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MAJA: A two-region DSGE model for Sweden and its main trading partners

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Abstract

The Swedish economy is strongly dependent on global economic developments, which is reflected in generally strong empirical relationships between Swedish and foreign macroeconomic variables. It is, however, difficult for standard open-economy dynamic stochastic general equilibrium (DSGE) models to generate substantial cross-country spillovers; see e.g. the seminal paper of Justiniano and Preston (2010).

We present a two-region DSGE model that better captures the dependence on global economic developments than previous models. It is estimated on data for Sweden and an aggregate of its main trading partners, the euro area and the United States, for the period 1995Q2–2018Q4. To capture the strong empirical relationships between Sweden and the foreign economy, we assume that global shocks to e.g. technology, real interest rates, financial risk, and firm and consumer sentiment directly affect both economies, while their impact on each economy may differ. We also allow for a flexible specification of the demand for Swedish exports to better account for the fluctuations in Swedish trade. Finally, headline and core inflation are distinguished by the introduction of consumption of energy goods, which allows for a more detailed and realistic analysis of inflation developments.

This new model, named MAJA (*Modell för Allmän JämviktsAnalys*), is used by the Riksbank for interpretation of economic developments, forecasting, scenarios, and policy analysis. It builds on the work of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003) and the Riksbank's previous models, Ramses I and Ramses II (see Adolfson et al. (2007) and Adolfson et al. (2013), respectively).

Keywords: DSGE model, Monetary Policy, Open economy, International spillovers, Bayesian estimation.
JEL classification: E30, E40, E37, E52, F41, F44, C11, C32, C52.

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Contents

1 Introduction

Sweden is a small economy with open goods and capital markets which are strongly influenced by global economic developments. This is reflected in the generally strong comovement between Swedish and foreign-economy macroeconomic variables. In this paper we develop an open-economy dynamic stochastic general equilibrium (DSGE) model, which can adequately capture those strong international dependencies.

The strong influence of global economic developments on Sweden is particularly evident for real variables such as GDP and its components, and various labour market indicators. Furthermore, the Swedish and foreign business cycles are well synchronised, that is, the effects of foreign developments on the Swedish economy are largely instantaneous.[1] Strong empirical relationships are also found among financial variables such as stock indices, interest rates and risk indicators, such as spreads. Nominal and real interest rates have declined globally in the past 20–30 years and the policy rate of the Riksbank, the repo rate, is comoving with the policy rates in other advanced economies.[2] While the common trend decline presumably 'exaggerates' these correlations, it is arguably the case that global factors have been the main driver behind the decline in Swedish interest rates.

Standard economic theory would suggest that the strong comovement of foreign and Swedish (real) business cycles and financial markets should also manifest itself in highly synchronised inflation movements. Previous research has also documented that global inflation fluctuations account for a large fraction of the variation in national inflation rates; see e.g. Ciccarelli and Mojon (2010). The relationship between Swedish and foreign inflation, however, is both weaker and apparently more complex than the relationships among real economic variables. While Swedish and foreign inflation are modestly positively correlated, the correlation appears to be largely driven by high frequency movements, mainly volatile global energy price changes, and comovement at low frequencies, while there is little evidence of comovement at the business cycle frequency.

These empirical regularities suggest that an understanding of Swedish economic developments generally needs to start out from an analysis of developments abroad and of the channels through which the global/foreign economic developments affect the Swedish economy. Swedish trade with other countries is significant, with a trade-to-GDP ratio of around 90%. However, the strong and apparently instantaneous relationships between many variables suggests that the trade channel alone may not be enough to account for the influence of foreign developments. There is reason to believe that other channels, e.g. working through confidence and financial markets, are also important.[3] In a macroeconomic model for Sweden, it is paramount that global economic movements are accounted for and that international spillovers are substantial.

At the same time, standard open-economy models tend to generate little international comovement. This was pointed out by Justiniano and Preston (2010), who show that standard New-Keynesian models assign most of the business cycle fluctuations to domestic shocks.[4] Their results have been

[1] Over the sample 1995-2018, Swedish GDP displayed a contemporaneous correlation of 0.9 with euro area GDP as well as with the weighted average of Sweden's 32 main trading partners, measured in annual percentage changes. The corresponding number for the correlation between Canadian and US GDP, to name one example of other highly correlated countries, is 0.8. We note that cross-country correlations of real variables generally increase with lower frequency, so that short-run movements are to a larger extent driven by domestic factors and medium to long-run movements by global ones.

[2] The OMXS stock index has a correlation of 0.7 with a global portfolio in the period 2001-2019. Swedish and German government bond yields are strongly correlated at both short and long maturities, e.g. for both 5- and 10-year bonds the correlation is 0.99. The correlations with US 5- and 10-year bond yields are around 0.9.

[3] Alpanda and Aysun (2014), among others, highlight the importance of international financial linkages in DSGE models. Consumer confidence as measured by the Economic Sentiment Indicator in Sweden and in the euro area has a correlation of 0.8 over the sample 1995–2018.

[4] In their study, the baseline structural model of the Canadian economy, estimated on Canadian and US data, implies that less than 3 percent of the variations in nominal and real Canadian variables are driven by US shocks, at any forecast horizon. Thus, the model-implied correlations between Canadian and US variables are essentially zero. Allowing shocks to be correlated across countries increases the influence of foreign factors, but even in this case the variance shares explained by US shocks are substantially below the shares observed in reduced-form models.

confirmed by other studies, for example Alpanda and Aysun (2014) and Jacob and Peersman (2013), and are also evident in earlier models of the Swedish economy such as Adolfson et al. (2008). It is not straightforward to pinpoint why cross-country spillovers and comovement are so limited in these models. It has been argued that it relates to the fact that foreign shocks have counterfactual implications for some domestic variables, which leads data to favour parameter values that virtually shut off international linkages, but the exact source of misspecification is not clear. Different improvements to the standard model have been proposed, including the multi-sector model by Bergholt (2015), but they are not easily incorporated into a fully-fledged quarterly-data model of the kind used for policy analysis at central banks. There are thus no obvious solutions to how to adequately capture the international dependencies of economies such as Sweden.

It is not only in models that the large influence of global factors is not entirely accounted for. Lindé and Reslow (2017) show that also official judgmental forecasts tend to underestimate the impact of economic developments abroad.[5] Judgmental forecasts are often to some extent influenced by macroeconomic models, and models are useful tools for disciplining discussions about economic developments. Having models that better capture the international spillovers observed in the data is thus important for improving forecasts in open economies.

The main strength of structural models relative to other tools, however, lies not in forecasting but in their storytelling abilities.[6] Structural models, in particular if they are estimated, can be used for decompositions, conditioning exercises and counterfactual scenarios. However, if international spillovers are not captured adequately in the model, the model interpretations of economic events may be misguided.[7] An apparent example of this is the global financial crisis in 2008–2009. For a country like Sweden, it is obvious that the source of the crisis was not domestic but that the crisis originated abroad, even if domestic factors may have contributed to the severity and transmission of these foreign or global shocks. A standard open-economy model would, however, tell a story quite similar to that of a closed economy model, namely that the movements in Swedish GDP in 2008–2009 were largely driven by domestic shocks. Relatedly, a model forecast or scenario for Swedish variables which takes the evolution of the foreign economy as given (i.e. a conditional forecast or a scenario) cannot be trusted if the spillovers in the model are much smaller than those observed in data. There is clearly a need for improvement of policy models along this dimension, and the primary goal of the analysis undertaken in this paper is to do precisely that in a macroeconomic model for Sweden.

This paper describes the new DSGE model MAJA, developed at Sveriges Riksbank.[8] It is a two-country model, where the large foreign economy represents the rest of the world. Both model economies consist of firms which produce goods using different inputs and sell the goods in markets characterised by monopolistic or perfect competition to maximise profits, households which obtain utility from consumption and leisure and offer their labour services to the firms, a government, and a central bank. In the small open economy, there is a larger variety of firm types, as some firms specialize in the production of exports or the distribution of imports, and final goods are produced through combining domestically produced and imported inputs. Moreover, households in the small open economy may save in both a domestic and a foreign asset. The model is estimated using Bayesian methods on data for Sweden and a trade-weighted measure of the foreign economy based on data for the euro area and the United States for the period 1995–2018.

The Riksbank has a rather long history of DSGE modelling for policy use. The models have

[5]Studying forecasts produced over the years 2007–2017 by a number of Swedish forecasters, Lindé and Reslow (2017) show that forecasts of Swedish GDP take too little account of foreign GDP developments at both short- and long-term horizons. At long-term horizons, the same holds true also for forecasts of inflation.

[6]It has been shown, though, that structural models can successfully be used for forecasting, as their forecasting performance is roughly on par with VAR models; see e.g. Smets and Wouters (2007).

[7]The same is also true for any other source of misspecification of our models. However, for small and very open economies the lack of spillovers may be an issue of particular importance given the large discrepancies between the cross-country correlations in models and in data.

[8]MAJA is an acronym for the Swedish '*Modell för Allmän Jämviktsanalys*', which means 'Model for General Equilibrium Analysis'.

traditionally been used for forecasting, scenarios, policy analysis, and interpretation of data and forecasts. Most importantly, the models have been key in structuring information and discussions about policy-relevant questions. MAJA was preceded by two earlier models: Ramses I developed by Adolfson et al. (2005), and Ramses II initially developed by Christiano, Trabandt, and Walentin (2011) and adapted for policy use by Adolfson et al. (2013).[9] These models were in use between 2005 and 2010, and 2010 and 2019, respectively. Compared to those earlier two models, MAJA differs in a few respects, where the following three are particularly important. First, the foreign economy is captured through a medium-sized structural model rather than a small structural vector autoregressive (SVAR) model. There are several reasons behind this. A richer modelling of the foreign economy opens up for a more detailed understanding of economic developments abroad. Moreover, a larger set of structural shocks, which to a large extent overlaps with the shocks in the small open economy, allows us to explore global factors and cross-country correlations between shocks. As global factors are found to be important drivers of the Swedish economy in empirical models, this allows for a better data fit and more realistic storytelling. Second, energy is included as a separate consumption good in the model, which allows us to analyze and observe both headline and core inflation in the two economies. Much of the cross-country comovement of inflation is driven by energy prices and shocks to energy prices account for a substantial share of inflation fluctuations, while core inflation (defined as headline inflation excluding energy) should be more closely linked to the real side of the model. Third, we include a global real interest rate trend in our model, to take into account the fact that interest rates have been trending down in our sample. The trend is modelled as a slow-moving process towards which actual interest rates converge in the medium run.[10]

The main overall result is that the model captures the empirical relationships between Swedish and foreign macroeconomic variables in a better way than standard open-economy New Keynesian models, among them the earlier models used at the Riksbank. This is illustrated in several ways: foreign/global shocks generally account for a substantial share of the variation in Swedish variables, cross-country correlations between variables are generally larger than in earlier models, and forecasts of Swedish variables conditional on foreign variables are generally more accurate.[11] Also, historical decompositions of Swedish variables during, for example, the financial crisis of 2008–09 and the euro crisis around 2012 accord with the common view that the downturns in the Swedish economy during those events were largely generated abroad. The assumption of shocks affecting the foreign and Swedish economies in similar ways and Swedish export demand being geared more towards foreign investment are important in increasing the cross-border spillovers to Sweden in the model.

In terms of key macroeconomic variables, our assessment is that the strong foreign dependencies of real economy variables and interest rates are reasonably well captured in the model, while the modelling of inflation appears more challenging. Real economy spillovers through trade, technology, confidence, and financial channels are substantial and larger than in standard open-economy models. The introduction of a time-varying persistent global real interest rate trend in the model makes it possible to capture the strong correlation between interest rates. Moreover, it improves foreign and Swedish policy rate forecast accuracy quite substantially. A more detailed modelling of inflation through the introduction of an energy component yields more direct interpretations of inflation developments. However, our estimates of key nominal rigidity parameters imply rather flat Phillips curves, suggesting a weak link between the real economy and inflation. Accounting for the low inflation period in Sweden, over the years 2012–2016, still poses a challenge. Just like earlier models, MAJA overpredicts inflation in this period. One consequence of the flat Phillips curves is also that the inflation response to a monetary policy shock is rather limited, with a magnitude of the effect similar to the one for Ramses II reported in Christiano, Trabandt, and Walentin (2011).

[9] Ramses I was documented and estimated on euro area data in Adolfson et al. (2005) and Adolfson et al. (2007). Estimates based on Swedish data are presented in Adolfson et al. (2008).

[10] We note also that MAJA has a different labour market modelling than both Ramses I and Ramses II, and that financial factors are captured in MAJA in a more simplistic way than in Ramses II.

[11] A staff memo focusing on the forecast performance of MAJA will be published during 2020.

The rest of the paper is organized as follows. In Section 2, we outline the model and provide some intuition behind the economic mechanisms. A detailed derivation of the model can be found in a separate Technical Appendix. In Section 3, we discuss the estimation of the model, calibration, the choice of priors and observables, the treatment of trends, and the estimation results. It also contains a discussion of important properties of the data which we are trying to match. Section 4 highlights selected model properties. Finally, Section 5 concludes. A Data and Estimation Appendix (henceforth referred to as the Appendix) contains additional material related to the data and empirical analysis.

2 The model

In this section, we present the structure of the model and discuss some of the main equations. Detailed derivations and further discussions about the underlying assumptions can be found in the Technical Appendix.

Our model consists of two economies. The domestic economy is given by a small open-economy medium-sized DSGE model, similar to the models in Adolfson et al. (2007) and the baseline version of Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). The foreign (large) economy is assumed to be closed, as the small economy is assumed to be of negligible size and thus not affect allocations in the large economy. It is given by a closed-economy version of the domestic economy. As such, our foreign economy model closely resembles those in Smets and Wouters (2003), and Smets and Wouters (2007). We here focus on describing the domestic economy model. The differences between the domestic and foreign economies are discussed further in Subsection 2.5.

The domestic economy model consists of a number of different firms, which produce, import or export different types of goods, households which obtain utility from consumption and leisure and offer their labour services to the firms, a government, and a central bank. The economy has two sources of secular growth: a positive drift in the state of the neutral technology shock z_t, and a positive drift in the state of an investment-specific technology shock Ψ_t. The former shock implies that GDP and its components, real wages, and labour productivity share a common trend. Introducing stochastic trends in the model allows us to estimate the model largely without pre-filtering the data.

In the rest of this section, we will present the log-linearized version of the model, equation by equation. We focus on the linear equations to provide intuition about model mechanisms, and discuss the underlying optimization problems in words in relation to each section below. All variables are log-linearized around their steady state.[12] A hat above a variable denotes its log deviation from steady state, e.g. $\hat{\pi}_t^d = \log\left(\pi_t^d\right) - \log\left(\pi^d\right)$, while the absence of a time subscript indicates the steady-state value of a variable.[13] Variables pertaining to the foreign economy are denoted by asterisks (*). We first present the model equations for the domestic economy, before moving on to the foreign-economy counterparts.

The remainder of this section is organized as follows. We first present the domestic economy block of the model. We begin by describing the firms, starting with domestic and imported intermediate goods firms, and then moving on to the production of final consumption and investment goods, and, finally, the production of export goods. We then consider the households' problem, including wage setting, where households are assumed to be represented by unions who determine the wages. We then present our assumptions regarding the central bank and the government, and derive the aggregate resource constraint and the evolution of net foreign assets. The model is then completed with a description of the foreign economy block. Each section begins with a brief discussion of the theoretical structure and the optimization problem of the respective agents. The derived equilibrium conditions have been scaled to express the model in stationary form, and finally log-linearized. We

[12] There are a few exceptions, which will be discussed further below.

[13] The log-linear deviation from steady state multiplied by 100 gives approximately the percentage deviation from steady state. Prior to log-linearization, all trending variables in the model have been stationarized, i.e. divided by the trend level of the neutral and, where applicable, investment-specific technologies. Log-deviations of these variables thus refer to deviations from their steady state trends.

present only the log-linearized equations, as the model is ultimately solved and estimated in log-linear form. The complete specification and solution of the different agents' optimization problems, scaling and log-linearization, as well as the steady-state solution of the model, are presented in detail in the accompanying Technical Appendix.

2.1 Firms

There are several types of firms in the model, producing or importing intermediate or final goods for consumption, investment and exports. Final goods are produced by combining the domestically produced goods with imported goods. The production structure in our model closely resembles the one in Adolfson et al. (2013), with the exception of energy consumption.

The domestic homogeneous good is produced by competitive retailers who buy their input — varieties of the domestic intermediate goods — from the domestic intermediate goods producers. The domestic intermediate goods producers are monopolistically competitive. Importing firms buy foreign goods and redistribute them in the domestic economy. Upon entering the domestic economy, imported goods are repackaged as specialized imported intermediate goods, and then supplied monopolistically to four different types of import retailers. These, in turn, produce the aggregate imported goods used in the production of final consumption, investment and exports. Exporting firms produce a specialized export good by combining domestic and imported intermediate inputs, and sell these to foreign competitive retailers, which create a homogeneous good that is eventually sold to foreign agents. The firms are owned by the households in the economy, and firm profits are paid out to the households as dividends.

2.1.1 Production of domestic intermediate goods

There is a large number of firms, each producing a differentiated domestic intermediate good. The intermediate goods producers use labour and capital as inputs in their production.[14] They choose capital and labour inputs such that enough goods are produced to meet demand. Intermediate goods demand is given by the production of the homogeneous good, which aggregates the differentiated intermediate goods using a Dixit-Stiglitz technology with a time-varying markup. We assume that firms need to borrow from financial intermediaries in order to finance a fraction of their wage bill in advance, following Christiano, Eichenbaum, and Evans (2005) and Adolfson et al. (2013), among others. Through this so called working capital channel higher interest rates will increase the firms' costs and hence inflation.[15]

While the domestic homogeneous goods producer takes prices as given, the producers of the differentiated intermediate goods are monopolistically competitive. Firm market power is a prerequisite for sticky prices. Price stickiness is introduced by assuming that firms cannot re-optimize their prices continuously, and instead set prices in a staggered fashion, following Calvo (1983). Each intermediate firm faces a probability $1 - \xi_d$ that it can re-optimize its price in any given period and, hence, with probability ξ_d it cannot choose an optimal price.[16] If the firm is not able to re-optimize in period t, the price in period $t + 1$ will be set according to a heuristic indexation rule, with weights on previous period's domestic inflation and the time-varying trend inflation rate, $\bar{\pi}_t^c$, which is given by an exogenous process. The purpose of including the inflation trend shock in the model is to be able to account for low frequency, persistent, movements in inflation that are otherwise difficult to capture. The trend is interpreted to capture movements in the medium-term inflation expectations of firms and

[14] The intermediate goods producers are also subject to a fixed production cost. The fixed cost is assumed to grow at the same rate as consumption, the real wage and output in steady state, to ensure that profits remain zero.

[15] The working capital channel has been shown to be important for the effects of monetary policy and the degree of price stickiness in estimations on US data (see Christiano, Eichenbaum, and Evans (2005)).

[16] The probability that the firm can set its price optimally in a given period does not depend on the time that has passed since it was last able to re-optimize its price. The number of periods until it can re-optimise its price again is a geometrically distributed stochastic variable (a distribution with the 'no memory' property).

wage-setters which, in turn, could reflect their perceptions of changes in policy makers' objectives. A positive (negative) trend shock increases (decreases) inflation temporarily but persistently for reasonable parameterization of the shock process. Empirical support for the inclusion of the inflation trend shock is provided in Section 3.8.2. The specifications of the exogenous shock processes in the model are provided and discussed in Section 2.6 below.

The intermediate firms' price setting problem results in the following hybrid price Phillips curve:

$$\begin{aligned} \hat{\pi}_t^d - \hat{\bar{\pi}}_t^c &= \Xi_d \left(\widehat{mc}_t^d + \hat{\lambda}_t^d \right) + \frac{\kappa_d}{1+\beta\kappa_d} \left(\hat{\pi}_{t-1}^d - \hat{\bar{\pi}}_t^c \right) \\ &\quad + \frac{\beta}{1+\beta\kappa_d} E_t \left(\hat{\pi}_{t+1}^d - \hat{\bar{\pi}}_{t+1}^c \right) - \frac{\beta\kappa_d}{1+\beta\kappa_d} E_t \left(\hat{\bar{\pi}}_t^c - \hat{\bar{\pi}}_{t+1}^c \right), \end{aligned} \tag{2.1}$$

where π_t^d denotes domestic inflation, mc_t^d denotes the real marginal cost of the domestic intermediate goods producer, and λ_t^d is the domestic price markup shock. The parameter Ξ_d denotes the slope of the domestic Phillips curve, and is given by the following combination of parameters:[17]

$$\Xi_d \equiv \frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d(1+\beta\kappa_d)}.$$

The slope of the Phillips curve is decreasing in ξ_d, as firms optimizing prices less frequently implies that inflation is less responsive to costs. Moreover, β denotes the discount factor, and κ_d the indexation parameter determining the weight on past inflation in indexation. We note that in the case κ_d is set to zero and in the absence of a time-varying inflation trend, the above Phillips curve simplifies to the following forward-looking New Keynesian Phillips curve:

$$\hat{\pi}_t^d = \Xi_d \left(\widehat{mc}_t^d + \hat{\lambda}_t^d \right) + \beta E_t \hat{\pi}_{t+1}^d. \tag{2.2}$$

If instead $\kappa_d = 1$, the coefficient on lagged inflation in the Phillips curve reaches its maximum value $\frac{1}{1+\beta}$, which is close to $\frac{1}{2}$ for any reasonable parameterization of the discount factor β. Note that, in contrast to reduced-form specifications of the Phillips curve, the coefficient on lagged inflation is thus restricted from above.

The Phillips curve implies that domestic inflation is determined by the current and anticipated future levels of real marginal costs. As long as $\kappa_d \neq 0$, even lagged real marginal costs play a role. The firm can increase production by increasing labour or capital input and, since the firm is assumed to use inputs efficiently, the marginal costs along each of these margins are equal. One expression for the marginal cost is given by

$$\widehat{mc}_t^d = \hat{\bar{w}}_t + \hat{R}_t^{wc,d} - \widehat{mpl}_t, \tag{2.3}$$

[17] We assume that the differentiated intermediate goods are aggregated into a homogenous good using a Dixit-Stiglitz aggregator. We could have, alternatively, assumed that they are aggregated using the aggregator developed in Kimball (1995), as in for example Smets and Wouters (2007) and Coenen et al. (2018). The Kimball aggregator implies that the demand elasticity depends on the relative price of the differentiated goods. In practice, it alters the expression for the slope of the Phillips curve in the following way:

$$\Xi_d \equiv \frac{1}{1+\epsilon_d/(\eta_d-1)} \frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d(1+\beta\kappa_d)},$$

where ϵ_d denotes the curvature of the Kimball goods market aggregator (calibrated to 10 in both of the referenced papers, and equalling zero in the Dixit-Stiglitz case) and η_d denotes the elasticity of substitution of the differentiated goods, which is related to the steady-state price markup through the following expression: $\lambda^d = \eta_d/(\eta_d - 1)$. As pointed out in Eichenbaum and Fisher (2007), the assumption of a Kimball aggregator (with $\epsilon_d > 0$) allows for a lower degree of nominal rigidities, more in line with micro data evidence. We note that the two assumptions result in models which are observationally equivalent. While we have opted for the Dixit-Stiglitz aggregator to keep the model simple, we report also what Calvo parameter our estimated slope of the Phillips curve implies under the alternative assumption of a Kimball aggregator.

Another way of reducing the estimated nominal rigitity in the model is the assumption that capital is firm-specific (rather than homogenous as in our model), as discussed in Eichenbaum and Fisher (2007) and Altig et al. (2011). We do not explore this assumption further, but note that it would result in a model observationally equivalent to ours.

where $\bar{w}_t$ denotes the real wage rate per unit of aggregate homogeneous labour common to all intermediate firms, $R_t^{wc,d}$ the gross effective nominal rate of interest faced by the firms, and $\widehat{mpl}_t$ the marginal product of labour. The presence of the term $R_t^{wc,d}$ stems from the assumption that firms finance part of their wage bill in advance. The interest rate faced by domestic goods firms is specified as follows:

$$\hat{R}_t^{wc,d} = \frac{\nu^{wc,d} R}{R^{wc,d}} \hat{R}_t + \frac{\nu^{wc,d}(R-1)}{R^{wc,d}} \hat{\nu}_t^{wc,d}, \tag{2.4}$$

reflecting the assumption that a (possibly time-varying) fraction $\nu_t^{wc,d}$ of the firms' wage bill has to be financed in advance at the gross nominal interest rate determined by the central bank and denoted by R_t. If the steady-state fraction, $\nu^{wc,d}$, is set to zero, $R_t^{wc,d}$ disappears from the expression for marginal costs in equation (2.3). We assume that $\nu_t^{wc,d}$ evolves according to an exogenous stochastic process.[18] The marginal product of labour, in turn, is provided by

$$\widehat{mpl}_t = \alpha \widehat{\left(\frac{k}{N}\right)}_t + \hat{\epsilon}_t, \tag{2.5}$$

where $\left(\frac{k}{N}\right)_t$ denotes the capital-labour ratio, and ϵ_t is a stationary neutral technology shock. The parameter α governs the share of capital in the intermediate goods producers' production function. The capital-labour ratio is given by the following expression:

$$\widehat{\left(\frac{k}{N}\right)}_t = \hat{k}_t - \hat{N}_t - \left(\hat{\mu}_{z^+,t} + \hat{\mu}_{\Psi,t}\right). \tag{2.6}$$

Here, N_t and k_t denote labour and capital inputs, respectively, which are used in the production of domestic homogenous goods. The terms $\mu_{z^+,t}$ and $\mu_{\Psi,t}$ denote the technological growth rates in the model. $\mu_{\Psi,t}$ is the growth rate of investment-specific technology, while $\mu_{z^+,t}$ is the evolution of the combination of investment-specific and neutral technology, z_t^+, given by

$$\hat{\mu}_{z^+,t} = \frac{\alpha}{1-\alpha} \hat{\mu}_{\Psi,t} + \hat{\mu}_{z,t}, \tag{2.7}$$

where $\mu_{z,t}$ is the growth rate of neutral technology. $\mu_{\Psi,t}$ and $\mu_{z,t}$ are assumed to evolve according to exogenous stochastic processes. All else equal, a positive temporary technology shock increases the marginal product of labour, decreases marginal cost and hence lowers inflation. A positive shock to the growth rate of technology, which increases the growth rate temporarily but the level of technology permanently, also lowers marginal cost and inflation while the effect on the marginal product of labour depends on the utilization of capital.

Optimizing with respect to capital, we get the following expression for marginal cost:

$$\widehat{mc}_t^d = \widehat{\bar{r}}_t^k - \widehat{mpk}_t, \tag{2.8}$$

where $\widehat{\bar{r}}_t^k$ is the rental rate of capital, and $\widehat{mpk}_t$ the marginal product of capital. The marginal product of capital is in turn given by

$$\widehat{mpk}_t = -(1-\alpha) \widehat{\left(\frac{k}{N}\right)}_t + \hat{\epsilon}_t. \tag{2.9}$$

From equations (2.3), (2.5), (2.8), and (2.9) the marginal cost can alternatively be expressed as

$$\widehat{mc}_t^d = (1-\alpha)\left(\widehat{\bar{w}}_t + \hat{R}_t^{wc,d}\right) + \alpha \widehat{\bar{r}}_t^k - \hat{\epsilon}_t. \tag{2.10}$$

[18] As discussed further in Section 3.4.2, when estimating the model, we eventually assume that $\nu_t^{wc,d} = 1$ and $\hat{\nu}_t^{wc,d} = 0$, which implies that expression 2.4 becomes $\hat{R}_t^{wc,d} = \hat{R}_t$. We make the same assumption also for the importing and exporting firms, implying that the strength of the working capital channel in our model is not time-varying.

We recall that α governs the share of capital in production. As α is generally set such that the share of labour exceeds the share of capital in production, it becomes clear from the above expression that the level of wages is a key driver of marginal costs. From the Phillips curve in equation (2.1), we see that wages then are a key driver also of domestic inflation. As we shall see further below, domestic inflation in turn forms the largest part of aggregate consumer price inflation, which implies that wages are an important determinant also of aggregate consumer price inflation. Wages in our model are set by the households, through their unions, and will be discussed in some more detail together with the households' optimization problem in Section 2.2 below.

A total of five shocks were introduced above. The price markup, λ_t^d, stationary technology, ϵ_t, and permanent technology, $\mu_{z,t}$ and $\mu_{\Psi,t}$, shocks can be broadly characterised as supply shocks since they have opposing effects on output and prices. The inflation trend shock, $\bar{\pi}_t^c$, can be characterised as a demand shock since it moves output and prices in the same direction.[19]

2.1.2 Production of imported intermediate goods

The import sector consists of domestic firms that buy a homogeneous good or, alternatively, energy from foreign firms. There are four different types of importing firms: (i) those that turn the imported product into a specialized non-energy consumption good, $C_{i,t}^m$, (ii) those that turn the imported product into a specialized investment good, $I_{i,t}^m$, (iii) those that turn the imported product into a specialized good used as input in production by exporting firms, $X_{i,t}^m$, and (iv) those that turn imported energy into a specialized energy consumption good, $C_{i,t}^{e,m}$. There is a large number of importing firms in each category. They sell their specialized output to import retailers, who produce the final imported goods. Consequently, there are also four types of import retailers.[20]

The setup in the import sector is similar to the one in the domestic sector. The import retailers take prices as given while the producers of the differentiated intermediate goods act under monopolistic competition and are assumed to set prices following Calvo (1983). The probability that an intermediate firm can set its price optimally in any given period is given by $1-\xi_{m,j},\ j=c,i,x,ce$. If the firm cannot re-optimize in period t, the price in period $t+1$ will be set according to an indexation rule, with weights on previous period's imported inflation and the domestic time-varying trend inflation rate. The assumption of nominal price rigidities in the import sectors allows for incomplete exchange rate pass-through to import and, in continuation, aggregate prices. As all firms do not immediately respond to changes in their marginal costs, including those driven by movements in the exchange rate, the pass-through to prices is limited in the short run. This is in line with the evidence from the data, in particular concerning aggregate prices, as we discuss further in Section 3.2 below. The importing intermediate firms' price setting problem then results in the following Phillips curves, one for each type of importing firm:

$$
\begin{aligned}
\hat{\pi}_t^{m,j}-\hat{\bar{\pi}}_t^c &= \Xi_{m,j}\left(\widehat{mc}_t^{m,j}+\hat{\lambda}_t^{m,j}\right)+\frac{\kappa_{m,j}}{1+\beta\kappa_{m,j}}\left(\hat{\pi}_{t-1}^{m,j}-\hat{\bar{\pi}}_t^c\right) \\
&\quad +\frac{\beta}{1+\beta\kappa_{m,j}}E_t\left(\hat{\pi}_{t+1}^{m,j}-\hat{\bar{\pi}}_{t+1}^c\right)-\frac{\beta\kappa_{m,j}}{1+\beta\kappa_{m,j}}E_t\left(\hat{\bar{\pi}}_t^c-\hat{\bar{\pi}}_{t+1}^c\right), \\
j &= c,\ i,\ x,\ ce.
\end{aligned} \tag{2.11}
$$

In line with the notation in the intermediate domestic Phillips curve above, $\pi_t^{m,j}$ denotes inflation in the import production sector j, $mc_t^{m,j}$ denotes the marginal cost of the intermediate import goods producer j, $\lambda_t^{m,j}$ is the price markup shock in sector j, and $\kappa_{m,j}$ the indexation parameter determining

[19] It should be noted that the categorisation of shocks as supply or demand shocks in a DSGE model is not obvious and depends e.g. on the assumed policy response of the central bank.

[20] The producers of the imported intermediate goods are subject to a fixed production cost, just as the domestic ones. The fixed cost is assumed to grow at the same rate as output in steady state, to ensure that profits remain zero. Given our setup with monopolistic power in the import and export sectors, the inclusion of fixed costs in these sectors is particularly important for the steady state solution, as zero net foreign assets in steady state otherwise do not imply that net exports derived from the aggregate resource constraint equal zero.

the weight on past inflation in indexation in sector j. The parameter $\Xi_{m,j}$ denotes the slope of the Phillips curve in import sector j, and is given by the following combination of parameters:

$$\Xi_{m,j} \equiv \frac{\left(1-\beta\xi_{m,j}\right)\left(1-\xi_{m,j}\right)}{\xi_{m,j}\left(1+\beta\kappa_{m,j}\right)},\ j=c,\ i,\ x,\ ce.$$

Note again that the slope of the Phillips curve is decreasing in $\xi_{m,j}$.[21]

The marginal cost of the importing firms is proportional to the price of the imported goods in domestic currency. The marginal cost is thus a function of the nominal exchange rate and of the price of the foreign homogenous goods purchased by the importing firms. Rewritten in terms of relative prices, the importing firms' real marginal costs are given by

$$\widehat{mc}_t^{m,j} = \hat{q}_t + \hat{p}_t^c - \hat{p}_t^{c,*} - \hat{p}_t^{m,j} + \hat{R}_t^{wc,m},\ j=c,\ x,\ i, \tag{2.12}$$

where q_t denotes the real exchange rate, p_t^c and $p_t^{c,*}$ the relative price of consumption to intermediate homogeneous goods in the home and foreign economies, respectively, and $p_t^{m,j}$ the relative price of homogeneous imported good j to the domestic homogeneous good. For the energy goods importers, the marginal costs are instead given by

$$\widehat{mc}_t^{m,ce} = \hat{q}_t + \hat{p}_t^c + \hat{p}_t^{ce,*} - \hat{p}_t^{c,*} - \hat{p}_t^{m,ce} + \hat{R}_t^{wc,m},\ j=ce, \tag{2.13}$$

including also the relative price of foreign energy to the foreign intermediate homogeneous good, $p_t^{ce,*}$. From equations (2.12) (or (2.13)) and (2.11) it is clear that the pass-through from a change in the exchange rate to import inflation, $\hat{\pi}_t^{m,j}$, is given by the slope of the Phillips curve, which in turn is governed by the Calvo parameter, $\xi_{m,j}$. $R_t^{wc,m}$ denotes the gross effective nominal interest rate faced by importing firms. Similar to the domestic firms, we assume that the importing firms have to finance a fraction $\nu_t^{wc,m}$ of their input costs in advance, according to the following expression:

$$\hat{R}_t^{wc,m} = \frac{\nu^{wc,m}R^*}{R^{wc,m}}\hat{R}_t^* + \frac{\nu^{wc,m}\left(R^*-1\right)}{R^{wc,m}}\hat{\nu}_t^{wc,m}, \tag{2.14}$$

where $\nu_t^{wc,m}$ is assumed to evolve according to an exogenous stochastic process. Note that the importing firms are assumed to borrow at the gross nominal interest rate R_t^*, determined by the foreign central bank, as their production input consists of the foreign homogeneous good and is thus purchased abroad.

Finally, the definitions of the relative prices of the four different types of imported goods imply the following restrictions across inflation rates, needed to close the model:

$$\hat{p}_t^{m,j} = \hat{p}_{t-1}^{m,j} + \hat{\pi}_t^{m,j} - \hat{\pi}_t^d,\ j=c,\ i,\ x,\ ce. \tag{2.15}$$

2.1.3 Production of final consumption goods

Final consumption goods are purchased by households and produced by a representative competitive firm that combines domestically produced and imported goods, c_t^d and c_t^m, into an aggregate non-energy good, c_t^{xe}, and domestically produced and imported energy, $c_t^{e,d}$ and $c_t^{e,m}$, into an aggregate energy good c_t^e.[22] The two aggregate goods c_t^{xe} and c_t^e are in turn combined into the final consumption good c_t through a nested CES (constant elasticity of substitution) production function. The introduction of energy consumption allows us to distinguish headline from core inflation in the model (with core inflation being defined as inflation excluding energy). The prices of the domestically produced

[21] Note also that a broad categorisation of the import markup shocks as supply or demand shocks based on their effects on economy-wide output and prices is less straightforward.

[22] We assume that production of energy requires some use of the domestic homogenous good, just as all the other goods. We think of this as there being an endowment of energy, which requires labour and capital in the same proportions as all other goods in order to be turned into consumable energy.

energy in the two countries are treated as stochastic processes.[23] Since positive energy price shocks increase inflation and decrease output these shocks can be broadly characterized as supply shocks.

Demand for the two intermediate non-energy goods is given by

$$\hat{c}_t^d = \eta_c \hat{p}_t^{cxe} + \hat{c}_t^{xe} \tag{2.16}$$

and

$$\hat{c}_t^m = -\eta_c \left(\hat{p}_t^{m,c} - \hat{p}_t^{cxe}\right) + \hat{c}_t^{xe}, \tag{2.17}$$

where p_t^{cxe} is the relative price of aggregate non-energy consumption to the domestic intermediate homogeneous good, and η_c is the elasticity of substitution between domestic and imported non-energy consumption goods. We can combine the above two equations to obtain the following expression:

$$\hat{c}_t^m - \hat{c}_t^d = -\eta_c \hat{p}_t^{m,c}. \tag{2.18}$$

From this, it is clear that the relative price of imported and domestic goods governs their relative demand, and that the sensitivity of demand to price changes is proportionate to the elasticity η_c. The higher the elasticity of substitution, the less import goods are demanded relative to domestic goods following an increase in the price of imports. A value of η_c above one implies that imported and domestic consumption are substitutes, while a value below one implies that they are complements. It is common to restrict η_c to be higher than one in estimation, as there are theoretical arguments for why different varieties of goods should reasonably be substitutes at the micro level. There is also a divergence in the estimates at the micro and macro level, where the former tend to be considerably higher than the latter. We note that the definitions of goods at the micro and macro level are often different.[24] The elasticities of substitution between imported and domestic goods in our model refer to the substitutability of entire baskets of goods, rather than single varieties of goods, in which case complementarity in production of final goods is a plausible assumption. Moreover, even though agents are in theory infinitely lived in our model, the time horizon of primary interest to us is the business cycle frequency. As production structures evolve over time only slowly, complementarity of imports and domestic goods may even be the preferred assumption. When the model is estimated we assign a prior for η_c centered close to unity and allow the parameter to take on values both above and below one.

Demand for the two intermediate energy goods is given by

$$\hat{c}_t^{e,d} = -\eta_{em} \left(\hat{p}_t^{d,ce} - \hat{p}_t^{ce}\right) + \hat{c}_t^e \tag{2.19}$$

and

$$\hat{c}_t^{e,m} = -\eta_{em} \left(\hat{p}_t^{m,ce} - \hat{p}_t^{ce}\right) + \hat{c}_t^e, \tag{2.20}$$

[23] Treating energy prices as stochastic processes implies that we are assuming that energy prices are driven by supply side factors. When it comes to Swedish energy, this assumption is warranted, as electricity prices are to a considerable extent determined by weather phenomena. As we discuss in Section 4.2, the foreign economy energy prices in our model are closely related to the movements of the price of oil. There is an ongoing discussion in the academic literature about the dominant drivers of oil prices (see Kilian (2009), Kilian (2014), and Hamilton (2003), for three prominent examples). We do not wish to take an explicit stand on the importance of demand and supply drivers in driving oil prices. Our present modelling choice, to let energy prices evolve exogenously, is guided by the ambition to keep the model relatively simple and tractable. Assessing the importance of demand effects on energy prices is left for future model development.

[24] The so called trade elasticities in the empirical literature usually refer to estimates of the reaction of aggregate trade volumes to changes in relative prices. Estimated on micro data, these elasticities tend to be rather high. Their estimation is usually based on the Armington (1969) assumption that the substitutability between two imported varieties is the same as the substitutability between an imported variety and a domestic one. As such, this elasticity of substitution is more closely related to (albeit not exactly the same as) our elasticity of substitution between differentiated intermediate import goods, which is related to the markup in the importing sector in the following way: $\lambda^{m,j} = \eta_{m,j} / \left(\eta_{m,j} - 1\right)$, $j = c, i, x, ce$. These elasticities in our model are calibrated to 6 or higher, depending on the sector. When aggregate data are employed for the estimation of these elasticities, obtained estimates tend to be much lower (see for example Mc Daniel and Balistreri (2003)). While the differences in estimates are related to different methodologies and the occurrence of biases in estimates on aggregate data, the concepts may be different if they concern single varieties or aggregate baskets of goods.

where $p_t^{d,ce}$, $p_t^{m,ce}$ and p_t^{ce} are the relative prices of domestic, imported and aggregate energy consumption, respectively, to the domestic intermediate homogeneous good, and η_{em} is the elasticity of substitution between domestic and imported energy consumption goods. Finally, demand for the two aggregate consumption goods is given by the following expressions:

$$\hat{c}_t^{xe} = -\eta_e \left(\hat{p}_t^{cxe} - \hat{p}_t^c \right) + \hat{c}_t \tag{2.21}$$

and

$$\hat{c}_t^{e} = -\eta_e \left(\hat{p}_t^{ce} - \hat{p}_t^c \right) + \hat{c}_t, \tag{2.22}$$

where η_e is the elasticity of substitution between energy and non-energy consumption in the final consumption aggregate.

The above described production structure gives rise to three different measures of aggregate inflation: the aggregate consumer price inflation π_t^c which corresponds to *headline* inflation, the aggregate non-energy price inflation π_t^{cxe}, which we refer to as *core* inflation, and the aggregate energy price inflation π_t^{ce}. These are given by the following three equations:

$$\hat{\pi}_t^c = \left(1 - \tilde{\omega}_e\right) \hat{\pi}_t^{cxe} + \tilde{\omega}_e \hat{\pi}_t^{ce}, \tag{2.23}$$

$$\hat{\pi}_t^{cxe} = \left(1 - \tilde{\omega}_c\right) \hat{\pi}_t^{d} + \tilde{\omega}_c \hat{\pi}_t^{m,c}, \tag{2.24}$$

and

$$\hat{\pi}_t^{ce} = \left(1 - \tilde{\omega}_{em}\right) \hat{\pi}_t^{d,ce} + \tilde{\omega}_{em} \hat{\pi}_t^{m,ce}, \tag{2.25}$$

where π_t^d and $\pi_t^{m,c}$ denote the domestic and imported non-energy price inflation, and $\pi_t^{d,ce}$ and $\pi_t^{m,ce}$ the domestic and imported energy price inflation, respectively. The energy share, $\tilde{\omega}_e$, is given by $\tilde{\omega}_e \equiv \omega_e \left(p^{ce}/p^c\right)^{1-\eta_e}$, and it is later calibrated to match the average energy share in aggregate consumption through calibration of the parameter ω_e. In the corresponding way, $\tilde{\omega}_c \equiv \omega_c \left(p^{m,c}/p^{cxe}\right)^{1-\eta_c}$ and $\tilde{\omega}_{em} \equiv \omega_{em} \left(p^{m,ce}/p^{ce}\right)^{1-\eta_{em}}$ are functions of the parameters determining the steady-state import share in non-energy and energy consumption, respectively. The relative prices of the aggregate goods are given by the following three expressions:

$$\widehat{p}_t^c = \left(1 - \tilde{\omega}_e\right) \widehat{p}_t^{cxe} + \tilde{\omega}_e \widehat{p}_t^{ce}, \tag{2.26}$$

$$\hat{p}_t^{cxe} = \tilde{\omega}_c \hat{p}_t^{m,c}, \tag{2.27}$$

and

$$\hat{p}_t^{ce} = \left(1 - \tilde{\omega}_{em}\right) \hat{p}_t^{d,ce} + \tilde{\omega}_{em} \hat{p}_t^{m,ce}. \tag{2.28}$$

Finally, in order to close the model, we need to include the following restriction across inflation rates implied by the definition of the relative price of domestic energy consumption:

$$\hat{p}_t^{d,ce} = \hat{p}_{t-1}^{d,ce} + \hat{\pi}_t^{d,ce} - \hat{\pi}_t^{d}. \tag{2.29}$$

2.1.4 Production of final investment goods

Just as the aggregate consumption goods, the aggregate investment goods are produced by a representative competitive firm that combines domestically produced and imported goods. The investment production technology includes the unit root process with a positive drift, Ψ_t.[25] Moreover, total investment is defined as the sum of investment goods used in the accumulation of physical capital, plus

[25] It is common to include an investment-specific unit root technology shock to account for a declining relative price of investment, following Greenwood, Hercowitz, and Krusell (1997) and Christiano, Eichenbaum, and Evans (2005), and this feature was also included in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). The empirial relevance of this trend depends on the country and sample studied. See also Christiano, Trabandt, and Walentin (2011). In our sample, Swedish investment growth has been higher than GDP growth. We therefore include a deterministic investment trend for Sweden in our estimation, but no trend for the trade-weighted average of Sweden's main trading partners. We discuss this further in Section 3.2.2.

investment goods used in capital maintenance. There is a distinction between physical and efficient capital in the model, as we allow for a variable capital utilization rate. As we shall see in Section 2.2 below, there is a trade-off between increasing the capital stock in the economy through investment in additional physical capital and increasing the utilization rate of already installed capital as adjustment along both of these margins is costly. The assumption of variable capital utilization matters, for example, for the effects of technology shocks.

Demand for the domestic and imported intermediate investment goods, i_t^d and i_t^m respectively, is given by

$$\hat{\imath}_t^d = \eta_i \hat{p}_t^i + \hat{\imath}_t + \frac{1}{i} \frac{\sigma_b k^p}{\mu_{z+} \mu_\Psi} \hat{u}_t, \tag{2.30}$$

and

$$\hat{\imath}_t^m = -\eta_i \left(\hat{p}_t^{m,i} - \hat{p}_t^i \right) + \hat{\imath}_t + \frac{1}{i} \frac{\sigma_b k^p}{\mu_{z+} \mu_\Psi} \hat{u}_t, \tag{2.31}$$

where p_t^i and $p_t^{m,i}$ are the relative prices of aggregate and imported investment to the domestic intermediate homogeneous good, i_t denotes aggregate investment, and η_i is the elasticity of substitution between domestic and imported investment goods. The last term in the two equations reflects the assumption that investment goods are used in the maintenance of physical capital, k_t^p, with the cost of the maintenance depending on the degree of capital utilization u_t. The parameter σ_b is a parameter of the capital utilization cost function.[26]

Finally, the relative prices of the different investment goods are related by the following expression:

$$\hat{p}_t^i = \tilde{\omega}_i \hat{p}_t^{m,i}, \tag{2.32}$$

where $\tilde{\omega}_i \equiv \omega_i \left(p^{m,i} / p^i \right)^{1-\eta_i}$ is a function of the parameter ω_i, which determines the steady-state import share of investment. The relative price of investment thus varies proportionately to the relative price of the imported investment input in production.

2.1.5 Production of export goods

Final export goods are produced by firms combining domestically produced and imported goods, just as final consumption and investment goods are. Unlike in the case of consumption and investment, however, there is a large number of final export goods producers, each producing a specialized export good. The producers of the specialized export goods act under monopolistic competition and are assumed to set prices following Calvo (1983). The homogenous export good, which is later used for consumption and investment in the foreign economy, is then assumed to be produced by a representative, competitive, foreign retailer using the specialized final export goods as inputs.[27] We note that the export producers set their prices in foreign currency, as their goods are purchased and consumed abroad.

Demand for the domestic and imported intermediate export goods, x_t^d and x_t^m respectively, is given by

$$\hat{x}_t^d = \eta_x \tilde{\omega}_x \hat{p}_t^{m,x} + \hat{x}_t, \tag{2.33}$$

and

$$\hat{x}_t^m = -\eta_x \left(1 - \tilde{\omega}_x \right) \hat{p}_t^{m,x} + \hat{x}_t, \tag{2.34}$$

where $p_t^{m,x}$ is the relative prices of intermediate import goods used in export production to the domestic intermediate homogeneous good, x_t denotes aggregate exports, and η_x is the elasticity of substitution

[26] The capital utilization cost function is an increasing, convex function of the utilization rate.

[27] The producers of the specialized export goods are subject to a fixed production cost, just as the producers of domestic and imported intermediate goods. The fixed cost is assumed to grow at the same rate as output in steady state, to ensure that profits remain zero. See further discussion under Section 2.1.2 above.

between domestic and imported inputs in export production. The parameter $\tilde{\omega}_x$ is defined as

$$\tilde{\omega}_x \equiv \frac{\omega_x \left(p^{m,x}\right)^{1-\eta_x}}{\omega_x \left(p^{m,x}\right)^{1-\eta_x} + (1-\omega_x)},$$

where ω_x determines the steady-state import share of exports. Allowing for imported inputs in the production of exports is important for generating the positive comovement between exports and imports observed in data. Around 40 percent of Swedish imports are used in the production of export goods.

The price setting setup in the exporting sector is analogous to the ones in the domestic and import sectors. Each specialized export producer faces a probability $1-\xi_x$ that it can re-optimize its price in any period, independent of when it was last able to re-optimize its price. If the firm is not able to re-optimize in period t, the price in period $t+1$ will be set according to an indexation rule, with weights on previous period's export inflation, π_t^x, and the foreign time-varying trend inflation rate, $\bar{\pi}_t^*$. The price setting problem results in the following Phillips curve:

$$\begin{aligned}\hat{\pi}_t^x - \hat{\bar{\pi}}_t^* &= \Xi_x \left(\widehat{mc}_t^x + \hat{\lambda}_t^x\right) + \frac{\kappa_x}{1+\beta\kappa_x}\left(\hat{\pi}_{t-1}^x - \hat{\bar{\pi}}_t^*\right) \\ &+ \frac{\beta}{1+\beta\kappa_x} E_t\left(\hat{\pi}_{t+1}^x - \hat{\bar{\pi}}_t^*\right) - \frac{\beta\kappa_x}{1+\beta\kappa_x} E_t\left(\hat{\bar{\pi}}_t^* - \hat{\bar{\pi}}_{t+1}^*\right),\end{aligned} \tag{2.35}$$

where mc_t^x denotes the marginal cost of the specialized export goods producer, λ_t^x is the price markup shock, κ_x the indexation parameter determining the weight on past inflation in indexation, and Ξ_d the slope of the export producers' Phillips curve, given by

$$\Xi_x \equiv \frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x(1+\beta\kappa_x)}.$$

The marginal costs of the exporting firms are a function of the prices of the domestic and imported input goods. In terms of relative prices, the marginal costs of the exporting firms are given by

$$\widehat{mc}_t^x = \hat{R}_t^{wc,x} + \hat{p}_t^{c,*} - \hat{q}_t - \hat{p}_t^x - \hat{p}_t^c + \tilde{\omega}_x \hat{p}_t^{m,x}, \tag{2.36}$$

where p_t^x denotes the relative price of exports to the foreign homogeneous goods and $R_t^{wc,x}$ the gross effective nominal interest rate faced by the exporting firms.[28] Just as domestic and importing firms, exporting firms are assumed to finance a fraction $\nu_t^{wc,x}$ of their input costs in advance, according to the following expression:

$$\hat{R}_t^{wc,x} = \frac{\nu^{wc,x} R}{R^{wc,x}} \hat{R}_t + \frac{\nu^{wc,x}(R-1)}{R^{wc,x}} \hat{\nu}_t^{wc,x}, \tag{2.37}$$

where $\nu_t^{wc,x}$ is determined by an exogenous stochastic process. We also need to include the following restriction implied by the definition of the relative price of exports in order to close the model:

$$\hat{p}_t^x = \hat{\pi}_t^x - \hat{\pi}_t^{d,*} - \frac{1}{\eta_f}\left(\hat{\mu}_{z^{+,*},t} - \hat{\mu}_{z^+,t}\right) + \hat{p}_{t-1}^x, \tag{2.38}$$

where $\pi_t^{d,*}$ denotes the rate of inflation of the foreign homogenous good. $\mu_{z^{+,*},t}$ and $\mu_{z^+,t}$ denote the evolution of the combination of investment-specific and neutral technology (also discussed in Section 2.1 above) in the foreign and domestic economies, respectively, included to make sure that exports are

[28] Note that real marginal costs in each sector are deflated by the price of the good produced in that same sector. Thus, the exporting firms' real marginal costs are deflated by the export price which is set in foreign currency. This explains the ocurrence of the relative price of exports, real exchange rate and relative prices of consumption in the domestic and foreign economies in equation (2.36). Note also that the price of the domestic homogeneous good is used as numeraire, which explains why the cost of the domestic production input is not explicitly seen in the expression for the exporting firms' marginal costs.

on a balanced growth path even in the case of different growth rates in the two economies.[29] Also, the parameter η_f denotes the elasticity of substitution between domestic and imported goods in the foreign economy.

Finally, total demand by the foreign economy for domestic exports is given by the following expression:

$$\hat{x}_t = -\eta_f \hat{p}_t^x + \omega_c^x \hat{c}_t^* + (1 - \omega_c^x)\,\widehat{i}_t^{d,*}, \tag{2.39}$$

where c_t^* denotes foreign aggregate consumption, and $i_t^{d,*}$ foreign total demand for investment goods.[30] The parameter ω_c^x determines the weights on foreign consumption and investment in total export demand. As the foreign economy is modelled as approximately closed and the foreign import sector is thus not explicitly modelled, we estimate ω_c^x and refrain from making assumptions on the underlying structural parameters.[31] As we will discuss further in Section 3.6.3 below, foreign investment turns out to be the dominating determinant of demand for Swedish exports in our estimations. One interpretation is that this reflects the fact that a relatively large share of Swedish exports is accounted for by investment goods.

2.2 Households

2.2.1 Household preferences and the labour market

We assume that there is a large representative household, with full risk sharing of consumption among household members as in Merz (1995). As preferences are separable in consumption and leisure, this implies that the consumption of all individual household members is equal, irrespective of their employment status, work specialization and skills. The household members attain utility from consumption and disutility from work. There is a large number of members who differ by the type of labour service they are specialized in and their disutility of work. The preferences with respect to consumption are specified as in Christiano, Trabandt, and Walentin (2011) and Adolfson et al. (2013). The modelling of the disutility from work instead relies on the setup in Galí (2011a) and Galí, Smets, and Wouters (2012), with the purpose of introducing unemployment into the model in a simple way. All variations in labour input take place at the extensive margin, that is, by varying the number of employed workers rather than their work intensity. This implies that labour input to production is modelled in terms of employment rather than hours. Unemployment in the model results from union market power in labour markets. As labour is monopolistically supplied, wages are set with a positive markup which generally results in wages that are too high to ensure full employment. There is habit persistence in consumption and shocks to household preferences. We include shocks to the discount rate (ζ_t^β), consumption preferences (ζ_t^c), and labour supply (ζ_t^n).[32]

We include also an endogenous preference shifter Θ_t as in Galí (2011a) and Galí, Smets, and Wouters (2012), which is a function of the aggregate consumption trend and allows for a potentially more limited wealth effect on labour supply.[33] The preference shifter is given by the following expres-

[29] In our baseline specification, technological growth is assumed to be global, which implies that $\mu_{z^+,t} = \mu_{z^{+,*},t}$ and that the term in parenthesis of equation (2.38) is zero.

[30] As in the domestic economy, foreign investment goods are assumed to be used both in the accumulation of physical capital (i_t^*) and in capital maintenance.

[31] A more standard way of modelling export demand is to assume that it moves proportionately with foreign GDP, in which case exports are implicitly assumed to be divided between the different uses according to the different components' share of GDP, as in Adolfson et al. (2013) and many other models. This assumption differs from how imports are treated in the domestic economy, where the import share is allowed to vary by component (and is assumed to be zero in public consumption). As we will see in Section 3.8 below, our more flexible specification is favoured by the data compared to the traditional modelling of aggregate export demand.

[32] All shocks are included to allow for the possibility of choosing any subset of the three. All three shocks cannot be active at once in the estimation of the model (see Section 3).

[33] The inclusion of the preference shifter is motivated in Galí, Smets, and Wouters (2012) as a way of reconciling the existence of a long-run balanced growth path with a small short-term wealth effect. This feature implies that the labour force moves procyclically in response to, for example, a monetary policy shock.

sion:

$$\hat{\Theta}_t = \hat{z}_t^C + \widehat{\bar{v}}_t^N, \tag{2.40}$$

where z_t^C denotes trend consumption and $\bar{v}_t^N$ the marginal utility of consumption. The trend consumption z_t^C evolves according to the following process:

$$\hat{z}_t^C = (1-\nu)\left(\hat{z}_{t-1}^C - \hat{\mu}_{z^+,t}\right) - \nu\widehat{\bar{v}}_t^N, \tag{2.41}$$

where the parameter $\nu \in 0, 1]$ (which we estimate) determines the importance of the wealth effect in the model. The higher the value of ν, the closer we are to standard preferences with no preference shifter. If $\nu = 1$, we have that $\hat{z}_t^C = -\widehat{\bar{v}}_t^N$ and $\hat{\Theta}_t = 0$. The marginal utility of consumption is given by the following equation:

$$\begin{aligned} \widehat{\bar{v}}_t^N \;=\;& \mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^\beta + \mu_{z^+}\left(\mu_{z^+} - b\right)\hat{\zeta}_t^c - \mu_{z^+}^2\hat{c}_t + b\mu_{z^+}\hat{c}_{t-1} - b\mu_{z^+}\hat{\mu}_{z^+,t} \\ & -\beta b\left[\left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^\beta + \left(\mu_{z^+} - b\right)E_t\hat{\zeta}_{t+1}^c - \mu_{z^+}E_t\hat{c}_{t+1} + b\hat{c}_t - \mu_{z^+}E_t\hat{\mu}_{z^+,t+1}\right], \end{aligned} \tag{2.42}$$

where b is a parameter determining the degree of habit persistence in consumption.[34] We note that without habits (that is setting $b = 0$), and in the absence of preference shocks and growth, the above equation boils down to

$$\widehat{\bar{v}}_t^N = -\hat{c}_t, \tag{2.43}$$

reflecting our assumption that utility is logarithmic in consumption.

The unemployment rate in the model is given by

$$\hat{U}_t = \hat{L}_t - \hat{N}_t, \tag{2.44}$$

where L_t denotes the labour force and N_t denotes employment.[35] With this definition, the unemployed include all the individuals who would like to work but are not currently employed. As argued in Galí, Smets, and Wouters (2012), it can thus be viewed as involuntary. Labour supply is determined by a participation constraint stating that the individuals find it optimal to participate in the labour market as long as the disutility of work does not exceed the utility to the household from the additional wage income.[36] This labour market participation constraint is given by

$$\widehat{\bar{w}}_t - \hat{p}_t^c = \hat{\zeta}_t^n + \hat{\zeta}_t^\beta + \hat{z}_t^C + \varphi\hat{L}_t, \tag{2.45}$$

where the parameter $\varphi \geq 0$ determines the shape of the distribution of work disutilities across the individual household members. In the case of standard preferences, i.e. when the parameter ν in equation (2.41) is set to one, we have that $\hat{z}_t^C = -\widehat{\bar{v}}_t^N$. In the absence of habits, preference shocks and growth, $\widehat{\bar{v}}_t^N = -\hat{c}_t$, and the labour market participation constraint is given by the following expression:

$$\widehat{\bar{w}}_t - \hat{p}_t^c - \hat{c}_t = \varphi\hat{L}_t. \tag{2.46}$$

As discussed by Galí, Smets, and Wouters (2012) and Galí (2011a), this equation is inconsistent with the empirical evidence on the effects of a monetary policy shock based on US data since a policy shock which reduces the policy rate decreases $\widehat{\bar{w}}_t - \hat{p}_t^c - \hat{c}_t$ but increases the participation rate. However,

[34] We include habit formation in our model as it has been shown to be important in generating hump-shaped responses of aggregate demand to shocks. See Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Adolfson et al. (2007), among many others.

[35] We note that $\hat{L}_t$ and $\hat{N}_t$ are log deviations from steady state while $\hat{U}_t$ is the deviation of the unemployment rate (in levels) from its steady state, rather than a log-deviation.

[36] Individual members take into account the utility of the household rather than their personal utility, as not internalizing the benefits to the household of an individual's unemployment would result in no participation. The reason is that the assumption of full consumption risk-sharing implies that unemployed individuals will enjoy a higher utility ex post than employed individuals.

in our model monetary policy shocks account for little of the variation in the economic variables and the importance of the endogenous preference shifter, as captured by the parameter ν, is presumably guided more by the responses of the labour force and the unemployment rate to other shocks.

As in Galí, Smets, and Wouters (2012), we define the natural rate of unemployment — i.e. the unemployment rate that would prevail in the absence of nominal wage rigidities — as

$$\hat{U}_t^n = \frac{1}{\varphi}\hat{\lambda}_t^w, \tag{2.47}$$

where λ_t^w is a time-varying wage markup inducing changes in workers' market power, which evolves according to an exogenous stochastic process.

2.2.2 The household's optimization

The households in the economy own the capital stock. They receive income from wages and return on their capital holdings. We distinguish the physical capital stock from the capital available to firms. The capital used by firms in production, 'efficient capital' or 'capital services', $\hat{k}_t$, can be increased either by investment in the physical capital stock, k_t^p, or through increased capital utilization, $\hat{u}_t$, as described by the following equation:[37]

$$\hat{k}_t = \hat{u}_t + \hat{k}_t^p. \tag{2.48}$$

Capital utilization is determined by the following expression:

$$\hat{p}_t^i = \hat{\bar{r}}_t^k - \sigma_a \hat{u}_t, \tag{2.49}$$

where σ_a is a parameter of the capital utilization cost function. Equation (2.49) shows that the relative price of investment, $\hat{p}_t^i$, depends positively on the rental rate on capital, $\hat{\bar{r}}_t^k$, and negatively on capital utilization, $\hat{u}_t$. This illustrates the trade-off between increasing the capital stock in the economy through investment in additional capital and increasing the utilization rate of existing capital which was mentioned in Section 2.1.4 above. The law of motion for physical capital is given by

$$\hat{k}_{t+1}^p = \frac{1-\delta}{\mu_{z^+}\mu_\Psi}\left(\hat{k}_t^p - \hat{\mu}_{z^+,t} - \hat{\mu}_{\Psi,t}\right) + \frac{\Upsilon i}{k^p}\left(\hat{\Upsilon}_t + \hat{i}_t\right), \tag{2.50}$$

where δ denotes the capital depreciation rate and Υ_t is a stationary investment-specific technology shock that affects the efficiency of transforming investment into capital.[38]

The households spend part of their resources on consumption and investment, purchasing the aggregate consumption and investment goods discussed in Sections 2.1.3 and 2.1.4 above. Moreover, as they own the economy's physical capital stock, they also pay for the capital utilization costs. Finally, the households invest in domestic bonds B_{t+1} (on which they earn interest in period $t+1$), denominated in domestic currency, and foreign bonds B_{t+1}^F, denominated in foreign currency. They earn interest, R_t and R_t^* on their domestic and foreign bond holdings, respectively, which is adjusted by a risk premium shock, χ_t, as in Smets and Wouters (2007). We assume that χ_t is given by an exogenous stochastic process.[39] A positive shock to this wedge increases the required return on assets and reduces current

[37] Variable capital utilization has been argued to be crucial for matching responses of inflation and output to a monetary policy shock, in Christiano, Eichenbaum, and Evans (2005) on US data. Later papers using a different estimation approach or data for other countries find that the inclusion of variable capacity utilization is less important; see, for example, Smets and Wouters (2007) and Adolfson et al. (2007).

[38] The transformation of investment into capital includes investment adjustment costs, as in Christiano, Eichenbaum, and Evans (2005), who argue that these costs are important in generating hump-shaped responses of investment and other variables to a monetary policy shock. Investment adjustment costs have been shown to be among the empirically most important frictions in models using Bayesian estimation techniques, such as Smets and Wouters (2007) and Adolfson et al. (2007). While investment adjusment costs do not show up in the log-linear version of equation (2.50), they will appear in the investment Euler equation (2.59) below.

[39] This shock induces a wedge between the interest rate controlled by the central bank and the return on assets held by the households, and has similar effects as a net-worth shock in Bernanke, Gertler, and Gilchrist (1999). A structural interpretation of the shock is provided in Fisher (2015).

consumption. The foreign bonds are, in addition, adjusted by the premium on foreign bond holdings, Φ_t, which depends on the real aggregate net foreign asset position of the domestic economy, $\breve{a}_t$, the current and anticipated growth rates of the exchange rate, and a time-varying mean-zero shock to the (country) risk premium, $\tilde{\phi}_t$.[40] Specifically, the additional premium on foreign bond holdings is given by

$$\hat{\Phi}_t = -\tilde{\phi}_a \breve{a}_t - \tilde{\phi}_s \left(E_t \hat{s}_{t+1} + \hat{s}_t\right) + \widehat{\tilde{\phi}}_t, \tag{2.51}$$

where $\tilde{\phi}_a$ and $\tilde{\phi}_s$ are parameters, and s_t denotes the change in the nominal exchange rate between periods $t-1$ and t. Note that the exchange rate is defined as domestic currency per unit of foreign currency, and that a positive change thus implies a depreciation of the domestic currency. The households' optimization with respect to domestic and foreign bonds gives rise to the following uncovered interest rate parity condition:

$$\hat{R}_t = \hat{R}_t^* + \hat{\Phi}_t + E_t \hat{s}_{t+1}. \tag{2.52}$$

This condition states that a positive (negative) difference between the domestic interest rate and the foreign interest rate plus the premium on foreign bond holdings is expected to be offset by a corresponding depreciation (appreciation) of the nominal exchange rate. We note that this interest parity condition can be rewritten in terms of real interest rates and the real exchange rate as follows:

$$\widehat{\bar{\bar{R}}}_t = \widehat{\bar{\bar{R}}}_t^* + \hat{\Phi}_t + E_t \hat{q}_{t+1} - \hat{q}_t, \tag{2.53}$$

where we have defined the domestic and foreign ex-ante real interest rates as

$$\widehat{\bar{\bar{R}}}_t = \hat{R}_t - E_t \hat{\pi}_{t+1}^c, \tag{2.54}$$

$$\widehat{\bar{\bar{R}}}_t^* = \hat{R}_t^* - E_t \hat{\pi}_{t+1}^{c,*}, \tag{2.55}$$

and where q_t denotes the real exchange rate. We note that the real exchange rate in our model is defined in terms of aggregate consumer prices, as the foreign CPI expressed in domestic currency over the domestic CPI, as commonly defined in data. We can write the real exchange rate in terms of inflation rates and changes in the nominal exchange rate, as follows:

$$\hat{q}_t = \hat{q}_{t-1} + \hat{s}_t + \hat{\pi}_t^{c,*} - \hat{\pi}_t^c, \tag{2.56}$$

where q_t denotes the real exchange rate and s_t the change in the nominal exchange rate from $t-1$ to t.

The utility maximization of the household with respect to consumption (and domestic bond holdings) results in the following consumption Euler equation:

$$\widehat{\bar{v}}_t^N = E_t \widehat{\bar{v}}_{t+1}^N + \widehat{\bar{\bar{R}}}_t - E_t \hat{\mu}_{z^+,t+1} + \hat{\chi}_t, \tag{2.57}$$

where $\bar{v}_t^N$ is the marginal utility of consumption as given by equation (2.42). We note that in the absence of habits (that is setting $b = 0$), preference shocks and growth, we can use equations (2.42) and (2.54) to rewrite the above equation into the standard textbook consumption Euler equation (adjusted by a risk premium shock), as given by

$$\hat{c}_t = E_t \hat{c}_{t+1} - \left(\widehat{\bar{\bar{R}}}_t + \hat{\chi}_t\right). \tag{2.58}$$

[40] The dependence of Φ_t on $\bar{a}_t$ ensures that the steady state of the model is well-defined. If the domestic economy is a net borrower, domestic households must pay a premium on the foreign interest rate. If the domestic economy is a net lender, they instead recieve lower interest on their savings.

The dependence of Φ_t on the exchange rate is included to allow the model to reproduce two empirical observations regarding the uncovered interest parity (UIP) and the output response to a monetary policy shock, namely that the assessment of risk is affected by movements in interest rates, with partly offsetting effects on exchange rate movements, and that output responses to monetary policy shocks are hump-shaped in data, which requires slower responses of demand to the shock. Our specification follows Adolfson et al. (2013). See the Technical Appendix for a more detailed discussion.

This states that the utility obtained from consuming today, on the one hand, and the expected utility from saving and instead consuming in the future, on the other, are equal.

Household optimization with respect to investment yields the following investment Euler equation:

$$\widehat{\Delta i_t} = \beta E_t \widehat{\Delta i_{t+1}} + \frac{1}{(\mu_{z+}\mu_\Psi)^2 \tilde{S}''} \left[\widehat{\breve{p}}_{k',t} + \hat{\Upsilon}_t - \frac{p^i}{\breve{p}_{k'}\Upsilon} \hat{p}^i_t \right], \tag{2.59}$$

where

$$\widehat{\Delta i_t} = \widehat{i}_t - \widehat{i}_{t-1} + \hat{\mu}_{z+,t} + \hat{\mu}_{\Psi,t}.$$

This implies that current investment growth is a function of next period's expected investment growth, the real value of a unit of installed capital, $\breve{p}_{k',t}$, the relative price of investment to domestic intermediate homogeneous goods, p^i_t, and the stationary investment-specific technology shock, Υ_t. The parameter $\tilde{S}''$ belongs to the function of the technology that transforms current and past investment into installed capital for use in the following period. The higher $\tilde{S}''$, the lower is the sensitivity of investment to the real value of capital and to movements in the relative price or investment-specific technology. The growth terms in the equation reflect the inclusion of investment adjustment costs in our model.

The real value of capital, in turn, is determined by the following equation:

$$\begin{aligned} \widehat{\breve{p}}_{k',t} \quad = \quad & \frac{\beta(1-\delta)}{\mu_{z+}\mu_\Psi} E_t \widehat{\breve{p}}_{k',t+1} + \frac{\beta \bar{r}^k}{\mu_{z+}\mu_\Psi \breve{p}_{k'}} E_t \left(\widehat{\bar{r}}^k_{t+1} + \hat{u}_{t+1} \right) \\ & - E_t \hat{\mu}_{\Psi,t+1} - E_t \left(\hat{R}_t - E_t \hat{\pi}^d_{t+1} + \hat{\chi}_t \right), \end{aligned} \tag{2.60}$$

where we recall that δ denotes the capital depreciation rate. The current real value of capital is thus a function of next period's expected real value, the expected real rental rate on capital and capital utilization, expected investment trend growth, and the ex-ante real interest rate adjusted by a risk premium shock (and relative prices).[41]

2.2.3 Wage setting

We assume that households are monopoly suppliers of differentiated labour services hired by the firm. Thus, households can determine their wages. After having set their wages, households agree to supply the firm's demand for labour at the going wage rate. Differentiated labour is sold by households to labour contractors, who combine it into a homogeneous input good purchased by the domestic intermediate goods producers.

The households supply labour under monopolistic competition. They set wages as a markup over the (real) marginal rate of substitution between consumption and employment. The marginal rate of substitution captures the trade-off between working, and hence receiving income that can be used for consumption, and leisure. Just as the monopolistic firms setting prices, households are assumed to set wages in a staggered fashion subject to Calvo wage setting frictions, as in Erceg, Henderson, and Levin (2000). Once the wage has been set, households meet the demand for workers at the ongoing wage, which is determined by the firms. In every period, each labour type — or each union representing that labour type — faces a probability $1 - \xi_w$ that it can re-optimize its nominal wage. This probability is independent of when the union was last able to re-optimize its wage, as well as independent across labour types. When the nominal wage is not re-optimized, it is set according to an indexation rule with (estimated) weights on the previous period's aggregate consumer price inflation, π^c_t, and the time-varying trend inflation rate, $\bar{\pi}^c_t$.[42] The household's wage setting problem results in the following

[41] The definition of the ex-ante real interest rate given by equation (2.54) inlcudes aggregate consumer price inflation, while equation (2.60) includes a domestic inflation term. Rewriting (2.60) in terms of the real interest rate defined in (2.54) would therefore, in addition to $\widehat{R}_t$, introduce also relative price terms in the expression for $\widehat{\breve{p}}_{k',t}$.

[42] The growth rate of wages is also assumed to be tied to steady state productivity growth.

wage Phillips curve:

$$\hat{\pi}_t^w = \widehat{\bar{\pi}}_t^c + \beta E_t\left(\hat{\pi}_{t+1}^w - \widehat{\bar{\pi}}_{t+1}^c\right) + \kappa_w\left(\hat{\pi}_{t-1}^c - \widehat{\bar{\pi}}_t^c\right) - \beta\kappa_w E_t\left(\hat{\pi}_t^c - \widehat{\bar{\pi}}_{t+1}^c\right) - d_w\varphi\left(\hat{U}_t - \hat{U}_t^n\right), \quad (2.61)$$

where

$$d_w \equiv \frac{(\lambda^w - 1)(1-\beta\xi_w)(1-\xi_w)}{\xi_w\lambda^w(1+\varphi) - \xi_w},$$

and where π_t^w denotes wage inflation and κ_w the indexation parameter determining the weight on past consumer price inflation in indexation. Also, λ^w denotes the wage markup in steady state and φ is a parameter determining the household disutility from work. We note that in the case κ_w is set to be zero and in the absence of a time-varying inflation trend, the above wage Phillips curve simplifies to the following purely forward-looking wage Phillips curve, where current wage inflation is determined by next period's expected wage inflation and the deviation of unemployment from its natural rate:[43]

$$\hat{\pi}_t^w = \beta E_t\hat{\pi}_{t+1}^w - d_w\varphi\left(\hat{U}_t - \hat{U}_t^n\right). \quad (2.62)$$

Iterating this expression forward, it is easy to see that wage inflation is proportional to the discounted sum of expected deviations of current and future unemployment from its natural level.

Finally, wage inflation in the model is defined according to the following equation:

$$\hat{\pi}_t^w = \widehat{\bar{w}}_t - \widehat{\bar{w}}_{t-1} + \hat{\pi}_t^d + \hat{\mu}_{z^+,t}, \quad (2.63)$$

where $\bar{w}_t$ denotes the real wage at time t, defined as the nominal wage over the price of domestic intermediate homogeneous goods. Note that $\bar{w}_t$ is also assumed to grow with the productivity in the economy, which explains the occurrence of the term $\mu_{z^+,t}$ in the above equation.

2.3 Monetary and fiscal authorities

We assume that monetary policy is conducted by setting the policy rate according to a rule. The policy maker is assumed to adjust the short-run interest rate in response to deviations of CPI inflation from the inflation target and deviations of the unemployment rate from its long-run (steady-state) value. Since quarterly inflation is a rather volatile variable we choose to instead include annual inflation in the rule. We choose the unemployment rate as our measure of resource utilization following the reasoning in Adolfson et al. (2013) that, unlike the output gap, unemployment is an observed variable and judgments of it can thus directly affect monetary policy.[44] The policy maker also takes into account the rate of change in inflation and in unemployment. We allow for interest rate smoothing by including the lagged interest rate as an argument in the rule. Monetary policy is thus determined by the following rule:

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\left[\widehat{\bar{\pi}}_t^c + r_\pi\left(\hat{\pi}_{t-1}^{c,a} - \widehat{\bar{\pi}}_t^c\right) + r_{RU}\hat{U}_{t-1}\right] + r_{\Delta\pi}\Delta\hat{\pi}_t^c + r_{\Delta RU}\Delta\hat{U}_t + \hat{\varepsilon}_{R,t}, \quad (2.64)$$

where $\hat{R}_t$ is the short-term interest rate in deviation from a time-varying trend, as further explained below, $\pi_t^{c,a}$ the annual CPI inflation rate, U_t the unemployment rate, and $\hat{\varepsilon}_{R,t}$ an interest rate shock.[45]

[43] Recall that the natural rate of unemployment is defined as $\hat{U}_t^n = \frac{1}{\varphi}\hat{\lambda}_t^w$. We note that, in our model (and in Galí, Smets, and Wouters (2012)), unlike in Christiano, Trabandt, and Walentin (2011), Adolfson et al. (2013) and Erceg, Henderson, and Levin (2000)), the wage Phillips curve includes only shocks to the wage markup and not preference (labour supply) shocks, which allows us to separately identify both of these shocks.

[44] Adolfson et al. (2013) include hours worked in their Taylor rule. Due to a different labour market setup in our model, we do not observe variations in hours but focus on the extensive margin instead, including employment and unemployment as observables.

[45] The annual inflation rate is given by the following moving average:

$$\hat{\pi}_t^{c,a} = \frac{1}{4}\left[\hat{\pi}_t^c + \hat{\pi}_{t-1}^c + \hat{\pi}_{t-2}^c + \hat{\pi}_{t-3}^c\right].$$

We include the annual inflation rate in the policy rule as a smoother alternative to the (very volatile) quarterly rate.

Following Adolfson et al. (2013) the policy rule incorporates the inflation trend shock, $\bar{\pi}_t^c$, which was introduced in Section 2.1.1. [46] Empirical evidence on the inflation trend shock is reported in Section 3.8.2. While there is strong support for the incorporation of the time-varying inflation trend in the indexation rule applied by the firms that do not re-optimise their prices there is no clear guidance from marginal likelihood comparisons on whether to include the trend also in the policy rule.

Government consumption expenditures are assumed to follow an exogenous stochastic process, described in Section 2.6.

2.3.1 Time-varying neutral rate

In the past 20-30 years nominal interest rates have trended down globally. The decline in nominal rates is mainly associated with a decline in real interest rates, as there is no apparent (downward) trend in inflation expectations in this period.[47] The structural decline in real rates has been attributed to demographic factors, low trend productivity growth and an increased demand for safe assets; see e.g. Rachel and Smith (2015).

The standard assumption in DSGE models is that the relevant interest rate gap, $\hat{R}_t$, (and the real interest rate gap, $\hat{\bar{R}}_t$) is expressed in deviation from a time-invariant steady state, i.e. $\hat{R}_t = \log R_t - \log R$ (just as is the case for many other variables in the model). Motivated by the empirical observations above, here we instead assume that the agents in the economy, including the central bank, consider deviations of the interest rate from a time varying trend, $\hat{R}_t^t$. The policy rate gap is therefore defined as

$$\hat{R}_t = \hat{R}_t^{dev,ss} - \hat{R}_t^t, \tag{2.65}$$

where $\hat{R}_t^{dev,ss} = \log R_t - \log R$ and where the nominal policy rate trend, $\hat{R}_t^t$, is assumed to depend on a real interest rate trend, $\hat{\bar{R}}_t^t$, and a component intended to capture shifts in (long-run) inflation expectations, $\hat{\bar{\pi}}_t^t$, as follows:

$$\hat{R}_t^t = \hat{\bar{R}}_t^t + \hat{\bar{\pi}}_t^t. \tag{2.66}$$

We alternately refer to $\hat{R}_t^t$ and $\hat{\bar{R}}_t^t$ as the 'trend interest rates' or 'neutral rates'. Simplifying the monetary policy reaction function in equation (2.64) by assuming $\rho_R = 0$ (no interest rate smoothing), $r_{\Delta\pi} = r_{\Delta RU} = 0$ (no response to changes in inflation and unemployment) and $\hat{\bar{\pi}}_t^c = 0$ (no inflation trend) the rule can be written as

$$\hat{R}_t = \hat{R}_t^{dev,ss} - \hat{R}_t^t = r_\pi \hat{\pi}_{t-1}^{c,a} + r_{RU} \hat{U}_{t-1} + \hat{\varepsilon}_{R,t}. \tag{2.67}$$

This simplified expression illustrates directly that the neutral policy rate, $\hat{R}_t^t$, can be interpreted as the rate that is neither expansionary ($\hat{\varepsilon}_{R,t} < 0$) nor contractionary ($\hat{\varepsilon}_{R,t} > 0$) when the economy operates near its potential, i.e. when $\hat{\pi}_{t-1}^{c,a} \approx 0$ and $\hat{U}_{t-1} \approx 0$.[48]

We specify a simple model for $\hat{R}_t^t$ which harbours two, empirically motivated, simplifying assumptions. First, we assume that the real interest rate trend, $\hat{\bar{R}}_t^t$, is almost exclusively determined by global factors, as described further below. Sweden is a small economy with open capital markets, implying that Swedish real interest rates are to a large extent determined by the global supply and demand

[46] An alternative would be to exclude the trend shock from the rule which would imply that the central bank reacts more strongly to inflation trend shocks. Note that $\hat{\bar{\pi}}_t^c + r_\pi \left(\hat{\pi}_{t-1}^{c,a} - \hat{\bar{\pi}}_t^c\right) = r_\pi \hat{\pi}_{t-1}^{c,a} - (r_\pi - 1)\, \hat{\bar{\pi}}_t^c$ such that the difference between the two rules is given by the term $(r_\pi - 1)\, \hat{\bar{\pi}}_t^c$. Note also that the inflation response coefficient satisfies $r_\pi > 1$.

[47] Inflation increased globally in the 1970s (the great inflation) and decreased in the 1980s and beginning of the 1990s (the great disinflation) but has been stationary in Sweden in the inflation targeting period since the mid-90s. Inflation expectations have also been stable, and close to the inflation target.

[48] Note that the terminology 'expansionary'/ 'contractionary' is used in a model-specific context here. An expansionary (contractionary) policy means that the policy rate is set lower (higher) than what is suggested by the systematic part of the policy rule. Obviously, different models and perspectives can lead to different conclusions on whether policy is 'expansionary' or 'contractionary' at a given point in time.

of savings. This assumption is further supported by the strong dependence of Swedish interest rates on foreign interest rates (see Section 3.2). Second, we take the view that de-anchoring of inflation expectations has not played a key role in the trend decline in nominal interest rates and assume $\widehat{\bar{\pi}}_t^t = 0$.[49]

Following standard theory the real interest rate trend is allowed to depend on global technology growth, $\mu_{z^+,t}$. We also allow for a dependence on the risk premium shock, χ_t. The latter is intended to capture the effects of shifts in the demand for safe assets on policy rates, and could also be viewed as an attempt to endogenize part of the variation in the real neutral rate, which would otherwise be attributed to exogenous factors. The resulting specification of the real interest rate trend is provided by

$$\widehat{\bar{R}}_t^t = r_{\mu_{z^+}} \hat{\mu}_{z^+,t} - r_\chi \hat{\chi}_t + \hat{z}_t^R, \tag{2.68}$$

where z_t^R is a shock to the real interest rate trend which is included to capture non-modelled factors, e.g. effects of demographic changes on the real interest rate. An innovation to $\hat{z}_t^R$ affects the policy rate but no other variables in the model since it does not affect the policy rate gap. This specification of the real neutral rate is similar to its modelling in semi-structural models aimed to provide estimates of the neutral interest rate; see e.g. Laubach and Williams (2015), Holston, Laubach, and Williams (2016) and Pescatori and Turunen (2015). The modelling of foreign monetary policy is identical to the modelling of domestic monetary policy; see Section 2.5 below. Since it is assumed that the global technology growth shock and the shock to the real interest rate trend are common to the foreign and Swedish economies , i.e. $\hat{\mu}_{z^+,t} = \hat{\mu}^*_{z^+,t}$ and $\hat{z}_t^R = \hat{z}_t^{R,*}$ (see further the discussion in Sections 2.6 and 3.3), the domestic real interest rate trend is largely determined by global factors. It thus approximately holds that $\widehat{\bar{R}}_t^t \approx \widehat{\bar{R}}_t^{t,*}$, where $\widehat{\bar{R}}_t^{t,*}$ is the foreign/global real interest rate trend.[50]

While Holston, Laubach, and Williams (2016) assume that the real neutral rate contains a unit root, here it is assumed to be stationary and the steady-state values of the real interest rate trend and the actual real interest rate are assumed to be identical.[51] Hence the steady states of the nominal neutral rate and the actual policy rates are equal, i.e. $R^t = R$. More fundamentally our assumption implies that the policy rate eventually reverts back to a long-run, or steady-state, level while the unit root specification of Holston, Laubach, and Williams (2016) implies random walk forecasts of the neutral rate. The possible dependence on the risk premium, χ_t, also reflects that, while the central bank determines the policy rate, it ultimately aims to influence the interest rates faced by households and firms. In estimating the model we let the data determine whether this dependence is warranted or not.

The interest rate trend is an unobserved variable and in estimating the model we do not provide 'external data' to directly identify the trend.[52] Instead the process for $\widehat{\bar{R}}_t^t$ is estimated jointly with the cyclical component, $\widehat{\bar{\bar{R}}}_t$, and its identification is based mainly on data on foreign and Swedish resource utilization, inflation and policy rates, i.e. the variables which enter the policy rules. This could be considered a type of 'indirect inference' and it is described in a simple and intuitive way by e.g. Taylor and Wieland (2016).

Finally, one may note that our specification of the real interest rate trend nests the more 'standard' specification with a constant intercept in the monetary policy rule (equal to the long-run, or steady-

[49] An alternative would be to assume $\widehat{\bar{\pi}}_t^t = \widehat{\bar{\pi}}_t^c$, but this assumption results in a downward trending, and therefore counterfactual, estimate of the inflation expectations component, $\widehat{\bar{\pi}}_t^t \left(= \widehat{\bar{\pi}}_t^c\right)$. Since we do not use data on inflation expectations in estimation to directly identify $\widehat{\bar{\pi}}_t^t$, it appears instead to be indirectly identified by the policy rates.

[50] The dependence on the risk premium shock introduces a difference between the Swedish and foreign real interest rate trends but, as will made be clear in the empirical part of the paper, the domestic and foreign risk premium shocks are strongly correlated, reflecting a strong co-movement of foreign and Swedish spreads in data.

[51] In the DSGE model steady state, $\bar{R}^t = \bar{R} = \mu_{z^+}/\beta$, and is thus roughly equal to the steady-state rate of technological growth.

[52] The interest rate trend is often estimated using small semi-structural models; see e.g. the references in the text, as well as Armelius, Solberger, and Spånberg (2018) for an estimate of the Swedish neutral rate. It is, however, difficult to motivate the use of such estimates as data input to our model.

state level of the policy rate, R), i.e. a rule where $\hat{R}_t^t = 0$ such that $\hat{R}_t = \hat{R}_t^{dev,ss}$.[53] In the empirical part of the paper, we compare the two specifications and show that there is very strong support for the time-varying trend specification described above.

2.4 Market clearing conditions

The aggregate resource constraint, equalising the uses of the domestic homogeneous good to total production, is given by the following equation:

$$\hat{y}_t = \frac{g}{y}\hat{g}_t + \frac{c^d}{y}\hat{c}_t^d + \frac{c^{e,d}}{y}\hat{c}_t^{e,d} + \frac{i^d}{y}\hat{i}_t^d + \frac{x^d}{y}\hat{x}_t^d. \tag{2.69}$$

Output is used as input in the production of aggregate consumption of non-energy and energy goods, c_t^d and $c_t^{e,d}$, respectively, as input in the production of aggregate investment, i_t^d, as well as input in the production of the aggregate export good, x_t^d. It is also absorbed by government consumption, g_t.

From the production function of the producers of intermediate domestic goods, we in turn get the following expression of total production from the supply side:

$$\hat{y}_t = \left(1 + \frac{\phi^d}{y}\right)\left[\hat{\epsilon}_t + \alpha\left(\hat{k}_t - \hat{\mu}_{\Psi,t} - \hat{\mu}_{z^+,t}\right) + (1-\alpha)\hat{N}_t\right]. \tag{2.70}$$

Using equations (2.5) and (2.6), we obtain the following alternative expression to equation (2.70):

$$\hat{y}_t = \left(1 + \frac{\phi^d}{y}\right)\left[\widehat{mpl}_t + \hat{N}_t\right], \tag{2.71}$$

which shows how the deviations of output, productivity, and employment from their steady states are related.

We further need to specify an expression linking net exports and the current account. Expenses on imports and net new purchases of foreign assets must equal income from exports and from previously purchased net foreign assets. Focusing first on the expenses on imports, we note that the relevant measure here is the total value of all imports that cross the border, that is the marginal cost times the gross imports entering the domestic economy. There is an important distinction between the value of the imports crossing the border and the value of the imports used in the domestic economy, as the latter is higher due to positive markups from the monopolistic importing firms. This explains the inclusion of fixed costs for the import producing firms.[54] Moving on to the receipts from exports, we note again that the relevant measure is the value of all exports that cross the border. In other words, we include in the equation what remains of exports once the fixed costs of production are covered times the aggregate export price. The evolution of net foreign assets is thus given by the following expression:

$$\breve{a}_t + \widehat{\text{(real) expenses on imports}}_t = \widehat{\text{(real) receipts from exports}}_t + \frac{1}{\beta}\breve{a}_{t-1}, \tag{2.72}$$

where $\breve{a}_t$ denotes the real aggregate net foreign asset position. With the expressions for expenses on imports and receipts from exports substituted in, we get

$$\begin{aligned}
& \breve{a}_t + \tilde{q}R^{wc,m}\left(c^m\hat{c}_t^m + i^m\hat{i}_t^m + x^m\hat{x}_t^m + p^{ce,*}c^{e,m}\hat{c}_t^{e,m}\right) \\
& +p^x\tilde{q}\left(x - \phi^x\right)\hat{R}_t^{wc,m} + p^{ce,*}\tilde{q}R^{wc,m}\left(c^{e,m} + \phi^{m,ce}\right)\hat{p}_t^{ce,*} \\
= \quad & p^x\tilde{q}\left[x\hat{x}_t + \left(x - \phi^x\right)\hat{p}_t^x\right] + \frac{1}{\beta}\breve{a}_{t-1},
\end{aligned} \tag{2.73}$$

[53] Note that $r_{\mu_{z^+}} = r_\chi = 0$ and $\hat{z}_t^R = 0$ together imply $\hat{\bar{R}}_t^t = 0$.

[54] In the Riksbank's earlier models (see Adolfson et al. (2005) and Adolfson et al. (2013)), this distinction was not made, why the implications for net exports from the GDP identity and from the evolution of net foreign assets were not consistent. This, in turn, implied that the consumption-to-output ratios were not in line with data.

where $\tilde{q} \equiv qp^c/p^{c,*}$ and q denotes the steady-state value of the real exchange rate, and $\phi^{m,ce}$ and ϕ^x denote the fixed costs of the imported intermediate consumption goods producers and export goods producers, respectively.[55] We note that the assumption that firms in the import sector finance part of their input costs in advance gives rise to the inclusion of the term $R_t^{wc,m}$ in the above equation. Also, as the foreign energy price is possibly different from the price of the foreign homogeneous good, expenses on imports (and thereby also net foreign assets) are a function also of the relative price $p_t^{ce,*}$.

2.5 Foreign economy

We model the foreign economy as a structural model. We thus depart from the modelling strategies in e.g. Adolfson et al. (2007) and Adolfson et al. (2013) where the foreign economy was represented by a small VAR model. There are several benefits of the structural modelling approach for the foreign economy. It allows us to use the model as a storytelling device when it comes to foreign economic developments to a larger degree than a VAR model does. It enables the identification of a larger number of foreign structural shocks. As we further discuss in Section 3.2.1 below, we also include a larger set of foreign observables compared to many other small open economy models. Moreover, and most importantly for our purposes, it opens up for a more thorough analysis of international spillovers. As discussed in the introduction to our paper, it is an established fact in the academic literature that small open economy DSGE models — that is, models of the class that our model belongs to — have a hard time generating the international comovement we observe in data. In order to understand why and have a chance of improving the degree of spillovers through existing channels or through introducing additional spillover channels, we need a more detailed model of the foreign economy. The structural model setup has enabled us to do both: we improve on spillovers through trade as well as introduce new transmission channels through the assumption of correlated shocks.

The structure of the foreign economy is analogous to that of the domestic economy. All functional forms (utility function, production technologies, various costs) are unchanged. Nonetheless, given our small open-economy assumption, the influence of the domestic economy is negligible, and the foreign economy is thus approximately closed. Most of this block's equilibrium conditions are identical to those of the domestic block.[56] The structure of the foreign economy's corporate sector is analogous to that of the domestic economy's homogeneous goods sector. Final consumption and investment goods are produced by foreign, competitive, representative firms using the same production technologies as the domestic final goods producers, but with the shares of imported inputs set to zero. The household problem in the foreign economy is very similar to the domestic economy one, but for the following exception: the small open-economy assumption implies that foreign households do not have access to the bond market of the small-open economy. The wage setting problem is analogous to the domestic one. We assume that foreign monetary policy is also conducted according to an instrument rule, which is analogous to the one in the domestic economy above. Finally, we note that the market clearing conditions consist of the resource constraint from the user and supply side, where the foreign homogeneous good is allocated to the same uses as in the domestic economy except for the production of exports, due to the assumption of the foreign economy being closed.

We thus do not discuss in detail the full set of equations pertaining to the foreign-economy model in this section, but only list the equations below. More detailed derivations can be found in the Technical Appendix.

[55] Note that the net foreign asset position is zero in steady state, why level (rather than log) deviations must be used for deviations of $\bar{a}_t = \frac{S_t B_{t+1}^F}{P_t^d z_t^+}$ from steady state. We have therefore defined $\breve{a}_t = \bar{a}_t - \bar{a}$.

[56] The foreign economy is very close to the model in Smets and Wouters (2003), on which the Riksbank's first DSGE model, Ramses, also was based (see Adolfson et al. (2005) and Adolfson et al. (2007)).

2.5.1 Summary of the foreign-economy model equations

The foreign-economy model consists of equations corresponding to expressions (2.1), (2.3)–(2.9), (2.21)–(2.24), (2.26), (2.29), (2.30), (2.32), (2.40)–(2.42), (2.44), (2.45), (2.47)–(2.50), (2.54), (2.57), (2.59)–(2.61), (2.63), (2.64), (2.66), and (2.68)–(2.70) in the model of the domestic economy. Notation is consistent with the one in the domestic-economy model. Foreign-economy variables are distinguished from their domestic-economy counterparts by asterisks.

From the foreign intermediate goods firms' problem, we have the price Phillips curve (corresponding to equation (2.1) in the domestic-economy model)

$$\begin{aligned}\hat{\pi}_t^{d,*} - \widehat{\bar{\pi}}_t^{c,*} &= \Xi^* \left(\widehat{mc}_t^* + \hat{\lambda}_t^*\right) + \frac{\kappa^*}{1+\beta^*\kappa^*}\left(\hat{\pi}_{t-1}^{d,*} - \widehat{\bar{\pi}}_t^{c,*}\right) \\ &+ \frac{\beta^*}{1+\beta^*\kappa^*} E_t\left(\hat{\pi}_{t+1}^{d,*} - \widehat{\bar{\pi}}_{t+1}^{c,*}\right) - \frac{\beta^*\kappa^*}{1+\beta^*\kappa^*} E_t\left(\widehat{\bar{\pi}}_t^{c,*} - \widehat{\bar{\pi}}_{t+1}^{c,*}\right),\end{aligned} \tag{2.74}$$

where

$$\Xi^* \equiv \frac{(1-\beta^*\xi^*)(1-\xi^*)}{\xi^*(1+\beta^*\kappa^*)},$$

the expression for marginal costs (corresponding to equation (2.3) in the domestic-economy model)

$$\widehat{mc}_t^* = \widehat{\bar{w}}_t^* + \hat{R}_t^{wc,*} - \widehat{mpl}_t^*, \tag{2.75}$$

the gross effective nominal interest rate faced by the foreign firms (corresponding to equation (2.4) in the domestic-economy model)

$$\hat{R}_t^{wc,*} = \frac{\nu^{wc,*}R^*}{R^{wc,*}}\hat{R}_t^* + \frac{\nu^{wc,*}(R^*-1)}{R^{wc,*}}\hat{\nu}_t^{wc,*}, \tag{2.76}$$

the marginal product of labour (corresponding to equation (2.5) in the domestic-economy model)

$$\widehat{mpl}_t^* = \alpha^* \widehat{\left(\frac{k}{N}\right)}_t^* + \hat{\epsilon}_t^*, \tag{2.77}$$

the capital-labour ratio (corresponding to equation (2.6) in the domestic-economy model)

$$\widehat{\left(\frac{k}{N}\right)}_t^* = \hat{k}_t^* - \hat{N}_t^* - \left(\hat{\mu}_{z^{+,*},t} + \hat{\mu}_{\Psi^*,t}\right), \tag{2.78}$$

the evolution of the combination of investment-specific and neutral technology (corresponding to equation (2.7) in the domestic-economy model)

$$\hat{\mu}_{z^{+,*},t} = \frac{\alpha^*}{1-\alpha^*}\hat{\mu}_{\Psi^*,t} + \hat{\mu}_{z^*,t}, \tag{2.79}$$

the alternative expression for marginal costs determining the real rental rate of capital (corresponding to equation (2.8) in the domestic-economy model)

$$\widehat{mc}_t^* = \widehat{\bar{r}}_t^{k,*} - \widehat{mpk}_t^*, \tag{2.80}$$

and the marginal product of capital (corresponding to equation (2.9) in the domestic-economy model)

$$\widehat{mpk}_t^* = -(1-\alpha^*)\widehat{\left(\frac{k}{N}\right)}_t^* + \hat{\epsilon}_t^*. \tag{2.81}$$

From the final goods firms' problem, we have the expression for non-energy consumption demand (corresponding to equation (2.21) in the domestic-economy model)

$$\hat{c}_t^{xe,*} = \eta_e^*\hat{p}_t^{c,*} + \hat{c}_t^*, \tag{2.82}$$

and the expression for energy consumption demand (corresponding to equation (2.22) in the domestic-economy model)

$$\hat{c}_t^{e,*} = -\eta_e^* \left(\hat{p}_t^{ce,*} - \hat{p}_t^{c,*}\right) + \hat{c}_t^*. \tag{2.83}$$

We also include the expression for the CPI inflation (corresponding to equation (2.23) in the domestic-economy model)

$$\hat{\pi}_t^{c,*} = (1-\omega_e^*)\left(\frac{1}{p^{c,*}}\right)^{1-\eta_e^*} \hat{\pi}_t^{d,*} + \omega_e^* \left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*} \hat{\pi}_t^{ce,*}. \tag{2.84}$$

as well as the expressions for non-energy price inflation (corresponding to equation (2.24) in the domestic-economy model)

$$\hat{\pi}_t^{cxe,*} = \hat{\pi}_t^{d,*}. \tag{2.85}$$

Moreover, we need to include the expression of the relative prices of the aggregate consumption good (corresponding to equation (2.26) in the domestic-economy model)

$$\widehat{p}_t^{c,*} = \omega_e^* \left(\frac{p^{ce,*}}{p^{c,*}}\right)^{1-\eta_e^*} \widehat{p}_t^{ce,*}, \tag{2.86}$$

and energy (corresponding to equation (2.29) in the domestic-economy model)

$$\hat{p}_t^{ce,*} = \hat{p}_{t-1}^{ce,*} + \hat{\pi}_t^{ce,*} - \pi_t^{d,*}. \tag{2.87}$$

We further include the demand for domestic investment goods (corresponding to equation (2.30) in the domestic-economy model)

$$\hat{\imath}_t^{d,*} = \hat{\imath}_t^* + \frac{1}{i^*}\frac{\sigma_b^* k^{p,*}}{\mu_{z+,*}\mu_{\Psi^*}}\hat{u}_t^*. \tag{2.88}$$

and the relative price of the aggregate investment good (corresponding to equation (2.32) in the domestic-economy model)

$$\hat{p}_t^{i,*} = 0. \tag{2.89}$$

From the household problem, we have the endogenous preference shifter (corresponding to equation (2.40) in the domestic-economy model)

$$\hat{\Theta}_t^* = \hat{z}_t^{C,*} + \widehat{\upsilon}_t^{N,*}, \tag{2.90}$$

the evolution of trend consumption (corresponding to equation (2.41) in the domestic-economy model)

$$\hat{z}_t^{C,*} = (1-\nu^*)\left(\hat{z}_{t-1}^{C,*} - \hat{\mu}_{z+,*,t}\right) - \nu^* \widehat{\upsilon}_t^{N,*}, \tag{2.91}$$

and the marginal utility of consumption (corresponding to equation (2.42) in the domestic-economy model)

$$\begin{aligned} \widehat{\upsilon}_t^{N,*} \;=\; & \mu_{z+,*}\left(\mu_{z+,*}-b^*\right)\hat{\zeta}_t^{\beta,*} + \mu_{z+,*}\left(\mu_{z+,*}-b^*\right)\hat{\zeta}_t^{c,*} - \mu_{z+,*}^2\hat{c}_t^* + b^*\mu_{z+,*}\hat{c}_{t-1}^* - b^*\mu_{z+,*}\hat{\mu}_{z+,*,t} \\ & -\beta^* b^*\left[\left(\mu_{z+,*}-b^*\right)E_t\hat{\zeta}_{t+1}^{\beta,*} + \left(\mu_{z+,*}-b^*\right)E_t\hat{\zeta}_{t+1}^{c,*} - \mu_{z+,*}E_t\hat{c}_{t+1}^* + b^*\hat{c}_t^* - \mu_{z+,*}E_t\hat{\mu}_{z+,*,t+1}\right]. \end{aligned} \tag{2.92}$$

We also have the the unemployment rate (corresponding to equation (2.44) in the domestic-economy model)

$$\hat{U}_t^* = \hat{L}_t^* - \hat{N}_t^*, \tag{2.93}$$

the expression determining the labour participation rate (corresponding to equation (2.45) in the domestic-economy model)

$$\widehat{\overline{w}}_t^* - \hat{p}_t^{c,*} = \hat{\zeta}_t^{n,*} + \hat{\zeta}_t^{\beta,*} + \hat{z}_t^{C,*} + \varphi^*\hat{L}_t^*, \tag{2.94}$$

and the natural rate of unemployment (corresponding to equation (2.47) in the domestic-economy model)

$$\hat{U}_t^{n,*} = \frac{1}{\varphi^*}\hat{\lambda}_t^{w,*}. \tag{2.95}$$

We moreover include the relationship between efficient and physical capital (corresponding to equation (2.48) in the domestic-economy model)

$$\hat{k}_t^* = \hat{u}_t^* + \hat{k}_t^{p,*}, \tag{2.96}$$

the expression for the capital utilization (corresponding to equation (2.49) in the domestic-economy model)

$$\hat{p}_t^{i,*} = \widehat{\bar{r}}_t^{k,*} - \sigma_a^* \hat{u}_t^*, \tag{2.97}$$

the law of motion for capital (corresponding to equation (2.50) in the domestic-economy model)

$$\hat{k}_{t+1}^{p,*} = \frac{1-\delta^*}{\mu_{z+,*}\mu_{\Psi^*}}\left(\hat{k}_t^{p,*} - \hat{\mu}_{z+,*,t} - \hat{\mu}_{\Psi^*,t}\right) + \frac{\Upsilon^* i^*}{k^{p,*}}\left(\hat{\Upsilon}_t^* + \hat{i}_t^*\right), \tag{2.98}$$

the definition of the ex-ante real interest rate (corresponding to equation (2.54) in the domestic-economy model)

$$\widehat{\bar{R}}_t^* = \hat{R}_t^* - E_t\hat{\pi}_{t+1}^{c,*} \tag{2.99}$$

the log-linearized consumption Euler equation (corresponding to equation (2.57) in the domestic-economy model)

$$\widehat{\upsilon}_t^{N,*} = E_t\widehat{\upsilon}_{t+1}^{N,*} + \widehat{\bar{R}}_t^* - E_t\hat{\mu}_{z+,*,t+1} + \chi_t^*, \tag{2.100}$$

the first-order condition for investment (corresponding to equation (2.59) in the domestic-economy model)[57]

$$\widehat{\Delta i_t^*} = \beta^* E_t \widehat{\Delta i_{t+1}^*} + \frac{1}{\left(\mu_{z+,*}\mu_{\Psi^*}\right)^2 \tilde{S}''^{,*}}\left[\widehat{\breve{p}}_{k',t}^* + \hat{\Upsilon}_t^* - \frac{p^{i,*}}{\breve{p}_{k'}^*\Upsilon^*}\hat{p}_t^{i,*}\right], \tag{2.101}$$

where

$$\widehat{\Delta i_t^*} = \hat{i}_t^* - \hat{i}_{t-1}^* + \hat{\mu}_{z+,*,t} + \hat{\mu}_{\Psi,*,t},$$

and the first-order condition for capital (corresponding to equation (2.60) in the domestic-economy model)

$$\begin{aligned}\widehat{\breve{p}}_{k',t}^* \;=\; & \frac{\beta^*\left(1-\delta^*\right)}{\mu_{z+,*}\mu_{\Psi^*}} E_t\widehat{\breve{p}}_{k',t+1}^* + \frac{\beta^*\bar{r}^{k,*}}{\mu_{z+,*}\mu_{\Psi^*}\breve{p}_{k'}^*} E_t\left(\widehat{\bar{r}}_{t+1}^{k,*} + \hat{u}_{t+1}^*\right) \\ & - E_t\hat{\mu}_{\Psi^*,t+1} - E_t\left(\hat{R}_t^* - E_t\hat{\pi}_{t+1}^{d,*} + \chi_t^*\right).\end{aligned} \tag{2.102}$$

Finally, from the wage setting problem, we include the wage Phillips curve (corresponding to equation (2.61) in the domestic-economy model)

$$\begin{aligned}\hat{\pi}_t^{w,*} \;=\; & \widehat{\bar{\pi}}_t^{c,*} + \beta^* E_t\left(\hat{\pi}_{t+1}^{w,*} - \widehat{\bar{\pi}}_{t-1}^{c,*}\right) \\ & + \kappa_w^*\left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^{c,*}\right) - \beta^*\kappa_w^* E_t\left(\hat{\pi}_t^{c,*} - \widehat{\bar{\pi}}_{t+1}^{c,*}\right) - d_w^*\varphi^*\left(\hat{U}_t^* - \hat{U}_t^{n,*}\right),\end{aligned} \tag{2.103}$$

where

$$d_w^* \equiv \frac{\left(\lambda^{w,*}-1\right)\left(1-\beta^*\xi_w^*\right)\left(1-\xi_w^*\right)}{\xi_w^*\lambda^{w,*}\left(1+\varphi^*\right) - \xi_w^*},$$

[57] Note that the second derivative of the function S is a function of steady state variables only and therefore treated as a parameter. This explains why is has a superscript $*$, even though the function S itself doesn't.

and the expression for the wage inflation (corresponding to equation (2.63) in the domestic-economy model)

$$\hat{\pi}_t^{w,*} = \widehat{\bar{w}}_t^* - \widehat{\bar{w}}_{t-1}^* + \hat{\pi}_t^{d,*} + \hat{\mu}_{z+,*,t}. \tag{2.104}$$

We next need to include the central bank policy rule (corresponding to equation (2.64) in the domestic-economy model)

$$\hat{R}_t^* = \rho_{R^*}\hat{R}_{t-1}^* + (1-\rho_{R^*})\left[\widehat{\bar{\pi}}_t^{c,a,*} + r_{\pi^*}\left(\hat{\pi}_{t-1}^{c,*} - \widehat{\bar{\pi}}_t^{c,*}\right) + r_{RU^*}\hat{U}_{t-1}^*\right] + r_{\Delta\pi^*}\Delta\hat{\pi}_t^{c,*} + r_{\Delta RU^*}\Delta\hat{U}_t^* + \hat{\varepsilon}_{R^*,t}, \tag{2.105}$$

noting also that the foreign nominal neutral rate is given by the following expression (corresponding to equation (2.66) in the domestic-economy model)

$$\hat{R}_t^{t,*} = \widehat{\bar{R}}_t^{t,*} + \widehat{\bar{\pi}}_t^{t,*}, \tag{2.106}$$

where the real neutral rate evolves as follows (corresponding to equation (2.68) in the domestic-economy model)

$$\widehat{\bar{R}}_t^{t,*} = r_{\mu_{z+,*}}\hat{\mu}_{z+,*,t} - r_{\chi^*}\hat{\chi}_t^* + \hat{z}_t^{R,*}, \tag{2.107}$$

and the following two equations for the aggregate resource constraint (corresponding to equations (2.69) and (2.70), respectively, in the domestic-economy model):

$$\hat{y}_t^* = \frac{g^*}{y^*}\hat{g}_t^* + \frac{c^{xe,*}}{y^*}\hat{c}_t^{xe,*} + \frac{c^{e,*}}{y^*}\hat{c}_t^{e,*} + \frac{i^{d,*}}{y^*}\hat{\imath}_t^{d,*}, \tag{2.108}$$

and

$$\hat{y}_t^* = \left(1 + \frac{\phi^*}{y^*}\right)\left[\hat{\epsilon}_t^* + \alpha^*\left(\hat{k}_t^* - \hat{\mu}_{z+,*,t} - \hat{\mu}_{\Psi^*,t}\right) + (1-\alpha^*)\hat{N}_t^*\right]. \tag{2.109}$$

2.6 Shock processes

The model contains a large number of foreign and domestic shocks and most of these could be considered 'standard', i.e. they are commonly included in medium-scale DSGE models. The choice of the exact set of shocks to include in the baseline estimated model and the modelling of the shocks is to a large extent based on empirical considerations; see Section 3 where estimation results are discussed. The shocks and their stochastic processes in the baseline specification of the model are listed in Table 1. The baseline specification contains 29 exogenous shocks — 13 foreign and 16 domestic.

An important feature in increasing the influence of foreign economic developments on the Swedish economy is the introduction of dependencies between foreign and domestic shocks. The usual practice with DSGE models, including the previous DSGE models developed by the Riksbank, has been to assume independent shock processes, and typically AR(1) processes. Independence implies that the shocks can be given a structural interpretation and the usual assumption of AR(1) processes means that the shock may be persistent while the number of parameters in the model does not become too large. In other words, it is a relatively parsimonious specification.[58] However, given a structural model the independence assumption may deliver unreasonable implications for the endogenous variables. In our context, it is well-known that a standard small open-economy DSGE model without foreign-domestic shock dependencies cannot properly account for the substantial cross-country spillovers observed in the data; see e.g. Justiniano and Preston (2010). While ideally one would prefer to capture these comovements among variables endogenously, it has proven difficult to do so in an open-economy model setting.

Curdia and Reis (2010) discuss the modelling of the process for the vector of shocks at a general level and motivate the use of correlated shocks mainly as a tool to discover misspecification and to

[58] Simultaneous equation reduced form macro-econometric models usually allow for richer shock dynamics, both across time and across shocks.

Table 1: Shock processes in the baseline specification of the model

Shock		Stochastic process	Parameters
Foreign			
$\hat{\epsilon}^*_t$	Temporary technology	AR(1)	ρ_{ϵ^*}, σ_{ϵ^*}
$\hat{\Upsilon}^*_t$	Temporary marginal efficiency of inv.	AR(1)	ρ_{Υ^*}, σ_{Υ^*}
$\hat{\mu}_{z^*,t}$	Permanent technology	AR(1)	$\rho_{\mu_z^*}$, $\sigma_{\mu_z^*}$
$\hat{\lambda}^*_t$	Price markup	iid	σ_{λ^*}
$\hat{\lambda}^{w,*}_t$	Wage markup	ARMA(1,1)	$\rho_{\lambda^{w,*}}$, $\theta_{\lambda^{w,*}}$, $\sigma_{\lambda^{w,*}}$
$\hat{\zeta}^{c,*}_t$	Consumer preference	Correlated with $\hat{\Upsilon}^*_t$, eq. 2.110	$\rho_{\zeta^{c,*}}$, $\sigma_{\zeta^{c,*}}$, $c_{\zeta^{c,*},\Upsilon^*}$
$\hat{\zeta}^{n,*}_t$	Labour supply	AR(1)	$\rho_{\zeta^{n,*}}$, $\sigma_{\zeta^{n,*}}$
$\hat{\chi}^*_t$	Household risk premium	AR(1)	ρ_{χ^*}, σ_{χ^*}
$\hat{\varepsilon}^*_{R,t}$	Monetary policy	iid	$\sigma_{\varepsilon^*_R}$
$\hat{p}^{ce,*}_t$	Relative price of energy	AR(1)	$\rho_{p^{ce,*}}$, $\sigma_{p^{ce,*}}$
$\hat{\bar{\pi}}^*_t$	Inflation target/trend	AR(1)	$\rho_{\bar{\pi}^{c,*}}$ (cal), $\sigma_{\bar{\pi}^{c,*}}$
$\hat{g}^*_t$	Government consumption	AR(1)	ρ_{g^*}, σ_{g^*}
$\hat{z}^{R,*}_t$	Real interest rate trend	ARMA(1,1)	$\rho_{z^{R,*}}$, $\theta_{z^{R,*}}$, $\sigma_{z^{R,*}}$
$\hat{\mu}_{\Psi^*,t}$	Permanent inv spec techn.	Inactive	-
$\hat{\nu}^{wc,*}_t$	Work. cap. frac., firms	Inactive	-
$\hat{\zeta}^{\beta,*}_t$	Discount rate	Inactive	-
Domestic			
$\hat{\epsilon}_t$	Temporary technology	Correlated with $\hat{e}^*_t$, eq. 2.110	ρ_{ϵ}, σ_{ϵ}, c_{ϵ,ϵ^*}
$\hat{\Upsilon}_t$	Temporary marginal efficiency of inv.	Correlated with $\hat{\Upsilon}^*_t$, eq. 2.110	σ_{Υ}, c_{Υ,Υ^*}
$\hat{\mu}_{z,t}$	Permanent technology	Common, $\hat{\mu}_{z,t} = \hat{\mu}^*_{z,t}$	-
$\hat{\lambda}^d_t$	Domestic price markup	iid	σ_{λ^d}
$\hat{\lambda}^{m,c}_t$	Import consumption markup	iid	$\sigma_{\lambda^{m,c}}$
$\hat{\lambda}^{m,i}_t$	Import investment markup	Correlated with $\hat{\mu}^*_{z,t}$, eq. 2.110	$\sigma_{\lambda^{m,i}}$, $c_{\lambda^{m,i},-\mu_z^*}$
$\hat{\lambda}^{m,x}_t$	Import-in-export markup	iid	$\sigma_{\lambda^{m,x}}$
$\hat{\lambda}^x_t$	Export markup	iid	σ_{λ^x}
$\hat{\lambda}^w_t$	Wage markup	AR(1)	ρ_{λ^w}, σ_{λ^w}
$\hat{\zeta}^c_t$	Consumer preference	Correlated with $\hat{\zeta}^{c,*}_t$, eq. 2.110	σ_{ζ^c}, $c_{\zeta^c,\zeta^{c,*}}$
$\hat{\zeta}^n_t$	Labour supply	Correlated with $\hat{\zeta}^{n,*}_t$, eq. 2.110	ρ_{ζ^n}, σ_{ζ^n}, $c_{\zeta^n,\zeta^{n,*}}$
$\hat{\chi}_t$	Household risk premium	Correlated with $\hat{\chi}^*_t$, eq. 2.110	ρ_{χ}, σ_{χ}, c_{χ,χ^*}
$\hat{\tilde{\phi}}_t$	Country exch. rate. risk premium	Correlated with $\hat{\mu}^*_{z,t}$, eq. 2.110	$\rho_{\tilde{\phi}}$, $\sigma_{\tilde{\phi}}$, $c_{\tilde{\phi},-\mu_z^*}$
$\hat{\varepsilon}_{R,t}$	Monetary policy	iid	σ_{ε_R}
$\hat{\bar{\pi}}^c_t$	Inflation target/trend	AR(1)	$\rho_{\bar{\pi}^c}$ (cal), $\sigma_{\bar{\pi}^c}$
$\hat{p}^{d,ce}_t$	Relative price of energy	Correlated with $\hat{p}^{ce,*}_t$, eq. 2.110	$\rho_{p^{d,ce}}$, $\sigma_{p^{d,ce}}$, $c_{p^{d,ce},p^{ce,*}}$
$\hat{g}_t$	Government consumption	Correlated with $\hat{g}^*_t$, eq. 2.110	ρ_g, σ_g, c_{g,g^*}
$\hat{z}^R_t$	Real interest rate trend	Common, $\hat{z}^R_t = \hat{z}^{R,*}_t$	-
$\hat{\mu}_{\Psi,t}$	Permanent inv spec techn.	Inactive	-
$\hat{\nu}^{wc,d}_t$	Work. cap. frac., dom. goods firms	Inactive	-
$\hat{\nu}^{wc,m}_t$	Work. cap. frac., import firms	Inactive	-
$\hat{\nu}^{wc,x}_t$	Work. cap. frac., export firms	Inactive	-
$\hat{\zeta}^{\beta}_t$	Discount rate	Inactive	-

robustify inference. They consider a fairly general shock correlation structure — the process for the vector of shocks is described by an unrestricted VAR model and they also discuss special cases within this framework. Re-estimating the model of Smets and Wouters (2007) using i) independent shocks and ii) a VAR(1) model for the shocks, they find that most of the parameter estimates and the model fit (as judged by the marginal likelihood) are similar in the two specifications while forecast error variance decompositions differ. Other researchers have instead focused on *particular* shock dependencies, where one example is the relationship between productivity and government spending shocks.[59] We focus on foreign-domestic 'natural pairs' shock correlations in an open-economy setting and in this regard our paper is most closely related to the work of Justiniano and Preston (2010), and also De Walque et al. (2017) and Alpanda and Aysun (2014). In our context, the modelling of shocks should meet three objectives. First, introducing shock dependencies should meaningfully increase the spillovers from the foreign to the domestic economy in the model, i.e. it should help in better reflecting the strong cross-country correlations between Swedish and foreign variables. Second, it should be possible to provide some degree of interpretation of the shock dependencies, e.g. in terms of non-modelled data. And third, the modelling of shock dependencies should still be reasonably parsimonious, which implies that it cannot be entirely data-driven.

Regarding the interpretation of shock correlations, the basic idea is that many shocks appear to affect the foreign/global and Swedish economies in similar ways. Considering that Sweden is a small open economy and that we treat the foreign economy as the rest of the world, it is rather intuitive that, when a shock hits the rest of the world, Sweden should typically not be exempted. As an example, euro area and Swedish consumption growth are rather strongly correlated in the data. In DSGE models, a substantial fraction of the variability in consumption is typically attributed to consumer preference shocks. A positive preference shock increases households' current consumption and decreases their savings. If it is assumed in our model that the foreign and domestic consumer preference shocks are independent, smoothed estimates of the two shock processes turn out to be correlated nevertheless. This follows since the shock is a key driver of consumption in the respective economies and consumption growth rates in the two economies are correlated. Hence the model is misspecified, i.e. it lacks an internal mechanism that can explain the positive foreign-domestic consumption correlation in the data. Alternatively, if artificial data is simulated using the model assuming independent foreign and domestic preference shocks, the consumption correlation will be too low in comparison with what is observed in the data (in fact close to zero).

One interpretation of the consumer preference shock is that it captures the 'mood swings' of households — waves of optimism and pessimism — which are needed to explain variations in consumption beyond those accounted for by e.g. changing incomes and interest rates. DSGE model based estimates of the consumer preference shock are therefore presumably related to consumer confidence indicators (CCI). The correlation between euro area and Swedish CCIs is high. A likely explanation is that foreign and Swedish confidence are to a large extent influenced by the same news, i.e. Swedish consumer confidence is strongly affected by foreign economic developments. Allowing for a dependence of the Swedish consumer preference shock on the corresponding foreign shock in our model could be viewed as a shortcut to capture such dependencies in non-modelled data — in this case the consumer sentiment.

Similar motivations could be provided for other foreign-domestic shock pairs. The interdependence of national financial markets has increased over time and foreign and Swedish risk indicators, e.g. stock prices or bond spreads, are rather strongly correlated. In the model this is accounted for by a dependence of the domestic risk premium shock on the corresponding foreign shock. This assumption is similar to Alpanda and Aysun (2014) who assume that US and euro area credit spread (and also

[59] Smets and Wouters (2007) allow for contemporaneous correlation between the productivity and government spending shocks, a choice that is supported by the analysis of Curdia and Reis (2010). Chari, Kehoe, and McGrattan (2010) estimate a VAR model for the structural shocks of a DSGE model and find that most of the cross-correlations are large. Ireland (2004) estimates a small DSGE model using maximum likelihood assuming a VAR model for the observation errors.

net worth and monetary policy) innovations are correlated in a two-region DSGE model with a more elaborate financial sector, and find support for substantial cross-country financial shock correlations. In estimating our model we generally allow for correlations between most 'natural pairs' of foreign and domestic shocks and let the data determine whether a dependency is warranted or not.

A generic pair of *correlated shocks* are modelled using the submodel

$$\begin{cases} x_{1,t} = \rho_{11} x_{1,t-1} + s_{\varepsilon_1} \sigma_{11} \tilde{\varepsilon}_{1,t} \\ x_{2,t} = \rho_{21} x_{1,t} + \rho_{22} x_{2,t-1} + s_{\varepsilon_2} \sigma_{22} \tilde{\varepsilon}_{2,t} \end{cases}, \tag{2.110}$$

a simple dynamic structural model where $x_{1,t}$ and $x_{2,t}$ are the shocks and where $\tilde{\varepsilon}_{1,t}$ and $\tilde{\varepsilon}_{2,t}$ are independent and standard normally distributed innovations. The most common pair we consider is foreign ($x_{1,t}$) and domestic ($x_{2,t}$) shocks of the same type, e.g. the foreign and domestic consumer preference shocks, but we also allow for a few other pairs of correlated shocks, mainly on empirical grounds. The foreign shock affects the domestic shock via the parameter ρ_{21}. It is important to note that $\varepsilon_{1,t} = s_{\varepsilon_1} \sigma_{11} \tilde{\varepsilon}_{1,t}$ and $\varepsilon_{2,t} = s_{\varepsilon_2} \sigma_{22} \tilde{\varepsilon}_{2,t}$ are assumed to be independent which means they can be given structural interpretations as the foreign/global and domestic *innovations*, respectively.[60] The standard deviations of the innovations are given by $s_{\varepsilon_1}\sigma_{11}$ and $s_{\varepsilon_2}\sigma_{22}$ where σ_{11} and σ_{22} are estimated and where s_{ε_1} and s_{ε_2} are scaling parameters which are calibrated.[61] When $\rho_{21} \neq 0$, the domestic shock is affected by the foreign shock while if $\rho_{21} = 0$ the model reduces to the standard assumption of independent AR(1) shock processes. The submodel for the shocks for foreign-domestic pairs also embeds the small open economy assumption since $x_{1,t}$ is exogenous with respect to $x_{2,t}$. The parameters of the submodel are ρ_{11}, ρ_{22}, σ_{11}, σ_{22}, and ρ_{21}. When the model is taken to the data we estimate the first four of these parameters and the correlation between the two shocks $c_{x_2,x_1} = corr\,(x_{1,t}, x_{2,t})$ and then solve for the 'loading' parameter of the foreign shock onto the domestic shock, ρ_{21}. Our specification is somewhat more flexible than the one used by Justiniano and Preston (2010) since it contains one more parameter, i.e. the shock correlation is not a function of the other parameters. More technical details on the modelling of shocks and a discussion of alternative approaches is provided in a separate technical documentation.[62]

A more restrictive assumption is to assume that a shock is common to the foreign and Swedish economies, i.e. $x_{2,t} = x_{1,t}$. We refer to such shocks as *common shocks*. In the main specification of the model, we assume that there are two shocks that are common to the foreign and domestic economies: the technology shock, $\hat{\mu}_{z^*,t}$ and the real interest rate trend shock, $\hat{z}_t^{R,*}$. The assumption of a common global technology trend is quite standard and follows e.g. Christiano, Trabandt, and Walentin (2011). It captures the idea that long-run total factor productivity developments in Sweden should follow those in other advanced economies. The assumption of a common shock to the real interest rate trend is motivated by the fact that Sweden is a small open economy with free movement of capital which implies that long-term real interest rates are largely determined abroad. While our assumption of a common real interest rate trend for the foreign economy and Sweden may be viewed as extreme, it is, we believe, a simplification that broadly captures the global trend in interest rates in our sample covering the past 25 years.

For the remaining domestic shocks in the model we allow for global and local components through the specification in (2.110), and let the data determine the relative importance of the global component. We find empirical support for modelling two foreign shocks as ARMA(1,1) processes, the real interest rate trend shock and the wage markup shock. We note that the wage markup shock is modelled as an ARMA(1,1) process also by Smets and Wouters (2007) and Galí (2011b) where the MA term captures high frequency variation in wage growth. Further details on the specification and estimation of the shock processes is provided in Section 3 below.

[60] Note that the assumption of orthogonal innovations differs from De Walque et al. (2017) and Alpanda and Aysun (2014) who model correlated *innovations*. In order to identify the innovations Alpanda and Aysun (2014) assume e.g. that the financial shock innovation in the euro area has no contemporaneous effect on the US financial innovation, i.e. a Cholesky identification.

[61] The scaling parameters are inconsequential for the discussion here and their role is instead discussed in Section 3.5.

[62] The documentation of the modelling of shock dependencies is available upon request.

3 Taking the model to the data

The model is estimated using Bayesian methods with data for Sweden, the euro area and the United States. We use the Matlab package Dynare 4.5.0 to obtain all the results in this paper. In Section 3.1, we outline the method for taking the model to the data. These methods are, by now, fairly standard among macroeconomists and a reader familiar with the Bayesian econometrics of DSGE models may skip reading this section. In Section 3.2, the data series used for estimation are described and the properties of the data are summarised through a set of stylized facts. In Section 3.3, the trend assumptions in the model and how they relate to trends in the data are discussed. In Sections 3.4 and 3.5, the calibrations and prior distributions of the parameters in the model are discussed and motivated. Estimation results are presented in Section 3.6 and in Section 3.7 model fit is examined by comparing model-implied standard deviations and correlations of variables with those in the data. Finally, in Section 3.8 the baseline empirical specification of the model is assessed through marginal likelihood comparisons with alternative specifications.[63]

3.1 Bayesian estimation of DSGE models

3.1.1 The posterior distribution

The model parameters are estimated using Bayesian inference methods. In this section we review the Bayesian estimation procedure for DSGE models; see e.g. Herbst and Schorfheide (2015) for a more comprehensive review. The vector containing the model parameters, θ, is partitioned into two groups of parameters, calibrated and estimated parameters, θ_{cal} and θ_{est} respectively, and $y_{1:T} = (Y_1, ..., Y_T)$ denotes the data where Y_t is the vector of observed variables at time t. Calibrated parameters are parameters that are kept fixed throughout the estimation, i.e. the prior probability that they attain their assigned value equals one.[64] The objective is to characterise the posterior distribution of the estimated parameters provided by

$$p\left(\theta_{est}|y_{1:T}\right) = \frac{p\left(y_{1:T}|\theta_{est}\right) p\left(\theta_{est}\right)}{p\left(y_{1:T}\right)} \propto p\left(y_{1:T}|\theta_{est}\right) p\left(\theta_{est}\right), \tag{3.1}$$

where $p\left(y_{1:T}|\theta_{est}\right)$ is the likelihood function, $p(\theta_{est})$ is the prior density of the parameters and $p\left(y_{1:T}\right)$ is the marginal likelihood.[65] While the conditioning on the set of calibrated parameters, θ_{cal}, is not explicitly stated it should be implicitly understood. Once a representation of the posterior distribution of the parameters is available, it is straightforward to obtain probability distributions for other objects of interest, e.g. means and standard deviations of model variables, impulse responses etc.

Two modifications to the standard Bayesian estimation method are considered in this paper. The first alternative estimation method allows us to assess how Swedish data affects the estimation of foreign economy parameters when all parameters are estimated jointly using both foreign and Swedish data. The two-country DSGE model is based on the assumption that foreign variables are exogenous with respect to the domestic variables (the small open-economy assumption), which implies that domestic shocks are assumed by construction not to affect foreign variables. However, inference on foreign economy parameters based on the posterior, equation (3.1), are, in general, affected by the inclusion of Swedish data series in the dataset. This is perhaps most apparent for the parameters governing long-run productivity growth and the trend interest rate, i.e. the processes for the shocks that

[63] The new notation introduced in this chapter may, in a few instances, conflict with the notation in previous chapters. It should, however, be clear from the context what is intended.

[64] Note that the distinction between calibrated and estimated parameters is not always clear in the process of estimating a model, i.e. it may be decided based on preliminary estimations, hypothesis testing and model comparisons that a parameter should be calibrated in order to simplify the model.

[65] The density of the observables conditional on the parameters, $p\left(y_{1:T}|\theta_{est}\right)$, describes the model, e.g. for a given parameter vector, we can simulate data and compare the properties of the simulated data to actual data. Bayesian analysis instead interprets the likelihood function as a density function for the parameters given the data, i.e. given the data it asks which parameter vector is most likely to have generated the data.

are assumed to be common for the two economies. Therefore, it is of interest to also consider a conditional two-step estimation approach where, first, the foreign model parameters are estimated using only foreign data series and, second, the parameters of the domestic model are estimated conditional on the foreign model parameter estimates. By comparing the results from the joint and conditional estimation approaches it is then possible to assess to what extent the inclusion of Swedish data series affects the inference on foreign parameters in the joint estimation case.

Formally, we partition the estimated parameters, θ_{est}, into two groups containing the parameters of the foreign and domestic parts of the model, θ_{est}^{for} and θ_{est}^{dom}. In the first step, the foreign parameters are estimated using only foreign data, i.e. we characterise the marginal foreign parameters posterior distribution

$$p\left(\theta_{est}^{for}|y_{1:T}^{for}\right)=\frac{p\left(y_{1:T}^{for}|\theta_{est}^{for}\right)p\left(\theta_{est}^{for}\right)}{p\left(y_{1:T}^{for}\right)}. \tag{3.2}$$

In the second step, the domestic parameters are estimated conditional on the foreign posterior mode parameter estimate

$$p\left(\theta_{est}^{dom}|y_{1:T};\hat{\theta}_{est}^{for}\right)=\frac{p\left(y_{1:T}|\theta_{est}^{dom};\hat{\theta}_{est}^{for}\right)p\left(\theta_{est}^{dom}\right)}{p\left(y_{1:T}\right)}, \tag{3.3}$$

where $\hat{\theta}_{est}^{for}$ denotes the posterior mode of the foreign parameters. Henceforth, we refer to this procedure in short as *conditional estimation*. The first step corresponds to estimating the foreign, closed-economy part of the model in separation, as in e.g. Smets and Wouters (2003). Moreover, note that the procedure is similar to, but somewhat simpler than, the two-step estimation strategy outlined by Justiniano and Preston (2010).[66]

The second modification to the standard Bayesian estimation approach relates both to the main objective of our paper and, more generally, to how DSGE models are used in a policy context. It may be argued that it is more important for the model to capture certain aspects of the data, while other aspects are relatively less important. Examples of more important dimensions are cross-correlations between key Swedish and foreign variables, e.g. GDP growth, inflation and policy rates. Relatively less important dimensions could involve the various properties of some variables in the set of observables that may be considered 'non-core'.[67] Our approach consists of adding a penalty term to the objective function, thereby penalising parameter values which imply a large discrepancy between the variances and covariances of selected variables in the model and data. Since our focus is on cross-country relationships between variables we choose to target covariances of cross-country pairs of variables which are strongly correlated in the data. By contrasting the results from this estimation approach with the standard Bayesian approach we can assess e.g. whether the model-implied cross-country correlations can be brought closer to the data sample correlations.

The modified posterior density takes the form

$$\frac{p\left(\theta_{est}|y_{1:T}\right)}{f((S_m-S_d)^2\,|\theta_{est};y_{1:T})}, \tag{3.4}$$

where S_m is a vector containing a set of model-implied population second moments and S_d is the vector containing the corresponding data sample moments. Our approach generalizes the 'endogenous prior' approach by Christiano, Trabandt, and Walentin (2011) to allow for covariance matching in addition to the matching of variances. Since we use the sample (rather than pre-sample) data to compute the sample second moments in S_d, this approach is, strictly speaking, not Bayesian and it explains

[66] In the second step, these authors assign tight priors for the foreign parameters which are centered on the estimates obtained in the first step. They then estimate all parameters, foreign and domestic, in the second step. We instead calibrate (i.e. impose infinitely strict priors for) the foreign parameters in the second step.

[67] An example of such a variable could be capacity utilization, which is included in the set of observables to enable identification of the utilization cost parameter. Matching the statistical properties of this variable could be considered less relevant than matching the properties of, for example, GDP.

why we prefer to use the terminology 'penalty' instead of 'prior'. We generally choose to include the covariances of variables which are strongly correlated in our data sample. In our estimation, half of the matched covariances involve cross-country pairs of variables. The full set of variances and covariances matched in estimation are reported in the Appendix.

In summary we characterise i) the joint posterior distribution, ii) the marginal foreign posterior and conditional domestic posterior distributions, and iii) a modified posterior distribution. The objective here is not mainly to argue in favour of one of the estimation approaches, but rather to learn from the differences in the results produced. Standard Bayesian estimation of the joint posterior distribution is the baseline estimation approach. In Section 3.6.6 a brief discussion of the advantages and disadvantages of the respective estimation methods is provided.

The respective objective functions are optimized to obtain the parameter mode and the Metropolis-Hastings algorithm is used to sample from the posterior, or modified posterior, distribution.

3.1.2 State-space representation of the model

The equilibrium of a DSGE model is described by a set of nonlinear expectation equations, or by the corresponding (log-)linear approximations to these equations; see Section 2. The solution to these equations is given by a policy function

$$X_t = g\left(X_{t-1}, \epsilon_t; \theta\right), \tag{3.5}$$

which relates the vector of state variables, X_t, to its lagged value, a vector of innovations, ϵ_t, and the parameter vector, θ. The vector of state variables contains all the variables in the model, including the shocks. The policy function cannot typically be derived in closed form. This implies that numerical approximation methods must be used to obtain an approximation, $\hat{g}$, to g. The evaluation of the likelihood function, $p\left(y_{1:T}|\theta_{est}\right)$, for a DSGE model consists of two parts. First, given a parameter vector, θ, an approximation to the policy function is obtained (the solution) and represented as a state-space model. Second, Kalman (linear) or particle filtering (non-linear) methods are used to evaluate the likelihood function.

Several methods are available to solve DSGE models linearly, e.g. Anderson and Moore (1985), Sims (2002) and Klein (2000). In the (log-)linear approximation case, which is the standard case and the one considered in this paper, the resulting state-space representation of the DSGE model is

$$X_t = \hat{g}\left(X_{t-1}, \epsilon_t; \theta\right) = T\left(\theta\right) X_{t-1} + R\left(\theta\right) \epsilon_t, \tag{3.6}$$

and

$$Y_t = c_t + d\left(\theta\right) + Z X_t + v_t,\ t = 1, ..., T\ , \tag{3.7}$$

where (3.6) is the state equation, and (3.7) is the observation equation, which maps the data series to the variables in the model. Here X_t (dimension n_x) is the vector containing the state variables, in the econometric sense, and Y_t (dimension n_y) is a vector containing the observed variables. The parameters of the model are collected in the vector θ (dimension n_θ) and the coefficient matrices, T (which is typically dense) and R, and the vector d are nonlinear functions of θ. The matrix Z is a selector matrix which does not typically depend on θ. The innovations, ϵ_t (dimension n_ϵ), and the measurement errors, v_t (dimension n_v), are assumed to be independent and normally distributed, $\epsilon_t \backsim N\left(0, \Sigma_\epsilon\right)$ and $v_t \backsim N\left(0, \Sigma_v\right)$. We include the (possibly) time-varying parameter c_t in order to capture special assumptions related to the trends or levels of the observable variables; see Section 3.2. Henceforth, we will refer to $d\left(\theta\right)$ as the *model-implied steady state* of the observables while the term *steady state* will refer to $c_t + d\left(\theta\right)$.

The resulting model is therefore a particular type of linear and Gaussian state-space (LGSS) model. Importantly, what makes DSGE models 'special' is that the functions $T\left(\theta\right),\ R\left(\theta\right),\ d\left(\theta\right)$ are not available analytically (except in very special cases), but are obtained for a given θ by numerically solving for the rational expectations equilibrium. This implies that Gibbs sampling, which is otherwise the standard posterior sampling method for LGSS models, is not feasible in general for DSGE models.[68]

The distribution of the initial state vector, X_0, is typically assumed to be equal to the stationary distribution of the state vector $N(0, \Sigma_x)$, where Σ_x is the solution to the Lyapunov equation[69]

$$\Sigma_x = T\Sigma_x T^T + R\Sigma_\epsilon R^T. \tag{3.8}$$

Also note that Σ_x contains the model-implied population, or stationary distribution, variances and covariances which are used to construct the modified posterior.

3.1.3 The likelihood function

The likelihood function is given by

$$p(Y_{1:T}|\theta) = \prod_{t=1}^{T} p(Y_t|Y_{1:t-1};\theta) = \prod_{t=1}^{T} \int p(Y_t|X_t;\theta)p(X_t|Y_{1:t-1};\theta)dX_t, \tag{3.9}$$

where $Y_{1:T} = (Y_1, ..., Y_T)$ is the data. For the linear and Gaussian state-space model, the likelihood is evaluated using the prediction error decomposition and the Kalman filter; see e.g. Harvey (1989).

3.1.4 Prior distribution

The kernel of the posterior density of the parameter vector θ_{est} is given by

$$p(\theta_{est}|Y_{1:T}) \propto p(Y_{1:T}|\theta_{est})p(\theta_{est}), \tag{3.10}$$

where attention is usually restricted to the determinacy region of the parameter space, Θ_D (see Lubik and Schorfheide (2004) for an exception). The determinacy region is the subset of the parameter space where the model has a unique and stable solution.[70] Here, we interpret the restriction as being part of the formulation of the prior distribution, i.e. the prior density is truncated at the boundary of the indeterminacy region. The effective prior distribution is typically formulated as a set of marginal prior distributions:[71]

$$p(\theta_{est}) = I(\theta_{est} \in \Theta_D)\tilde{p}(\theta) = I(\theta_{est} \in \Theta_D)\Pi_{j=1}^{n_\theta} p_j(\theta_{est}^j), \tag{3.11}$$

where θ^j is the j^{th} element of the vector θ and p_j denotes its marginal, univariate, prior distribution.

There are essentially three categories of parameters in DSGE models: unbounded parameters, parameters which are bounded from below (or above), and parameters which are bounded both from below and above. In the DSGE literature, parameters belonging to these classes are typically given normal, gamma (or inverse gamma) and beta prior distributions, respectively, and we follow this practice. The marginal prior distributions of the estimated parameters, θ_{est}, are presented in Section 3.5 below.

[68] Gibbs sampling implies that one can sample directly from the conditional posterior distributions of blocks/subsets of parameters. It is, however, possible to use a block Metropolis sampler where a subset of the parameters, those related to the shock processes, are sampled using a Gibbs step; see e.g. Curdia and Reis (2010) for such a Gibbs-within-Metropolis algorithm.

[69] We use the doubling algorithm to solve the Lyapunov equation since it is faster than the standard method using Dynare; see the function lyapunov_symm.m in Dynare. Solving the equation analytically, i.e. $vec(\Sigma_x) = \left(I_{n_x^2} - T \otimes T\right)^{-1} vec\left(RQR^T\right)$, is not feasible in estimating a model of the size considered here. A computationally less expensive alternative is to let $\Sigma_x = diag\left(\Sigma_x\right)$ and assume that the variances are 'large', possibly combined with using a training sample.

[70] DSGE models typically admit an infinite number of explosive solutions.

[71] The marginal priors are not, strictly speaking, independent due to the indeterminacy constraint.

3.1.5 Optimizing the posterior density and sampling from it

The posterior density is non-standard, i.e. it is not a known functional form. The objective of sampling algorithms is to generate a sequence of draws, θ_i, $i = 1, ...R$, from the posterior kernel, $p(\theta_{est}) = p(\theta_{est}|Y_{1:T})$, where R is the length of the chain. These draws provide a *representation* of the posterior density. In the context of Bayesian estimation of DSGE models, the single-block random walk Metropolis (RWM) algorithm has been the preferred sampling method. In the RWM algorithm, a proposal, $\theta_{est,p}$, is generated using a symmetric proposal density, $q(.|\theta_{est,i})$, where $\theta_{est,i}$ is the current state of the chain. The proposal is then accepted with probability

$$\alpha_{i+1} = \min\left\{1, \frac{p(\theta_{est,p})}{p(\theta_{est,i})}\right\}. \tag{3.12}$$

The proposal distribution is typically chosen to be a normal distribution, $q(\theta_{est,p}|\theta_{est,i}) = N(\theta_{est,i}, \Sigma)$, with covariance matrix Σ proportional to the inverse of the Hessian at the posterior mode, $\theta_{est,m}$, $\Sigma = -\varkappa H_m^{-1}$, where $\varkappa > 0$ is a scaling parameter.[72] A crucial step in this approach is then the optimization of the posterior density, since the quality of the RWM sampler will rely on the quality of the estimated inverse Hessian at the posterior mode. Moreover, policy analysis and forecasting using central bank DSGE models is often conditioned on the posterior mode estimate, i.e. the uncertainty about parameters which is reflected in the posterior distribution is not always taken into account, with the implication that the optimization step is very important 'in its own right' and not merely as a prelude to sampling. More advanced sampling methods in the context of DSGE models are discussed by e.g. Chib and Ramamurthy (2010).

3.1.6 Model comparison

The DSGE model presented in Section 2 contains many features. In the empirical work our goal is to assess which of these features are important, or not important, for model fit and to contrast alternative model specifications. The main tool for model comparison is the difference between the log marginal likelihoods for two alternative model specifications, the log Bayes factor, provided by

$$\log BF_{12} = \log p_1\left(y_{1:T}\right) - \log p_2\left(y_{1:T}\right), \tag{3.13}$$

where BF_{12} denotes the Bayes factor. A positive value of $\log BF_{12}$ indicates support for model specification 1 relative to model specification 2 and larger positive values implies stronger support. However, for various reasons one needs to be careful in interpreting the Bayes factor and, if possible, it is preferable to frame the model comparison (or model selection) problem as Bayesian hypothesis testing on the model's parameters.[73] Scales for interpretation of the Bayes factor have been provided by e.g. Jeffreys (1961) and Kass and Raftery (1995). The latter authors provide the following guidelines: if $\log BF_{12}$ is in the range of 1-3 the evidence is 'positive', in the range of 3-5 it is 'strong', and above 5 it is 'decisive'.[74] In this paper, we broadly follow this scale.

[72] A good choice, based on the theory of optimal mixing, is often to let $\varkappa = 2.38/\sqrt{n_{\theta_{est}}}$ where $n_{\theta_{est}}$ is the number of estimated parameters.

[73] If an individual parameter θ^i is estimated a Baysian test of the hypothesis $\theta^i = \theta^{i,0}$ involves checking whether $\theta^{i,0}$ is included in the, say, a 95% posterior probability interval of the parameter. A necessary condition for this to be valid is that there is positive prior probability mass on the point $\theta^{i,0}$ since otherwise the posterior probability mass will be zero by construction. In DSGE models extreme values of parameters are often assigned zero prior probability. In these instances the fact that the extreme value is outside the posterior probability interval cannot be used to conclude that the extreme value should be rejected.

[74] The corresponding ranges for the Bayes factor BF_{12} provided by these authors are $3-20$ (positive), $20-150$ (strong) and > 150 (very strong).

3.2 Data

3.2.1 Observables

The model is estimated on data for Sweden, the euro area and the United States. The variables in the model, collected in the state vector X_t, are related to their observed counterparts, Y_t, through the observation equation (3.7). The sample period begins shortly after inflation targeting was introduced in Sweden, in 1995Q2, and ends in 2018Q4. A total of $n_y = 25$ data series are used for estimation — 10 foreign data series and 15 Swedish data series. The foreign data series are constructed by weighting data for the euro area and the United States, according to Swedish trade weights. While the trade weights vary slightly across time, the weight on the euro area is close to 85% throughout the sample period, and hence the weight on the US is around 15%. We will refer to this weighted aggregate of euro area and US data as the KIX20 aggregate.[75] The Riksbank usually considers a broader representation of the trade-weighted foreign economy, including 32 countries (KIX32).[76] There are two main reasons why we have opted for a narrower representation, including 20 countries. The first concerns the availability of reliable data on all of the 10 observed foreign variables. The broader representation, KIX32, usually focuses on a smaller set of key variables, i.e. GDP, inflation and the policy rate. Second, since the KIX20 is 'nested' within the broader KIX32, the properties of the data series are quite similar. For example, the correlations between narrow (KIX20) and broad (KIX32) foreign data series are usually large and typically above 0.95, which suggests that the exact choice of data representation of the foreign economy is perhaps not a key issue.

Where relevant, the data series have been seasonally adjusted. The series are displayed in Figures 1, 2 and 3, and some descriptive statistics are collected in Table 2. The table also includes the steady-state values of the observables to facilitate a comparison with the data sample mean. A more complete description of the data series, including definitions, sources and treatment of outliers, and the observation equations for all of the observed variables, is provided in the Appendix.

The inclusion of a stochastic unit root technology shock in the model allows us to use data series that have not been pre-processed (detrended). The majority of the data series involve only basic transformations of the raw data series to make the series stationary, e.g. the foreign and Swedish real GDP series are divided by population size to yield per capita terms and then transformed into annualised quarterly growth rates. In Table 2 the transformations applied to the data series are reported. Here, we briefly comment on those series where *particular assumptions are made in constructing the data series*, i.e. where the data has been pre-processed in some way.

The foreign, KIX20-weighted, employment rate series displays a trend, stemming mainly from an upward trend in the euro area employment rate in the sample period. We detrend the foreign employment rate using a piece-wise linear trend, assuming a trend annual growth rate of 0.3 percent until the end of 2008 and 0.15 percent thereafter.[77] Similarly, the Swedish employment rate displays an upward trend in the latter part of the sample, e.g. due to various economic reforms introduced around 2006, which increased the incentives to participate in the labour force. Here, we have chosen to detrend the series using the trend measure that is used in forecasting and policy work at the Riksbank, obtained with a demographic model for labour market variables, KAMEL; see NIER (2012) for a description of the model. For both foreign and Swedish employment, the data series used for estimation are therefore employment (or equivalently employment rate) gaps, $N_t^{*,obs,gap}$ and $N_t^{obs,gap}$, measured as percentage deviations from trend.

Swedish aggregate price inflation is measured using CPIF inflation, which is the CPI inflation with mortgage rates held fixed. The CPIF has been the Riksbank's operational target variable for several years and as of 2017 also the formal inflation target variable for monetary policy. The measure of price changes for imported consumer goods excluding energy, $\pi_t^{m,cxe,obs}$, is constructed based on an

[75] The KIX (krona index) refers to an aggregate of countries that are important for Sweden's international transactions.

[76] The weight of the euro area in the KIX32 index is 47.6 percent in 2018 and the US weight is 8.5 percent.

[77] Coenen et al. (2018) make a similar assumption. They detrend the euro area employment rate using a linear trend consistent with an annual increase in the labour force of 0.3%.

Table 2: Data series used in estimation. Basic statistics and model steady state.

Variable	Transf. and unit	Data: statistic and sample period Mean 95Q2-18Q4	Mean 95Q2-08Q2	Mean 08Q3-18Q4	Std 95Q2-18Q4	ADF 95Q2-18Q4	Model Steady state
Foreign (KIX20)							
Consumption	Per cap, 4qq, perc	1.2	1.7	0.6	1.5	R,1	1.3
CPI	4qq, perc	1.8	2.2	1.3	1.3	R,1	2.0
CPI excluding energy	4qq, perc	1.7	1.9	1.3	0.7	R,1	2.0
Employment	Per cap, perc	−1.6	−0.5	−2.9	1.9	A,10	0.0
GDP	Per cap, 4qq, perc	1.3	1.8	0.7	2.1	R,1	1.3
Investment	Per cap, 4qq, perc	1.5	2.6	0.0	5.2	R,1	1.3
Policy rate	Perc	2.2	3.7	0.3	2.0	A,10	3.0
Corporate spread	Perc points	1.8	1.4	2.2	0.5	A,10	1.8
Unemployment rate	Perc	9.0	8.5	9.7	1.2	A,10	8.0
Wage	4qq, perc	2.2	2.6	1.7	0.9	R,1	2.5
Sweden							
Capacity utilisation	Perc	83.8	85.3	82.0	3.4	A,10	85.0
Consumption	Per cap, 4qq, perc	1.7	2.2	1.1	2.8	R,1	1.7
CPIF	4qq, perc	1.5	1.7	1.4	1.2	R,1	2.0
CPIF excl. energy	4qq, perc	1.3	1.3	1.3	0.9	R,1	2.0
CPIF imp. excl. energy	4qq, perc	−0.4	−0.4	−0.4	1.9	R,1	0.0
Employment	Per cap, perc	−0.8	−0.8	−0.7	1.9	A,10	0.0
Exports	Per cap, 4qq, perc	2.7	3.8	1.4	9.6	R,1	2.7
GDP	Per cap, 4qq, perc	1.8	2.5	0.9	3.6	R,1	1.7
Imports	Per cap, 4qq, perc	2.7	3.4	1.7	9.1	R,1	2.7
Investment	Per cap, 4qq, perc	2.8	4.1	1.2	10.6	R,1	2.5
Real exchange rate	4qq, perc	0.7	0.3	1.3	9.8	R,1	0.0
Policy rate	Perc	2.4	3.8	0.6	2.2	A,10	3.0
Corporate spread	Perc points	1.8	1.5	2.1	0.4	A,10	1.8
Unemployment rate	Perc	7.6	7.7	7.5	1.4	A,10	7.2
Wage	4qq, perc	3.3	3.8	2.6	1.1	R,1	3.7

Per cap = per capita. Perc = percent. 4qq=annualised quarterly change. The null hypothesis of the augemented Dickey-Fuller (ADF) test is H_0: the data series contains a unit root. 'A' denotes that H_0 is not rejected and 'R' denotes that it is rejected at the stated significance level, which is expressed in percent.

Figure 1: Data: GDP and components of GDP. Annualised quarterly growth (aqg). Units: see Table 2. Swedish data in blue and KIX20-weighted data in red.

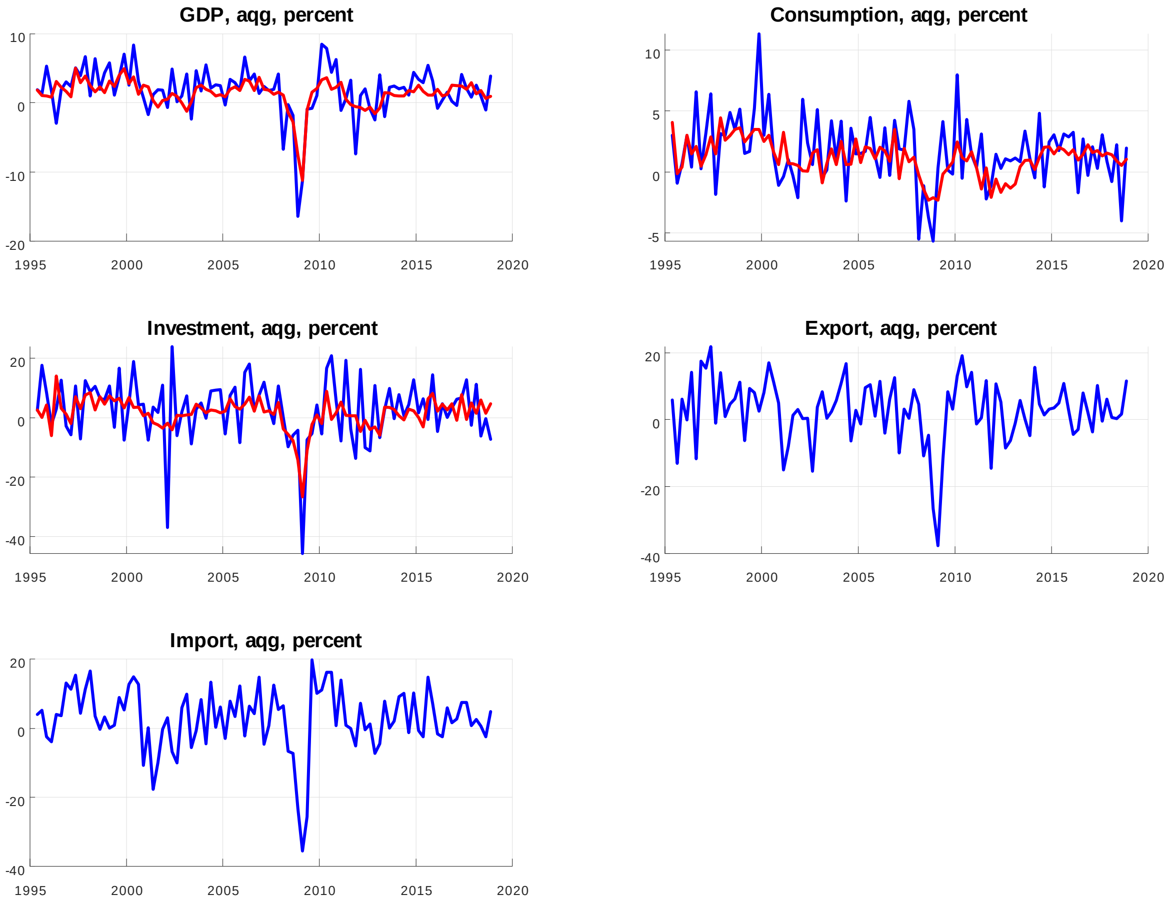


Figure 2: Data: labour market, wages and the real exchange rate. Annualised quarterly change (aqc). Units: see Table 2. Swedish data in blue and KIX20-weighted data in red.

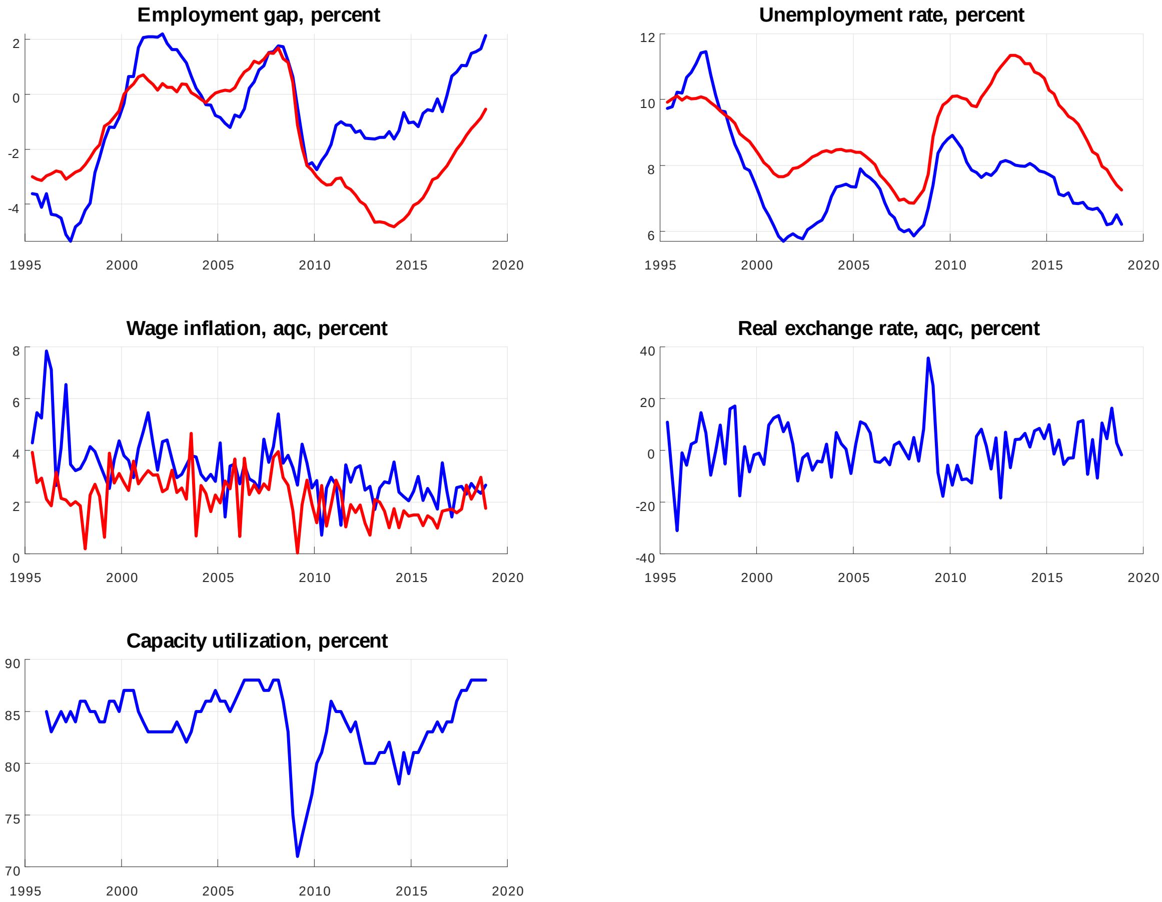
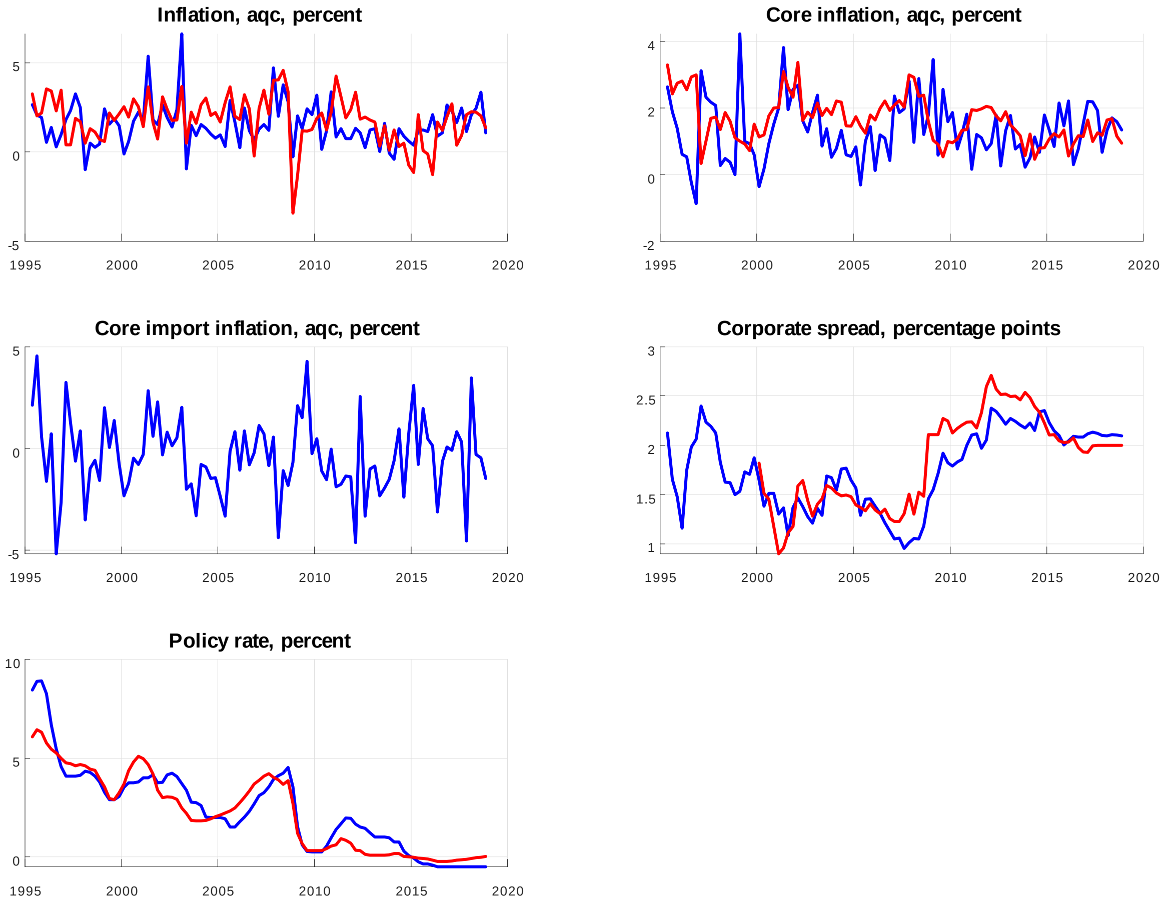


Figure 3: Data: inflation and interest rates. Annualised quarterly change (aqc). Units: see Table 2. Swedish data in blue and KIX20-weighted data in red.



assessment of the import content of the various goods and services in the CPI, and covers roughly 26% of the consumption basket.[78] It should be noted that this measure of import inflation is very similar to the change in the price of the goods component of the CPI ('CPI goods') — both measures average at -0.4 in the sample period and their correlation is 0.9.[79,80]

The choice of observed series for inflation is important for the estimated degree of exchange rate pass-through. As pointed out in De Walque et al. (2017), standard open-economy models have a hard time simultaneously matching the pass-through to both import and aggregate consumer prices. The pass-through to import prices is generally found to be higher than the pass-through to aggregate consumer prices; see e.g. Ortega and Osbat (2020). Here, it is also important whether import prices are measured as producer prices (at the border) or as final consumer prices (which are less commonly observed). In a standard model, the direct exchange rate pass-through to aggregate consumer prices is proportional to the pass-through to import prices and scaled by the share of imports in final goods production.[81] If the price rigidities in the import sector are matched to import prices at the border, the resulting pass-through to aggregate consumer prices becomes too high. Ultimately, this reflects misspecification of the production and price setting in our models.[82] However, within the present setup the problem can be alleviated through a more appropriate choice of inflation measures. Our measure of imported inflation corresponds to final goods prices and should, as such, generate a lower degree of exchange rate pass-through than if import prices at the border were used as the import inflation measure. Consequently, we should expect to find a relatively high value of the Calvo parameter in the import consumption goods sector when estimating the model.[83]

3.2.2 Some key properties of the data

Sweden is a small economy with open goods and capital markets. One indicator of openness is the trade-to-GDP ratio (the sum of exports and imports as a share of GDP). In line with globalization trends, this so called trade openness index for Sweden roughly doubled from 46% in 1960 to 91% in 2018. In the sample period the ratio increased from 70% in 1995 to 93% in 2008 while in the past decade it has been fairly stable.

In this section we briefly characterise the data used for estimation through a set of 'stylized facts'. These facts are based on sample means, standard deviations, unit root inference and pairwise cross-correlations of the data series. We generally focus on describing strong relationships in the data rather than on describing weak correlations or the absence of relationships. Contemporaneous sample correlations for all the observed variables in the model, $corr\left(Y_{i,t}, Y_{j,t}\right)$, are reported in the Appendix. A subset of the correlations, which are more directly of relevance for the discussion below, are reported in Table 3. Below, in a few instances we also refer to data regularities revealed by the sample cross-correlations, $corr\left(Y_{i,t}, Y_{j,t-s}\right)$, while these are not reported in the paper. With $n_y = 25$ observed variables there are $n_y\left(n_y - 1\right) = 25\left(25 - 1\right) = 300$ pairs of variables and dimension reduction suggests a focus on the contemporaneous relationships. We have also noted that, when there are strong

[78] Clothes, electronic equipment, vehicles and foreign travel are some of the more important product groups which are classified as 'largely imported'.

[79] Around 84% of the products that are classified as imported are goods, 8% are food and 8% are services. Hence, there is a large overlap between imports excluding energy and the goods component in the CPI, which explains the strong correlation between the two series.

[80] The Riksbank's previous model, Ramses II, did not include a measure of import inflation among the observed variables. Instead, the change in the GDP deflator was used to measure the domestic component of CPI inflation and the import component of inflation was implicitly identified.

[81] There are, of course, also indirect effects that can make aggregate consumer prices move more or less than proportionally to the weight of imports in the aggregate. These include price effects of changes in demand and inflation expectations.

[82] Suggestions for ways to resolve this problem are proposed by Corsetti and Dedola (2005) and De Walque et al. (2017), among many others. Some have already been implemented in our setup, such as imports entering the production of exported goods.

[83] As we do not observe prices in the investment and export goods sectors, it is not straighforward to form expectations about the degree of price stickiness and exchange rate pass-through in these sectors.

Table 3: Contemporaneous sample correlations. Selected pairs of foreign (KIX20) and Swedish variables. Sample 1995Q2-2018Q4. Absolute value of correlations sorted in descending order. Units of variables: see Table 2.

KIX20	Sweden	Corr
Policy rate	Policy rate	0.92
Corp. spread	Corp. spread	0.86
GDP	GDP	0.68
Unemp. rate	Unemp. rate	0.67
GDP	Imports	0.66
CPIxe	Repo rate	0.65
Employment	Employment	0.63
Investment	Imports	0.63
GDP	Exports	0.62
Investment	Exports	0.58
Consumption	Consumption	0.52
Investment	Investment	0.47
CPIxe	Wages	0.46
Consumption	Exports	0.45
Consumption	Imports	0.42
CPI	CPIF	0.40
GDP	Real exch. rate	−0.35
Wages	Wages	0.27
CPIxe	CPIFxe	0.07

KIX20	KIX20	Corr
Employment	Unemp. rate	−0.95
Corp spread	Policy rate	−0.84
GDP	Investment	0.82
GDP	Consumption	0.71
Consumption	Investment	0.60
CPIxe	Policy rate	0.58
CPI	Wage	0.51
Employment	Wage	0.50
Unemp. rate	Policy rate	−0.32
CPI	Policy rate	0.30

Sweden	Sweden	Corr
Employment	Unemp. rate	−0.98
Wage	Policy rate	0.71
Exports	Imports	0.70
Exports	GDP	0.65
Imports	GDP	0.51
Consumption	GDP	0.50
Investment	GDP	0.49
Corp spread	Policy rate	−0.49
Exports	Investment	0.40
GDP	Real exch. rate	−0.39
Imports	Real exch. rate	−0.31
Unemp. rate	CPIFxe	−0.19
Consumption	Investment	0.11
CPIF	Policy rate	0.09

correlations among variables in the data, the maximum correlations are typically contemporaneous or with short lags/leads.

Weaker economic development after the financial crisis 2008

In Table 2 we contrast the means of the data series for the pre- and post financial crisis subsamples, 1995Q2–2008Q2 and 2008Q3–2018Q4 respectively. Foreign and Swedish average GDP growth, wage and price inflation and policy rates have all been lower in the post-financial crisis subsample, when compared to the pre-crisis subsample.[84] These sub-sample differences make it challenging to calibrate a (time-invariant) steady state for many of the observed variables. Here, we have generally chosen to base calibrations on the full sample of data, 1995Q2 to 2018Q4, with the interpretation that economic developments in the decade following the financial crisis have been 'weaker than normal', while developments prior to the financial crisis were 'stronger than normal'. It is important to acknowledge that

[84] Note that the post-financial crisis sample includes the crisis quarters in 2008 and 2009. The average growth rates of real variables, e.g. foreign and Swedish GDP growth, obviously depend on the sample period. However, the overall picture of significantly higher real growth rates and interest rates in the pre-crisis period is not affected by the exact starting point of the later subsample.

the sub-sample differences could reflect structural breaks or time-varying trends in some of the data series, but a careful treatment of these issues is beyond the scope of this paper.

Swedish GDP growth is more volatile than foreign growth

The volatility of Swedish GDP and its components, measured by the standard deviation, is almost double the volatility of its foreign counterparts; see Table 2. In cyclical upturns the Swedish economy tends to grow faster than the foreign economy, and in downturns the reverse holds — a fact sometimes referred to as Sweden being a 'high beta economy'.[85] Investment and trade (exports and imports) are the most volatile components of Swedish GDP, while consumption has the lowest volatility. For the remaining observables, the standard deviations of the foreign and Swedish variables are of a similar magnitude.

Foreign and Swedish unemployment rates and policy rates display persistent behaviour

Likelihood-based inference is based on the assumption of stationarity of the observed variables. The augmented Dickey-Fuller test is applied to test for a unit root in the data series to provide an indication of the trending behaviour of the observed series, while acknowledging that the sample period is short and that the power of the test is low. We find that the null hypothesis of a unit root is strongly rejected for most of the observables — for 16 out of the 25 series the null is rejected at the 1% significance level; see Table 2. All these data series have been first differenced, i.e. they appear as growth rates. On the other hand, for the level variables the hypothesis of a unit root cannot be rejected at the 10% level. These variables are the foreign and Swedish employment gaps, unemployment rates, policy rates and spreads, as well as Swedish capacity utilization. The unit root, or near unit root, behaviour of some data series is reflected in very persistent estimates of some of the shock processes; see Section 3.6. The labour market variables are very persistent with only a few peaks and troughs in the sample period, but visually they do not display apparent upward or downward trends. In order to handle the downward trends in the foreign and Swedish policy rates the model has been extended with real interest rate trends; see Section 2.3.1.

Strong contemporaneous correlations between GDP and its components

The correlations among GDP and its components are generally strong and the largest correlations are typically recorded contemporaneously, i.e. without a lead or a lag. For the foreign variables the contemporaneous correlations range from 0.6 for consumption and investment to 0.8 between GDP and investment. For the Swedish data the correlation between exports and GDP is particularly strong while consumption is somewhat less strongly correlated with the other components. Once again, it should be noted that the correlations are computed for the variables in quarterly growth rates, i.e. as they enter the dataset used for estimation of the model. The correlations between the variables in annual growth rates are typically larger than the correlations between variables in (annualised) quarterly growth rates.

GDP leads labour market variables

Cross-correlations between GDP and various labour market variables suggest that there is a positive comovement between GDP and employment or hours, and that GDP leads the labour market

[85] Here, beta refers to the coefficient in the capital asset pricing models which describes the movementes of an asset's returns in response to overall market swings. An asset with a positive beta moves in the same direction as the rest of the market. A high beta implies a high degree of volatility or systematic risk. In a macroeconomic setting, the fact that Swedish GDP has a positive comovement with foreign GDP, and in addtion is more volatile, makes Sweden a 'high beta economy'.

variables by a few quarters.[86] These observations are valid both for foreign and Swedish data, and for variables expressed in growth rates or levels/gaps. Employment and unemployment are almost perfectly negatively correlated, reflecting that business cycle fluctuations in the labour force are of relatively minor importance.

Strong correlation between foreign and Swedish GDP

The strong relationship between foreign and Swedish GDP growth is a key stylized fact, and a feature that we would like our DSGE model to capture better than standard open-economy DSGE models. The contemporaneous correlation between the quarterly growth rates of foreign and Swedish GDP per capita is 0.7 and the correlation between the annual growth rates is 0.9, which we consider a remarkably strong relationship. It is not the case that foreign real developments lead the Swedish cycle but instead the maximum correlations are contemporaneous. We also note that the correlations between the Swedish GDP components and foreign GDP are generally large and largely contemporaneous. Also, foreign and Swedish employment gaps and unemployment rates are rather strongly correlated and the maximum correlations are recorded contemporaneously. *Overall, for the real quantities in the model there are strong relationships between foreign and Swedish variables.*

However, this does not imply a strong relationship between foreign and Swedish inflation rates. The contemporaneous correlation between foreign and Swedish headline inflation is 0.4 and the relationship is mainly due to fluctuations in the oil price. The empirical relationship between foreign and Swedish inflation is discussed further in Section 3.8.2.

Foreign and Swedish policy rates share a common downward trend

Foreign and Swedish interest rates have trended down in the sample and are very strongly correlated. The trend is mostly attributed to a downward trend in *real* interest rates, while inflation expectations in Sweden, the euro area and the US have been largely stationary in the sample period. The fact that policy rates have trended down in many countries simultaneously suggests that common, or global, factors play a key role in the decline. This observation also motivates the introduction of a common, near unit root, real interest rate trend in the model. One may also note that, while Swedish inflation has been somewhat lower than foreign inflation on average in the sample, the levels of the nominal policy rates have been very similar. Hence the (ex post) real interest rate has been higher on average in Sweden than in the foreign economy, which is consistent with a somewhat higher level of productivity growth in Sweden in the sample period.

The Krona depreciates in bad times, appreciates in good times, and leads import inflation

Changes in the real exchange rate, which are primarily driven by changes in the nominal exchange rate, are modestly negatively correlated with foreign and domestic real growth. The Krona tends to appreciate in good times and depreciate in bad times. The contemporaneous correlation between the change in the real exchange rate and import inflation is low but the correlation is rather strong with a lag, where a weaker exchange rate tends to be followed in a few quarters by higher import inflation. The correlation between the change in the real exchange rate and core inflation is also moderately strong while the correlation with headline inflation is weak. These correlations suggest that energy price and exchange rate movements are negatively correlated, i.e. energy price developments tend to be weak in downturns when the Krona depreciates.

Note also that the rather strong correlation between the exchange rate and import price inflation does not translate into a large exchange rate pass-through. Regression-based estimates of the average pass-through from the exchange rate to import inflation or inflation for Sweden are instead quite low.[87]

[86] The empirical relationships between GDP and the labour market variables are not evident from the contemporaneous correlations reported here. These relationships are more evident if one studies the cross-correlation function between the annual growth rates of GDP and employment (or hours).

[87] In the article 'The impact of the exchange rate on inflation', Sveriges Riksbank (2016), the estimated effect on CPIF

Stronger relationships between real and nominal variables in the foreign economy than in the Swedish economy

The level of foreign employment is moderately strongly correlated with foreign wage inflation and foreign core inflation. Moreover, foreign wage and price inflation are rather strongly correlated. In Swedish data, on the other hand, these relationships are generally much weaker. It is particularly interesting to note the weak correlations between the employment gap, or the unemployment rate, and wage inflation in Sweden. Swedish wage inflation is instead somewhat more strongly correlated with foreign variables, and in particular with foreign core inflation (while only weakly positively correlated with foreign wage inflation). These observations could perhaps suggest that Swedish wage negotiations are generally more directly influenced by foreign developments; see also the evidence in Westermark (2019). However, wage inflation has been substantially higher on average in Sweden than in the foreign economy according to the wage measures we use, with an average difference of around one percentage point; see Table 2.

Wages, productivity and the labour share

In New Keynesian models inflation is related to current and future real marginal costs through the Phillips curve; see equation (2.1). Real marginal cost is often measured by the labour share (or real unit labour cost), the ratio of real wages to labour productivity (or alternatively the ratio of the wage sum to nominal GDP). The foreign labour share has declined markedly in our sample. Average nominal wage growth has been very low while average price inflation has been broadly in line with the inflation targets of the Federal Reserve and the ECB. Average foreign real wage growth has been lower than average productivity growth resulting in a declining labour share.

The Swedish labour share instead displays a slight upward trend in the sample, which could be interpreted partly as a recovery after the deep crisis in the Swedish economy in the beginning of the 1990s, which led to a sharp fall in the labour share. These trends arguably create difficulties when the model is taken to the data and require additional treatment.

In estimating our model we do not use direct measures of foreign or Swedish real marginal costs but they are instead identified by data on wage inflation, price inflation, GDP, and employment. Since it is difficult to reconcile a trending labour share with stationary inflation, in the case of the foreign economy we choose to incorporate an excess trend for wage inflation to account for the low wage growth in the sample. A consequence of this is that the model estimate/measure of foreign real marginal cost does not display the downward trend otherwise apparent in the data.[88]

3.3 Long-run assumptions and treatment of trends

3.3.1 Trend and cycle

The modelling of trends in DSGE models is very important, not least from a forecasting perspective, but also genuinely difficult. We work with a sample of limited length and it is thus very likely that some of the theoretical trend assumptions in the model will not hold in data, even if the assumptions are theoretically plausible. Our ambition in this context is to try to incorporate some features in the data, which we believe are particularly important, into the model while simultaneously avoiding imposing a too complex trend structure (overfitting).

The observed variables, where some are in levels and others in growth rates, can be decomposed into a trend (growth) component, a cyclical (growth) component and an observation error

$$Y_{i,t} = Y_{i,t}^{trend} + Y_{i,t}^{cycle} + v_{i,t}. \tag{3.14}$$

inflation of a lasting 10 percent depreciation of the nominal exchange rate is reported to be 0.2–0.8 percentage points after one year and 0.4–0.7 percentage points after two years.

[88] Model estimates of foreign and domestic real marginal costs, as well as other unobserved variables, are reported in the Appendix.

where $Y_{i,t}$ denotes the i^{th} observed variable in the vector of observed variables, Y_t; see the observation equation (3.7). The trend component $Y_{i,t}^{trend}$ consists of two parts: a component capturing the trends which are included within the DSGE model, and an exogenous component which captures trends in the data which cannot be easily accounted for by the DSGE model trends. The latter trends are incorporated through the (possibly) time-varying intercept, c_t, in the observation equation. While these 'excess trends' could, equivalently, be handled by pre-filtering/de-meaning of the data we prefer to make these assumptions explicit through parameters included in the model, i.e. through c_t, in order to be as transparent as possible about the trend assumptions.[89]

3.3.2 DSGE model trends

The DSGE model contains two stochastic trends for each of the two economies, labour-augmenting technology, z_t and z_t^*, and investment-specific technology, Ψ_t and Ψ_t^*. Since we do not find strong empirical support for the inclusion of the latter stochastic trend it is inactive in the baseline version of the model, i.e. $\hat{\mu}_{\Psi,t} = 0$ and $\hat{\mu}_{\Psi^*,t} = 0$, such that $\hat{\mu}_{z^{+,*},t} = \hat{\mu}_{z^*,t}$ and $\hat{\mu}_{z^+,t} = \hat{\mu}_{z,t}$; see equation (2.7). The labour-augmenting technology trend, z_t, reflects the assumption of a balanced growth path and implies common long-run growth rates and pairwise cointegration among the following observed variables: GDP, consumption, investment, government consumption, and real wages for both the Swedish and foreign economies, and in addition exports and imports for Sweden. The foreign/global labour-augmenting technology trend is modelled as a random walk with drift process, with a growth rate given by the AR(1) process

$$\log\left(z_t^*/z_{t-1}^*\right) \equiv \log\left(\mu_{z,t}^*\right) = \left(1 - \rho_{\mu_z}^*\right)\log\left(\mu_z^*\right) + \rho_{\mu_z}^* \log\left(\mu_{z,t-1}^*\right) + \sigma_{\mu_z}^* \varepsilon_{\mu_z,t}^*. \tag{3.15}$$

We note that $400\log\left(\mu_z^*\right)$ is the annualised quarterly growth rate of global technology in percent. We calibrate the steady-state growth parameter, μ_z^*, and estimate the persistence parameter, $\rho_{\mu_z}^*$, and the standard deviation of the innovation to technology growth, $\sigma_{\mu_z}^*$. Since we assume that steady-state foreign investment-specific technology growth equals zero, i.e. $\log\left(\Psi^*\right) = 0$, the model-implied annualised quarterly percent steady-state growth rate of foreign per capita GDP, consumption and investment equals $d^{\Delta Y^*} = 400\log\left(\mu_z^*\right)$, where the notation refers to the intercept in the observation equation (3.7).

In the baseline specification of the model we further assume that the growth rate of Swedish labour-augmenting technology equals the global growth rate,

$$\mu_{z,t} = \mu_{z,t}^*, \tag{3.16}$$

which implies that foreign and Swedish GDP are assumed to share a common stochastic trend. However, average Swedish GDP growth has been higher than foreign growth in the sample, and to account for this difference we make two additional assumptions. First, we note that Swedish investment growth has been higher than GDP growth in the sample. We therefore allow for a positive investment-specific *deterministic* trend $\log\left(\mu_\Psi\right) > 0$.[90] This means that the model-implied steady-state percent growth rate of Swedish investment per capita, given by $d^{\Delta I} = \frac{1}{1-\alpha}400\log\left(\mu_\Psi\right) + 400\log\left(\mu_z\right)$, is somewhat higher than steady-state GDP per capita growth, which is given by $d^{\Delta Y} = \frac{\alpha}{1-\alpha}400\log\left(\mu_\Psi\right) + 400\log\left(\mu_z\right)$ where α is the capital share in the production function. The difference, given by $d^{\Delta I} - d^{\Delta Y} = 400\log\left(\mu_\Psi\right)$, is calibrated to be in line with the difference between the average investment and GDP growth rates in the sample period. It further implies that the model-implied Swedish steady-state GDP per capita growth is higher than the corresponding foreign growth rate —

[89] We expect these assumptions to be updated on a continuous basis when the model is used for forecasting and policy analysis.

[90] Again, note that the unit root investment-specific technology shock, Ψ_t, is not active in the baseline specification of the model (i.e. $\sigma_\Psi = 0$) while we allow for a positive drift, i.e. $\log\left(\mu_\Psi\right) > 0$, in Swedish investment-specific technology growth. In other words we assume that the trend is deterministic instead of stochastic.

the difference between the model-implied growth rates is $d^{\Delta Y} - d^{\Delta Y^*} = \frac{\alpha}{1-\alpha} 400 \log(\mu_\Psi)$. Second, we allow for an excess trend parameter to capture the remaining growth difference between Sweden and the foreign economy in the sample.

While it is common to include unit root technology shocks in DSGE models it is important to assess whether the trend assumptions hold in the data. In the Appendix, the ratios of real consumption, investment, exports, imports, and the real wage, respectively, to real GDP are displayed for Swedish data in the period 1980–2018. These graphs can be used for a rough assessment of the balanced growth assumption.[91] The consumption ratio, c/y, has been fairly stable while the government consumption ratio has declined in the sample period. The investment ratio i/y and in particular the export and import shares, x/y and m/y, have arguably increased. We also note that investment declined sharply during the crisis in the beginning of the 1990s and that investment has recovered and grown faster than GDP since 1995. These observed deviations from balanced growth motivate the incorporation of 'excess trends' for some variables which is discussed in the next subsection.

3.3.3 Additional trends and calibration of the steady states of key observables

The incorporation of unit root technology trends within the DSGE model provides structure on the trend assumptions of variables through the implied cointegrating relationships and, more generally, it increases the transparency of the modelling of trends.[92] It is, however, almost invariably the case that these assumptions are too restrictive and at odds with the data. In addition to the DSGE model trends we therefore allow for additional deterministic trends — 'excess trends'. These assumptions affect the steady states of the foreign policy rate and wage growth, and the Swedish policy rate, GDP per capita growth, trade growth and import price inflation. The excess trends are incorporated through the (possibly time-varying) parameter c_t in the observation equation (3.7).[93] It is important to note that these excess trends/parameters typically increase the in-sample fit of the model but also that they yield unreasonable implications asymptotically. For example, as we saw already above the steady-state Swedish GDP growth rate is assumed to be higher than the foreign growth rate, which is in line with the sample averages but the difference is obviously not sustainable in the very long run. As a consequence, it is vital to re-assess trend assumptions on a continuous basis when the model is used for forecasting and policy work (out of sample).

Below, the discrepancies between what is implied by the model and what is observed in the data are briefly summarised, and each excess trend is reported.

- Average annual Swedish GDP per capita growth has been higher than average foreign per capita growth (1.8 percent versus 1.3 percent). This difference is partly captured by an excess growth parameter for Swedish GDP and its components, $c^{\Delta Y} = 0.15$ p.p.
- Swedish trade — export and import — growth has been higher than average GDP growth (2.7 percent versus 1.8 percent). The difference is captured by an excess growth parameter for exports and imports, $c_t^{\Delta X} = c_t^{\Delta M} = 1.0$ p.p.
- Foreign and Swedish interest rates have trended down and are historically low at the end of the sample. We incorporate excess parameters, c^{R^*} and c^R respectively, and calibrate them such that the foreign and Swedish steady-state policy rates equal 3%.
- Average foreign real wage growth has been significantly lower than foreign labour productivity and GDP per capita growth (0.5 percent versus 1.3 percent). An excess parameter, $c^{\Delta w^*} = -0.8$ p.p., accounts for this difference.

[91] Here we refer to ratios between real variables, e.g. real consumption over real GDP in order to assess the relative real growth rates in the sample. In calibrating the expenditure shares we will later refer to the nominal shares.

[92] It is not uncommon in the DSGE model estimation literature to remove trends from the data in a rather unsystematic way, e.g. by HP filtering the data series individually.

[93] In log levels these trends are (possibly piece-wise) linear excess trends, and in growth rates they feature as (possibly time-varying) excess growth parameters.

Table 4: Calibrated foreign parameters and implied expenditure shares

Parameter	Description	Value	Short motivation/comment
α^*	Capital share	0.21	Investment share
β^*	Discount factor	0.999	Real policy rate
δ^*	Depreciation rate	0.015	Investment share
λ^*	Markup, price	1.2	Standard in literature
$\tilde{\varpi}_e^*$	Energy share, consumption	0.09	Match data
η_e^*	Substitution, energy and non-energy	0.5	Complementarity
U^*	Unemployment rate	0.08	15–74 years
l^*	Labour force participation rate	0.64	Match data, 15–74 years
n^*	Employment rate	0.59	Match data, 15–74 years
$400\log(\mu_z^*)$	Technology growth, quarterly	1.3%	Real variables growth
$400\log(\mu_\Psi^*)$	Inv. spec. tech. growth, quarterly	0.0%	Real variables growth
$\rho_{\bar{\pi}^{c,*}}$	Inflation trend, persistence	0.95	
$r_{\mu_{z^+}}$	Tech. param., real int. rate trend	0.0	Prel. estimated close to 0
Steady-state expenditure shares			
c^*/y^*	Consumption	0.59	Match data share
i^*/y^*	Investment	0.20	Match data share
$\eta_g^* = g^*/y^*$	Government consumption	0.21	Match data share

- Swedish import inflation has been substantially lower on average than CPIF inflation, and the difference is accounted for by the excess parameter $c^{MCXE} = -2.0$ p.p.

The steady states of the observed variables in the model are provided in Table 2. A more detailed discussion of the calibration of the steady state is provided in the Appendix.

3.4 Calibration of model parameters

3.4.1 Calibration of parameters and key shares

In this section we discuss the calibration of model parameters and the calibrated or inferred steady-state values of expenditure shares. Foreign and domestic parameter calibrations are summarised in Tables 4 and 5. While these parameters have been calibrated in the baseline estimated model, in preliminary work many of the parameters have been estimated to assess the sensibility of the calibrated values.[94] For most of the parameters we have added a short motivation or comment in the table such that the reader can quickly grasp the motivation behind the calibration.

The net price markups of the various goods producing firms are generally calibrated to 20%, which is a standard calibration in the literature. An exception is the Swedish export goods producing firms where a lower steady-state markup, $\lambda_x = 1.05$, is assumed in order to avoid a double markup for these goods, given that the prices of the production inputs are also subject to markups.

The foreign and Swedish discount factors are calibrated to $\beta^* = \beta = 0.999$, where an important consideration is the implied foreign and Swedish steady-state nominal interest rates, i.e. a lower calibration of the discount factors would imply higher steady-state values of the policy rates. The foreign and Swedish capital shares in production are calibrated to $\alpha^* = 21\%$ and $\alpha = 25\%$, respectively, and the quarterly depreciation rates are calibrated to $\delta^* = \delta = 1.5\%$, in order to broadly match the average investment-output ratios in the sample. The calibrations of the capital shares and depreciation rates further imply capital-to-annual output ratios for the foreign and Swedish economies equal to 2.7

[94] Model comparisons based on the Laplace approximation to the marginal likelihood are reported in the Appendix.

Table 5: Calibrated domestic parameters and implied expenditure shares

Parameter	Description	Value	Short motivation/comment
α	Capital share	0.25	Investment share
β	Discount factor	0.999	Real policy rate
δ	Depreciation rate	0.015	Investment share
$\tilde{\omega}_c = \omega_c \left(\frac{p^{m,c}}{p^{cxe}}\right)^{1-\eta_c}$	Import share, non-energy cons.	0.27	Match data share
$\tilde{\omega}_i = \omega_i \left(\frac{p^{m,i}}{p^{i}}\right)^{1-\eta_i}$	Import share, investment	0.40	Match data share
$\tilde{\omega}_x$	Import share, export	0.40	Match data share
$\tilde{\omega}_e = \omega_e \left(\frac{p^{ce}}{p^{c}}\right)^{1-\eta_e}$	Energy share, consumption	0.075	Match data share
$\tilde{\omega}_{em} = \omega_{em} \left(\frac{p^{m,ce}}{p^{ce}}\right)^{1-\eta_{em}}$	Import share, energy consumption	0.5	Match data share
λ^d	Markup, domestic	1.2	Standard in literature
λ^{mc}	Markup, import consumption	1.2	Standard in literature
λ^{mi}	Markup, import investment	1.2	Standard in literature
λ^{mx}	Markup, import-to-export	1.2	Standard in literature
λ^{me}	Markup, import energy	1.2	Standard in literature
λ^x	Markup, export	1.05	Avoid double mark up
η_{em}	Substitution, dom. and imp. energy	0.5	Low substitutability
η_e	Substitution, energy and non-energy	0.5	Low substitutability
$\xi_{m,e}$	Calvo, imported energy	0.1	No price rigidity
$\tilde{\phi}_a$	Weight on NFA in ER risk premium	10^{-5}	
U	Unemployment rate	0.072	15–74 years
l	Labour force participation rate	0.72	15–74 years
n	Employment rate	0.67	15–74 years
$400\log(\mu_z)$	Technology growth, quarterly	1.3%	$\mu_z = \mu_z^*$
$400\log(\mu_\Psi)$	Inv. spec. tech. growth, quarterly	0.8%	Excess invest. growth
Steady-state expenditure shares			
$p_c c/y$	Consumption	0.48	Match data share
$p_i i/y$	Investment	0.24	Match data share
$\eta_g = g/y$	Government consumption	0.28	Match data share
	Export	0.39	Match data share
	Import	0.39	Match data share

and 2.0, respectively. The government consumption ratios are calibrated to $\eta_g^* = g^*/y^* = 21\%$ and $\eta_g = g/y = 28\%$ to be broadly in line with the corresponding sample averages. Reasonable calibrations of the investment and government expenditure shares imply values for the consumption shares which are broadly in line with the data. One may, however, note that Sweden has had sizeable trade and current account surpluses during most of the sample period while the Swedish net export share of GDP is calibrated close to zero in the model. The domestic expenditure shares in the model are therefore somewhat larger than the corresponding shares in data. Time series for the Swedish expenditure shares are displayed in the Appendix. The calibration of the foreign expenditure categories' shares are also broadly in line with the calibrations in the NAWM II model for the euro area; see Coenen et al. (2018).

The Swedish import shares in consumption excluding energy, investment and exports are calibrated to $\tilde{\varpi}_c = 27\%$, $\tilde{\varpi}_i = 40\%$ and $\tilde{\varpi}_x = 40\%$ based on time series of import weights in the CPI (for consumption) and import shares from input-output tables (for all three parameters).[95] Our calibration implies that the value of imported goods used for consumption and investment as a share of GDP equals 23% and that the value of imported goods used in the production of export goods, as a share of GDP, equals 16%. This implies that the total import share in GDP equals 39% which is also roughly equal to the export share.[96]

The foreign and Swedish energy component weights in CPI and CPIF, $\tilde{\varpi}_e^* = 9\%$ and $\tilde{\varpi}_e = 7.5\%$, respectively, are calibrated based on time series of these weights. The average weight of the energy component in Swedish CPIF has been around 8% in the sample period, but has declined in the latter part of the sample. The import share in Swedish energy consumption has also declined over time, from 60% in 1995 to 50% in 2017, and it is calibrated to $\tilde{\varpi}_{em} = 50\%$.[97] The elasticities of substitution between energy and non-energy goods are calibrated as $\eta_e^* = \eta_e = 0.5$ reflecting an assumption of limited substitutability between these classes of goods.

The parameters governing the dynamics of imported energy price inflation have been calibrated based on support from preliminary estimation results. The Calvo parameter for the energy importing firms, $\xi_{m,e}$, is calibrated to a very low value (0.10), reflecting that the prices of these goods are essentially flexible. This calibration is also supported by preliminary estimation of the parameter which yields an estimate quite close to zero. The calibration of the elasticity of substitution between domestic and imported energy, $\eta_{em} = 0.5$, implies limited substitutability between these classes of goods.[98]

The steady-state gross wage markup, λ_w, the labour disutility parameter, φ, and the steady-state unemployment rate, U, are related in steady state through $U = \log(\lambda_w)/\varphi$, i.e. a higher degree of market power of unions is associated with a higher unemployment rate in the steady state. Again, note that the modelling of the labour market is identical for the foreign economy. We choose to calibrate the steady-state unemployment rate and to estimate the labour disutility parameter, which implies that the markup is implicitly estimated, i.e. it is solved for using the relationship above. The foreign unemployment rate is calibrated to 8.0% which is a percentage point lower than the sample average but higher than the calibration of Coenen et al. (2018) for the euro area. The Swedish unemployment rate is calibrated to 7.2% which is somewhat lower than the sample average of 7.6%. A measure of

[95] The average shares of imports in consumption, investment, and exports in the period 1993–2014 from input-output tables are 27%, 39%, and 39%, respectively, and the most recent values are slightly larger. In solving for the steady state, restrictions are imposed on the import shares $\tilde{\varpi}_c$, $\tilde{\varpi}_i$ and $\tilde{\varpi}_x$, and *the quasi-shares* ϖ_c, ϖ_i and ϖ_x are solved for.

[96] One may note that the share of imports in government consumption has been 11% on average 1993–2014 in Sweden such that its share of GDP has been roughly 3%. Since the government in the model is assumed not to use imported goods in its production (a simplifying assumption), this is a source of discrepancy between the model-implied and data import shares of GDP.

[97] As a robustness check we have estimated the share parameters, $\tilde{\varpi}_c$, $\tilde{\varpi}_i$, $\tilde{\varpi}_x$, $\tilde{\varpi}_e^*$, $\tilde{\varpi}_e$, and $\tilde{\varpi}_{em}$, using tight normal priors centered on the calibrated values. The posterior mode estimates of the shares are all somewhat lower than the respective calibrated values but the marginal likelihood criterion favours our main specification slightly.

[98] Swedish households' imported energy consumption mainly consists of motor fuel and fuel oil for heating which require imports of crude oil. Their consumption of domestically produced energy goods mainly consists of electricity for heating of appartments. Water and nuclear power are the main sources of electricity in Sweden.

trend unemployment used by the Riksbank is provided by the KAMEL model; see also the discussion above in relation to the construction of the employment gap. This measure of the unemployment trend has moved between 6.8% and 7.6% in the sample period and it has averaged at 7.1%. At the end of the sample it is around 7.3%. Our calibration of the (constant) steady-state Swedish unemployment rate is therefore reasonably well in line with the (time-varying) KAMEL trend.

The foreign and Swedish participation and employment rates have trended upwards in the sample period, which implies that it is difficult to calibrate time-invariant steady states for these variables. We assume that the foreign steady-state employment rate is $n^* = 59\%$, which implies that the participation rate is $l^* = n^* / (1 - U^*) = 0.59 / (1 - 0.08) = 64.1\%$. The Swedish steady-state employment rate is assumed to be $n = 67\%$, which implies a participation rate of $l = 72.2\%$. The average Swedish employment and participation rates for persons in the age 15 to 74 years in our sample period are 66% and 71%, respectively, and in the last year of our sample, in 2018, the rates are 69% and 73% respectively.

The persistence parameters for the inflation trend shocks, $\rho_{\bar{\pi}^c}$ and $\rho_{\bar{\pi}^{c,*}}$, are calibrated to 0.95. This value implies that half of the effect of the shock vanishes in roughly 3 years' time, which is the typical length of the forecast period at the Riksbank.[99] The parameter on technology in the equation for the real interest rate trend (equation (2.68)) is calibrated to $r_{\mu_{z^+}} = 0$ based on preliminary estimation of the model.

3.4.2 Simplifying calibrations

A set of parameters have been calibrated based on results from preliminary estimation of the model; see Table 6. These parameters are usually poorly identified when estimated and/or not important for the overall fit of the model, i.e. calibrating the parameter does not affect the marginal likelihood. We have therefore chosen to simplify the model in some dimensions by calibrating such parameters to extreme values. It is important to note that we still estimate a large number of parameters — around 90 — and that including too many poorly identified parameters in the set of estimated parameters creates additional difficulties in obtaining reliable estimates.[100]

The foreign and Swedish parameters governing the short-term wealth effect on labour supply are calibrated to $\nu^* = \nu = 1$, which means that household preferences have the standard form. Galí (2011b) and Smets, Warne, and Wouters (2013) estimate this parameter close to zero on US and euro area data, respectively. A low value of the parameter induces a pro-cyclical response of the labour force to some shocks but otherwise the effects on shock responses are marginal. Based on marginal likelihood comparisons ($\nu^* = \nu = 1$ against $\nu^* = \nu = 0$), its value appears largely unimportant for overall model fit. Robustness analysis with various model specifications suggest that both ν^* and ν are typically estimated closer to 1 than to 0 and this motivates our choice of calibration.[101]

The parameter σ_a^*, governing foreign capital utilization costs, is calibrated to a large number such that variable utilization of capital is effectively not allowed for in the foreign part of the model. Thus, $\hat{u}_t^* \approx 0$ such that efficient capital equals physical capital, $\hat{k}_t^* = \hat{k}_t^{p,*}$. Note again that we do not include data on foreign capacity utilization in estimating the model.

Firms are assumed to borrow money from financial intermediaries to finance part of their wage bill. We calibrate the fractions of the wage bill that are financed through loans to $\nu^{wc,*} = \nu^{wc,d} = \nu^{wc,m} = \nu^{wc,x} = 1$ for the foreign firms and the domestic, import and export firms, respectively, which implies that the working capital channel is active. Since the shocks to the working capital interest rates are not active in the main specification of the model, these assumptions imply that the working capital rates simply equal the relevant short-term interest rates, so that e.g. the borrowing rate for

[99] Adolfson et al. (2005) assume an inflation trend persistence coefficient of 0.975 and Smets and Wouters (2003) assume a unit root inflation trend shock.

[100] The fact that most of these calibrations do not substantially affect the marginal likelihood can also be interpreted as meaning that our 'simplifying calibrations' are not rejected by the data.

[101] Note that this implies that equation (2.41) is given by $\hat{z}_t^C = -\hat{\bar{v}}_t^N$ and equation (2.40) then becomes $\hat{\Theta}_t = 0$, i.e. the endogenous preference shifter is not active.

Table 6: Simplifying calibrations

Parameter	Description	Value
Foreign		
ν^*	Wealth effect, labour supply	1
$\nu^{wc,*}$	Working capital	1
κ_w^*	Indexation, wage	0
σ_a^*	Capital utilization	Large
Sweden		
ν	Wealth effect, labour supply	1
$\nu^{wc,d}$	Working capital, domestic good	1
$\nu^{wc,m}$	Working capital, import	1
$\nu^{wc,x}$	Working capital, export	1
κ_{mc}	Indexation, import consumption	0
κ_{mi}	Indexation, import investment	0
κ_{mx}	Indexation, import-to-export	0
κ_{me}	Indexation, import energy	0
κ_x	Indexation, export	0
κ_w	Indexation, wage	0
$\xi_{m,e}$	Calvo, import energy	0.1

.

the producers of the domestic goods becomes $\hat{R}_t^{wc,d} = \hat{R}_t$.[102] Again, preliminary estimation results suggest that these parameters are weakly identified and that their values are largely unimportant for the overall model fit.[103]

Firms that do not re-optimize their prices are assumed to increase their prices by a weighted average of past inflation and the time-varying inflation trend, where the weight, or indexation, parameter κ ranges between full indexation to the trend ($\kappa = 0$) and full indexation to past inflation ($\kappa = 1$).[104] If $\kappa = 0$, the corresponding Phillips curve is purely forward-looking, while if $\kappa > 0$, past inflation enters in the Phillips curve. Estimating these indexation parameters, we typically find that the parameters are weakly identified and that they tend to be estimated close to 0, and, finally, that their values do not affect the marginal likelihood substantially.[105] In the Swedish case, the low values found for the indexation parameters are probably related to the very low persistence of Swedish quarterly inflation series in our sample. We therefore calibrate most of these indexation parameters to zero, $\kappa_{mc} = \kappa_{mi} = \kappa_{mx} = \kappa_{me} = \kappa_x = 0$. On similar grounds the wage indexation parameters are calibrated to $\kappa_w = \kappa_w^* = 0$. Fundamentally, these calibrations imply that the corresponding Phillips curves become purely forward-looking. The only remaining indexation parameters that are estimated are the domestic and foreign price indexation parameters, κ_d and κ^*, where we do find some, albeit weak, support for positive values.

[102] Since the shocks to the working capital interest rates are not active, e.g. $\hat{\nu}_t^{wc,m} = 0$, and since $R^{wc,d} = R$ holds in steady state, equation (2.4) reduces to $\hat{R}_t^{wc,d} = \hat{R}_t$, i.e. the working capital interest rate equals the risk-free short-term interest rate. Similarily, equations (2.14) and (2.37) reduce to $\hat{R}_t^{wc,m} = \hat{R}_t^*$ and $\hat{R}_t^{wc,x} = \hat{R}_t$, respectively.

[103] The log marginal likelihood of a specification without the working capital channel is 7 units lower than that of the baseline specification.

[104] Note that we do not consider the case of partial indexation which would imply price dispersion in steady state.

[105] The log marginal likelihood of a specification where all 8 indexation parameters are estimated is 12 units *lower* than that of the baseline specification. The min, max and average estimates of the indexation parameters equal 0.08, 0.54 and 0.25. The likelihood function is rather flat around the posterior mode for all these parameters. The result that the indexation parameters generally tend to be estimated low is in line with e.g. Adolfson et al. (2007) on euro area data and Adolfson et al. (2013) on Swedish data.

3.4.3 Observation errors

The individual observation equations for the 25 observed variables are provided in the Appendix. The observation errors are assumed to be distributed as $v_t \backsim N(0, \Sigma_v)$; see section 3.1.2. The errors are assumed to be independent, $\Sigma_v = diag(\Sigma_v)$. Let $\Sigma_{v,ii}^{1/2}$ denote the standard deviation of the observation error of observed variable i. We calibrate $\Sigma_{v,ii}^{1/2} = 0$ for the following eight observed variables: the foreign and domestic policy rates, corporate spreads, employment gaps and unemployment rates. For the remaining variables we calibrate $\Sigma_{v,ii}^{1/2} = 0.1 s_i$ where s_i is the estimation sample standard deviation of the observed variable. The calibrations of the standard deviations of the observation errors imply that these errors account for a very small fraction of the variation in the observed variables, e.g. their contribution in historical decompositions of the variables are typically negligible.

3.5 Prior distributions

3.5.1 Structural parameters

The estimated parameters, collected in the vector θ_{est}, are mostly related to the nominal and real frictions in the model, the parameters in the monetary policy rule and the shock processes. Their marginal prior distributions, $p_j(\theta_{est}^j)$, are largely based on the existing literature and could in most cases be considered 'standard'. The foreign and domestic estimated parameters and their prior distributions are presented in Tables 7, 9 and 10. Parameters which are bounded between 0 and 1 are assigned Beta prior densities, and parameters which are bounded only from below are assigned Gamma, or inverse Gamma, densities. Most parameters are given rather uninformative priors. We also choose to assign *identical* priors for foreign and domestic structural parameters of the same type to simplify prior elicitation.

The densities of the eight (two foreign and six domestic) estimated Calvo parameters are all centered on 0.75, which implies that firms (unions) re-optimize prices (wages) on average once per year. This is a fairly standard assumption in the literature. In the case of the domestic price Calvo parameters, this is consistent with the empirical evidence on price stickiness in Apel, Friberg, and Hallsten (2005). Concerning the domestic wage Calvo parameter, the industrial collective wage agreement in Sweden is negotiated for a period of 1 to 3 years and the average length in 1997–2014 was two and a half years, while individual wages are usually re-negotiated once per year for most employees; see Björklund, Carlsson, and Nordström Skans (2019). Our prior on wage stickiness is also in line with Ramses II where it is assumed that wages are reset once per year. Since we later estimate rather high values for the Calvo parameters, the importance of these priors is further assessed through sensitivity analysis in Section 3.8.3. Foreign and domestic price indexation parameters, κ_d and κ^*, are given agnostic priors centered on 0.5.

The habit formation and capital adjustment cost parameters, b and $\tilde{S}''$, are centered on 0.75 and 5 respectively. These values are broadly in line with those in other papers. As an example, Coenen et al. (2018) obtain a posterior mode estimate of $\tilde{S}''$ close to 5.

The labour disutility parameter, the inverse Frisch elasticity φ, has a prior mean of 3, which is line with studies based on micro data.[106] This value implies that the implicit priors for the foreign and domestic gross wage markups derived using $\lambda_w = e^{\varphi U}$ have prior means of 1.27 and 1.24, respectively. The prior location of the gross wage markups are therefore in line with the priors used by Galí, Smets, and Wouters (2012) and Coenen et al. (2018) (1.25 and 1.3 respectively) while our prior location for φ is somewhat larger.[107]

[106] For example, the Congressional Budget Office in the US assumes an interval for the Frisch elasticity ranging from 0.27 to 0.53 based on the agency's assessment of the research literature. Estimates of the Frisch elasticity for the *extensive margin* (which is the relevant margin in our model) based on micro data range from 0.2 to 0.7 for men and 0.1 to 0.4 for women; see Reichling and Whalen (2012) and the references therein. Our prior is centered on 0.33.

[107] The prior means for the gross wage markup, 1.25, and the inverse Frisch elasticity, 2.0, in Galí, Smets, and Wouters (2012) imply a prior mean for the steady-state US unemployment rate of roughly 11 percent. Their posterior mean

The priors for the intratemporal elasticities of substitution between domestic and imported goods in the production of final goods — η_c, η_i, and η_x — and the foreign demand elasticity, η_f, have prior means close to 1. Note that we do not rule out values below 1 a priori, unlike the Riksbank's earlier DSGE models where the prior distribution was truncated from below at 1. We thus do not a priori take a stand on whether aggregate domestic and imported goods — or baskets of goods — are substitutes or complements. As discussed in Section 2, at the micro level there are theoretical arguments why individual goods or varieties of goods should be substitutes, which they are also in our model. At the macro level, however, complementarity may not be an implausible assumption. Moreover, these elasticities have often been estimated to be rather low in the macro literature. Adolfson et al. (2007) was an exception where the elasticity η_c was estimated to be high. However, for the estimates of this elasticity, it is of relevance that Adolfson et al. (2007) did not feature imported inputs in the production of export goods.

3.5.2 Shock process parameters

In preliminary work we have allowed for ARMA(1,1) shock processes and in the baseline specification two shocks — the foreign wage markup shock and the neutral rate shock — are modelled as ARMA(1,1) with a rather uninformative prior on the MA coefficient, centered at zero.[108] Foreign AR(1) shock persistence parameters are typically centered on 0.75, while many of the domestic shock persistence parameters are centered on 0.5. An exception is the persistence of the real interest rate trend shock where the prior is instead centered on 0.85 reflecting the prior view that this process needs to be very persistent to capture the downward trend in real interest rates in the sample. The lower prior persistence of the domestic shocks is related to the introduction of correlations between shocks, where it should be noted that domestic shocks which have a foreign/global component will (at least partly) inherit the persistence of the foreign shock. This motivates the lower prior mean on the domestic persistence parameters.

The standard deviations of the innovations in the model are parameterised as $s_\sigma \sigma$, where σ typically has an inverse gamma prior distribution centered on 0.2. The scaling parameter s_σ is calibrated to avoid that the posterior estimate of the innovation becomes restricted from below. The scaling parameters are included mainly to avoid upward biases in the standard deviation estimates which would imply that the importance of some shocks are exaggerated. This could be seen as a way to handle the concerns related to prior densities for innovation standard deviations which do not include the point zero in the support, raised by Ferroni, Grassi, and Leon-Ledesma (2017). In robustness analysis (not reported in this paper), we have also experimented with truncated normal prior densities (which include zero in the support) for the standard deviation parameters to validate that our elicitation approach is overall fine. The scaling parameters, s_σ, are reported in Tables 7 (foreign parameters) and 10 (domestic parameters).

The shock correlation parameters are assigned disperse Beta priors centered on 0.5, which means that we assume positive shock correlations a priori, while not as large as in Justiniano and Preston (2010). However, some shock correlations have been calibrated to zero based on preliminary estimation work and in order to simplify the model. Later, we perform sensitivity analysis using a normal prior centered on zero for the shock correlation parameters.

3.6 Posterior distribution

In this section we present the posterior estimates of the model parameters and discuss the estimates of key parameters in the context of a few related studies. The DSGE model is estimated using three approaches (see also Section 3.1.1):

estimate of the inverse Frisch elasticity is 4.0, which implies a more plausible steady-state unemployment rate of roughly 4.5 percent.

[108] The MA term for the wage markup shock can pick up high frequency variation in wages. See Smets and Wouters (2007) and Galí, Smets, and Wouters (2012), who also assume an ARMA(1,1) wage markup shock.

1. marginal posterior distribution for the foreign parameters

$$p\left(\theta_{est}^{for}|y_{1:T}^{for}\right), \tag{3.17}$$

and conditional posterior distribution for the domestic parameters

$$p\left(\theta_{est}^{dom}|y_{1:T};\hat{\theta}_{est}^{for}\right); \tag{3.18}$$

2. joint posterior distribution

$$p\left(\theta_{est}|y_{1:T}\right); \tag{3.19}$$

3. joint modified posterior distribution

$$\frac{p\left(\theta_{est}|y_{1:T}\right)}{f((S_m - S_d)^2\,|\theta_{est})}. \tag{3.20}$$

The penalty function in the denominator of equation (3.20) contains the variances of 16 variables and the covariances between 20 pairs of variables. For a pair of observed variables, matching the variances and the covariance between the variables implies that the contemporaneous correlation is implicitly matched. The set of variables for which the variance in the model is matched to the variance in the data is guided by the unit root tests reported in Table 2 — we choose to match the variances of those variables for which a unit root is clearly rejected. Concerning the set of covariances to match, we choose to match pairs of variables for which there are strong and significant correlations in the data, and our ambition is to try to better capture some of the stylized facts reported in section 3.2.2.

The main estimation approach is the standard Bayesian estimation of all — foreign and domestic — parameters jointly. The marginal-conditional approach is included to assess to what extent Swedish data series affect the estimation of the foreign parameters. The modified posterior approach is studied to assess to what extent the direct targeting of cross-country covariances in estimation can improve the model's ability to fit the strong correlations in the data discussed above.

3.6.1 Posterior estimates of foreign model parameters

The posterior mode, median and the 5^{th} and 95^{th} percentile estimates of the parameters of the foreign model are provided in Table 7. These estimates are obtained by maximising and sampling from the marginal posterior distribution, (expression (3.17), case 1), i.e. the estimation is based only on foreign data. We choose to focus on these estimates initially because it corresponds directly to the 'standard' case of estimating a closed-economy DSGE model using Bayesian methods. In the following section, we contrast these estimates to those obtained using the joint estimation approach, but the comparison is then restricted to the posterior mode estimates. The comparison shows that the marginal and joint estimates of the foreign parameters are overall very similar.

As described earlier, the foreign data series are obtained as (time-varying) trade weighted averages of euro area and US data series, where the weight on the euro area is dominant (around 85%). We therefore choose to contrast our parameter estimates with estimates obtained from estimating DSGE models on euro area data, while acknowledging that the comparison is imperfect since the foreign data series are constructed in the way described above.

The data generally suggest a high degree of nominal rigidities in the model. The various Calvo parameters are generally well identified by the data and the posterior medians are substantially larger than the prior medians. The posterior mode estimate of the foreign price Calvo parameter, ξ^*, equals 0.92 which implies that firms re-optimize prices roughly every third year ($\frac{1}{1-0.92} \approx 12$ quarters). Similar estimates of the Calvo parameter were obtained by Smets and Wouters (2003), Adolfson et al. (2007) and Christoffel, Coenen, and Warne (2008) with euro area data and these estimates imply a flat slope of the Phillips curve. The implied slope of the Phillips curve equals $\Xi^* = 0.0033$. In other

Table 7: Prior and marginal posterior distribution of estimated foreign parameters

Parameter		Prior Dist	Mean	Std	Scale	Posterior Mode	Median	Std	5%	95%
b^*	Habit	B	0.75	0.10		0.64	0.66	0.05	0.57	0.74
ξ^*	Calvo, price	B	0.75	0.075		0.92	0.92	0.01	0.90	0.95
ξ_w^*	Calvo wage	B	0.75	0.075		0.86	0.87	0.02	0.84	0.90
φ^*	Labour disutility	G	3.0	1.5		6.00	6.66	1.17	5.04	8.86
$\tilde{S}''^{,*}$	Inv adj cost	N	5.0	2.5		3.99	4.51	1.72	2.06	7.70
κ^*	Indexation, price	B	0.5	0.2		0.55	0.51	0.12	0.29	0.70
Monetary policy										
ρ_{R^*}	Smoothing	B	0.85	0.10		0.93	0.93	0.02	0.90	0.96
r_{π^*}	Inflation	N	1.75	0.15		1.75	1.75	0.15	1.51	1.99
r_{RU^*}	Unemp. rate	N	0.125	0.125		0.12	0.13	0.04	0.07	0.21
$r_{\Delta RU^*}$	Unem. rate, change	N	0.15	0.075		0.24	0.24	0.03	0.19	0.29
r_{χ^*}	Spread	N	0	1		0.57	0.59	0.17	0.30	0.87
Shock persistence										
ρ_{ϵ^*}	Temp. technology	B	0.75	0.10		0.81	0.79	0.09	0.62	0.91
$\rho_{p^{ce,*}}$	Energy, rel. price	B	0.75	0.10		0.91	0.90	0.03	0.85	0.95
ρ_{g^*}	Gov. cons.	B	0.75	0.10		0.98	0.97	0.01	0.94	0.99
ρ_{χ^*}	Risk premium	B	0.75	0.10		0.91	0.91	0.03	0.87	0.95
$\rho_{\lambda^{w,*}}$	Wage markup	B	0.75	0.10		0.90	0.80	0.10	0.60	0.92
$\rho_{\mu_z^*}$	Perm. techn.	B	0.75	0.10		0.55	0.57	0.05	0.48	0.64
ρ_{Υ^*}	Temp. inv.	B	0.75	0.10		0.66	0.66	0.05	0.57	0.74
$\rho_{\zeta^{c,*}}$	Cons. pref.	B	0.75	0.10		0.73	0.73	0.08	0.57	0.84
$\rho_{\zeta^{n,*}}$	Labour supply	B	0.75	0.10		0.99	0.99	0.01	0.97	0.99
$\rho_{z^{R,*}}$	Real int. rate trend	B	0.85	0.10		0.99	0.98	0.01	0.96	1.00
Shock, MA										
$\theta_{\lambda^{w,*}}$	Wage markup	N	0.0	0.5		0.85	0.72	0.12	0.48	0.88
$\theta_{z^{R,*}}$	Real int. rate trend	N	0.0	0.5		−0.75	−0.76	0.20	−1.12	−0.48
Innovation std.										
σ_{ϵ^*}	Temp technology	IG	0.2	∞	0.01	0.06	0.07	0.01	0.05	0.10
$\sigma_{p^{ce,*}}$	Energy, rel price	IG	0.2	∞	0.10	0.29	0.29	0.02	0.26	0.33
$\sigma_{\varepsilon_R^*}$	Monetary policy	IG	0.1	∞	0.01	0.03	0.03	0.01	0.02	0.03
σ_{g^*}	Gov. cons.	IG	0.2	∞	0.1	0.10	0.10	0.01	0.09	0.11
σ_{χ^*}	Risk premium	IG	0.2	∞	0.001	0.33	0.33	0.03	0.29	0.38
σ_{λ^*}	Price markup	IG	0.2	∞	1	0.18	0.21	0.09	0.12	0.39
$\sigma_{\lambda^{w,*}}$	Wage markup	IG	0.2	∞	10	0.10	0.13	0.05	0.08	0.24
$\sigma_{\mu_z^*}$	Perm. techn.	IG	0.2	∞	0.01	0.42	0.42	0.04	0.37	0.48
σ_{Υ^*}	Temp inv.	IG	0.2	∞	0.1	0.26	0.29	0.10	0.16	0.47
$\sigma_{\zeta^{c,*}}$	Cons. pref.	IG	0.2	∞	0.1	0.06	0.07	0.01	0.05	0.10
$\sigma_{\zeta^{n,*}}$	Labour supply	IG	0.2	∞	0.1	0.09	0.10	0.02	0.08	0.14
$\sigma_{z^{R,*}}$	Real int. rate trend	IG	0.1	∞	0.01	0.04	0.04	0.01	0.03	0.05
$\sigma_{\bar{\pi}^{c,*}}$	Inflation trend	IG	0.1	∞	0.001	0.05	0.17	0.15	0.04	0.49
Shock, correlation										
$c_{\zeta^{c,*},\Upsilon^*}$	Cons. pref., inv.	B	0.5	0.2		0.59	0.53	0.11	0.34	0.69

Based on 1,000,000 draws from the marginal posterior distribution for the vector of foreign economy parameters in the two-country model. Foreign data series used in estimation. Prior densities: Beta (B), Normal (N), Gamma (G), and Inverse Gamma (IG).

words, inflation is fairly insensitive to fluctuations in marginal costs and resource utilization. A lower estimate of the price Calvo parameter was reported by Coenen et al. (2018).[109]

The posterior mode estimate of the wage Calvo parameter is $\xi_w^* = 0.86$ which implies that households (through their unions) re-optimize wages roughly every second year. Notably lower estimates of this parameter are reported by Christoffel, Coenen, and Warne (2008), Smets, Warne, and Wouters (2013) and Coenen et al. (2018) on euro area data. The price and wage indexation parameters, κ^* and κ_w^*, which govern whether the Phillips curves are purely forward-looking or not, are not important for overall model fit and we choose to estimate κ^* and to calibrate $\kappa_w^* = 0$.

The estimates of the parameters governing the degree of real rigidities — the habit formation and investment adjustment cost parameters — appear to be broadly in line with those reported in the papers mentioned above. The labour supply elasticity parameter, φ^*, which determines the shape of the distribution of work disutilities across individuals, is estimated to 6.0. Since we assume a steady-state unemployment rate of $U^* = 8.0$ percent, this yields an implied estimate of the steady-state gross wage markup equal to $\lambda^{w,*} = \exp(\varphi^* U^*) = 1.6$. For comparison, Smets, Warne, and Wouters (2013) estimate a DSGE model with the Galí, Smets, and Wouters (2012) modelling of the labour market on euro area data and obtain quite similar estimates: $\varphi^* = 5.7$ and $\lambda^{w,*} = 1.5$, such that the steady-state rate of unemployment equals 7.1% in their case.

The trend real interest rate is estimated to be highly persistent, a near unit root, which is not surprising given the downward trends in the real and nominal interest rates in the sample period. The dependence of the policy rule intercept on the risk premium, i.e. the spread between risky and safe assets, is also supported by the data, i.e. r_{χ^*} is estimated to be significantly larger than zero.[110] The inflation trend (or target) shock is estimated to be unimportant, i.e. its variance is very low.[111] The estimate of the smoothing parameter in the policy rule is in line with the estimate of Coenen et al. (2018), while the inflation response coefficient, r_{π^*}, is significantly lower than their estimate. The difference in the estimates of this parameter could be due to the incorporation of the real interest rate trend in our model, which implies that the difference between the actual policy interest rate and the policy rule intercept becomes smaller. We also note that r_{π^*} is not well identified by the data.

The labour supply shock is estimated to be very persistent, which follows from the high persistence of the observed labour market variables, even though the employment rate series has been detrended as described above. The relative price of energy and risk premium shocks are also fairly persistent which is, again, in line with what is observed in the data, where both the relative price of energy and spreads are fairly persistent. The choice to model the wage markup and real interest rate trend shocks as ARMA(1,1) processes is validated by the posterior probability intervals for the respective MA terms, $\theta_{\lambda^{w,*}}$ and $\theta_{z^{R,*}}$, which do not include the value zero. Finally, the correlation between the marginal efficiency of investment and consumption preference shocks is estimated to be significant. Loosely speaking the correlation is in line with the generally positive comovement between household and firm sentiment indicators and it allows the model to better capture the rather strong positive correlation between consumption and investment growth observed in the data.

3.6.2 Marginal versus joint estimation of the foreign parameters

In the previous section, the posterior distribution of the foreign parameters was presented and discussed. It is interesting to compare these marginal parameter estimates (expression (3.17), case 1) with those obtained when all parameters, foreign and domestic, are estimated jointly (expression (3.19),

[109] Re-computing the Calvo parameter under the assumption that intermediate goods are instead aggregated using the Kimball aggregator with curvature parameter $\epsilon_d = 10$ yields $\xi^* = 0.865$. For comparison Coenen et al. (2018) who use the Kimball aggregator obtain the estimate 0.82 on euro area data.

[110] Note that the prior of r_{χ^*} is centered on zero while the posterior 90% interval ranges from 0.30 to 0.87. The terminology 'significant' means that the interval does not cover zero. A simple intuition for the positive estimate is that the policy rate has trended downwards in the sample while the spread increased around the time of the financial crisis.

[111] The unimportance of the foreign inflation trend shock is clearly illustrated by the smoothed estimate of the foreign inflation trend; see the Appendix.

case 2) using both foreign and domestic data. Remember that the foreign model is exogenous in the sense that domestic shocks do not influence foreign variables. However, the addition of Swedish data in estimation affects the foreign parameter estimates when the model's parameters are estimated jointly, i.e. inference on the foreign parameters is not 'exogenous'. While in principle it is natural that Swedish data carry information also on the workings of the foreign/global economy, it would appear problematic if the Swedish data are 'too influential' for the inference on the foreign parameters.

The posterior mode estimates of the foreign parameters in these two cases are contrasted in Table 8. The overall impression is that the marginal and joint posterior mode estimates of most of the foreign parameters are very similar. Therefore, the discussion and interpretation of the parameter estimates do not depend crucially on which of the two sets of estimates that is considered. An exception concerns the specification of the foreign and domestic inflation trends, which is discussed in more detail later. In the table we also include the posterior mode estimates of foreign parameters when it is assumed that the inflation trend is common to the Swedish and foreign economies. The specification of inflation trends is discussed further in the sensitivity analysis; see Section 3.8.2.

3.6.3 Posterior estimates of the domestic parameters

The posterior estimates of domestic parameters are provided in Table 9 (structural parameters) and Table 10 (shock process parameters). These estimates are obtained by optimizing and sampling from the joint posterior density (3.19).

Similar to what was found for the foreign economy, the data generally suggest a high degree of nominal rigidities in the domestic economy — the various Calvo parameters are rather tightly estimated and the posterior medians are substantially higher than the prior medians. The posterior mode estimate of the domestic price Calvo parameter, ξ_d, equals 0.94, and a re-computation using the Kimball aggregator with standard values (see above) yields a Calvo parameter equal to 0.90. The corresponding slope of the Phillips curve is $\Xi^* = 0.0027$. Our estimate of the Calvo parameter is higher than previous estimates using Riksbank models and Swedish data; see Adolfson et al. (2008) and Christiano, Trabandt, and Walentin (2011).[112] The estimate of the Calvo parameter associated with the import of consumption goods excluding energy, $\xi_{m,c}$, which is a key parameter for the pass-through of the exchange rate to consumer prices, equals 0.92. This estimate is quite close to those reported for Ramses I but somewhat higher than the estimate in Ramses II. As discussed in Section 3.2.1, the fact that our observed import price measure corresponds to final consumer prices leads us to expect a high degree of nominal rigidities in the import consumption sector and thereby a low degree of exchange rate pass-through in the short to medium run.[113] Note that the pass-through is higher in the very long run when the nominal rigidities have dissipated.[114] The remaining domestic price Calvo parameters, $\xi_{m,i}$, $\xi_{m,x}$ and ξ_x, are estimated just below 0.80. The estimate of the wage Calvo parameter implies that wages are re-optimized once every seven quarters, which is more frequently than the average time between collective wage bargainings in Sweden but less frequently than the usual time between individual wage negotiations (see discussion in Section 3.4.1 above). The estimate

[112] Recall that Adolfson et al. (2008) estimate Ramses I on Swedish data, Christiano, Trabandt, and Walentin (2011) develop and estimate Ramses II on Swedish data and Adolfson et al. (2013) estimate a version of Ramses II for use in the Riksbank's policy process.

[113] We note also that modelling energy price contributions explicitly allows us to better capture the different degree of exchange rate pass-through associated with different types of goods. As discussed in Section 3.4.1, our low estimates of the Calvo parameter in the energy import sector indicate that exchange rate pass-through to energy prices is high and immediate. Excluding energy from the consumption imports aggregate may then well result in lower pass-through to prices of the remaining import goods, i.e. a higher estimate for $\xi_{m,c}$. Imported non-energy consumption is itself composed of a large variety of goods, all of which are associated with a different degree of pass-through. For a more complete assessment of pass-through differences across sectors, further disaggregation would be needed. Still, separating energy from imports of other goods is an important step, as pass-through to to import prices is generally found to be higher for energy compared to goods and services (see e.g. Campa and Goldberg (2008), Ben Cheikh and Rault (2017), and Ortega and Osbat (2020)).

[114] This does not necessarily imply that observed aggregate consumer prices always comove positively with the exchange rate in the model, as there are other factors influencing price setting, not least domestic ones.

Table 8: Marginal and joint posterior mode estimates of foreign parameters

Parameter		Posterior mode 1. Marginal	2. Joint posterior Ind. inf. trends	Common inf. trends
b^*	Habit	0.64	0.63	0.64
ξ^*	Calvo, price	0.92	0.93	0.95
ξ_w^*	Calvo wage	0.86	0.85	0.86
φ^*	Labour disutility	6.00	5.64	6.19
$\tilde{S}''^{,*}$	Inv. adj. cost	3.99	4.06	2.79
κ^*	Indexation, price	0.55	0.57	0.45
Monetary policy				
ρ_{R^*}	Smoothing	0.93	0.96	0.96
r_{π^*}	Inflation	1.75	1.79	1.77
r_{RU^*}	Unemp. rate	0.12	0.15	0.22
$r_{\Delta RU^*}$	Unem. rate, change	0.24	0.22	0.21
r_{χ^*}	Spread	0.57	0.54	0.60
Shock persistence				
ρ_{ϵ^*}	Temp. technology	0.81	0.87	0.86
$\rho_{p^{ce,*}}$	Energy, rel. price	0.91	0.91	0.91
ρ_{g^*}	Gov. cons.	0.98	0.98	0.97
ρ_{χ^*}	Risk premium	0.91	0.90	0.90
$\rho_{\lambda^{w,*}}$	Wage markup	0.90	0.88	0.78
$\rho_{\mu_z^*}$	Perm. techn.	0.55	0.53	0.54
ρ_{Υ^*}	Temp. inv.	0.66	0.60	0.63
$\rho_{\zeta^{c,*}}$	Cons. pref.	0.73	0.78	0.78
$\rho_{\zeta^{n,*}}$	Labour supply	0.99	0.99	0.99
$\rho_{z^{R,*}}$	Real int. rate trend	0.99	0.99	0.99
Shock, MA				
$\theta_{\lambda^{w,*}}$	Wage markup	0.85	0.83	0.75
$\theta_{z^{R,*}}$	Real int. rate trend	−0.75	−0.81	−0.80
Innovation std.				
σ_{ϵ^*}	Temp technology	0.06	0.07	0.07
$\sigma_{p^{ce,*}}$	Energy, rel price	0.29	0.29	0.29
$\sigma_{\varepsilon_R^*}$	Mon. pol.	0.03	0.03	0.03
σ_{g^*}	Gov. cons.	0.10	0.10	0.10
σ_{χ^*}	Risk premium	0.33	0.33	0.33
σ_{λ^*}	Price markup	0.18	0.26	0.46
$\sigma_{\lambda^{w,*}}$	Wage markup	0.10	0.10	0.13
$\sigma_{\mu_z^*}$	Perm. techn.	0.42	0.40	0.41
σ_{Υ^*}	Temp inv.	0.26	0.29	0.21
$\sigma_{\zeta^{c,*}}$	Cons. pref.	0.06	0.07	0.07
$\sigma_{\zeta^{n,*}}$	Labour supply	0.09	0.09	0.10
$\sigma_{z^{R,*}}$	Real int. rate trend	0.04	0.04	0.04
$\sigma_{\bar{\pi}^{c,*}}$	Inflation trend	0.05	0.05	0.44
Shock, correlation				
$c_{\zeta^{c,*},\Upsilon^*}$	Cons. pref., inv.	0.59	0.46	0.49

Posterior mode of foreign economy parameters for i) marginal posterior, based on foreign data series, ii) joint posterior, based on foreign and domestic data series, and iii) joint posterior for a version of the model where the inflation trend shock is assumed to be common for the domestic and foreign economies.

Table 9: Prior and posterior distributions of estimated domestic structural parameters

Parameter		Prior Dist	Mean	Std	Posterior Mode	Median	Std	5%	95%
b	Habit	B	0.75	0.10	0.75	0.77	0.04	0.69	0.82
ξ_d	Calvo, dom. price	B	0.75	0.075	0.94	0.95	0.01	0.93	0.96
$\xi_{m,c}$	Calvo, imp. cons.	B	0.75	0.075	0.92	0.92	0.01	0.91	0.94
$\xi_{m,i}$	Calvo, imp. inv.	B	0.75	0.075	0.79	0.79	0.03	0.73	0.84
$\xi_{m,x}$	Calvo, imp. exp.	B	0.75	0.075	0.80	0.80	0.03	0.75	0.85
ξ_x	Calvo, exp.	B	0.75	0.075	0.79	0.83	0.05	0.74	0.88
ξ_w	Calvo, wage	B	0.75	0.075	0.86	0.87	0.02	0.83	0.90
η_c	Subst, dom and imp, cons	G	1.01	0.5	0.87	1.02	0.48	0.41	1.97
η_i	Subst, dom and imp, inv	G	1.01	0.5	0.27	0.30	0.13	0.12	0.55
η_f	Subst, dom and imp goods	G	1.01	0.5	0.37	0.51	0.27	0.20	1.04
η_x	Subst, dom and imp, exp.	G	1.01	0.5	1.53	1.53	0.40	0.93	2.24
$\tilde{\phi}_s$	UIP, risk premium	B	0.5	0.2	0.16	0.20	0.06	0.10	0.31
σ_a	Capital util.	IG	0.5	∞	0.17	0.19	0.06	0.12	0.31
$\tilde{S}''$	Inv. adj. cost	N	5.0	2.5	8.39	8.95	1.22	7.09	11.10
κ_d	Indexation, price	B	0.5	0.2	0.33	0.32	0.11	0.15	0.51
φ	Labour disutility	G	3.0	1.5	3.65	4.09	0.79	3.05	5.59
ω_c^x	Exp., weight on cons.	B	0.5	0.2	0.27	0.28	0.12	0.11	0.49
Monetary policy									
ρ_R	Smoothing	B	0.85	0.10	0.92	0.92	0.02	0.88	0.95
r_π	Inflation	N	1.75	0.15	1.71	1.71	0.15	1.45	1.96
r_{RU}	Unemp.rate	N	0.125	0.125	0.25	0.25	0.07	0.15	0.38
$r_{\Delta RU}$	Unemp.rate, change	N	0.15	0.075	0.17	0.17	0.02	0.13	0.21
r_χ	Spread	N	0	1	0.59	0.61	0.16	0.35	0.87

Based on 1,000,000 draws from the joint posterior distribution for the vector of foreign and domestic economy parameters in the two-country model. Foreign and domestic data series used in estimation. Prior densities: Beta (B), Normal (N), Gamma (G), and Inverse Gamma (IG).

Table 10: Prior and posterior distributions of estimated domestic shock process parameters

Parameter		Prior Dist	Mean	Std	Scale	Posterior Mode	Median	Std	5%	95%
Shock persistence										
ρ_{ϵ}	Temp. techn.	B	0.5	0.2		0.87	0.87	0.04	0.79	0.92
$\rho_{p^{d,ce}}$	Energy, rel. price	B	0.5	0.2		0.88	0.86	0.06	0.74	0.94
ρ_g	Gov. cons.	B	0.5	0.2		0.66	0.64	0.02	0.44	0.78
ρ_{χ}	Risk premium	B	0.5	0.2		0.69	0.70	0.05	0.61	0.78
ρ_{ζ^n}	Labour supply	B	0.5	0.2		0.83	0.83	0.08	0.67	0.92
$\rho_{\tilde{\phi}}$	UIP risk premium	B	0.5	0.2		0.84	0.79	0.08	0.64	0.88
ρ_{λ^w}	Wage markup	B	0.5	0.2		0.44	0.44	0.10	0.27	0.59
Innovation std										
σ_{ϵ}	Temp. techn.	IG	0.2	∞	0.01	0.59	0.60	0.05	0.52	0.68
$\sigma_{p^{d,ce}}$	Energy, relative price	IG	0.2	∞	0.10	0.51	0.52	0.04	0.46	0.59
σ_{ε_R}	Monetary policy	IG	0.2	∞	0.01	0.05	0.06	0.01	0.05	0.07
σ_g	Gov. cons.	IG	0.2	∞	0.1	0.26	0.26	0.02	0.23	0.30
σ_{χ}	Risk premium	IG	0.2	∞	0.01	0.04	0.04	0.003	0.03	0.04
σ_{λ^d}	Markup, dom.	IG	0.2	∞	10	0.07	0.08	0.02	0.05	0.12
$\sigma_{\lambda^{m,c}}$	Markup, imp. cons.	IG	0.2	∞	10	0.07	0.08	0.02	0.05	0.12
$\sigma_{\lambda^{m,i}}$	Markup, imp. inv.	IG	0.2	∞	1	0.91	0.98	0.31	0.63	1.67
$\sigma_{\lambda^{m,x}}$	Markup, imp. exp.	IG	0.2	∞	10	0.10	0.11	0.04	0.07	0.19
σ_{λ^x}	Markup, exp.	IG	0.2	∞	10	0.11	0.12	0.06	0.07	0.28
$\sigma_{\tilde{\phi}}$	UIP,risk premium	IG	0.2	∞	0.01	0.30	0.38	0.11	0.23	0.59
σ_{Υ}	Temp. inv.	IG	0.2	∞	1	0.22	0.23	0.03	0.19	0.30
σ_{ζ^c}	Cons. preference	IG	0.2	∞	0.1	0.22	0.23	0.05	0.17	0.32
σ_{ζ^n}	Labour supply	IG	0.2	∞	0.1	0.10	0.11	0.02	0.08	0.16
σ_{λ^w}	Wage markup	IG	0.2	∞	10	0.09	0.11	0.04	0.07	0.18
$\sigma_{\bar{\pi}^c}$	Inflation trend	IG	0.2	∞	0.001	0.50	0.50	0.19	0.25	0.87
Shock correlations										
c_{ϵ,ϵ^*}	Temp. techn.	B	0.5	0.2		0.41	0.39	0.15	0.15	0.65
$c_{p^{d,ce},p^{ce,*}}$	Energy, rel. price	B	0.5	0.2		0.34	0.34	0.13	0.13	0.57
c_{g,g^*}	Gov. cons.	B	0.5	0.2		0.59	0.56	0.14	0.31	0.76
c_{Υ,Υ^*}	Inv.	B	0.5	0.2		0.07	0.08	0.04	0.03	0.16
$c_{\zeta^c,\zeta^{c,*}}$	Cons. pref.	B	0.5	0.2		0.35	0.39	0.12	0.20	0.59
$c_{\zeta^n,\zeta^{n,*}}$	Labour supply	B	0.5	0.2		0.44	0.43	0.16	0.17	0.71
c_{χ,χ^*}	Risk premium	B	0.5	0.2		0.68	0.66	0.09	0.49	0.78
$c_{\tilde{\phi},\,-\mu_z^*}$	UIP risk premium	B	0.5	0.2		0.16	0.18	0.06	0.08	0.28
$c_{\lambda^{m,i},\,-\mu_z^*}$	Markup, imp. inv.	B	0.5	0.2		0.31	0.31	0.06	0.21	0.41

Based on 1,000,000 draws from the joint posterior distribution for the vector of foreign and domestic economy parameters in the two-country model. Foreign and domestic data series used in estimation. Prior densities: Beta (B), Normal (N), Gamma (G), and Inverse Gamma (IG).

of the domestic indexation parameter, $\kappa_d = 0.33$, implies that the weight on previous period inflation in the Phillips curve for the domestic goods equals $\frac{\kappa_d}{1+\beta\kappa_d} = 0.25$, while the weight on the expected next period inflation equals $\frac{\beta}{1+\beta\kappa_d} = 0.75$.

The elasticities of substitution are generally imprecisely estimated, and it is only the investment elasticity, η_i, which is estimated to be significantly below 1 (i.e. the 90 percent posterior probability interval does not contain the value 1). In general, therefore, the data does not provide conclusive evidence on whether goods are complements ($\eta < 1$) or substitutes ($\eta > 1$) . The posterior mode of the elasticity of substitution between domestic and imported consumption goods, η_c, is 0.87 implying that the goods are complements rather than substitutes. The 90% posterior probability interval for the parameter, however, includes values both below and above 1. The foreign elasticity of substitution between domestic and imported goods, η_f, which enters our model through the aggregate export demand, is also estimated to be rather low, but again it is not significantly below 1. For the elasticity of substitution in the production of export goods, η_x, however, we obtain a posterior mode estimate above unity. Our results are in line with earlier Riksbank models, as both Adolfson et al. (2008) and Christiano, Trabandt, and Walentin (2011) obtain low estimates for most sectors, although above unity as their priors did not allow for values below 1. The exception is the high consumption elasticity, η_c, in Ramses I, which was partly explained by the high volatility of imports in combination with a smooth processes for aggregate consumption. When imports were included as input in the production of exports in Ramses II, this problem was alleviated as a substantial part of imports entered directly into exports, resulting in a low value also for η_c.[115]

The estimates of the parameters governing the degree of real rigidities — the habit formation and investment adjustment cost parameters — are broadly in line with those reported by Adolfson et al. (2008) and suggest that these rigidities are important. The inverse Frisch elasticity, φ, is estimated to 3.65, which implies a gross steady-state wage markup equal to $\lambda^w = \exp(\varphi U) = \exp(3.65 * 0.072) = 1.30$. A lower steady-state unemployment rate in Sweden in comparison to the foreign economy (which is dominated by the euro area) is thus rationalised partly through lower market power of households/unions in wage setting in Sweden compared to the foreign economy, i.e. $\lambda^w < \lambda^{w,*}$, and partly as a larger responsiveness of the labour supply to changes in wages, i.e. a larger Frisch elasticity, in Sweden.

The weight on foreign consumption in the export demand function is estimated as $\omega_c^x = 0.25$, which means that the weight is lower than the share of consumption in foreign GDP (note again that the foreign consumption share is calibrated to 59%). Swedish export demand therefore depends to a larger extent on foreign investment. The large weight of foreign investment, $1 - \omega_c^x = 0.75$, can be explained through the properties of the data series involved. First, foreign investment is more volatile than foreign consumption and the investment volatility is closer to the volatility of Swedish trade; see Table 2. Second, the correlation between Swedish exports and foreign investment is somewhat larger than the correlation between exports and foreign consumption; see the Appendix. Furthermore, the 90% posterior probability interval for ω_c^x does not contain the value for the foreign consumption share in private absorption, $c^*/(c^* + i^*) = 0.74$. This can be interpreted as indicating that a more 'standard' export demand function, where exports are related to foreign GDP, is rejected by the data and that our more flexible specification is preferred instead.[116] The importance of foreign investment for Swedish exports is also in line with the observation that a large part of Swedish exports consists of investment goods.

The estimates of the monetary policy rule parameters are fairly similar to those obtained for the foreign economy. In particular, the response coefficients for the level and change of the unemployment rate are fairly large in comparison with the real variable gap coefficients reported by Adolfson et al.

[115] Christiano, Trabandt, and Walentin (2011) also mention problems with some of their estimations as the posterior mode of η_c was driven to its lower bound of 1. This supports the finding in our estimation that rather low elasticities of substitution are favoured by the data.

[116] However, it does not mean that our specification of export demand is the best possible specification. Note again that we have opted for a fairly simple export demand function. Further improvements are left for future model development.

(2008) and Christiano, Trabandt, and Walentin (2011) for the Riksbank's previous models. However, neither Ramses I nor Ramses II included unemployment as the measure of resource utilization in the monetary policy reaction function, which makes their estimates less comparable to ours. The response coefficient for inflation in the rule does not appear to be well identified.

The persistence parameters of the domestic shocks are generally estimated to be lower than the corresponding parameters for the foreign shocks. One reason is that the shocks that are correlated with their foreign counterparts partly inherit the persistence of the foreign shock, which reduces the need for additional persistence through the domestic persistence parameter. The foreign and Swedish risk premium shocks are rather strongly correlated, which is explained by the positive correlation between foreign and domestic interest rate spreads in the data. A higher global demand for safe assets, or a 'risk off' mode in the global financial markets, has similar effects on Swedish financial markets, with increasing risk spreads as one manifestation. Reasonably large correlations between the foreign and Swedish shocks are also recorded for some other 'natural shock pairs', e.g. the consumer preference shocks. Consumer preference shocks capture changes in consumer sentiment and Swedish sentiment indicators are related to foreign. In other words, there is a global component affecting the mood of consumers in different countries in similar ways. Based on more detailed forecast error variance decompositions reported in the Appendix, it appears that these two shock correlations, the risk premium c_{χ,χ^*} and the consumer preference $c_{\zeta^c,\zeta^{c,*}}$, along with the energy price shock correlation, $c_{p^{d,ce},p^{ce,*}}$, are among the key parameters for increasing the importance of foreign shocks for Swedish variables. While some of the other shock correlation parameters are estimated to be substantial, e.g. the one between government consumption shocks c_{g,g^*}, the limited importance of the government consumption shock in the foreign economy implies that it cannot matter a whole lot for the fluctuations in Swedish variables.

Finally, we allow for an empirically motivated correlation between the country risk premium shock and the global permanent technology shock. While the estimated correlation is low, it is still helpful in accounting for the empirical relationship between the exchange rate and Swedish and foreign real economic developments.

3.6.4 Conditional estimation of the domestic parameters

Estimating the domestic parameters conditional on the foreign marginal parameter estimates, i.e. estimating equation (3.18), yields posterior mode estimates of the domestic parameters which are very similar to those obtained when estimating all parameters jointly. This result is anticipated since the marginal and joint posterior mode estimates of the foreign parameters are similar; see Section 3.6.2. The median (mean) percentage difference between the joint and conditional posterior mode estimates of the 54 domestic parameters is 1.3% (3.3%). Our assessment based on these results, and those reported for the foreign parameters in Section 3.6.2, is that the question of whether to estimate the foreign and domestic parameters separately (marginal foreign and then conditional domestic) or jointly is not a key issue. The conditional posterior estimates of the domestic parameters are provided in the Appendix.

3.6.5 Modified posterior distribution

The modified posterior adds a penalty term to the posterior, penalising deviations between a set of model-implied variances and covariances and their counterparts in data. We include the variances of the 16 observed variables for which the ADF test rejects the null hypothesis of a unit root; see Table 2. The 20 covariances included in the penalty term are generally those of pairs of variables which are strongly correlated in the data. The following ten covariances of cross-country pairs of variables are included.

- The covariance of foreign GDP growth with Swedish GDP growth, export growth and import growth, respectively.

- The covariance between foreign and Swedish 'same variable' pairs for consumption growth, investment growth, policy rates, core and headline inflation, wage inflation and the unemployment rate.

The full set of matched moments, i.e. including ten within-country covariances, and the modified posterior mode estimates are provided in the Appendix. Next we briefly summarise the main differences between the posterior and modified posterior estimates.

The modified posterior parameter estimates imply a larger degree of both real and nominal rigidities than the posterior mode estimates, and the shock correlations are generally estimated to be larger. In particular, the key foreign and domestic Calvo parameters are estimated to be even larger than those reported above.

As should be anticipated the modified posterior estimates yield a better model fit in the dimensions targeted, i.e. the targeted variances and covariances are generally better aligned with the data. However, the 'gains', e.g. the ability to better reproduce some of the strong correlations in the data, may not be considered that large overall. The model's ability to fit the standard deviations in the data when estimated using standard Bayesian methods is already good (see Section 3.7 below), and improves only marginally with the modified posterior mode estimate. The average absolute deviation between the sample correlations in the data and the population correlations in the model drops from 0.22 to 0.16 for the 20 targeted pairs of variables, when computed for the posterior and modified posterior mode parameters, respectively. Some of the largest gains are recorded for relationships between Swedish GDP and its components, and between Swedish GDP and foreign GDP and its components. For pairs of variables from these groups, which, again, are strongly correlated in the data, the model correlations increase by 0.10–0.20 with the modified posterior. For example, the correlation between Swedish and foreign GDP per capita growth in the sample data is 0.68. The model-implied population correlation for the posterior mode parameter is 0.31, while the model-implied correlation for the modified posterior mode estimate equals 0.44, for a 'gain' of $0.44 - 0.31 = 0.13$.

3.6.6 Assessment of estimation approaches

In this section, we briefly summarise the experiences from the alternative estimation approaches discussed above. We also discuss some potential advantages and disadvantages of using the different estimation approaches in a central bank context.

Estimation of the marginal posterior distribution of the foreign parameters followed by estimation of the conditional posterior distribution of the domestic parameters produces parameter estimates which are similar to those obtained when all parameters are estimated jointly. The inclusion of Swedish data when estimating the foreign and domestic parameters jointly does not imply that the foreign parameter estimates change substantially. Hence the choice between the marginal-conditional and joint approaches is not crucial for our inferences. This result is, we believe, reassuring since it would be difficult to motivate large differences in foreign parameter estimates resulting from the two approaches.

A practical advantage of the marginal-conditional estimation approach is that estimation is much simplified since fewer parameters are estimated in each step. A key disadvantage is simply that it does not provide us with a joint probability distribution for all the parameters in the model. One cannot use the marginal and conditional posterior distributions as defined here to infer the posterior distribution of objects of interest which depend both on foreign and domestic parameters, e.g. correlations between foreign and domestic variables.

The modified posterior allows us to emphasise certain features in the data when estimating the model. Since we are primarily interested in better capturing the dependencies between foreign and domestic variables, we choose to match covariances of cross-country variable pairs. Our results show that it is possible to increase the model-implied correlations but the increases, or gains, could be characterised as being modest. In Section 4.1, using forecast error variance decompositions we provide further evidence that the modified posterior approach yields parameter estimates which imply larger spillovers from the foreign to the domestic economy.

The main advantage of the modified posterior approach is that it allows us to put a greater emphasis on aspects of the data that could be considered more important in a direct way. Achieving the same objective e.g. by considering alternative prior distributions for individual parameters would arguably be more difficult. However, the approach also has some potential disadvantages. First, the exact choice of dimensions in the data to target is necessarily arbitrary and is therefore likely to be contested. Second, in our experience, an already cumbersome estimation exercise involving a large number of parameters, is further complicated by the addition of the penalty to the objective function. One manifestation of this is that it is generally more difficult to find a good proposal density for the Metropolis-Hastings sampler.

3.7 Model fit

3.7.1 Comparing posterior model statistics to the data

An advantage of the Bayesian approach is that it is relatively straightforward to compute the posterior distributions of various statistics of interest, and compare these to the corresponding statistics in the data (which were discussed in Section 3.2). In this section we study the posterior distributions of the observed variables' volatility and contemporaneous correlations.[117] Comparing the posterior distributions for these statistics to the corresponding data statistics provides an assessment of the model's ability to capture the volatilities of the observed variables and the correlations among the variables. This provides a partial assessment of the model's ability to fit the data.

We consider the baseline model where all the model's parameters are jointly estimated using standard Bayesian methods (approach 2 in Section 3.6 above). In sampling from the posterior distribution of the parameters of the model $R = 1,000,000$ draws are obtained. Here we keep every 200^{th} draw and consider the thinned chain consisting of $R = 5,000$ parameter draws. For each parameter draw, an artificial data sample of length $T = 95$ is generated using the model, where T is chosen to match the length of the sample period in quarters. For each of these simulated datasets various statistics of interest are computed, e.g. standard deviations and correlations among the variables. This yields a posterior distribution for the statistic of interest which can be summarised e.g. through percentile values of the distribution or a histogram. We refer to this as the *posterior distribution of the model sample statistic* and it is the distribution that should be compared to the data statistic. Note that the posterior distributions of the statistics obtained using this procedure reflect both parameter uncertainty and the uncertainty stemming from the shocks.

An alternative would be to compute the *posterior distribution of the theoretical/population statistic*, i.e. computing the population standard deviations and correlations for the posterior distribution of the parameters by solving the Lyapunov equation; see equation (3.8). Since this distribution only takes into account parameter uncertainty, the posterior probability intervals for the statistic in question are narrower. In the Appendix, we report also the distributions of the population standard deviations and correlations.

3.7.2 Standard deviations

In Table 11, the data sample standard deviations of the observed variables are reported along with the model's posterior 5^{th}, 12.5^{th}, 50^{th} (median), 87.5^{th} and 95^{th} percentiles for the statistic (such that 75% and 90% posterior probability intervals can be inferred from the table). As a summary statistic, we also report the probability interval with the lowest probability that contains the data sample

[117] We choose to focus on the contemporaneous correlation, rather than the entire cross-correlation function, for two reasons: first, in order to reduce the dimensionality, and, second, since most of the strong correlations in the data are found contemporanelously or at short leads/lags. Therefore, we believe, the contemporaneous correlations will also be informative on the model's ability to fit dynamic correlations.

standard deviation, where a lower probability implies that the data statistic is closer to the center of the model's posterior distribution (see the column 'cover' in the table).[118] For 14 of the 25 variables, the data sample standard deviation is contained in the posterior 50% probability interval. For 20 of the variables it is contained in the 90% interval, and for 23 in the 95% interval. Only the standard deviations of the foreign spread and Swedish GDP growth are outside the respective posterior 95% intervals. Overall, the volatilities of the variables are well captured by the model.[119]

3.7.3 Contemporaneous correlations

Next, we consider how well the model captures the contemporaneous correlations among the observed variables. With $n_y = 25$ observed variables there are $n_y (n_y - 1)/2 = 300$ pairs of variables. The data sample contemporaneous correlations for all pairs are reported in the Appendix. We first present some aggregate metrics to provide an idea of how well the contemporaneous correlations are captured by the model overall. We then focus on a subset of observed variable pairs, which are strongly correlated in the data.

The first metric considered is the overall fraction of observed variable pairs for which the signs of the data sample correlation and the posterior median sample correlation are equal. This fraction equals 75%. However, a large number of the pairwise sample correlations could be considered 'insignificant' in that they are close to zero.[120] Restricting attention to data sample correlations that exceed a threshold which is set to 0.2 (in absolute value), the fraction of correlations for which the model generates the right sign increases to 88%.[121] If one further restricts attention to 'strong correlations', where 'strong' means above the arbitrarily selected threshold of 0.5, the fraction of posterior median correlations with the correct sign increases to 100% (45 out of 45).[122] A second metric is the median of the deviations between the data sample correlation and the corresponding model posterior median correlation for all the pairs, which equals 0.13. The median deviation for correlations among foreign variables, among domestic variables, and cross-country are 0.12, 0.12 and 0.15. Hence, the ability to capture cross-country correlations appears to be only slightly worse than the ability to capture within-country correlations.

Since our main focus in developing the new DSGE model is to better capture foreign dependencies, we next study the contemporaneous correlations between Swedish and foreign variables in some more detail. Of the 45 'strong correlations' among the observed variables, around half are cross-country variable pairs. In Table 12 we choose a subset of these pairs, which are among the most strongly correlated pairs in the dataset, and report the data sample correlation along with the posterior distribution of the sample correlation in the model.[123] Note that the pairs are sorted based on the absolute size of the data sample correlation. We also note that many of the selected pairs are 'same-variable pairs', e.g. Swedish and foreign GDP per capita growth.

A comparison of the correlations in the data and in the model leads to the following observations.

[118] We consider probability intervals with mass 50%, 75%, 90% and 95%.

[119] For comparison, Adolfson et al. (2005) report the posterior probability intervals for the standard deviations of 10 domestic observed variables in Ramses I. For half of the variables, the 95% intervals cover the data sample standard deviation.

[120] For example, the sample correlation between Swedish GDP per capita quarterly growth and quarterly CPIF inflation is 0.11 while the posterior median model correlation equals -0.16. In this case the model presumably captures the *abscence* of a correlation in the data in a good way.

[121] An asymptotic normal test of the hypothesis that the correlation is zero against the two-sided alternative that the correlation is different from zero at the 5% level typically uses the critical value $1.96/\sqrt{T} = 1.96/\sqrt{95} = 0.20$. However, this test is misleading when the data series involved are persistent. For very persistent series, e.g. the policy rates, the small sample critical value is substantially higher. Here we use the terminology 'significant' mainly for descrpitive purposes.

[122] Of the 300 pairs, 141 have a correlation above 0.2 and 45 have a correlation above 0.5 (in absolute value). Again, note that most of the observed variables in the model are in (annualised) quarterly growth rates, which motivates the perhaps seemingly low threshold of 0.5.

[123] We choose to exclude pairs which are strongly correlated by construction, e.g. employment and unemployment or headline and core inflation. These correlations are usually very well matched by the model.

Table 11: Posterior distribution of observed variables' sample standard deviation

Variable	Transf. and unit	Data	Posterior distribution Percentile					
			5	12.5	50	87.5	95	Cover
Foreign (KIX20)								
Consumption	Per cap, 4qq, perc	1.45	1.21	1.29	1.48	1.71	1.81	50
CPI	4qq, perc	1.33	1.11	1.16	1.31	1.46	1.54	50
CPI excluding energy	4qq, perc	0.67	0.54	0.58	0.70	0.85	0.93	50
Employment	Per cap, perc	1.91	0.80	0.90	1.21	1.61	1.81	95
GDP	Per cap, 4qq, perc	2.13	1.48	1.56	1.76	2.01	2.13	90
Investment	Per cap, 4qq, perc	5.23	4.19	4.48	5.15	5.91	6.30	50
Policy rate	Perc	2.01	0.77	0.88	1.22	1.72	2.01	90
Corporate spread	Perc points	0.46	0.19	0.21	0.27	0.36	0.41	Outside 95
Unemployment rate	Perc	1.22	0.65	0.74	1.00	1.35	1.52	75
Wage	4qq, perc	0.87	0.76	0.81	0.95	1.14	1.24	50
Sweden								
Capacity utilization	Perc	3.36	2.65	2.93	3.79	4.93	5.57	50
Consumption	Per cap, 4qq, perc	2.76	2.31	2.43	2.72	3.04	3.20	50
CPIF	4qq, perc	1.20	1.23	1.30	1.46	1.64	1.72	95
CPIF excl. energy	4qq, perc	0.92	0.89	0.94	1.09	1.28	1.39	90
CPIF imp. excl. energy	4qq, perc	1.90	1.63	1.72	1.93	2.17	2.29	50
Employment	Per cap, perc	1.95	1.59	1.72	2.05	2.46	2.68	50
Exports	Per cap, 4qq, perc	9.56	7.65	8.05	9.04	10.15	10.67	50
GDP	Per cap, 4qq, perc	3.64	5.02	5.24	5.80	6.38	6.66	Outside 95
Imports	Per cap, 4qq, perc	9.13	7.74	8.11	8.99	9.91	10.32	50
Investment	Per cap, 4qq, perc	10.61	9.59	10.08	11.35	12.83	13.57	50
Real exchange rate	4qq, perc	9.81	8.51	8.98	10.09	11.30	11.89	50
Policy rate	Perc	2.19	1.10	1.21	1.54	2.02	2.30	90
Corporate spread	Perc points	0.40	0.21	0.22	0.28	0.35	0.39	95
Unemployment rate	Perc	1.41	1.35	1.45	1.74	2.10	2.28	90
Wage	4qq, perc	1.14	0.94	1.00	1.18	1.40	1.52	50

Per cap = per capita. Perc = percent. 4qq=annualised quarterly change. Based on 5,000 thinned parameter draws from the joint posterior distribution (every 200th draw from a chain of length 1,000,000). For each parameter draw an artificial data sample of size T=95 is simulated using the model and the sample standard deviation is computed. In the table, the distribution of model sample standard deviations is characterised through the percentiles of the distribution. 'Cover' indicates the mass of the smallest probability interval which includes the corresponding data sample standard deviation, selected among the 50, 75, 90 and 95 percent intervals.

First, the posterior probability intervals for the correlations indicate that the correlations are in most cases significantly different from zero and the model-implied median correlations always have the 'right sign'. This contrasts with the complete inability of the standard open-economy DSGE model in Justiniano and Preston (2010) to generate meaningful cross-country correlations. Second, however, our model cannot fully capture these strong correlations in the data. For most of the correlations the data sample correlation exceeds the 95^{th} percentile in the posterior distribution, which means that the model underestimates the correlation. Third, it appears more difficult for the DSGE model to capture strong correlations in the data, i.e. the deviation between data and model correlations reported in the table is larger than the median deviation of 0.13 reported above. The difficulty of capturing strong correlations is further illustrated and discussed in the Appendix.

In the table, we also report a few correlations between selected within-country pairs of strongly correlated variables. Again the model always gets the sign of the correlation right, but it appears difficult for the model to fully capture strong relationships in the data, i.e. the data correlation is typically residing in the tail of the model distribution.

In summary, the model does a reasonably good job in accounting for the correlations in the data. Significant positive (negative) correlations in the data are in most cases matched by significantly positive (negative) model-implied correlations. However, strong correlations in data, whether within- or cross-country, are typically underestimated by the model.

3.8 Model comparison and sensitivity analysis

In this section, we report sensitivity analysis of the model specification through model comparisons. The main purpose is to further motivate how we arrived at the baseline specification described above. The analysis focuses on the modelling of domestic-foreign dependencies (Section 3.8.1), inflation trends (Section 3.8.2) and the Calvo parameters which determine the slopes of the Phillips curves in the model (Section 3.8.3). The alternative model specifications are obtained by using alternative prior distributions for a subset of the estimated parameters. Each of the specifications is estimated using Bayesian methods to obtain the posterior mode, and the main tool for comparison is the marginal likelihood, which is computed using the Laplace approximation at the posterior mode. We note again that if possible, i.e. if one of the models is nested within the other, model comparisons should preferably be framed as hypothesis tests on model parameters; see e.g. the discussion of the export demand parameter ω_c^x above for an example.

We view the results reported in this section as indicative rather than decisive. A more thorough comparison of alternative specifications could e.g. be based on modified harmonic mean (MHM) estimates of the marginal likelihood, which would require obtaining the respective posterior distributions of each of the model specifications.

3.8.1 Domestic-foreign dependencies

In this section, we report the results of experiments that shed light on the modelling of spillovers from the foreign to the domestic economy in the model. The baseline specification, with a rather flexible export demand function (equation (2.39)) and domestic-foreign shock dependencies through common or correlated shocks (see Section 2.6), is compared to alternative specifications where these two features of the model are varied. We are primarily interested in contrasting the baseline specification with alternatives where either, or both of these features are altered or shut down. The alternative specifications are briefly described below and the log marginal likelihood difference of each specification relative to the baseline, i.e. the log Bayes factor, is reported in Table 13. A positive (negative) value of the log Bayes factor means that the alternative (baseline) specification is preferred.

In summary, the experiments reported below lend support to the flexible modelling of export demand and the introduction of cross-country shock dependencies through common and correlated shocks in the baseline specification.

Table 12: Posterior distributions of contemporaneous sample correlations, foreign and Swedish variables

Variable		Data	Posterior distribution Percentile 5	12.5	50	87.5	95	Width 90%
Foreign (KIX20)	Sweden							
Policy rate	Policy rate	0.92	0.29	0.42	0.67	0.83	0.87	0.58
Corporate spread	Corporate spread	0.86	0.23	0.35	0.59	0.77	0.82	0.59
GDP	GDP	0.68	0.14	0.19	0.30	0.41	0.45	0.31
Unemployment rate	Unemployment rate	0.67	−0.05	0.08	0.36	0.58	0.66	0.71
Employment	Employment	0.63	0.01	0.14	0.42	0.63	0.70	0.69
GDP	Imports	0.66	0.13	0.18	0.30	0.41	0.45	0.32
GDP	Exports	0.62	0.10	0.16	0.28	0.40	0.45	0.35
Consumption	Consumption	0.52	0.15	0.22	0.36	0.49	0.53	0.38
Investment	Investment	0.47	−0.08	−0.01	0.14	0.29	0.35	0.43
CPI	CPIF	0.40	0.10	0.14	0.28	0.40	0.45	0.35
Foreign (KIX20)	Foreign (KIX20)							
GDP	Investment	0.82	0.49	0.54	0.65	0.74	0.77	0.28
GDP	Consumption	0.71	0.43	0.48	0.60	0.70	0.74	0.29
CPIxe	Policy rate	0.58	−0.18	−0.05	0.24	0.49	0.58	0.76
CPI	Wage	0.51	−0.14	−0.08	0.07	0.22	0.28	0.42
Employment	Wage	0.50	−0.06	0.04	0.28	0.51	0.59	0.65
Sweden	Sweden							
Wage	Policy rate	0.71	−0.25	−0.16	0.07	0.30	0.39	0.64
Exports	Imports	0.70	0.34	0.39	0.49	0.58	0.62	0.28
GDP	Exports	0.65	0.24	0.29	0.40	0.50	0.54	0.30
GDP	Investment	0.49	0.22	0.27	0.38	0.48	0.52	0.30
GDP	Real exchange rate	−0.39	−0.40	−0.35	−0.22	−0.09	−0.04	0.36

Based on 5,000 thinned parameter draws from the joint posterior distribution (every 200th draw from a chain of length 1,000,000). For each parameter draw an artificial data sample of size T=95 is simulated using the model and the sample correlation is computed. The distribution of the model sample correlations are characterised through percentiles. 'Width, 90%' is the length of a 90% probability interval computed as the difference between the 95^{th} and 5^{th} percentile values.

Table 13: Model comparison: the export demand function and shock dependencies

Model	Assumptions	Log Bayes factor
Baseline		0
'Standard' export demand	$\omega_c^x = 0.75$	−5
Export demand related only to for. inv.	$\omega_c^x = 0$	10
No shock corr.	$c_{i,j} = 0$ (9 par)	−23
No shock corr., no common shocks	$c_{i,j} = 0$ (9 par), no common shocks	−66
Shock corr. priors centered at zero	$c_{i,j}$ prior $N(0,1)$ (9 par)	−15
No time-varying neutral rate	$\rho_{z^{R,*}} = \theta_{z^{R,*}} = \sigma_{z^{R,*}} = r_\chi = r_\chi^* = 0$	−71

Export demand

It was noted above that the 90% posterior probability interval for the weight on foreign consumption in the export demand equation (2.39), ω_c^x, does not contain the value $\frac{0.59}{0.59+0.20} = 0.75$, a value which corresponds to a 'standard' export demand function where exports are related to foreign GDP. Therefore the 'standard' export demand function is rejected in favour of the more flexible specification. We verify this result by considering a model specification where the weight on foreign consumption in the export demand function is calibrated according to the consumption share in private absorption, $\omega_c^x = 0.75$, instead of being estimated using a $B(0.5, 0.2)$ prior (as in the baseline model). The marginal likelihood comparison, again, suggests that the baseline specification is preferred; see Table 13.

Since the baseline model posterior mode estimate of the weight on foreign consumption in export demand, ω_c^x, is estimated low we also consider a specification where export demand is related solely to foreign investment demand. The consumption weight is calibrated as $\omega_c^x = 0$, instead of being estimated using a $B(0.5, 0.2)$ prior. This leads to a further improvement in the log marginal likelihood compared to the baseline; see Table 13.

Shock correlations

Next, we consider alternative model specifications where the priors on the parameters governing the domestic-foreign shock dependencies are altered. In the baseline specification we estimate nine shock correlations, $c_{i,j}$, which are assigned $B(0.5, 0.2)$ priors; see Table 10. We first consider an alternative specification where the shock correlation parameters are instead assigned a truncated $N(0, 1)$ prior density, implying that the prior is centered on zero shock correlations. Estimating the model with the alternative prior we find that the estimates of the other parameters are largely unchanged when compared to those obtained with the baseline beta prior specification for the shock correlations (the parameter estimates are not reported here). Most of the shock correlations are, naturally, estimated somewhat closer to zero but most of them are still significantly different from zero, i.e. the 90 percent posterior probability interval does not contain the value zero. The marginal likelihood is lower than in the baseline specification but judging from the similarity of parameter estimates it appears that a comparison of the marginal likelihoods is not relevant in this case, i.e. the marginal likelihoods appear to be influenced by the different priors.

A model specification where the nine shock correlations are instead calibrated to zero, $c_{i,j} = 0$, rather than being estimated yields a substantially lower marginal likelihood than for the baseline; see Table 13. Next, it is further assumed that the foreign and domestic technology and real interest rate trend shocks are modelled as independent processes, instead of as being common shocks to the foreign and domestic economies. In this alternative specification the nine shock correlations are again calibrated to zero and there are no common shocks, i.e. the restrictions $\hat{\mu}_{z,t} = \hat{\mu}_{z,t}^*$ and $\hat{z}_t^R = \hat{z}_t^{R,*}$ are relaxed and $\hat{\mu}_{z,t}$ and $\hat{z}_t^R$ are instead assumed to follow independent AR(1) processes, where the priors of the parameters governing the domestic shock processes, $\hat{\mu}_{z,t}$ and $\hat{z}_t^R$, are identical to those for the corresponding foreign shocks. The Bayes factor of this specification relative to the baseline suggests that it is strongly rejected by the data; see Table 13. This lends support to our choice to model the technology and real interest rate trend shocks as common shocks to the foreign and domestic economies.

The neutral rate

Finally, a specification without a time-varying real interest rate is considered. We calibrate $\rho_{z^{R,*}} = \theta_{z^{R,*}} = \sigma_{z^{R,*}} = r_\chi = r_\chi^* = 0$, which implies that the foreign and domestic nominal and real interest rate trends are zero, i.e. $\widehat{R}_t^t = \widehat{R}_t^{t,*} = 0$ and $\widehat{\bar{R}}_t^t = \widehat{\bar{R}}_t^{t,*} = 0$. The marginal likelihood of this specification is much lower than the baseline one; see Table 13. This illustrates that the real interest rate trend in the model is important in capturing the downward trends in the domestic and foreign policy rates observed in the data.

3.8.2 Inflation trends

In the baseline specification of the model, the foreign and domestic inflation trends are assumed to be independent. The modelling of these trends is important both for the interpretation of inflation developments and for the inflation forecasts. Focusing on Sweden, a practical motivation for including the inflation trend shock is that there appear to be low frequency trends and/or persistent deviations of inflation from target, which are not easily accounted for by the other shocks in the model. For example, in Ramses II, a large part of the persistently low Swedish inflation in the latter part of the sample is attributed to non-persistent but recurring negative markup shocks, with the interpretation that low inflation is associated with low profit margins of firms. But this explanation of low inflation appears unsatisfactory given the rather persistent deviations of inflation from target. Since markup shocks typically have rather non-persistent effects on inflation, it also implies that the inflation forecasts revert back to the inflation target rather quickly.

Empirical research has suggested that national inflation rates have a substantial global component, i.e. variations in (some measure of) global inflation are a large contributor to variations in national inflation rates; see e.g. Ciccarelli and Mojon (2010). Analysis along the lines of these authors suggests that the modestly positive correlation between foreign (KIX20) and Swedish inflation is due to reasonably strong relationships between the respective high- and low-frequency components of the inflation series, while the relationship between foreign and Swedish inflation at the business cycle frequency is rather weak. The high-frequency component of inflation mainly captures energy price movements, while the comovement of the low-frequency components could be due to e.g. global productivity developments or similarities in the conduct of monetary policy. However, while the low frequency components display some degree of comovement, it is also the case that average Swedish inflation has been significantly lower than average foreign inflation in our sample; see Table 2.

In the model development phase, these empirical observations have led us to consider alternative specifications of the inflation trends, where the alternatives to the baseline model specification are no trends (which could perhaps be considered the 'standard' case), correlated inflation trend shocks or a common inflation trend. In Table 14, the marginal likelihoods of a number of model specifications, which differ in the modelling of the inflation trends, are compared. In the column 'firms' it is indicated whether firms that cannot re-optimize their prices index to the inflation target (the inflation trend shock is not active, $\bar{\pi}_t^c = 0$, 'no') or the time-varying inflation trend (the shock $\bar{\pi}_t^c$ is active, 'yes'). In the column 'policy rule' we indicate whether $\bar{\pi}_t^c$ features in the monetary policy rule ('yes') or not ('no').[124]

Three main results emerge from this comparison. First, there is strong support for the inclusion of foreign and/or domestic inflation trends in the model. The increase in the marginal likelihood when inflation trends are added to the model should imply that the ability to forecast inflation is, at least somewhat, improved. Second, however, there is no specification of the inflation trends which is very strongly preferred to the other specifications. Third, while our results suggests that the indexation rule of firms should feature a time-varying inflation trend, $\bar{\pi}_t^c$, there is no strong evidence on whether to include $\bar{\pi}_t^c$ in the monetary policy rule or not.

In Table 8, the parameter estimates obtained in the cases of independent foreign and domestic inflation trends (baseline) and a common inflation trend are contrasted. Assuming independent trends, the estimate of the foreign inflation trend, i.e. the shock process, essentially implies that the shock is not important in the foreign economy part of the model. When it is instead assumed that the inflation trend is common to the foreign and Swedish economies, the estimate of its volatility is an order of magnitude larger. Assuming a common trend and using both foreign and Swedish inflation series to identify the trend, the estimated trend component in the latter part of the sample is low and substantially below the foreign and domestic inflation targets. This common inflation trend component estimate could be interpreted as being a 'compromise' between foreign and Swedish low frequency inflation developments. In order not to let Swedish data have a non-proportional influence on the

[124] In the latter case the policy rule is $\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\left[r_\pi \hat{\pi}_{t-1}^{c,a} + r_{RU}\hat{U}_{t-1}\right] + r_{\Delta\pi}\Delta\hat{\pi}_t^c + r_{\Delta RU}\Delta\hat{U}_t + \hat{\varepsilon}_{R,t}$.

Table 14: Model comparison: inflation trends

Model	Firms	Policy rule	Log Bayes factor
No inflation trends	No	No	0 (benchmark)
Independent inflation trends (baseline)	Yes	Yes	16
Independent inflation trends	Yes	No	15
Common inflation trend	Yes	Yes	17
Common inflation trend	Yes	No	15
Common inflation trend, calibrated	Yes	Yes	12
Correlated inflation trends, JP	Yes	Yes	21
Correlated inflation trends, JP	Yes	No	22

estimated foreign/global inflation trend, we have therefore opted for the assumption of independent inflation trends in the baseline specification of the model. The drawback associated with this choice is that the influence of foreign/global low frequency inflation developments on Swedish inflation are potentially underestimated.

We also consider a specification where the domestic and foreign inflation trends are correlated, using the setup of Justiniano and Preston (2010). While this specification obtains the largest marginal likelihood among the specifications the posterior mode estimate of the foreign Calvo parameter increases from $\xi^* = 0.92$ in the baseline specification (with independent inflation trends) to $\xi^* = 0.96$ in the correlated shocks specification, which could be viewed as implausibly high.

In summary, the data supports the inclusion of an inflation trend in the domestic part of the model. Estimating the foreign model separately, or estimating a separate independent inflation trend for the foreign economy in the full model, suggests that the inflation trend is largely unimportant in the foreign economy. This is further verified by the variance decompositions reported in the Appendix. We have therefore decided to incorporate independent foreign and domestic inflation trends in the model. The estimation results then suggest that it is only the domestic shock that plays a meaningful role.

3.8.3 Calvo parameters

The slopes of the Phillips curves in the model are important for the propagation of shocks, e.g. the monetary policy shock, and the output-inflation trade-off faced by policy-makers. An important result in our estimations is the rather high estimates of Calvo parameters, and consequently low estimates of Phillips curves' slope coefficients. The estimated Calvo parameters are also higher than in the Riksbank's previous DSGE models, Ramses I and Ramses II. This mainly concerns the following five 'key' Calvo parameters: ξ^*, ξ_w^*, ξ_d, $\xi_{m,c}$ and ξ_w. A similar result is reported for Norges Bank's model NEMO where the parameters related to the costs involved in changing prices are higher when the model is estimated including post-financial crisis data; see Motzfeldt Kravik and Mimir (2019).

Here, we therefore assess the sensitivity of the Calvo parameter posterior mode estimates and the model fit, as measured by the marginal likelihood, to different priors for these parameters. In the spirit of Del Negro and Schorfheide (2008) we estimate the model using three different priors for the Calvo parameters: a low-rigidities prior, Beta(0.45,0.075), a high-rigidities prior, Beta(0.75,0.075) and a medium-rigidities prior, Beta(0.6,0.075).[125] There are eight Phillips curves, and hence eight Calvo parameters, in the model and to simplify the analysis we assume that they all share the same prior density. Note that the high-rigidities prior is the one used in the baseline specification of the model.

[125] The low-, medium- and high-rigidities priors are centered at the same values as in Del Negro and Schorfheide (2008) but the standard deviations are lower. We have chosen tighter priors in order to obtain meaningful differences between the posterior estimates in the three cases.

Table 15: Model comparison: Calvo parameters with low, medium and high priors on nominal rigidities

Model		High rigidities (Baseline)	Medium rigidities	Low rigidities
Prior on Calvo		Beta(0.75,0.075)	Beta(0.6,0.075)	Beta(0.45,0.075)
Parameter				
ξ^*	Calvo, for. price	0.93	0.91	0.86
ξ_w^*	Calvo, for. wage	0.85	0.83	0.80
κ^*	Index., for. price	0.57	0.60	0.67
ξ_d	Calvo, dom. price	0.94	0.93	0.86
$\xi_{m,c}$	Calvo, imp. cons.	0.92	0.91	0.86
$\xi_{m,i}$	Calvo, imp. inv.	0.79	0.75	0.69
$\xi_{m,x}$	Calvo, imp. exp.	0.80	0.75	0.70
ξ_x	Calvo, exp.	0.79	0.72	0.67
ξ_w	Calvo, dom. wage	0.86	0.83	0.81
κ_d	Index., dom. price	0.33	0.34	0.87
Log Bayes factor		0	-57	-228
Max posterior diff.		0	-56	-169

The posterior mode estimates of the Calvo and indexation parameters, and the (Laplace approximation of the) log marginal likelihood difference from the high-rigidities benchmark when estimating the model using the three different priors are reported in Table 15. The results of these experiments can be summarised as follows. First, centering the Calvo parameter priors on lower values yield lower posterior mode estimates of these parameters, and consequently larger estimates of the Phillips curve slopes. It is also noted that the slope of the hybrid Phillips curves are positively related to the coefficients on past inflation. Second, however, even with lower-rigidities priors the posterior mode estimates are large, in the sense that they are far out in the right tail of the prior distribution. Thus, the data speaks clearly in favour of a high degree of price and wage stickiness. Third, the marginal likelihood comparison indicates that the baseline high-rigidities alternative is strongly favoured to the other two alternatives.

4 Model properties

A main objective in developing the third generation DSGE model at the Riksbank has been to better capture the importance of foreign/global economic developments for the evolution of the Swedish economy. In this section we zoom in on some key aspects of the model. In Section 4.1, we study the influence of foreign developments on Sweden in the model. In Section 4.2, we discuss our modelling of inflation and the implications it has for the interpretations of previous inflation developments. Finally, in Section 4.3, we shed some light on the monetary policy mechanisms in our model.

4.1 The influence of foreign developments on Sweden

A central empirical regularity in Swedish macroeconomic data is the strong dependence on global economic developments (see Section 3.2.2). In this section, the model's ability to reproduce the observed comovement in the foreign and Swedish business cycles is investigated further. In Section 3.7, we studied the contemporaneous correlations between foreign and Swedish variables in the model and in the data. While these correlations generally have the right signs and are quantitatively meaningful in the model, they are not as strong as in the data. The analysis in this section focuses on a small set

Table 16: Foreign shocks forecast error variance share, baseline DSGE model

	Bayesian					Modified Bayesian				
Forecast horizon	1	4	8	20	∞	1	4	8	20	∞
Variable										
GDP growth	9	37	34	34	34	17	52	49	50	50
CPIF inflation	12	19	22	21	20	16	26	25	24	24
Repo rate	6	27	40	57	82	11	38	51	64	78

of key variables — Swedish GDP per capita growth, headline inflation and the repo rate — and the statistics considered are the foreign variance shares in Swedish variables and historical decompositions.

4.1.1 The share of the variation in Swedish variables attributed to foreign shocks

The forecast error variance decomposition (FEVD) is a device commonly used in multivariate studies to assess how important the various innovations in the model are in driving different variables. When the forecast of a variable, say GDP growth, deviates from the outcome, there is a forecast error and, if this error can be attributed to a particular innovation, it is reasonable to conclude that this innovation is important in driving the variable.

The innovations in the model are divided into two groups containing the foreign and domestic innovations, respectively. Following Justiniano and Preston (2010), the main statistic of interest here is *the fraction of the forecast error variance in the Swedish data series attributed to foreign shocks*, the 'foreign variance share' for short. The foreign variance shares in Swedish GDP growth, inflation and the repo rate in the baseline model, and at four different forecast horizons, are reported in Table 16. The overall impression is, first, that the foreign variance shares are quite substantial and, second, that the foreign variance shares are larger at longer horizons. With the modified estimation approach, which attaches a larger weight to the strong correlations between foreign and Swedish variables in the data, the foreign variance shares become larger, in particular for GDP growth. Again, it appears that the main gains of the modified Bayesian approach is that it brings the properties of Swedish GDP growth closer in line with the data.

Next, the foreign variance shares in two specifications of the DSGE model, the Riksbank's previous DSGE model Ramses II, and a standard open economy Bayesian vector autoregressive (BVAR) model are compared.[126] The foreign variance shares for the Swedish variables in these models, at forecast horizon $h = 8$ quarters, are provided in Table 17. In addition to the baseline DSGE model, estimated using the Bayesian and modified Bayesian approaches, we consider a version of the model which does not allow for common or correlated shocks.[127] Contrasting the foreign variance shares in the DSGE model with and without common and/or correlated shocks provides an assessment of how important this feature is for obtaining realistic foreign spillovers in data. Contrasting the baseline DSGE model with the VAR model provides a coarse assessment of how well the DSGE model captures foreign spillovers. We also include the foreign variance shares in the Riksbank's previous model, Ramses II, for comparison.[128]

[126] The BVAR model contains seven variables: Swedish GDP growth, CPIF inflation, real exchange rate growth and repo rate, and foreign (KIX20-weighted) GDP growth, CPI inflation and policy rate. The foreign variables are assumed to be exogenous with respect to the domestic variables, i.e. Swedish shocks do not influence foreign variables.

[127] The Bayes factor of this model relative to the baseline model was reported in Section 3.7.

[128] In Christiano, Trabandt, and Walentin (2011), two alternative numbers for the foreign variance shares are reported for Ramses II. The first number is the variance share of the five foreign shocks in the model, and the second (higher number) is the combined variance share of the 5 foreign shocks as well as the markup shocks on imports and exports (an additional 4 shocks). We here report the first number as it is the one most closely comparable to the reported numbers for our model.

Table 17: Foreign shocks forecast error variance share at horizon two years: comparison of models

Variable	GDP growth	CPIF inf	Repo rate
Model			
DSGE, baseline, Bayesian	34	22	40
DSGE, baseline, modified Bayesian	49	25	51
DSGE without shock dependencies	3	5	3
Ramses II	8	5	9
BVAR	46	29	75

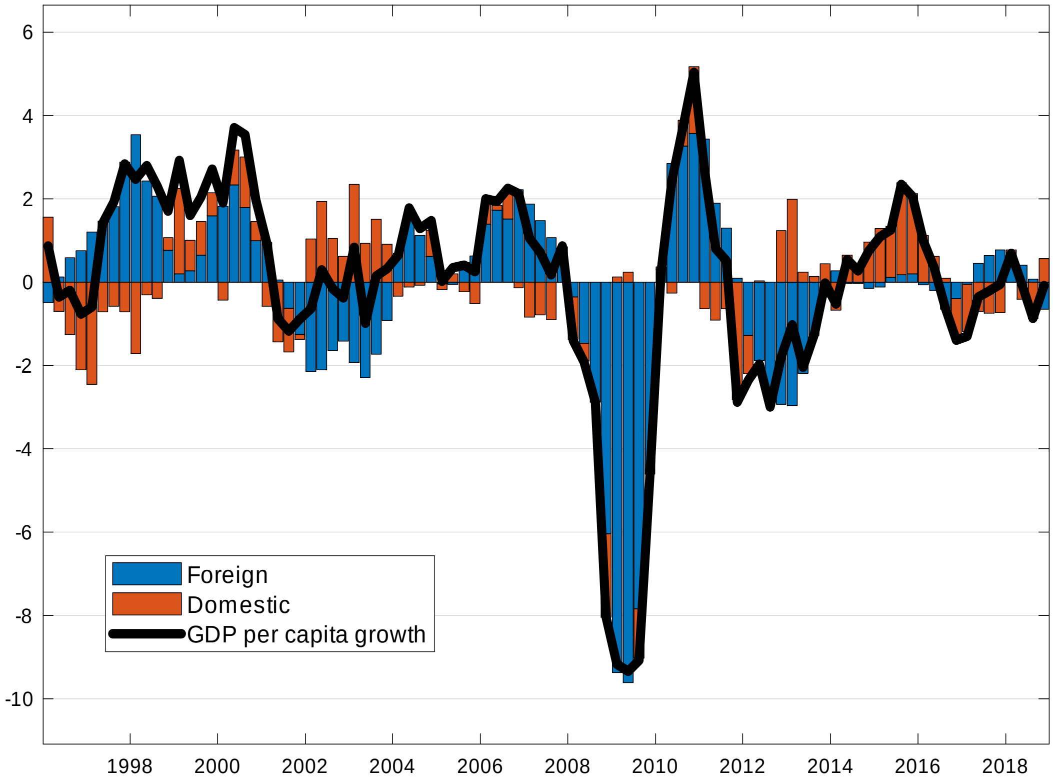
First, it is clear that a 'standard' specification of the DSGE model, without common or correlated shocks, cannot account for the influence of foreign shocks. The foreign variance shares for all three variables at the two year horizon are below 5%. This result is also in line with, while not as extreme as, the results reported by Justiniano and Preston (2010) for a standard open economy DSGE model estimated on US and Canadian data. It is also corroborated by the rather small (below 10%) foreign variance shares reported by Christiano, Trabandt, and Walentin (2011) for Ramses II. Second, the baseline model, which allows for common and correlated shocks, does a much better job in accounting for the importance of foreign shocks. The foreign variance shares are also reasonably well in line with the corresponding BVAR model shares, in particular when the DSGE model is estimated using the modified Bayesian approach.

Two shocks account for the bulk of the influence of foreign/global shocks on Swedish GDP (and its components) and labour market variables: the global permanent technology shock and the shock to the marginal efficiency of investment. The former shock is a supply shock, whereas the latter shock could be characterised as a demand shock with very limited effects on inflation. Detailed variance decompositions, i.e. the variance shares of all individual shocks in the model, and the impulse responses to these two shocks are provided in the Appendix. Modified Bayesian estimation generally increases the degree of real and nominal rigidities and attributes a larger share of the variation in Swedish real variables to the foreign investment shock. A positive shock to foreign investment increases the demand for Swedish exports and thereby increases GDP. Since we have allowed for correlations between i) the investment and consumer preference shocks in the foreign economy and ii) the consumer preference shocks in the two economies, it also affects the Swedish economy through a consumer confidence channel. In particular, one may note that the foreign investment shock induces a positive correlation between investment and consumption in both economies, which means that it can replicate the positive data correlations between these variables.

The foreign variance share in Swedish CPIF inflation is mainly accounted for by the global permanent technology shock and the energy price shock. The latter shock affects the foreign and domestic economies as a supply shock, i.e. a positive shock increases inflation and reduces real activity. While the DSGE model foreign variance share is largely in line with the BVAR share, we stress that the DSGE model share is sensitive to the assumption we have made on the inflation trend. If the domestic inflation trend is allowed to depend on the corresponding foreign trend, the foreign variance share becomes larger.

The foreign variance share in the repo rate is mainly attributed to the global real interest rate trend shock and the foreign investment shock, and to a smaller extent also to the foreign risk premium shock. The real interest rate trend shock affects the foreign and domestic policy rates but has no effects on the other variables in the economies, whereas the risk premium shock is a demand shock. While the foreign variance share in the repo rate is lower than in the BVAR model at the two-year forecast horizon, the results for longer horizons suggest that the model in fact does a good job in accounting for the influence of foreign shocks on the repo rate; see Table 16.

Figure 4: Historical decomposition of Swedish GDP per capita into contributions from foreign and domestic shocks. Annual percentage change.



4.1.2 Historical decomposition of Swedish GDP growth

Forecast error variance decompositions summarise how important various shocks are 'on average' in the sample. Historical decompositions of variables into contributions from innovations/shocks complement this picture by showing how important different shocks have been during particular episodes in the sample. In Figure 4, Swedish GDP per capita growth is decomposed into contributions from foreign and domestic shocks, displayed as blue and red bars, respectively. The figure illustrates that foreign developments have generally been important for Swedish real economic developments, which is also in line with the variance decompositions reported above. The large drop in GDP growth in the financial crisis 2008–2009 and the weak growth associated with the euro crisis around 2012 are both largely explained by foreign/global factors, which is arguably in line with most people's prior views.[129] On the other hand, the strong developments in 2015–2016 are largely attributed to domestic factors. Again, this is broadly in line with the common view, as e.g. Swedish housing investment was particularly strong during this period, driven by a strong domestic demand for housing.

In summary, the baseline model captures the spillovers from the foreign economy to the Swedish economy in a better way than standard open DSGE economy models, and in particular the Riksbank's previous model Ramses II. In separate work, we also document that the ability to forecast most domestic variables, and in particular real variables, conditional on forecasts of foreign variables is substantially improved in our model when compared to the Riksbank's previous DSGE models.

[129]While we do not show the historical decomposition of GDP growth for the model version without common or correlated shocks, or for Ramses II, it is clear from the variance decompositions reported earlier that in those models domestic shocks will be dominant at all times, including the financial and euro crisis episodes.

Finally, central bank staff are used to working 'creatively' with models to bridge gaps between model interpretations of economic developments and interpretations based on alternative approaches. For example, during the financial crisis, Swedish exports declined dramatically and a large part of the 'literal explanation' using a standard open-economy model was that this was due to large positive export markup shocks — sharp export price increases caused the sharp decline in export quantities. While these shocks could perhaps more loosely (and based on information external to the model) be interpreted as being due to 'lower foreign demand' it is arguably preferable to have a model with a more direct interpretation. In our baseline model, the dominant explanation is instead that a negative shock mainly affecting foreign investment — a type of negative foreign demand shock — decreased the demand for Swedish exports. Another example of the need for indirect interpretations using Ramses II are the strong correlations between the smoothed estimates of various import markup shocks and the oil price (a non-modelled variable in Ramses II). Movements in these markups can therefore often be interpreted as movements in the oil price, while this is not directly apparent from e.g. a historical decomposition of headline inflation. In the present model, energy prices are instead explicitly modelled, allowing for a more direct interpretation, as will be furthered discussed in the following section.

4.2 Inflation

The modelling of inflation in the present model is more detailed than in the previous Riksbank DSGE models due to the introduction of energy components in both foreign and domestic consumption. The Swedish energy component is further decomposed into domestic and imported components. In this section, we first discuss the modelling and measurement of inflation through the equations in the model that relate the various inflation rates. We then consider a historical decomposition of inflation and, in particular, the way the model attributes the low inflation in Sweden in 2012–2017 to various factors.

4.2.1 Decomposing headline inflation

A subset of the inflation variables in the model are observed and the remaining ones are 'implicitly observed' or 'indirectly identified' through the identities that link the inflation variables in the model. For example, CPIF inflation (headline inflation), $\hat{\pi}_t^c$, is decomposed into CPIF excluding energy inflation (core inflation), $\hat{\pi}_t^{cxe}$, and energy inflation, $\hat{\pi}_t^{ce}$; see equation (2.23). Headline and core inflation in the model have direct observable counterparts ($\pi_t^{c,obs}$ and $\pi_t^{cxe,obs}$), and energy inflation can therefore be solved for as

$$\hat{\pi}_t^{ce} = \frac{1}{\tilde{\omega}_e} \left(\hat{\pi}_t^c - (1 - \tilde{\omega}_e) \, \hat{\pi}_t^{cxe} \right), \tag{4.1}$$

which means that energy inflation is implicitly observed. It is, however, important to note that the model measure of energy inflation may deviate from the change in the CPI energy component in data, since the weight on energy in the CPI basket, given by the parameter $\tilde{\omega}_e$, is a constant in the model and the actual energy share in the CPI is varying over time. The model measure of energy inflation could therefore be viewed as an approximation of the true energy price inflation since the model does not capture the time variation in the energy share. Foreign energy inflation, $\hat{\pi}_t^{ce,*}$, is similarly implicitly identified since foreign headline, $\hat{\pi}_t^{c,*}$, and core, $\hat{\pi}_t^{cxe,*}$, inflation are observed variables ($\pi_t^{f,c,obs}$ and $\pi_t^{f,cxe,obs}$).

Core inflation, in turn, is decomposed into domestic inflation (excluding energy), $\hat{\pi}_t^d$, and import price inflation (excluding imported energy inflation), $\hat{\pi}_t^{m,c}$; see equation (2.24). Finally, energy inflation is decomposed into domestic, $\hat{\pi}_t^{d,ce}$, and import price, $\hat{\pi}_t^{m,ce}$, components; see equation (2.25). Import price inflation (excluding imported energy) is an observed variable ($\pi_t^{m,cxe,obs}$), which implies that domestic inflation (excluding energy) is implicitly observed.

Swedish imported energy inflation is related to foreign energy inflation through the Phillips curve in equation (2.11) (for $j = ce$), and since the change in the nominal exchange rate is observed, it follows that Swedish imported energy inflation (in domestic currency) is also implicitly observed. Since nominal rigidities are virtually absent in the imported energy sector, i.e. the Calvo parameter $\xi_{m,ce}$ is very low, imported energy inflation is essentially determined by current marginal cost, such that

$$\hat{\pi}_t^{m,ce} \approx \hat{\pi}_t^{ce,*} + s_t, \tag{4.2}$$

where s_t is the change in the nominal exchange rate.[130] The pass-through from the exchange rate to headline inflation through the imported energy channel equals $\tilde{\omega}_{em}\tilde{\omega}_e = 0.0375$, i.e. a 1 percent increase in the exchange rate increases headline inflation by roughly 0.04 percentage points through this direct channel. Finally, since $\hat{\pi}_t^{ce}$ and $\hat{\pi}_t^{m,ce}$ are implicitly observed, domestic energy inflation, $\hat{\pi}_t^{d,ce}$, is also identified through equation (2.25).

The smoothed estimates of foreign energy inflation (in foreign currency), $\hat{\pi}_t^{ce,*}$, and the change in the nominal exchange rate, s_t, are negatively correlated.[131] This reflects the movements typically observed in the data, that in good times global energy price inflation tends to be high and the krona tends to appreciate.[132] A limitation of our approach is that the energy prices are exogenous, i.e. they cannot respond endogenously to changes in global demand, and therefore have the character of supply shocks. In any case, the typical pattern is therefore that global energy price movements are somewhat dampened by krona movements working in the opposite direction. The counteracting exchange rate movements imply that the positive correlation between imported energy inflation (in domestic currency), $\hat{\pi}_t^{m,ce}$, and foreign energy inflation (in foreign currency), $\hat{\pi}_t^{ce,*}$, is lower than 1, but quite large nevertheless.

The model measure of domestic energy inflation, $\hat{\pi}_t^{d,ce}$, turns out to be positively correlated with foreign energy inflation, $\hat{\pi}_t^{ce,*}$, but not correlated with imported energy inflation, $\hat{\pi}_t^{m,ce}$, as $corr(\hat{\pi}_t^{d,ce}, \hat{\pi}_t^{m,ce}) \approx 0$. In the estimation, we allow for a correlation between the foreign and domestic shocks to the relative price of energy, $\hat{p}_t^{ce,*}$ and $\hat{p}_t^{d,ce}$, and the shock correlation, $c_{p^{d,ce},p^{ce,*}}$, is estimated to be positive and well in line with the correlation between the smoothed series $corr(\hat{\pi}_t^{d,ce}, \hat{\pi}_t^{ce,*})$. The estimated shock correlation thus captures a positive comovement between the change in domestic energy prices (in domestic currency), i.e. domestic electricity prices, and the change in foreign energy prices (in foreign currency), which is dominated by changes in the oil price.

In summary, all the components of foreign and Swedish consumer price inflation are either observed directly or implicitly observed. This provides a more detailed account of inflation developments and opens up for a better understanding of the drivers of inflation.

4.2.2 Historical decomposition of CPIF inflation

Swedish inflation was low in the years 2012–2016 but has increased since then following a range of expansionary monetary policy measures, including purchases of government bonds. Interpretations of the low inflation in this period has been discussed in several articles; see Andersson, Corbo, and Löf (2015) for an example. A historical decomposition of annual CPIF inflation using the baseline model is provided in Figure 5. We choose to focus on the shocks that are the most important in explaining the evolution of inflation according to the model. These are the shocks to energy prices, the exchange rate, and trend inflation.[133]

The contribution of global energy price movements to CPIF inflation, the red bars in the figure, show a pattern similar to the broad oil price movements in the sample period. The oil price increased

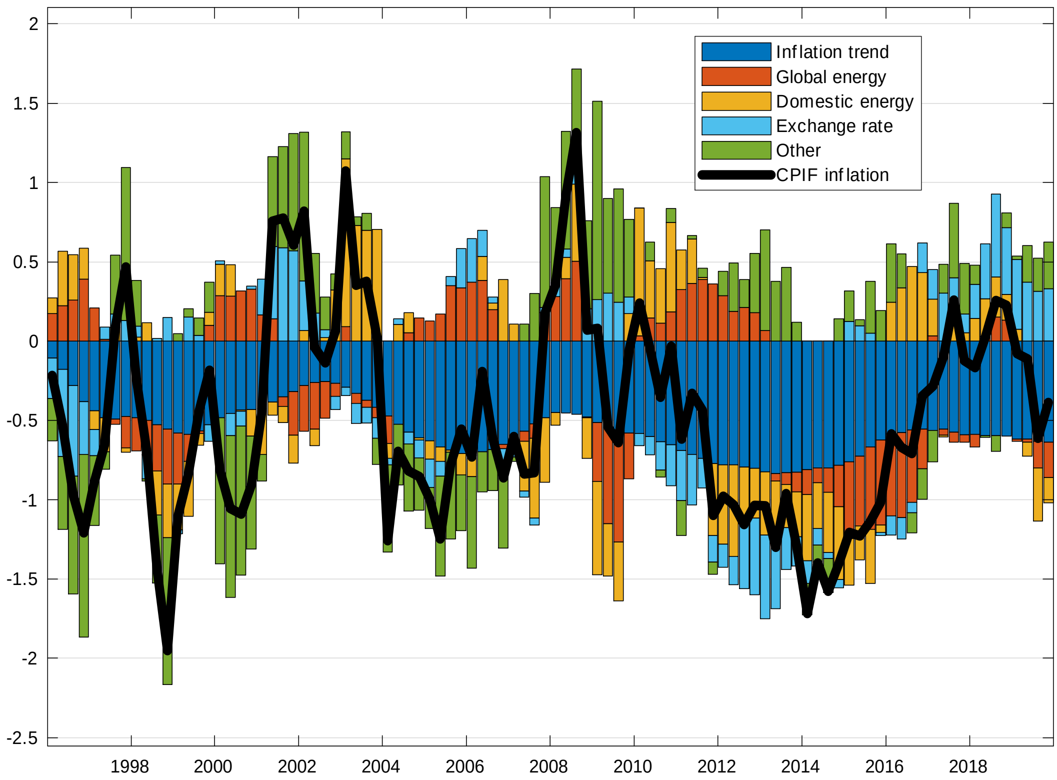
[130] The markup shock in the energy import sector is inactive since it is not possible to identify the shock. Note also that the Calvo parameter is calibrated to $\xi_{m,ce} = 0.1$ based on preliminary estimation results.

[131] We refer to the smoothed estimates of model variables computed for the posterior mode of the parameters.

[132] Alternatively, when the oil price increases, the dollar tends to depreciate against other currencies.

[133] We do not show contributions from all of the individual shocks in the model, since they are too many to be clearly visible in one single figure. Instead, we select a small number of shocks which are dominant in explaining the evolution in inflation over our sample, and group the remaining shocks into one single category. The decomposition shows the extent to which each shock (or group of shocks) has caused inflation to deviate from its steady state at 2 percent.

Figure 5: Historical decomposition of CPIF inflation 1996Q1–2019Q4. Annual percentage change.



before the outbreak of the financial crisis in 2008, only to fall sharply in the years immediately after. The oil price subsequently recovered and stayed above 100 USD/barrel between 2010 and 2013. While CPIF inflation started to increase in 2014, the sharp decline in the oil price, largely driven by a global excess supply of oil, provided a headwind for headline inflation and delayed the return to target. Global energy price fluctuations affect both foreign and Swedish headline inflation and are a key driver of the correlation between the two series; see Figure 3.

The contribution from Swedish energy prices, shown by the yellow bars in the figure, developed in a similar way to the contribution from global energy prices during and immediately after the financial crisis. But in 2011, Swedish electricity prices fell and the contribution of domestic energy price movements to inflation remained negative for several years. Their fall roughly coincided with the beginning of the low inflation period in Sweden, during which total energy price contributions (red and yellow) were consistently negative.

The contributions of energy price movements to the fluctuations in foreign and Swedish headline inflation have been large in the sample period. In a model without energy consumption these contributions are instead captured by markup shocks; see e.g. the decomposition of inflation using Ramses II in Andersson, Corbo, and Löf (2015).

Another important contributor to the fall and rise in Swedish inflation in the years after the financial crisis has been the exchange rate. Contributions from exchange rate shocks, i.e. shocks to the country risk premium, are shown by the light blue bars in the figure. The Swedish krona appreciated in 2010–2011, and in the years 2011–2014 it was relatively strong. A relatively low country risk premium implied a strong exchange rate which contributed negatively to inflation. The weakening trend of the krona from 2014 and onwards is also visible in the figure, as the contributions from the exchange rate become positive towards the end of the sample. Note that the figure only shows the contributions from exogenous shocks, and not the aggregate effect that movements in the exchange rate — or any other variable for that matter — have on inflation.[134]

Movements in energy prices and the exchange rate appear important in explaining the fluctuations in inflation, but they cannot explain the low level of inflation on average, i.e. that the average inflation rate has been below the inflation target of 2%. The largest contribution to inflation, according to the model, is instead provided by inflation trend shocks (dark blue bars). A key question then becomes how to interpret the persistently large contributions from these shocks. This is not straightforward but we consider three possible explanations. First, these shocks could reflect that monetary policy has been too contractionary on average. However, the average longer-run inflation expectations from surveys do not indicate a de-anchoring of expectations. Second, they could reflect structural changes such as digitalization or increased competition from low-cost producing emerging economies. The effects of these changes are difficult to fully grasp, but we note at least that the average inflation of imported consumption goods (excluding energy) has been negative in the sample period. And, third, they could reflect the effects of persistently weak demand not captured by the channels in the model.

All other drivers of inflation deviations have been grouped together and appear as green bars in the figure. These include global and domestic supply and demand factors, out of which supply factors turn out to be more important in explaining inflation than demand factors. Global supply, or technology, has predominantly contributed positively to inflation since the financial crisis. Domestic supply factors, mainly in form of markups, have on the other hand had a small negative contribution through much of the recent decade, including the low inflation period. Finally, while monetary policy is clearly of importance for the interpretations of economic developments through its systematic component, exogenous monetary policy shocks drive a rather limited share of the variation in inflation and other variables.

We summarise the above discussion by noting that inflation modelling is challenging and most models fail to provide a thorough understanding of the drivers and determinants behind aggregate price developments. We note that our more detailed modelling of inflation has improved our understanding of *headline* inflation. However, several challenges, related to our understanding of *core* inflation, still remain.[135] There is a weak relationship between inflation and real economic developments at the business cycle frequency. This is illustrated by the very flat Phillips curves — in other words, inflation reacts little to fluctuations in marginal costs and resource utilization. This is perhaps not unexpected, given the weak relationships between real and nominal variables found in Swedish data discussed in Section 3.2.2. A real-nominal disconnect at the business cycle frequency may not be the sole explanation, however. The relatively large contribution of the exogenous inflation trend movements needed to explain Swedish inflation could be a signal of the presence of underlying trends or structural changes which our models fail to capture. While we have tried to deal with some of these trends, through the inclusion of excess trends in the observation equations for foreign wages and the inflation trend shock, understanding the reasons behind them — and thus the adequate way of modelling them — remains a challenge.

4.3 Monetary policy

Real interest rates have been trending down in many countries, including Sweden, in the past 30 years. By the end of the 2010s, policy rates in many countries had reached historically low levels,

[134] In the case of the exchange rate, however, a large share of the movements are driven by the country risk premium shock. This reflects the well-established fact that exchange rate movements in the short to medium term are not explained particularly well by the interest rate parity condition; see e.g. Engel (2014) for an overview of the findings related to the empirical plausibility of the UIP condition. Variance decompositions at the 8-quarter horizon show that nearly 70 percent of exchange rate movements in our model are accounted for by the risk premium shock. That the shock is a dominant driver of the exchange rate is a standard finding in DSGE models. Itskhoki and Mukhin (2017) provide a lengthy discussion about potential fundamental interpretations of this shock.

[135] These are not specific to our model, but our more detailed modeling may contribute to disentangling the potential reasons behind the difficulties of adequately describing inflation developments.

close to zero. To account for the downward trends in policy rates the modelling of foreign and Swedish monetary policy has been modified in comparison with the previous model, Ramses II, through the incorporation of a common global real interest rate trend, or neutral rate; see Section 2. The objective of this extension is, first, to better capture the trend in foreign and Swedish policy rates and, second and related, to improve domestic and foreign policy interest rate forecasts.

The introduction of the globally determined real interest rate trend in the model implies that the share of the variation in the repo rate that is attributed to global/foreign developments increases significantly in comparison with a model without this feature. In Table 16, we show that the foreign shocks' forecast error variance share of the repo rate is around 80% in the very long run, while it is quite close to zero in a model without the common real interest rate trend. The data sample correlation between the repo rate and the foreign, i.e. KIX20-weighted, policy rate equals 0.92 and the posterior median model sample correlation equals 0.67; see Table 12. While the downward trends in Swedish and foreign rates presumably 'exaggerate' the relationship between the interest rates in the data it is arguably not a case of a spurious correlation. The large foreign variance share in the repo rate, and the model correlation between the Swedish and foreign policy rates indicate that the strong connection between the repo rate and foreign policy rates in the data is largely reflected in the model.

4.3.1 Repo rate forecasts

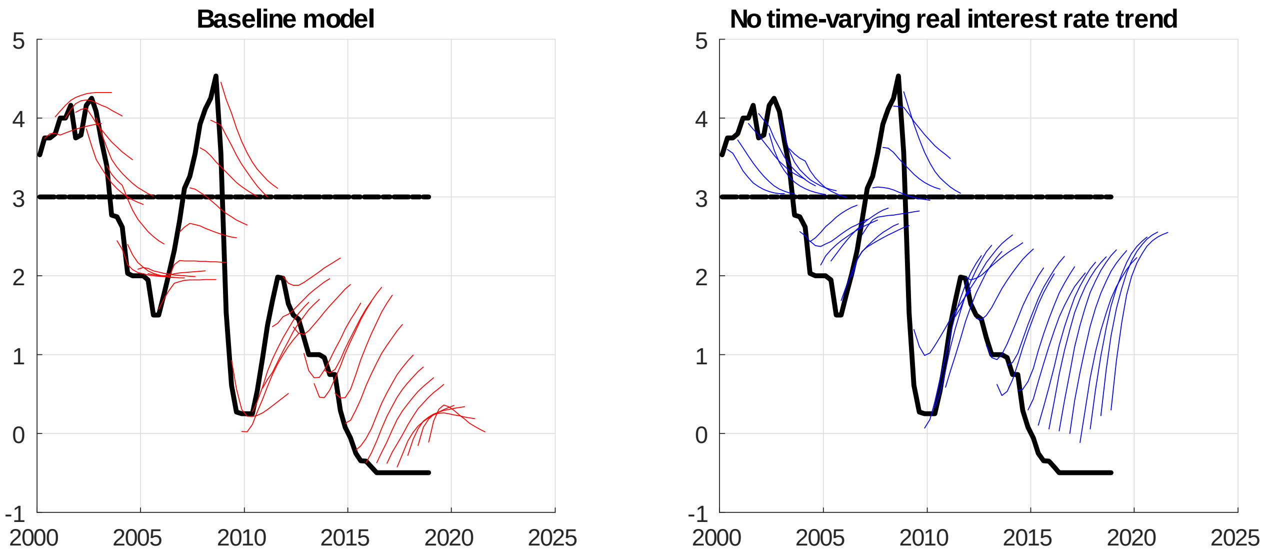
A key advantage of our modelling of monetary policy with a real interest rate trend is that the accuracy of both the foreign and domestic policy rate forecasts from the model improve quite substantially. In Figure 6, three-year-ahead pseudo real-time out-of-sample repo rate (i.e. Swedish policy rate) forecasts from the baseline model and a version of the model without the real interest rate trend are contrasted.[136] Starting in 2000Q1, two repo rate forecasts per year are displayed in the figure. The steady-state level of the repo rate in both model specifications is 3.0%. The baseline model produces repo rate forecasts with smaller forecast errors, in particular in the later part of the sample where the repo rate is historically low. For example, while the baseline three-year-ahead forecasts of the repo rate after 2014 are around or below 1 percent, the 'no trend' model forecasts are instead around or above 2 percent. The time-varying real interest rate trend is estimated to be very persistent, where the persistence mainly derives from the persistence of the trend shock z_t^R, which is estimated to be a 'near unit root'.[137] Therefore, the persistence of the policy rates is better captured and the forecasts are generally more conservative. The policy rate forecast converges towards the forecast of its trend, which is changing slowly, and the repo rate is therefore typically predicted to change relatively little in the forecast period. In the model without the trend, the repo rate forecasts instead converge more quickly towards the assumed — and in this case constant — steady state of 3%, as the forecasts for the unemployment rate and inflation (i.e. the arguments in the policy rule) approach their respective steady-state levels.

In summary, we view the incorporation of the time-varying real interest rate into the model as a pragmatic way of bridging the gap between the actual low policy rates observed in the past decade and the levels which would be implied by standard, constant-intercept, policy rules. While the model does not provide a full *explanation* of why interest rates are low, this new feature helps us to obtain more accurate and more realistic policy rate forecasts.

[136] The forecasts are 'pseudo real-time' for two reasons. First, we use only one vintage of data — the data available in April 2019. Second, the model estimated on the full sample, 1995Q2–2018Q4, is used to generate all recursive forecasts, i.e. the model is not re-estimated for each forecast. A more extensive forecast evaluation of the model, including also conditional forecasts, is provided in a separate paper.

[137] The model's estimates of the foreign and domestic nominal and real interest rate trends and the associated policy rate gaps are provided in the Appendix.

Figure 6: Repo rate forecasts. Baseline model and the model without a time-varying real interest rate trend. Percent.



4.3.2 Effects of a domestic monetary policy shock

Figure 7 shows the effects of a positive (i.e. contractionary) domestic monetary policy shock on key macroeconomic variables.[138] The shock has been scaled to yield an increase in the repo rate of 100 basis points. A one standard deviation policy shock increases the repo rate by roughly 15 basis points. The policy shock affects the economy by reducing aggregate demand and strenghtening the real exchange rate. GDP and employment decrease, and wage inflation falls. At the same time, the exchange rate appreciates and CPIF inflation falls. In comparison with the estimates for Ramses II in Christiano, Trabandt, and Walentin (2011), the quantitative effects on GDP and inflation are quite similar while we obtain larger effects on the exchange rate and employment.[139] It should be noted that monetary policy shocks are much less important in our model as judged by forecast error variance decompositions. For example, the variance share of the policy shock for the repo rate at the two year forecast horizon is 26% in Ramses II while it is a mere 3% in our model. Instead, the shock to the real interest rate trend, which is not present in Ramses II, accounts for 28% of the forecast error variance of the repo rate at the two year horizon in our model. Comparing the peak effects of a policy shock with those in Norges Bank's model NEMO, it appears that the effects are overall quite similar; see Motzfeldt Kravik and Mimir (2019).[140]

5 Conclusions

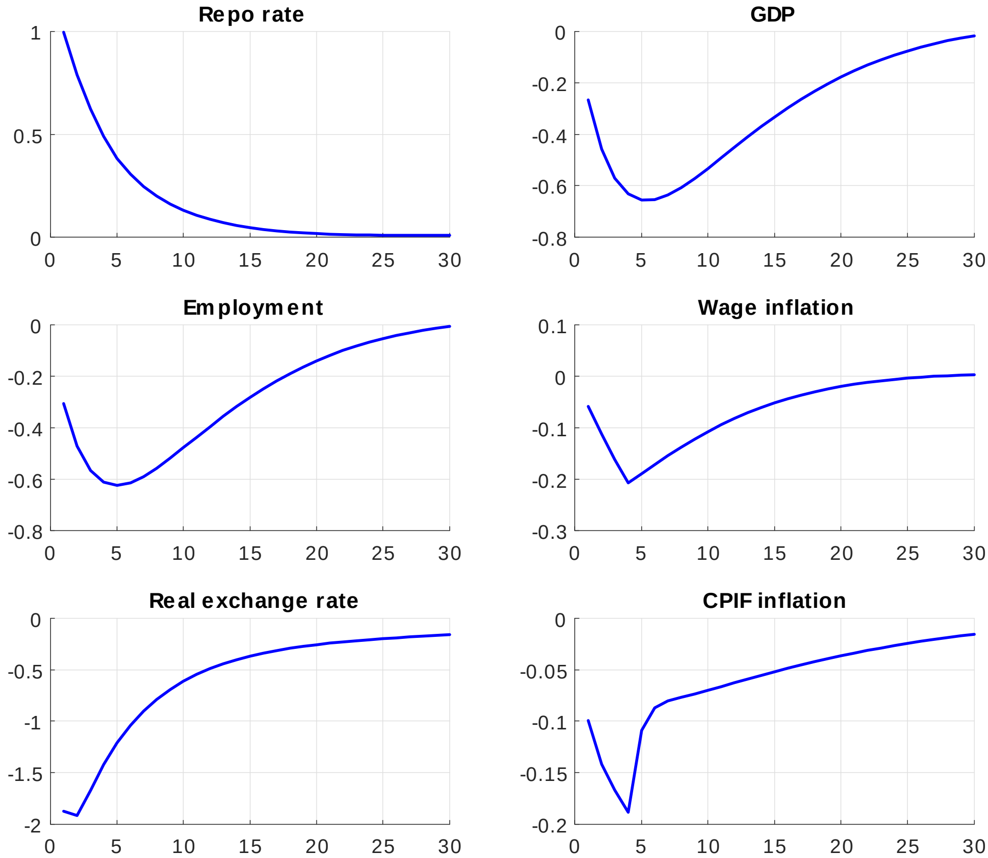
In this paper we have outlined MAJA, a two region DSGE model for Sweden and its main trading partners. The importance of foreign economic developments for the Swedish economy has motivated the modelling of the foreign economy using a fully-fledged DSGE model, and the model used here is based on the closed economy models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003). The model of the Swedish economy is similar to the Riksbank's previous DSGE models, Ramses I and Ramses II, documented in Adolfson et al. (2007) and Adolfson et al. (2013),

[138] The responses to the monetary policy shock for a larger set of variables are provided in the Appendix.

[139] The effects on inflation in our estimation are however smaller than the ones in Adolfson et al. (2013).

[140] A casual comparison suggests that we have somewhat larger effects on GDP, employment and the exchange rate, while the effects on inflation of a monetary policy shock are very similar.

Figure 7: Posterior mode impulse responses to a monetary policy shock. Repo rate in percentage points; GDP, employment and real exchange rate in percentage deviations; wage and CPIF inflation in annual percentage changes.



respectively. To increase the foreign spillovers in the model, Swedish export demand is allowed to be geared more towards foreign investment. We also equip the model with global shocks, which affect Sweden and its trading partners simultaneously and in similar ways — these are shocks to technology, real interest rates, energy prices, firm and consumer confidence, and financial risk. The foreign spillovers in the resulting empirical model go a long way, while not all the way, in matching the strong relationships between foreign and Swedish macroeconomic variables found in the data. This is manifested in foreign variance shares for Swedish variables of a similar magnitude to those obtained with a block exogenous small-open-economy VAR model, substantial correlations between foreign and domestic variables, and more accurate and reasonable forecasts of domestic variables conditional on foreign developments in the model. In particular, the foreign drivers of Swedish GDP and the repo rate are, we believe, better captured in MAJA in comparison with the Riksbank's previous models.

A DSGE model used for forecasting and policy work needs to be continuously updated and adapted to changing circumstances. One manifestation of this is that we have attempted to take the modelling of trends seriously in MAJA, trying to bridge some apparent gaps between theory and data while being transparent about the assumptions we have made, but more could certainly be done to make the model even more empirically realistic. We conclude the paper by highlighting some areas where we believe that further empirical and theoretical work would be interesting.

While MAJA introduces comovement between Swedish and global inflation mainly through global energy price movements there also appears to be a relationship between the low frequency, or 'trend', components of the inflation series. Understanding these components and their relationship, and hence the factors that have contributed to low inflation in Sweden in the past decade, better should be a priority. In MAJA, this could involve using data on foreign and Swedish longer-term inflation expectations to identify the inflation trends. It would also be interesting to further investigate how the treatment of trends in the foreign and Swedish labour shares (real marginal costs) affect the estimates of the main Phillips curve slope parameters in the model.

The discussion on the model's ability to match the strong correlations among real variables in the data suggests that further improvements in this dimension should center on the modelling of investment. In the model, Swedish export demand is mostly related to foreign investment, and in the data there is a strong relationship between Swedish exports and investment (which is also often used as a guide in forecasting investment). The model-implied correlation between Swedish and foreign investment is, however, weak, and the correlations between exports and both foreign and Swedish GDP are not as large as in the data. This suggests that it could be worthwhile considering exports of capital (or durable) goods in a DSGE model for Sweden in order to better capture these relationships in the data. Work along these lines is ongoing at the Riksbank.

DSGE models necessarily entail a simplified description of monetary policy and, needless to say, cannot capture all the fine nuances of actual policy-making. In MAJA, the repo rate responds rather strongly to variations in the unemployment rate, which probably reflects the rather strong correlation between the two variables in the data. Assessing the empirical and normative properties of a broader set of alternative monetary policy reaction functions could further improve the description of policy and the understanding of how policy is best conducted. These alternatives could involve alternative measures of resource utilization and forward-looking rules.

The Swedish labour force, and also employment, has increased quite substantially in the past decade. This is mainly because of an increase in the population due to immigration, but it can also be attributed to an increase in the labour force participation rate driven by various labour market reforms. Using MAJA to study the relationship between the increase in the labour supply and the weak wage and price inflation in this period could contribute to the discussion on the effects of structural factors on inflation.

Finally, in their seminal work, Justiniano and Preston (2010) showed that DSGE models cannot capture the strong cross-country comovements found in the data — in their example the strong relationships between US and Canadian data. Our results illustrate that DSGE models have difficulties in reproducing strong correlations between macroeconomic variables more generally, i.e. in our model the ability to fit strong cross-country correlations does not appear to be much worse than the ability to fit strong within-country correlations. DSGE models are equipped with many drivers (shocks), with different implications for the cross-correlations between variables, while the 'clusters' of strong correlations among sets of variables in the data appear to suggest a small set of key drivers. Using a VAR approach, Angeletos, Collard, and Dellas (2019) identify a 'main business cycle shock' on US data — a shock which explains a large fraction of the business cycle variation in GDP, consumption, investment, unemployment, and hours worked. We find strong correlations between these variables both in Sweden and the KIX20-weighted foreign economy, and furthermore strong correlations across borders for these variables. We further note that the responses to their 'main business cycle shock' and a shock to the marginal efficiency of investment in the foreign economy in MAJA share many similarities. In particular, both shocks generate strong positive comovement among the variables just mentioned, and both have small effects on inflation, i.e. they could be characterised as 'non-inflationary demand shocks'. Applying the method of Angeletos, Collard, and Dellas (2019) to data for a small open economy and its trade-weighted foreign economy, e.g. Sweden and the KIX20 aggregate used in MAJA, could be interesting in order to assess the importance of a 'main *global* business cycle shock' in accounting for the variation in the macroeconomic variables of the small economy.

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