alpha = 0.4579$ and $B = 1.8722$ by matching (23) and (24) with their empirical counterpart. Given the calibrated values of the parameters and the expression for (21), we can find all the endogenous variables:

(i) $n_B = (1-\alpha)n/A$ from (20);

(ii) $N = n + n_B \Rightarrow N = (A+1-\alpha)n/A$;

(iii) $\ell^B = (1-\alpha)n_B$ from labor market clearing;

(iv) $q = (1-\alpha)n/\alpha$ from (6);

(v) $X = B$;

(vi) $L = (1-\alpha)l$;

(vii) $D = \alpha d$;

(viii) $p\phi = c'(N) \Rightarrow p\phi = N^{\psi-1}$;

(ix) $\zeta = p[F(\ell_B)/F'(\ell_B) - \ell_B]$.

[6] The log of real money balances on each date t (M_t/P_t) is regressed on the date t log of real GDP, nominal interest rates, and one-period lagged real money balances: $\ln m_t = \gamma_0 + \gamma_1 \ln y_t + \gamma_2 \ln i_t + \gamma_3 \ln m_{t-1} + v_t$. To account for first-order autocorrelation in the residuals v_t the Cochrane-Orcutt procedure is used.

8 Robustness check: DRS technology

Here we calculate the welfare cost of inflation when the technology has decreasing returns to scale. We consider $F(\ell_B) = A\ln(1+\ell_B)$. This, together with (12), allows us to find an expression for n_B in terms of n, which is:

$$n_B = \frac{e^{(1-\alpha)^2 n/A} - 1}{1-\alpha} \tag{26}$$

Using the functional forms we chose for $u(q)$, $c(N)$ and $F(\ell_B)$, the Euler equation in (16) for $r > 0$ becomes:

$$\left[\frac{(1-\alpha)n}{\alpha} + b\right]^{-\delta} = (n+n_B)^{\psi-1}\left[1 + i + \frac{(1-\alpha)(1+(1-\alpha)n_B)}{A}\right] \tag{27}$$

Given (26) and $\alpha q = (1-\alpha)n$ from goods market clearing, (27) implicitly defines n as a function of only the parameters.
With DRS returns to scale the expression for elasticity in (22) becomes:

$$\varepsilon_m = -\frac{i\,[n+n_B]^{\psi-1}}{\delta(1-\alpha)n\,[(1-\alpha)n/\alpha + b]^{-(\delta+1)}} \tag{28}$$

where n and n_B are defined in (27) and (26) respectively. As before, the empirical counterpart is -0.1793.
Given the functional form for $F(\ell_B)$, the expression for velocity in equilibrium becomes:

$$v = 1 + \frac{Aln\,[1+(1-\alpha)n_B]}{(1-\alpha)n} + \frac{B}{(n+n_B)^{\psi-1}\,(1-\alpha)n} \tag{29}$$

where n_B and n are given by (26) and (27) respectively.

Last, given (8) the interest-rate-spread is:

$$r_H - r = \frac{1+(1-\alpha)n_B}{A} \tag{30}$$

where n_B is defined in (26). Given the the values we set for β, δ and b, ψ, once we match (28), (29) and (30) with their empirical counterparts we find $\alpha = 0.4580$, $B = 1.8723$ and $A = 53.4288$. Given the calibrated values of the parameters and the expression for (27), we can find all the endogenous variables of the model. Specifically:

(i) $n_B = (e^{(1-\alpha)^2 n/A} - 1)/(1-\alpha)$ from (26);

(ii) $N = n + n_B \Rightarrow N = n + (e^{(1-\alpha)^2 n/A} - 1)/(1-\alpha)$;

(iii) $\ell_B = (1-\alpha)n_B$ from labor market clearing;

(iv) $q = (1-\alpha)n/\alpha$ from (6);

(v) $X = B$;

(vi) $L = (1-\alpha)l$;

(vii) $D = \alpha d$;

(viii) $p\phi = c'(N) \Rightarrow p\phi = N^{\psi-1}$.

Table 2 reports the welfare cost of several inflation rates as opposed to the Friedman rule for a bank with a DRS production function. We use $F(\ell_B) = Aln(1 + \ell_B)$ and the calibrated values $A = 53.4288$, $\alpha = 0.4580$ and $B = 1.8723$.

Inflation:	1%	5%	10%	20%
Welfare Cost:	0.22	0.26	0.35	0.60

Table 2: Welfare costs of inflation relative to the Friedman rule (DRS technology)

Earlier Working Papers:

For a complete list of Working Papers published by Sveriges Riksbank, see www.riksbank.se

Estimation of an Adaptive Stock Market Model with Heterogeneous Agents *by Henrik Amilon*	2005:177
Some Further Evidence on Interest-Rate Smoothing: The Role of Measurement Errors in the Output Gap *by Mikael Apel and Per Jansson*	2005:178
Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through by Malin Adolfson, Stefan Laséen, Jesper Lindé and Mattias Villani	2005:179
Are Constant Interest Rate Forecasts Modest Interventions? Evidence from an Estimated Open Economy DSGE Model of the Euro Area *by Malin Adolfson, Stefan Laséen, Jesper Lindé and Mattias Villani*	2005:180
Inference in Vector Autoregressive Models with an Informative Prior on the Steady State *by Mattias Villani*	2005:181
Bank Mergers, Competition and Liquidity *by Elena Carletti, Philipp Hartmann and Giancarlo Spagnolo*	2005:182
Testing Near-Rationality using Detailed Survey Data *by Michael F. Bryan and Stefan Palmqvist*	2005:183
Exploring Interactions between Real Activity and the Financial Stance *by Tor Jacobson, Jesper Lindé and Kasper Roszbach*	2005:184
Two-Sided Network Effects, Bank Interchange Fees, and the Allocation of Fixed Costs *by Mats A. Bergman*	2005:185
Trade Deficits in the Baltic States: How Long Will the Party Last? *by Rudolfs Bems and Kristian Jönsson*	2005:186
Real Exchange Rate and Consumption Fluctuations follwing Trade Liberalization *by Kristian Jönsson*	2005:187
Modern Forecasting Models in Action: Improving Macroeconomic Analyses at Central Banks *by Malin Adolfson, Michael K. Andersson, Jesper Lindé, Mattias Villani and Anders Vredin*	2005:188
Bayesian Inference of General Linear Restrictions on the Cointegration Space *by Mattias Villani*	2005:189
Forecasting Performance of an Open Economy Dynamic Stochastic General Equilibrium Model *by Malin Adolfson, Stefan Laséen, Jesper Lindé and Mattias Villani*	2005:190
Forecast Combination and Model Averaging using Predictive Measures *by Jana Eklund and Sune Karlsson*	2005:191
Swedish Intervention and the Krona Float, 1993-2002 *by Owen F. Humpage and Javiera Ragnartz*	2006:192
A Simultaneous Model of the Swedish Krona, the US Dollar and the Euro *by Hans Lindblad and Peter Sellin*	2006:193
Testing Theories of Job Creation: Does Supply Create Its Own Demand? *by Mikael Carlsson, Stefan Eriksson and Nils Gottfries*	2006:194
Down or Out: Assessing The Welfare Costs of Household Investment Mistakes *by Laurent E. Calvet, John Y. Campbell and Paolo Sodini*	2006:195
Efficient Bayesian Inference for Multiple Change-Point and Mixture Innovation Models *by Paolo Giordani and Robert Kohn*	2006:196
Derivation and Estimation of a New Keynesian Phillips Curve in a Small Open Economy *by Karolina Holmberg*	2006:197
Technology Shocks and the Labour-Input Response: Evidence from Firm-Level Data *by Mikael Carlsson and Jon Smedsaas*	2006:198
Monetary Policy and Staggered Wage Bargaining when Prices are Sticky *by Mikael Carlsson and Andreas Westermark*	2006:199
The Swedish External Position and the Krona *by Philip R. Lane*	2006:200

Sveriges Riksbank
Visiting address: Brunkebergs torg 11
Mail address: se-103 37 Stockholm

Website: www.riksbank.se
Telephone: +46 8 787 00 00, Fax: +46 8 21 05 31
E-mail: registratorn@riksbank.se