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Liquidity, Capital Pledgeability and Inflation Redistribution*

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Abstract

We investigate the redistributive effects of expected inflation in a microfounded monetary model with heterogeneous discount factors and collateralized debt. In equilibrium, this heterogeneity leads to borrowing, lending and a tractable distribution of money and capital holdings. Even when inflation is fully anticipated and nominal lending rates adjust to it, redistribution can arise because inflation affects capital accumulation and, through collateral constraints, borrowing capacity. We find that the welfare cost of inflation is larger for lenders than for borrowers, that is, inflation acts as a progressive tax. Although inflation always reduces aggregate capital, it can generate a Mundell–Tobin effect for lenders when capital is productive more broadly in the economy and loan-to-value ratios are sufficiently high, increasing lenders' capital holdings and wealth inequality. However, compared with a model with uncollateralized debt, collateralization reduces net-wealth inequality by encouraging borrowers to accumulate more capital and by making debt adjust endogenously with collateral values.

Keywords: Money, Inequality, Collateral constraint, Welfare cost of inflation
JEL classifications: E40, E50, E44, E58, G51

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1 Introduction

Following the Covid pandemic, inflation surged worldwide, reaching 9.1% in the United States and 10.6% in the Eurozone in 2022. The subsequent persistence of inflation above central bank targets has reignited fundamental debates about the costs of high inflation, among them its distributional consequences. This paper studies how expected inflation[1] redistributes welfare and wealth between borrowers and lenders through collateralized debt—a channel of first-order importance given that collateralized loans account for 78% of U.S. household debt.[2]

We focus on expected inflation because its redistributive effects through the borrowing channel are not well understood, unlike the ones due to unexpected inflation. When nominal contracts are fixed in advance, an unanticipated increase in inflation reduces the real value of outstanding liabilities and transfers wealth from nominal creditors to nominal debtors, as in Doepke and Schneider (2006). This mechanism, however, relies on surprise inflation. When inflation instead is fully anticipated, nominal interest rates adjust ex ante, in principle insulating borrowers and lenders from redistribution through changes in the real value of debt. This argument, however, relies on borrowing capacity being fixed in real terms. When borrowing is instead limited by collateral constraints, as in Kiyotaki and Moore (1997)—that is, when borrowers must pledge capital as collateral—and inflation affects capital accumulation, expected inflation also affects the value of collateral and hence the amount borrowers are able to borrow. The focus of our paper is on this redistributive channel and on its welfare and wealth implications. We study this question in a microfounded general equilibrium monetary model based on Lagos and Wright (2005) and Aruoba, Waller, and Wright (2011), extended to incorporate heterogeneous time preferences, capital accumulation, and collateralized borrowing. This framework enables agents to endogenously choose their portfolios of nominal and real assets, as well as their borrowing and lending positions.

Our model features sequential markets. In the decentralized market (DM), agents face stochastic trading opportunities, capital is a production input, and money is the only means of payment. In the centralized market (CM), households consume, work, choose their portfolios, and borrow or lend. Heterogeneity in discount factors generates an endogenous distribution of money, capital, and debt: patient households are lenders and accumulate more money and capital, while impatient households hold less of both and borrow up to the value of the capital they can pledge as collateral.

We consider two versions of the model. In the baseline specification, capital is productive only in the DM. This simplified setting is analytically tractable and isolates the collateral channel, enabling a closed-form characterization of how inflation and capital

1. Throughout the paper, expected inflation refers to a deterministic inflation rate that is fully anticipated by agents, rather than to changes in agents' beliefs about future inflation.

2. Data are from the Survey of Consumer Finances, 1989 to 2022.

pledgeability affect consumption, capital accumulation, and borrowing. We then study an extended model in which capital is productive in both the DM and the CM. This richer specification introduces additional general-equilibrium effects of inflation on capital accumulation, factor prices, labor effort, and CM consumption, and allows inflation to induce portfolio reallocation between nominal assets and productive capital. Because these channels preclude an analytical solution, we analyze the extended model numerically. We calibrate both versions of the model to U.S. data.

Our analysis delivers four main results. First, collateralized debt creates a specific redistribution channel. With a fixed borrowing limit, rather than a collateral-based constraint, the nominal lending rate adjusts to anticipated inflation while the quantity of debt remains unchanged. Credit then has essentially no effect on the welfare cost of inflation. This does not mean that inflation does not matter, since it does affect economic activity in the DM and in turn welfare. However, the welfare cost of inflation is virtually identical in the case of a fixed borrowing limit and in the case of no borrowing and lending. With collateralized debt, by contrast, the amount that can be borrowed varies with the borrower's capital holdings. Inflation can therefore affect credit and redistribute welfare between borrowers and lenders even when the lending rate fully adjusts.

Second, inflation reduces aggregate capital but can also change its distribution, decreasing capital holdings for borrowers while increasing them for lenders. In the baseline model, capital is productive only in the DM, where money is required for trade. Inflation reduces money demand and DM expenditure, thereby lowering the return to capital associated with decentralized-market activity and discouraging investment by both borrowers and lenders. When capital is also productive in the CM, instead, an additional and potentially offsetting force emerges. As inflation reduces the aggregate capital stock relative to labor, capital becomes scarcer and its return in the centralized market rises. Thus, while weaker monetary activity reduces the return to capital in the DM, the higher return in the CM can make capital relatively more attractive for patient lenders. If debt is collateralized and the loan-to-value ratio is sufficiently high, this effect can dominate for lenders, inducing them to increase their capital holdings even as borrowers' holdings and aggregate capital fall. In this case, inflation increases wealth inequality. This contrasts with Aruoba, Waller, and Wright (2011), where inflation unambiguously discourages capital accumulation.

Third, expected inflation reduces welfare for everyone, but disproportionately burdens lenders, making it a progressive tax. In the baseline calibration, the welfare cost of 10% inflation relative to zero is 0.42% of consumption for borrowers, 1.16% for lenders, and 0.72% on average. The extended model shows a similar pattern, though the inflation tax is less progressive: 0.55% of consumption for borrowers, 1.06% for lenders, and 0.75% on average. Since the model is microfounded, we can decompose these welfare effects, which reflect several offsetting forces that differ across borrowers and lenders. When

inflation rises, lenders face greater exposure to the inflation tax since they hold more money than borrowers, while contraction in collateralized debt reduces their interest income. Borrowers partly benefit from monetary transfers (as the inflation tax subsidizes them) and lower debt-service costs. In the extended model, capital for lenders increases with inflation, raising their net capital payoff. The opposite is true for borrowers. In both specifications, lenders' greater exposure to money is central to inflation's progressive incidence. We show that broader access to inflation-protected payment methods, by reducing this exposure, can weaken or reverse the effect. Collateralization thus creates an additional redistribution channel, but does not have an unambiguous effect on the degree of progressivity.

Finally, capital pledgeability redistributes capital across agent types. Because pledgeability raises capital's collateral value, impatient borrowers accumulate more of it. The resulting increase in aggregate capital lowers its marginal return, which discourages investment by patient lenders and narrows the capital gap. Wealth inequality does not necessarily fall, however, because borrowers also take on more debt, and lenders hold the corresponding financial claims. In our calibrated models, collateralized borrowing is nonetheless associated with lower wealth inequality than uncollateralized borrowing.

Taken together, these findings show that the distributional consequences of expected inflation depend critically on the interaction between monetary frictions, capital accumulation, and collateralized credit. In particular, collateralized borrowing creates a redistribution channel that remains active even when inflation is fully anticipated and nominal interest rates adjust accordingly.

Our mechanism builds on the classic literature studying the interaction between expected inflation and capital accumulation (Tobin 1965; Sidrauski 1967; Stockman 1981; Cooley and Hansen 1989, 1991) and on more recent microfounded models of money and capital (Aruoba and Wright 2003; Lagos and Rocheteau 2008; Aruoba, Waller, and Wright 2011). We extend these frameworks by introducing heterogeneity and collateralized borrowing, which allow inflation to affect the capital holdings of borrowers and lenders differently.

Our work is also related to Kiyotaki and Moore (1997) and subsequent contributions such as Iacoviello (2005), Iacoviello and Neri (2010), and Kiyotaki and Moore (2019), which emphasize the role of collateral constraints in macroeconomic dynamics. We adopt a similar borrowing structure but focus on the long-run distributional effects of expected inflation. Related contributions by Ferraris and Watanabe (2008), Venkateswaran and Wright (2014), and Finocchiaro et al. (2018) study the interaction between collateralized credit and steady-state inflation, but do not focus on the resulting redistribution between borrowers and lenders.

Finally, a growing literature embeds household heterogeneity in New Keynesian models, highlighting the importance of redistribution for the transmission of monetary policy

(Auclert 2019; Kaplan, Moll, and Violante 2018). We complement this literature by focusing on ex-ante heterogeneity and the long-run effects of expected inflation in a flexible-price environment with collateralized lending. The endogenous heterogeneity in money, capital, and debt holdings generated by our model also improves on earlier monetary search frameworks such as Boel and Camera (2009), which features neither debt nor real assets.

The remainder of the paper is organized as follows. Section 2 presents the baseline model, and Section 3 characterizes the stationary monetary equilibrium. Sections 4 and 5 describe the calibration and quantitative results for the baseline and extended models. Section 6 introduces an inflation-protected means of payment, and Section 7 concludes. Proofs and figures are provided in the Appendix.

2 The baseline model

The model builds on Lagos and Wright (2005), Aruoba, Waller, and Wright (2011) and Kiyotaki and Moore (1997). Time is discrete, the horizon is infinite and there is a large population of infinitely-lived agents who consume perishable goods and discount only across periods. Agents are heterogeneous in their degree of patience. For convenience, we divide the population into two types $j = H, L$ in proportions ρ and $1-\rho$ respectively, with $1 > \beta_H > \beta_L > 0$. We refer to agents L as impatient and agents H as patient.

In each period, agents may visit two sequential rounds of trade—we refer to the first as the decentralized market (DM) and the second as the centralized market (CM). In the DM, agents draw i.i.d. trade shocks determining whether they can trade, i.e., agents can either produce, consume, or do neither (idle). Agents are idle with probability $1-\sigma$ and trade with probability σ, with consumption and production being equally likely. We refer to consumers as buyers and producers as sellers. In the CM, everybody can consume and work to produce a general good that can be used for either consumption or investment. For simplicity, we first consider an analytically-tractable environment in which goods in the CM are produced using labor only, and capital accumulated in the CM becomes productive in the following DM. In Section 5, we consider a version of the model where capital is also productive in the CM.

Notation is as follows. In the CM, an agent of type j consumes $x_j \geq 0$ goods and supplies $n_j \geq 0$ labor. As in Lagos and Wright (2005), preferences are quasi-linear with $U(x_j) - n_j$. We assume $U'(x) > 0$, $U''(x) \leq 0$, $U'(0) = +\infty$ and $U'(\infty) = 0$. In the DM, consumers of type j derive utility $u(q_j)$ from $q_j \geq 0$ consumption. We assume $u'(q) > 0$, $u''(q) \leq 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. An agent's capital k_j accumulated in the CM can be used in the following DM to produce a different good y_j with technology $f(k_j) = y_j$. We assume $f'(k) > 0$, $f''(k) \leq 0$, $f'(0) = +\infty$ and $f'(\infty) = 0$. Capital depreciates at a

rate δ with $\delta \in (0,1)$. Agents are price takers in both markets.[3]

We assume there is a government that is in charge of monetary policy and is the sole issuer of fiat money, with initial stock $M_0 > 0$. We denote the gross growth rate of the money supply by $\pi = M_t/M_{t-1}$, where M_t indicates the money stock in the CM in period t. The central bank implements its long-term inflation goal by providing deterministic lump-sum injections of money $\tau = (\pi - 1)M_{t-1}$, which are given to private agents at the beginning of the CM. If $\pi > 1$, agents receive lump-sum transfers of money; if $\pi < 1$, the central bank must be able to extract money via lump-sum taxes from the economy.

2.1 Information frictions, money, and credit

The preference structure generates a single-coincidence problem in the DM, since buyers cannot produce. Three additional frictions further constrain trade: (i) agents are anonymous (Kocherlakota 1998), ruling out trade credit; (ii) trading outcomes are private, preventing social punishments from supporting gift-giving; and (iii) buyers cannot carry capital to the DM (Aruoba, Waller, and Wright 2011), thus capital cannot serve as payment. Together with the single-coincidence problem, these frictions imply that sellers require immediate compensation, making money essential in the DM. By contrast, in the CM money is not essential, since agents can finance consumption through labor, money balances, or borrowing.

Borrowing is constrained by limited enforcement. We assume that borrowers can credibly force renegotiation before default, so lenders cannot obtain more than their outside option: the liquidation value $\theta k'$ of the pledged capital, where $\theta < 1$ captures liquidation inefficiencies, such as reallocation losses, legal costs, and other frictions that reduce the value lenders can recover from collateral.[4] The enforceable repayment therefore satisfies

$$a' \leq \theta k'.$$

Assuming repayment when the parties are indifferent, no default occurs in equilibrium. This formulation is similar to Kiyotaki and Moore (1997).

3 Stationary monetary allocations

In what follows, we focus on stationary monetary outcomes such that end-of-period real money, capital, and bond balances are time invariant. We simplify notation by omitting t subscripts and use a prime superscript ($'$) to denote next-period variables, when necessary.

3. See Rocheteau and Wright (2005) for a discussion of the role of money in Walrasian markets.

4. We interpret θ as an effective pledgeability parameter that incorporates both depreciation between origination and repayment and liquidation losses. Equivalently, if $\widetilde{\theta}$ denotes the fraction of surviving capital that lenders can recover, then $\theta = \widetilde{\theta}(1-\delta)$.

Accordingly, we let p_1 denote the nominal price of DM goods and p_2 the price in the CM. In addition, we choose to work with real variables normalizing all nominal variables by the unit of account p_2 so that $p = p_1/p_2$ denotes the relative price.

An agent of type $j = H, L$ starts each period with a portfolio $\zeta_j = m_j, k_j, a_j$, listing real holdings of money m_j, capital k_j, and nominal bonds a_j, all accumulated in the preceding period. Then, the idiosyncratic trading shock $z = b, s, o$ is realized, where b, s, and o denote a buyer, a seller, and an idle agent respectively. Subsequently, trade occurs and after the DM closes, the agent enters the CM with a portfolio $\zeta_j^z = m_j^z, k_j, a_j$ where m_j^z denotes money balances at the beginning of the CM. Real balances evolve within each period according to:

$$m_j^b = m_j - pq_j, \;\; m_j^s = m_j + pf(k_j) \;\text{ and }\; m_j^o = m_j \tag{1}$$

That is, when agents trade in the DM, real balances of a buyer decrease by pq_j whereas those of a seller increase by $pf(k_j)$. Cash left over is used to trade in the CM. Note that bonds a_j and capital k_j are not affected by trade in the DM, given that they cannot be used as a means of payment there. Therefore, $a_j^z = a_j$ and $k_j^z = k_j$ for $j = H, L$ and $z = b, s, o$. Figure 1 describes the timing of the events within each sub-period.

In a stationary economy real asset holdings must be constant, i.e., $m_j' = m_j$, $k_j' = k_j$ and $a_j' = a_j$. If M is cash at the start of a period and $M' = \pi M$ is cash available in the CM, then the gross growth rate of money supply is:

$$\frac{p_2'}{p_2} = \frac{M'}{M} = \pi \tag{2}$$

The money growth rate, i.e., the inflation rate, is controlled via lump-sum transfers τ in the CM so that:

$$\tau = \bar{m}(\pi - 1) \tag{3}$$

where the stationary real money stock $\bar{m} = M/p_2$ is defined by:

$$\bar{m} = \rho m_H + (1 - \rho) m_L \tag{4}$$

3.1 The CM problem

Given the recursive structure of the model, a dynamic programming approach is used to describe the problem faced by an agent of type j. Let $V_j(\zeta_j)$ be the agent's expected lifetime utility when she starts a period with a portfolio ζ_j before trade shocks are realized. Let instead $W_j(\zeta_j^z)$ be the expected lifetime utility when entering a CM with a portfolio

ζ_j^z.[5] The problem of a type j agent at the start of a CM then is:

$$W_j(\zeta_j^z) = \max_{x_j^z, n_j^z, m_j', k_j', a_j'} U(x_j^z) - n_j^z + \beta_j V_j'(\zeta_j') \tag{5}$$
$$\text{s.t. } x_j^z = n_j^z - k_j' + (1-\delta)k_j + m_j^z - \pi m_j' + \tau + p_a \pi a_j' - a_j$$
$$a_j' \leq \theta k_j'$$
$$k_j' \geq 0$$

where $a_j' \leq \theta k_j'$ is the collateral constraint. The resources available to the agent in the CM thus depend on the realization of the DM trading shock $z = b, s, o$ since she carries over m_j^z money balances. Other resources are labor income n_j^z, net capital holdings $(1-\delta)k_j$, lump-sum transfers τ, and a_j' borrowing at price p_a (lending if $a_j' < 0$). These resources can be used to finance current consumption x_j^z, to carry over real money holdings m_j' and capital k_j' into the next period, and to pay back loans a_j (or be paid back if $a_j < 0$). Note that the factor $\pi = p_2'/p_2$ multiplies m_j' and a_j' because the budget constraint is expressed in terms of real variables. The problem yields the following set of first-order conditions:[6]

$$1 = U_x(x_j) \tag{6}$$
$$\pi = \beta_j \frac{\partial V_j'}{\partial m_j'} \tag{7}$$
$$1 = \beta_j \frac{\partial V_j'}{\partial k_j'} + \mu_j^k + \theta \lambda_j^a \tag{8}$$
$$-p_a = \frac{\beta_j}{\pi} \frac{\partial V_j'}{\partial a_j'} - \frac{\lambda_j^a}{\pi} \tag{9}$$

where μ_j^k is the multiplier on the non-negativity constraint for capital, λ_j^a is the multiplier on the collateral constraint and $U_x \equiv \frac{\partial U(x)}{\partial x}$. The first-order conditions above have several implications. First, (6) implies that all agents consume the same amount in the CM, i.e., $x_H = x_L = x^*$. Second, (7), (8), and (9) imply that the savings and borrowing choices m_j', k_j' and a_j' are independent of trading histories but might be type dependent. To see exactly how, we will examine DM trades later. Also, note that the marginal valuations of money, capital and bonds are type independent as shown by the envelope conditions below:

$$\frac{\partial W_j}{\partial m_j^z} = 1, \quad \frac{\partial W_j}{\partial a_j} = -1 \text{ and } \frac{\partial W_j}{\partial k_j} = 1 - \delta.$$

5. Note that ζ_j^z depends on the trading shock z. Agents that received different trading shocks z in the DM will reach the CM with different money holdings and in turn exert different labor efforts n_j^z.

6. Conditions for $n_j^z > 0$ are in the Appendix.

CM goods market clearing implies:

$$X + I = N \tag{10}$$

where $X = x^*$, investment $I = K' - (1-\delta)K$, and aggregate labor and capital N, K satisfy:

$$N = \rho\left\{\frac{\sigma}{2}(n_H^s + n_H^b) + (1-\sigma)n_H^o\right\} + (1-\rho)\left\{\frac{\sigma}{2}(n_L^s + n_L^b) + (1-\sigma)n_L^o\right\} \tag{11}$$

$$K = \rho k_H + (1-\rho)k_L \tag{12}$$

3.2 The DM problem

The problem of an agent of type j starting a generic period t with a portfolio ζ_j is:

$$V_j(\zeta_j) = \frac{\sigma}{2}V_j^b(\zeta_j) + \frac{\sigma}{2}V_j^s(\zeta_j) + (1-\sigma)V_j^o(\zeta_j) \tag{13}$$

First, we determine DM consumption q_j. The buyer's problem can be written as:

$$\begin{aligned} V_j^b(\zeta_j) = &\max_{q_j} \quad u(q_j) + W_j(\zeta_j^b) \\ &\text{s.t. } pq_j \leq m_j \end{aligned} \tag{14}$$

Given (1) and (5), we have the following first-order condition:

$$u_q(q_j) = p(1+\lambda_j^b) \tag{15}$$

where λ_j^b is the multiplier on the buyer's budget constraint. If the buyer is constrained and $pq_j = m_j$, then $u_q(q_j) > p$. If the buyer is not constrained, then $pq_j < m_j$ and $u_q(q_j) = p$.

The seller's problem is trivial since capital has already been chosen in the previous CM and it is the only factor of production in the DM. Thus, the seller's problem is just the continuation value, i.e. $V_j^s(\zeta_j) = W_j(\zeta_j^s)$. Similarly, the problem for an idle agent is $V_j^o(\zeta_j) = W_j(\zeta_j)$.
The DM market-clearing condition is:

$$\rho q_H + (1-\rho)q_L = \rho f(k_H) + (1-\rho)f(k_L) \tag{16}$$

3.3 Savings and borrowing

decisions

To find the optimal portfolio of an agent, we must calculate the expected marginal

values $\partial V_j/\partial m_j$, $\partial V_j/\partial k_j$ and $\partial V_j/\partial a_j$. To do so, we combine (5) and (13) to get:

$$\begin{aligned} V_j(\zeta_j) =& m_j + (1-\delta)k_j + \frac{\sigma}{2}u(q_j) + U(x_j) - x_j + \beta_j V_j'(\zeta_j') \\ &+ \frac{\sigma p}{2}(f(k_j) - q_j) + \tau - \pi m_j' - k_j' + p_a \pi a_j' - a_j \end{aligned} \tag{17}$$

Therefore:

$$\frac{\partial V_j}{\partial m_j} = 1 + \frac{\sigma}{2p}\left[u_q(q_j) - p\right] \tag{18}$$

$$\frac{\partial V_j}{\partial k_j} = 1 - \delta + \frac{\sigma}{2} p f_k(k_j) \tag{19}$$

$$\frac{\partial V_j}{\partial a_j} = -1 \tag{20}$$

From (7), (8), (9), (18), (19), and (20), the agents' optimal portfolio choices must satisfy:

$$\pi = \beta_j \left[1 + \frac{\sigma}{2p}(u_q(q_j) - p)\right] \tag{21}$$

$$1 = \beta_j \left[(1-\delta) + \frac{\sigma}{2} p f_k(k_j)\right] + \mu_j^k + \theta \lambda_j^a \tag{22}$$

$$\pi p_a = \beta_j + \lambda_j^a \tag{23}$$

The central observation we derive from the Euler equations above is that money, bonds, and capital are valued differently in the economy by patient and impatient agents, since the expressions in (21), (22), and (23) all depend on the discount factor β_j. Of course, agents' decisions are also affected by asset-specific characteristics. First, the choice of real money balances described in (21) depends on the real yield on cash $1/\pi$ and on $u_q(q_j)/p$, which can be interpreted as the expected liquidity premium from having cash in the DM, given that money is necessary to trade in that market. This premium grows with the severity of the cash constraint, which is reflected in $u_q(q_j)$, and the likelihood of a consumption shock $\sigma/2$. Second, the choice of capital described in (22) depends on the gross rate of return on capital net of depreciation $1 - \delta + (\sigma/2)pf_k(k_j)$, the fraction of capital that can be collateralized θ, and, importantly, on inflation, given that π enters (21) and p enters both (21) and (22). Intuitively, inflation matters for capital choices because money is needed to trade in the DM, and therefore inflation affects consumption there. This in turn influences capital accumulation, since capital is a factor of production in that market. Last, the choice of bonds described in (23) is influenced by the real yield $1/(\pi p_a)$ and households' discount factors which will determine whether the collateral constraint is binding, as shown below. Note also that (23) reflects the fact that bonds have no liquidity premium, since they cannot be used to buy consumption in the DM.

At this point, we can provide a definition of equilibrium.

Definition 1. *A competitive stationary monetary equilibrium consists of* (m_j, a_j, k_j) *satisfying* (21), (22), *and* (23) *for* $j = H, L$.

The reason is that once the equilibrium stocks of money, bonds, and capital are determined, all other endogenous variables can be derived. We next characterize borrowing and lending in any stationary monetary equilibrium with collateralized credit. We find that the following result holds:

Lemma 1. *In any stationary monetary equilibrium with borrowing and lending, impatient agents borrow and patient agents lend at the price* $p_a = \beta_H/\pi$. *Specifically,*

$$a_L = \theta k_L \qquad \text{and} \qquad a_H = -\frac{1-\rho}{\rho}\theta k_L.$$

Proof. See the Appendix. □

Why are agents interested in borrowing and lending even if they are always consuming the same quantity x^* in the CM? The reason lies in the heterogeneity in discount factors. For impatient agents, borrowing at the rate π/β_H is cheaper than carrying money across periods at the cost π/β_L. Thus, with quasi-linear preferences, they would like to borrow an infinite amount of the general good if at all possible. Because of the collateral constraint, their borrowing is bounded by θk_L.

Once we know the price $p_a = \beta_H/\pi$ at which these bonds circulate in equilibrium, we can pin down the net borrowing rate:

$$i_a = \pi/\beta_H - 1 \tag{24}$$

which is affected directly by monetary policy via the inflation rate π.

We now want to determine the returns on money and bonds that are consistent with equilibrium.

Lemma 2. *Any stationary monetary equilibrium must be such that* $\pi \geq \beta_H$.

Proof. See the Appendix. □

This result derives from a simple no-arbitrage condition—in a monetary equilibrium, the value of money cannot grow too fast with $\pi < \beta_H$ or else type H agents will accumulate an infinite amount of it, which cannot be a stationary equilibrium. This, together with (24), implies that the borrowing rate $i_a = 0$ if $\pi \to \beta_H$. This is what we will refer to as the Friedman rule in the remainder of the paper.[7]

7. See also Boel and Camera (2006) and Boel and Waller (2019) for a similar approach.

Summing up, both policy π and discount factors β_H and β_L affect the expected returns on money, bonds, and capital, the latter of which influences agents' ability to borrow due to the collateral constraint. Since patient and impatient agents price future consumption differently, we expect their portfolio choices to differ, with effects on their consumption choices, labor effort and welfare. We investigate this next and we first show that the existence of borrowing and lending in equilibrium hinges on the concavity of the DM production function.

Lemma 3. *If $\pi \geq \beta_H$ and $f(k)$ is linear, $k_L = 0$ in a stationary monetary equilibrium.*

Proof. See the Appendix. □

Thus, when the production function is linear, there cannot be any collateralized borrowing in equilibrium—in this case, the return on capital is too low for impatient agents. We next characterize stationary monetary equilibria with borrowing and lending when the production function is strictly concave.

Proposition 1. *Let $\pi \geq \beta_H$ and $f''(k) < 0$. Any stationary monetary equilibrium with borrowing and lending satisfies*

$$m_H > m_L > 0 \qquad \text{and} \qquad k_H > k_L > 0.$$

Proof. See the Appendix. □

Patient and impatient agents value future consumption at different rates, and thus assets have different discounted returns for them. As a consequence, the equilibrium described in Proposition 1 and Lemma 1 is one in which patient agents are savers and hold more assets—both money and capital—than impatient ones, who are borrowers. This is different from Boel and Camera (2009) where there is asset specialization by type, no capital and no borrowing and lending.

Our ultimate goal here is to understand if expected inflation can have redistributive effects via borrowing and lending. We already know from Lemma 1 that the return on bonds adjusts perfectly for inflation. So, if any redistribution exists through the credit channel, it has to operate through the collateral constraint θk_L. We proceed as follows. We first identify how equilibrium allocations are affected by collateral constraints and we then show the role played by inflation. For the comparative statics that follow, we assume that u and f are strictly concave.

Proposition 2. *Let $\pi \geq \beta_H$, $a_L = \theta k_L$ and $a_H = -\theta k_L(1-\rho)/\rho$. Then, $dp/d\theta < 0$, $dq_H/d\theta > 0$, $dq_L/d\theta > 0$, $dk_H/d\theta < 0$, and $dk_L/d\theta > 0$.*

Proof. See the Appendix. □

From Proposition 2 we learn that collateral constraints induce borrowers to hold more capital and increase production. All else equal, the increase in supply creates a downward pressure on prices in the DM market, thereby reducing the incentive to invest in capital for the patient agents. Thus, collateral constraints narrow the gap between agents' capital holdings.

Next, we investigate how agents' portfolio choices are affected by inflation.

Proposition 3. *Let* $\pi \geq \beta_H$, $a_L = \theta k_L$ *and* $a_H = -\theta k_L(1-\rho)/\rho$. *Then,* $dp/d\pi < 0$, $dk_j/d\pi < 0$ *for* $j = H, L$ *and* $dq_H/d\pi < 0$.

Proof. See the Appendix. □

Inflation here reduces investment, just as in Aruoba, Waller, and Wright (2011). As inflation increases, the real value of money decreases together with DM total consumption, since money is necessary to buy in that market. As demand for consumption is reduced, less capital is necessary for production. While the effect on patient agents' consumption is always negative, in the following proposition we show that the effect on impatient agents' consumption depends on the coefficient of relative risk aversion $\mu = -\frac{u_{qq}(q)}{u_q(q)}q$ and the elasticity of the production function with respect to capital $\varepsilon = \frac{f_k(k)k}{f(k)}$.

Proposition 4. *Assume* $f(k)$ *is isoelastic. Let* $\pi \geq \beta_H$, $a_L = \theta k_L$, $a_H = -(1-\rho)\theta k_L/\rho$ *and* $\bar{\mu} = \frac{1-\varepsilon}{\varepsilon}\frac{\beta_H - \beta_L}{\beta_H}\left(\frac{2-\sigma}{\sigma}\right)$. *Then,* $dq_L/d\pi < 0$ *if* $\mu > \bar{\mu}$.[8]

Proof. See the Appendix. □

Summing up, expected inflation affects prices, consumption, money holdings, capital accumulation, and consequently borrowing and lending via the collateral constraint. The overall balance of these different effects on agents' welfare is ambiguous and must be determined quantitatively, as we do next.

4 Quantitative analysis

4.1 Calibration

In this section, we calibrate the model for the United States. Data are annual. The nominal price level P is CPI, aggregate nominal output PY is nominal GDP, and the nominal

8. This condition is not stringent for standard parameter values and is satisfied by our chosen calibration in the next section.

money supply M is M1.[9] For ease of comparison with other studies based on Lagos and Wright (2005), we consider the following functional forms: $U(x) = B\ln(x)$, $f(k) = k^{\alpha}$, $u(q) = \ln(q)$. Therefore, the vector of parameters to identify is $(\rho, \beta_H, \beta_L, \delta, \theta, \alpha, \sigma, B)$.

We proceed as follows. We set the proportion of patient agents ρ to 0.40, in line with Boel and Camera (2009). We consider discount factors $\beta_H = 0.965$ and $\beta_L = 0.91$, consistent with the empirical estimates of distributions of discount factors in Lawrance (1991), Carroll and Samwick (1997), and Samwick (1998). We use a depreciation rate $\delta = 0.07$ as in Aruoba, Waller, and Wright (2011). We set $\theta = 0.85$, which is consistent with the loan-to-value ratio in Iacoviello and Neri (2010), among others.

We calibrate the remaining parameters α, σ and B by matching three moments simultaneously. First, we fit the ratio $L = M/PY$, i.e., the inverse of money velocity. Second, we match the theoretical share of DM consumption with its empirical counterpart. The latter is calculated as the share of cash consumption transactions, using data for the period 2015–2019 from the Diary of Consumer Payment Choice from the Federal Reserve Bank of Atlanta. Last, we match the theoretical ratio of collateralized debt over GDP with its empirical counterpart. The latter is computed as the ratio of mortgages and auto loans originations to GDP, using data for the period 1999–2019 from the Quarterly Report on Household Debt and Credit from the Federal Reserve Bank of New York. Following this approach, we pin down $B = 3.12$, $\sigma = 0.689$ and $\alpha = 0.589$. Table 1 summarizes our calibration.

Table 1: Calibration

Parameters taken from the literature:		
β_H	0.965	Lawrance (1991), Carroll and Samwick (1997), Samwick (1998)
β_L	0.91	Lawrance (1991), Carroll and Samwick (1997), Samwick (1998)
δ	0.07	Aruoba, Waller, and Wright (2011)
θ	0.85	Iacoviello and Neri (2010)
Parameters calibrated simultaneously:		
B	3.12	
σ	0.689	
α	0.589	

Notes: Calibration results are for the baseline model, where capital is an input only in the DM production function. β_H and β_L are discount factors for lenders and borrowers respectively; θ is the loan-to-value ratio; δ is the depreciation rate; B, σ and α are calibrated to simultaneously match money velocity, share of cash consumption and the ratio of collateralized loans originations/GDP.

Figures 2, 3 and 4 illustrate the quantitative implications of the baseline model for capital distribution and wealth inequality, and how inflation affects these results. We

9. M1 is in billions of dollars, December of each year, not seasonally adjusted. For 1929–1958, it is from Friedman and Schwartz (1982) (pp. 708–718, col. 7). For 1959-2019, it is from the St. Louis Fed FRED Database. For 1929-2019, nominal GDP and CPI are from the St. Louis Fed FRED Database.

define net wealth for type j agent as:

$$\mathbb{W}_j \equiv m_j + k_j - a_j,$$

where $a_j > 0$ denotes borrowing and $a_j < 0$ lending. Hence, our measure of wealth includes money, capital and net bond positions. Since type H agents have higher income than type L agents, the Gini coefficient in this economy is:

$$G_{\mathbb{W}} = \frac{\rho(1-\rho)(\mathbb{W}_H - \mathbb{W}_L)}{\rho\mathbb{W}_H + (1-\rho)\mathbb{W}_L}.$$

Importantly, a narrower gap in capital holdings does not necessarily imply a narrower gap in net wealth, since net wealth also includes borrowing and lending positions. We therefore distinguish the effect of pledgeability on capital holdings from the comparison of wealth inequality across credit arrangements.

Figure 2 shows that greater capital pledgeability θ increases capital holdings for borrowers (L agents), while reducing those of lenders (H agents). The intuition is as follows. When pledgeability rises, credit-constrained impatient agents can obtain more loans for a given amount of collateral. This raises the value of capital for borrowers and strengthens their incentive to accumulate it. The resulting increase in aggregate capital, in turn, lowers its marginal return and reduces the incentive of patient agents to invest. Greater pledgeability therefore narrows the gap in capital holdings between borrowers and lenders.

To isolate the interaction between inflation and collateralization, we compare three credit arrangements: (i) collateralized borrowing, in which borrowing capacity is tied to capital holdings; (ii) uncollateralized borrowing, with a fixed borrowing limit A; and (iii) no borrowing or lending. In all three cases, inflation reduces capital holdings for both types of agents (Figure 3). Capital holdings coincide in the uncollateralized and no-credit economies.

The implications of inflation for wealth inequality (Figure 4), however, differ across the three credit arrangements. With uncollateralized debt, borrowing remains fixed while inflation reduces money and capital holdings. Borrowers' liabilities and lenders' corresponding assets therefore become larger relative to total wealth, raising the Gini coefficient. With collateralized debt, by contrast, borrowing shrinks together with borrowers' capital holdings. Since lenders' bond holdings are the counterpart of borrowers' debt, their asset position also contracts. This adjustment in debt positions cushions the effect of inflation on net wealth and leaves inequality nearly unchanged even as inflation increases. Inequality is also virtually unaffected in the no-credit economy, where the debt channel is absent. In our calibration, collateralized borrowing is therefore associated with lower wealth inequality than uncollateralized borrowing and with a weaker response of inequality to inflation. Relative to the no-credit economy, however, collateralized borrowing

generates higher net-wealth inequality in the baseline model.

In sum, debt collateralization affects both the distribution of capital holdings across borrowers and lenders and the distribution of net wealth. These two distributions need not move in the same direction because net wealth also includes money as well as borrowing and lending positions. In our quantitative exercise, collateralization narrows the gap in capital holdings by inducing borrowers to accumulate more capital. Wealth inequality is lower under collateralized than under uncollateralized borrowing, but remains higher than in the no-credit economy in the baseline calibration.

Lower wealth inequality due to debt collateralization does not imply that everyone is better off. As shown in Figure 5, collateralization redistributes welfare from lenders to borrowers. Meanwhile, inflation remains harmful to both groups under all credit arrangements. We now formally quantify these welfare costs.

4.2 The welfare cost of inflation

Our measure of the welfare cost of inflation follows Lucas (2000) and determines how much total consumption an agent of type $j = H, L$ would be willing to give up to have $\gamma - 1$ instead of $\pi - 1$ inflation.[10] Fixing π, we let $q_{j\pi}$, $k_{j\pi}$, p_π, $m_{j\pi}$ and $\bar{m}_\pi$ denote equilibrium quantities. Then, we use (5) and (13) to define equilibrium ex-ante welfare $\bar{V}_j$ for an agent of type j:

$$\begin{aligned}(1-\beta_j)\bar{V}_j(\zeta_{j\pi}) =& \frac{\sigma}{2}u(q_{j\pi}) + U(x^*) - x^* - \delta k_{j\pi} \\ &+ \frac{\sigma}{2}p_\pi(f(k_{j\pi}) - q_{j\pi}) + (\pi-1)(\bar{m}_\pi - m_{j\pi}) - a_{j\pi}(1-\beta_H)\end{aligned} \tag{25}$$

Inflation π affects ex-ante welfare in several ways. It impacts DM consumption, $\frac{\sigma}{2}u(q_{j\pi})$, and also capital accumulation in the CM, thus affecting the labor effort necessary to make up for the depreciated capital $\delta k_{j\pi}$. These are the only distortions induced by inflation in a representative-agent setting, since in that case the second line in (25) vanishes because $m_{j\pi} = m_\pi$, $k_{j\pi} = k_\pi$, $q_{j\pi} = q_\pi$ for $j = H, L$, $f(k_\pi) = q_\pi$ and there is no credit. Instead, with heterogeneity, inflation generally affects $\bar{V}_j$ through three additional channels. First, it impacts expected net earnings in the DM, $\frac{\sigma}{2}p_\pi(f(k_{j\pi}) - q_{j\pi})$, which can be nonzero since agents may produce and consume different amounts. Second, given that money balances are heterogeneous, inflation also redistributes monetary wealth because of inequalities in the inflation tax $(\pi-1)(\bar{m}_\pi - m_{j\pi})$. Third, which is new in our model, inflation affects the collateral constraint and thus the amount impatient agents borrow

10. In our analysis, we compare stationary equilibria and therefore abstract from transitional dynamics. In particular, welfare is evaluated for an agent entering each stationary equilibrium with the corresponding stationary portfolio ζ_j. The resulting welfare costs should thus be interpreted as long-run, steady-state welfare comparisons rather than as the welfare effects of an unanticipated transition from one inflation rate to another.

in equilibrium $a_{j\pi} = \theta k_{j\pi}$. Note that the cost of borrowing adjusts perfectly for inflation, given that $p_a = \beta_H/\pi$ and π is known to agents since we are considering expected inflation.

If we reduce inflation from π to γ, but also change the consumption of both CM and DM goods by a factor $\Delta_{j\gamma}$, ex-ante welfare for an agent j becomes:

$$(1-\beta_j)\bar{V}_j(\zeta_{j\gamma}) = \frac{\sigma}{2}u(\Delta_{j\gamma}q_{j\gamma}) + U(\Delta_{j\gamma}x^*) - n_{j\gamma} \tag{26}$$

$$= \frac{\sigma}{2}u(\Delta_{j\gamma}q_{j\gamma}) + U(\Delta_{j\gamma}x^*) - x^* - \delta k_{j\gamma} \tag{27}$$
$$+ \frac{\sigma}{2}[p_\gamma(f(k_{j\gamma}) - q_{j\gamma})] + (\gamma-1)(\bar{m}_\gamma - m_{j\gamma}) - a_{j\gamma}(1-\beta_H)$$

We measure the cost of $\pi - 1$ inflation as opposed to $\gamma - 1$ inflation for a type j agent as the value $\Delta_{j\gamma}$ that solves $\bar{V}_{j\pi}(\zeta_{j\pi}) = \bar{V}_{j\gamma}(\zeta_{j\gamma})$. That is, agents would be willing to give up $100(1-\Delta_{j\gamma})$ percent of consumption to have $\gamma - 1$ rather than $\pi - 1$ inflation.

Table 2 reports the welfare costs of 10 percent expected inflation as opposed to both zero inflation and the Friedman rule for both borrowers (type L) and lenders (type H), as well as the average welfare cost of inflation for that economy. Note that, given Lemma 1, the Friedman rule in our framework corresponds to $\pi = \beta_H$ and therefore, using the calibrated values in Table 1, to a net inflation rate $\pi - 1 = -3.5$ percent. The first row in Table 2 reports the results for the case when borrowing is subject to a collateral constraint, i.e., $a'_j \leq \theta k'_j$. The second row focuses on the case of a fixed borrowing limit, i.e., $a'_j \leq A$.[11] The last row displays results for the case where credit is shut down, i.e. $\theta = 0$. Under all scenarios, we use the calibrated parameters listed in Table 1 to quantify welfare costs.

Table 2: Percentage welfare cost of 10% inflation relative to zero inflation and the Friedman rule.

	Zero Inflation			Friedman Rule		
Model	Type L	Type H	Average	Type L	Type H	Average
Collateralized borrowing	0.42	1.16	0.72	0.53	1.62	0.96
Uncollateralized borrowing	0.55	1.14	0.79	0.72	1.59	1.07
No credit	0.55	1.14	0.79	0.72	1.59	1.07

Notes: Results are for the baseline model, where capital is an input only in the DM production function. Welfare costs are expressed in consumption percentage points. At the Friedman rule, the net inflation rate is $\beta_H - 1 = -3.5\%$. In the models with borrowing—both collateralized and uncollateralized—type H agents are lenders and type L agents are borrowers. The average welfare cost of inflation is the weighted average of the welfare costs for type H and type L agents.

Our quantitative analysis reveals a few key findings. First, the welfare cost of inflation is the same in a model with an uncollateralized borrowing limit A and in a model with no

11. For the calculations reported in Table 2, we set A to the level of the collateral constraint at 10 percent inflation. We can show that this specific level chosen for A is irrelevant for our calculations of the welfare cost of inflation. These results are available upon request.

credit. That is, expected inflation does not generate redistributive effects via the credit channel if borrowing is uncollateralized. When borrowing depends on an exogenous limit A, following the proof of Lemma 1 it is straightforward to show that type L agents borrow up to the limit A. Since in this case $p_a = \beta_H/\pi$, $a_L = A$ and $a_H = -A(1-\rho)/\rho$, neither the cost of borrowing nor the loan principal depend on inflation.

Second, collateralized borrowing reduces the welfare cost of inflation for borrowers and increases it for lenders, compared to the other credit arrangements in Table 2. Quantitatively, the effect is substantial. When a loan is collateralized, the welfare cost of 10 percent inflation relative to zero inflation decreases by 24 percent for borrowers and increases by 2 percent for lenders compared to the case of a fixed borrowing limit and to the case of no credit.[12]

Third, overall inflation acts as a progressive tax, meaning that, even if inflation decreases welfare for everyone, the burden of inflation falls more heavily on lenders. To understand why, Figure 6 decomposes the welfare effects of an increase from 0% to 10% inflation. Due to the structure of the model described in Section 2, we can separate the DM and CM effects. Inflation hurts borrowers and lenders similarly in the DM but affects them differently in the CM, and this difference makes its incidence progressive. In the DM, inflation reduces consumption and production, lowering the *DM surplus* $\frac{\sigma}{2}[u(q_j) + p(f(k_j) - q_j)]$ by comparable amounts for both groups. In the CM, three channels operate. First, the *inflation tax* $(\pi - 1)(\bar{m}_\pi - m_{j\pi})$ benefits borrowers, who hold less money than average, but hurts lenders, who hold more. Second, lower capital accumulation shrinks borrowers' stationary debt and thus their *interest expenses* $-a_{j\pi}(1-\beta_H)$, which raises their welfare, while lenders earn less interest income and lose by the same logic. Third, lower capital holdings reduce depreciation costs, $-\delta k_{j\pi}$, so both groups gain from what we call the *net capital payoff*.[13] For borrowers, all CM channels work in the direction of partly offsetting the DM loss. For lenders, both the inflation-tax loss and the reduced interest income reinforce it. Thus, lenders bear the larger net welfare loss, which is why inflation ends up being a progressive tax.

5 Extended model

Up to this point, we have assumed that capital is a factor of production only in the DM. This assumption, made mainly to retain analytical tractability, abstracts from additional general-equilibrium channels through which inflation can affect capital accumulation, factor prices, CM consumption and labor effort, and ultimately welfare. In this section, we relax this assumption and allow capital to be productive also in the CM, as in Aruoba,

12. Similar results are obtained when comparing 10 percent inflation to the Friedman rule: a 27 percent decrease for borrowers and 2 percent increase for lenders.

13. In our baseline model, capital is productive only in the DM, so its net CM payoff is just the depreciation cost. We relax this assumption in Section 5.

Waller, and Wright (2011).

We assume that in the CM a competitive, stand-in firm aggregates capital and labor to produce output according to a production function $F(K, N)$. Then, the returns on labor and capital are $w = F_N(K, N)$ and $r = F_K(K, N)$, where N and K are defined in (11) and (12), respectively. The problem in the CM in this case is analogous to the one in (5), except that now we need to take into consideration the returns on capital r and labor w:

$$\begin{aligned} W_j(\zeta_j^z) = \max_{x_j^z, n_j^z, m_j', k_j', a_j'} & U(x_j^z) - n_j^z + \beta_j V_j'(\zeta_j') \\ \text{s.t. } & x_j^z = w n_j^z - k_j' + (1 + r - \delta) k_j + m_j^z - \pi m_j' + \tau + p_a \pi a_j' - a_j \\ & a_j' \leq \theta k_j' \\ & k_j' \geq 0 \end{aligned} \tag{28}$$

We get the following first-order conditions:

$$\frac{1}{w} = U'(x_j) \tag{29}$$

$$\frac{\pi}{w} = \beta_j \frac{\partial V_j'}{\partial m_j'} \tag{30}$$

$$\frac{1}{w} = \beta_j \frac{\partial V_j'}{\partial k_j'} + \mu_j^k + \theta \lambda_j^a \tag{31}$$

$$-\frac{p_a}{w} = \frac{\beta_j}{\pi} \frac{\partial V_j'}{\partial a_j'} - \frac{\lambda_j^a}{\pi} \tag{32}$$

From (29), we get that $x_H = x_L = \bar{x}$. As in our baseline model, both types of agents consume the same amount in the CM market, but now monetary policy can affect their consumption via its effect on labor earnings. As before, the marginal valuations of money and capital are type independent, as shown by the envelope conditions below:[14]

$$\frac{\partial W_j}{\partial m_j^z} = \frac{1}{w}, \quad \frac{\partial W_j}{\partial a_j} = -\frac{1}{w} \text{ and } \frac{\partial W_j}{\partial k_j} = \frac{1}{w}(1 + r - \delta)$$

The DM problem is analogous to the one described in (13), except that now the continuation value will depend on w and r as well. From (1), (14) and (28), the first-order condition in the DM buyer's problem is:

$$u_q(q_j) = p\left(\frac{1}{w} + \lambda_j^b\right) \tag{33}$$

14. Assuming that factors of production are aggregated by a competitive firm is essential for this result.

While the envelope conditions for the DM problem are:

$$\frac{\partial V_j}{\partial m_j} = \frac{1}{w} + \frac{\sigma}{2}\left(\frac{u_q(q_j)}{p} - \frac{1}{w}\right) \tag{34}$$

$$\frac{\partial V_j}{\partial k_j} = \frac{1}{w}\left(1 + r - \delta + \frac{\sigma}{2} p f_k(k_j)\right) \tag{35}$$

$$\frac{\partial V_j}{\partial a_j} = -\frac{1}{w} \tag{36}$$

If we combine (30), (31), (32), (34), (35) and (36), the agents' optimal portfolio choices must satisfy:

$$\frac{\pi}{\beta_j} = 1 + \frac{\sigma}{2}\left(\frac{u_q(q_j)w}{p} - 1\right) \tag{37}$$

$$1 = \beta_j\left[(1 + r - \delta) + \frac{\sigma}{2} p f_k(k_j)\right] + w[\theta\lambda_j^a + \mu_j^k] \tag{38}$$

$$p_a \pi = \beta_j + w\lambda_j^a \tag{39}$$

The intuition behind equations (21), (22) and (23) carries over, with two additional elements. First, the liquidity premium in (37) now depends on the CM wage w. Since CM consumption satisfies $U'(x) = 1/w$, monetary policy can affect CM consumption through its general-equilibrium effect on wages.

Second, as shown by (38), capital return now consists of the CM return $1 + r - \delta$ and the expected DM return $(\sigma/2)pf_k(k_j)$. Inflation can affect these two components differently. In the DM, inflation reduces monetary expenditure and affects the relative price p, thereby changing the value of capital used in DM production. At the same time, changes in aggregate capital and labor affect the rental return r in the CM. The two components can therefore generate offsetting incentives for capital accumulation. For patient agents, whose collateral constraint does not bind, capital choice is governed by these productive returns. For impatient agents, capital has the additional value of relaxing the collateral constraint, as captured by the term $w\theta\lambda_L^a$ in (38).

At this point, we can provide a definition of equilibrium.

Definition 2. *A competitive stationary monetary equilibrium consists of* (m_j, a_j, k_j) *satisfying* (37), (38) *and* (39) *for* $j = H, L$.

Following the same argument as in Lemma 1, in any stationary monetary equilibrium with borrowing and lending, impatient agents borrow and patient agents lend at the price $p_a = \beta_H/\pi$. Specifically, $a_L = \theta k_L$, $a_H = -(1-\rho)\theta k_L/\rho$, $\lambda_L^a = (\beta_H - \beta_L)/w$ and $\lambda_H^a = 0$.

5.1 Quantitative analysis

We proceed with the quantitative analysis. We consider the same functional forms as in Section 4, with the addition of a standard Cobb–Douglas production function in the CM:

$$F(K, N) = K^{\psi} N^{1-\psi}$$

With this specification, the CM rental return depends on the aggregate capital–labor ratio: a decline in K/N raises the marginal product of capital. We keep all other parameters as in Section 4, and set $\psi = 0.33$, which corresponds to a capital share of approximately one third. This makes the general-equilibrium channel described above transparent.

Figures 7, 8, and 9 illustrate the effects of collateralization and inflation on capital accumulation and wealth inequality in the extended model. As in the baseline model, an increase in θ raises k_L while reducing k_H, indicating that greater pledgeability affects borrowers and lenders asymmetrically (Figure 7). The effects of inflation on the distribution of capital, however, differ in part from those in the baseline model. Although aggregate capital continues to decline with inflation, its distribution across household types changes. In particular, Figure 7 shows that the effect of inflation on patient households' capital holdings depends on the degree of pledgeability. Under uncollateralized borrowing or in the absence of credit, inflation reduces capital holdings for both types of households. Under collateralized borrowing, however, sufficiently high values of θ lead to an increase in k_H, even as k_L declines. At the calibrated value of θ, Figure 8 confirms that this reversal in the response of k_H to inflation occurs only under collateralized borrowing. This reversal, relative to the baseline model, reflects the interaction between the DM and CM components of the return to capital discussed above. Importantly, the decline in k_L more than offsets the increase in k_H, ensuring that aggregate capital continues to fall with inflation.

This mechanism differs from that in Aruoba, Waller, and Wright (2011). In their setting, inflation unambiguously discourages investment by acting as a tax on monetary transactions and reducing the demand for goods produced with capital. In our setting, heterogeneity and collateralized borrowing generate a richer interaction between inflation, collateral values, and portfolio choice.

While collateralization narrows the gap in capital holdings by encouraging borrowers to accumulate more capital, its effect on net wealth depends on the full portfolio of money, capital, and debt positions. As in the baseline model, under our calibration, we find that wealth inequality is lower with collateralized than with uncollateralized borrowing. In the extended model, however, wealth inequality is also lower with collateralized credit than in the no-credit economy, in contrast to the baseline (Figure 9). A further departure from the baseline is that inflation increases wealth inequality under collateralized borrowing. That's because inflation reduces capital holdings among impatient households but, when pledgeability is sufficiently high, increases those of patient households. This divergence

in capital accumulation widens the gap between the two groups and, together with the associated adjustments in their broader portfolio positions, increases the net-wealth Gini coefficient.

The compositional effects on capital accumulation translate into welfare outcomes (Figure 10). As in the baseline, collateralized lending increases borrowers' welfare but reduces lenders' welfare. Inflation again lowers welfare for both types, though through different channels as explained in what follows.

To quantify the welfare costs of inflation, we follow the same approach as the one in Section 4. When capital is a factor of production also in the CM market, the analogue of (27) becomes:

$$(1-\beta_j)\bar{V}_j(\zeta_{j\gamma}) = \frac{\sigma}{2}u(\Delta_{j\gamma}q_{j\gamma}) + U(\Delta_{j\gamma}\bar{x}_\gamma) - n_{j\gamma} \tag{40}$$

$$= \frac{\sigma}{2}u(\Delta_{j\gamma}q_{j\gamma}) + U(\Delta_{j\gamma}\bar{x}_\gamma) - \frac{1}{w_\gamma}\bar{x}_\gamma + \frac{1}{w_\gamma}(r_\gamma - \delta)k_{j\gamma} \tag{41}$$

$$+ \frac{\sigma p_\gamma}{2w_\gamma}[(f(k_{j\gamma}) - q_{j\gamma})] + \frac{1}{w_\gamma}(\gamma-1)(\bar{m}_\gamma - m_{j\gamma}) - \frac{1}{w_\gamma}a_{j\gamma}(1-\beta_H)$$

Table 3 reports the welfare costs of 10 percent expected inflation as opposed to both zero inflation and the Friedman rule for both borrowers (type L) and lenders (type H) as well as the average welfare cost of inflation for the economy.

Table 3: Percentage welfare cost of 10% inflation relative to zero inflation and the Friedman rule.

	Zero Inflation			Friedman Rule		
Model	Type L	Type H	Average	Type L	Type H	Average
Collateralized borrowing	0.55	1.06	0.75	0.78	1.54	1.08
Uncollateralized borrowing	0.08	1.66	0.71	0.12	2.40	1.03
No credit	0.06	1.70	0.71	0.08	2.46	1.03

Notes: Results are for the extended model, where capital is an input in both the DM and CM production functions. Welfare costs are expressed in consumption percentage points. At the Friedman rule, the net inflation rate is $\beta_H - 1 = -3.5\%$. In the models with borrowing—both collateralized and uncollateralized—type H agents are lenders and type L agents are borrowers. The average welfare cost of inflation is the weighted average of the welfare cost for type H and type L agents.

As in the baseline, inflation acts as a progressive tax, imposing a larger welfare cost on lenders than on borrowers. This effect is not unique to an economy with collateralized debt. Even without credit, lenders bear a substantially larger welfare cost when inflation increases, reflecting their greater exposure to money. Collateralization, however, shifts the incidence of inflation: unlike in the baseline, it raises the welfare cost for borrowers and lowers it for lenders.

To build intuition for these welfare effects, we decompose the welfare change associated with an increase from 0% to 10% in inflation in an economy with collateralized borrowing (Figure 11). Relative to the baseline, the extended model includes an additional component, $U(\bar{x}) - \bar{x}/w$, capturing *net benefits from CM consumption*. The *net capital payoff*,

$(r-\delta)k_j/w$, also changes, since it now includes rental income from CM production, net of depreciation, and therefore now depends on both capital holdings and endogenous factor prices. The remaining components (DM surplus, the inflation tax, and interest expenses) retain the same economic interpretation as in the baseline, but their monetary payoffs are now scaled by the endogenous wage to express them in units of labor effort.[15]

In the extended model, lenders' welfare costs of inflation are smaller than in the baseline. As seen in Figure 11, inflation reduces their DM surplus by less (this component accounts for 76% of their welfare change), while their net capital payoff rises. This rise reflects both higher capital holdings and changes in the return to capital, and it partly offsets their other welfare losses. Borrowers, by contrast, face larger welfare costs than in the baseline. When inflation rises, they reduce their capital holdings, which lowers their net capital payoff and contributes to the decline in their net benefit from the CM consumption component. These losses more than offset the gains from the inflation tax and lower debt-service costs.

6 Two means of payment

In the data, lower-income consumers are constrained in terms of the type of payment instruments they can use, while higher income consumers are not—see Shy (2020). To address this issue, we extend our baseline model in Section 2 and assume that patient agents can, with probability η, participate in an unsecured-credit segment of the DM, while impatient agents trade only in the cash market. In the credit segment, patient buyers purchase from patient sellers using unsecured credit, with transactions settled in the following CM.[16] We abstract from any issues related to default and revolving debt within the unsecured credit market, so that patient agents do not pay interest on the balances they carry from the DM to the CM. All algebra derivations can be found in the Appendix. The optimality conditions for the H-types are now:

$$\pi = \beta_H \left[1 + \frac{\sigma}{2p}(1-\eta)\left(u_q(q_H) - p\right)\right] \tag{42}$$

$$\pi p_a = \beta_H + \lambda_H^a \tag{43}$$

$$1 = \beta_H \left[(1-\delta) + \frac{\sigma}{2} f_k(k_H)\left((1-\eta)\, p + \eta \hat{p}\right)\right] + \mu_H^k + \theta \lambda_H^a \tag{44}$$

where $\hat{p}$ denotes prices in the unsecured credit market. Ceteris paribus, access to unsecured credit decreases money's liquidity premium for patient agents, and in turn their incentive to hold money. This shifts the burden of inflation towards impatient agents.

15. The formulas for the individual welfare components are reported in the notes to Figure 11 in the Appendix.

16. Our modeling of credit follows Aruoba, Waller, and Wright (2011).

Table 4 shows that, as long as type H households keep using money to pay for a sufficiently large share of their transactions, inflation remains a progressive tax. For higher values of η, inflation becomes a regressive tax.

Table 4: Percentage welfare cost of 10% inflation relative to zero inflation and the Friedman rule when type H has access to an inflation-protected means of payment.

	Zero Inflation			Friedman Rule		
Financial market access	Type L	Type H	Average	Type L	Type H	Average
$\eta = .1$	0.41	1.10	0.69	0.52	1.54	0.93
$\eta = .3$	0.42	0.86	0.60	0.52	1.17	0.78
$\eta = .5$	0.47	0.36	0.43	0.57	0.34	0.48

Notes: results are for the model where type H agents have access to an inflation-protected means of payment with probability η. Welfare costs are expressed in consumption percentage points. At the Friedman rule, the net inflation rate is $\beta_H - 1 = -3.5\%$. Type H agents are lenders and type L agents are borrowers in the collateralized lending market. The average welfare cost of inflation is the weighted average of the welfare cost for type H and type L agents.

7 Conclusion

Our paper contributes to the debate on the redistributive effects of expected inflation using a framework where money, capital and collateralized credit coexist. In our model, heterogeneity in discount factors generates endogenous differences in money, capital, and debt positions. Because borrowing is constrained by capital holdings, inflation affects the quantity of credit through its effect on capital accumulation. As a consequence, fully anticipated inflation can generate redistributive effects between borrowers and lenders through collateralized borrowing, even when nominal lending rates adjust to inflation.

We find that, in this environment, expected inflation reduces welfare for both borrowers and lenders, but the welfare cost is larger for lenders, so inflation acts as a progressive tax. At the same time, collateralization narrows the gap in capital holdings by inducing borrowers to accumulate more capital. In principle, this effect does not necessarily imply that greater pledgeability lowers wealth inequality, since net wealth also includes borrowing and lending positions. In our calibration, however, we do find that collateralized borrowing generates lower wealth inequality than an uncollateralized fixed borrowing limit. In our extended model, inflation can increase wealth inequality by reallocating capital toward patient lenders even as aggregate capital declines. Finally, we show that the progressive incidence of inflation depends on lenders remaining sufficiently exposed to money. Broader access to an inflation-protected means of payment can weaken, and eventually reverse, this result.

More broadly, our results show that the long-run distributional effects of expected inflation hinge on the interaction between monetary frictions, collateral constraints, and portfolio choice.

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Appendix

Throughout the Appendix, we adopt the notation $f'(x) = \partial f(x)/\partial x$ and $f''(x) = \partial^2 f(x)/\partial x^2$.

A. Proofs

Conditions for $n_j^z \geq 0$

We now derive a sufficient condition that guarantees $n_j^z \geq 0$ in the equilibrium described in Proposition 1.

Labor across DM states. In a stationary equilibrium, $m_j' = m_j$, $k_j' = k_j$, $a_j' = a_j$, $x_j^z = x^*$, $\tau = (\pi - 1)\bar{m}$, and $\pi p_a = \beta_H$ by Lemma 1. The budget constraint in (5) then implies

$$n_j^z = x^* + \delta k_j - m_j^z + \pi m_j - (\pi - 1)\bar{m} + (1 - \beta_H)a_j.$$

From (1), $m_j^o = m_j$, $m_j^b = m_j - pq_j$, and $m_j^s = m_j + pf(k_j)$. Hence

$$n_j^o = x^* + \delta k_j + (\pi - 1)(m_j - \bar{m}) + (1 - \beta_H)a_j,$$

$$n_j^b = n_j^o + pq_j, \qquad n_j^s = n_j^o - pf(k_j).$$

As shown in the proof of Proposition 1, $q_j > 0$ and $k_j > 0$. Under the additional condition $f(0) = 0$ imposed below, $f' > 0$ implies $f(k_j) > 0$. Therefore

$$n_j^s < n_j^o < n_j^b.$$

Thus, sellers work the least, and it is enough to establish $n_j^s \geq 0$ for $j = H, L$, where

$$n_j^s = x^* + \delta k_j - pf(k_j) + (\pi - 1)(m_j - \bar{m}) + (1 - \beta_H)a_j.$$

Notation. Equation (21) for $j = H$ can be written as

$$u'(q_H) = p\, c_H, \qquad c_H \equiv 1 + \frac{2}{\sigma}\left(\frac{\pi}{\beta_H} - 1\right) \geq 1,$$

where $c_H \geq 1$ because $\pi \geq \beta_H$. If $\pi > \beta_H$, then $c_H > 1$, so $u'(q_H) > p$ and, by (15), $\lambda_H^b > 0$. Hence the type-H cash constraint binds and

$$m_H = pq_H.$$

At $\pi = \beta_H$, where m_H is determined only up to $m_H \geq pq_H$, we focus on the equilibrium selection $m_H = pq_H$.

From (23) and Lemma 1,

$$\lambda_H^a = 0, \qquad \lambda_L^a = \beta_H - \beta_L.$$

Since $k_L > 0$, complementary slackness implies $\mu_L^k = 0$. Equation (22) for $j = L$ therefore gives

$$pf'(k_L) = A_L,$$

where

$$A_L \equiv \frac{2}{\sigma}\left[\frac{1-\theta(\beta_H-\beta_L)}{\beta_L} - (1-\delta)\right].$$

Since $\theta < 1$ and $\beta_H < 1$,

$$1-\theta(\beta_H-\beta_L) > 1-\beta_H+\beta_L > \beta_L.$$

Hence

$$\frac{1-\theta(\beta_H-\beta_L)}{\beta_L} > 1,$$

so that $A_L > 0$.

Finally, define

$$\bar{\Omega} \equiv \sup_{q>0} q\,u'(q),$$

and

$$\kappa_H \equiv \frac{1}{\rho} + (1-\rho)(1-\beta_H) + \frac{1-\rho}{\rho}\frac{(1-\beta_H)\theta}{A_L}, \qquad \kappa_L \equiv 1 + \frac{\rho\sigma\beta_H}{2}.$$

Lemma 4. *Let the assumptions of Proposition 1 hold, let $f(0) = 0$, and assume $\bar{\Omega} < \infty$. At $\pi = \beta_H$, consider the equilibrium selection $m_H = pq_H$. If*

$$x^* \geq \bar{\Omega}\max\{\kappa_H, \kappa_L\},$$

then $n_j^z > 0$ for all $j \in \{H, L\}$, $z \in \{b, s, o\}$, and $\pi \geq \beta_H$.

Proof. We first establish three bounds that will be useful below.

First, since $u'(q_H) = pc_H$ and $m_H = pq_H$ for the equilibrium selection considered above,

$$m_H = \frac{q_H u'(q_H)}{c_H} \leq \bar{\Omega},$$

because $c_H \geq 1$.

Moreover, for $\pi > 1$,

$$\frac{d}{d\pi}\left(\frac{\pi-1}{c_H}\right) = \frac{1+\frac{2}{\sigma}\left(\frac{1}{\beta_H}-1\right)}{c_H^2} > 0,$$

and

$$\lim_{\pi\to\infty}\frac{\pi-1}{c_H}=\frac{\sigma\beta_H}{2}.$$

Hence

$$(\pi-1)m_H<\frac{\sigma\beta_H}{2}\bar{\Omega}\qquad\text{for all }\pi>1. \tag{45}$$

Second, from the proof of Proposition 1,

$$0<q_L<q_H,\qquad 0<m_L<m_H\qquad\text{and}\qquad 0<k_L<k_H.$$

Since $f(0)=0$ and $f'>0$,

$$0<f(k_L)<f(k_H).$$

The DM market-clearing condition (16) therefore implies

$$\rho f(k_H)<\rho f(k_H)+(1-\rho)f(k_L)=\rho q_H+(1-\rho)q_L<q_H.$$

Moreover,

$$f(k_L)<\rho f(k_H)+(1-\rho)f(k_L)<q_H.$$

Multiplying by p gives

$$pf(k_H)<\frac{pq_H}{\rho}=\frac{m_H}{\rho}\le\frac{\bar{\Omega}}{\rho},\qquad pf(k_L)<pq_H=m_H\le\bar{\Omega}. \tag{46}$$

Third, strict concavity of f together with $f(0)=0$ implies

$$f(k_L)>f'(k_L)k_L.$$

Multiplying by $p>0$ and using $pf'(k_L)=A_L$, we obtain

$$pf(k_L)>A_Lk_L,$$

and hence

$$k_L<\frac{pf(k_L)}{A_L}<\frac{\bar{\Omega}}{A_L}. \tag{47}$$

Type L. Using $a_L=\theta k_L$, seller-state labor is

$$n_L^s=x^*+\delta k_L-pf(k_L)+(\pi-1)(m_L-\bar{m})+(1-\beta_H)\theta k_L.$$

Since $\delta k_L\ge 0$ and $(1-\beta_H)\theta k_L\ge 0$,

$$n_L^s\ge x^*-pf(k_L)+(\pi-1)(m_L-\bar{m}).$$

Moreover, from $\bar{m} = \rho m_H + (1-\rho)m_L$,

$$m_L - \bar{m} = -\rho(m_H - m_L).$$

If $\beta_H \leq \pi \leq 1$, then $(\pi - 1)(m_L - \bar{m}) \geq 0$, so by (46),

$$n_L^s > x^* - \bar{\Omega}.$$

Since $\kappa_L > 1$, this also implies

$$n_L^s > x^* - \kappa_L \bar{\Omega}.$$

If $\pi > 1$, since $0 < m_H - m_L < m_H$,

$$(\pi - 1)(m_L - \bar{m}) > -\rho(\pi - 1)m_H.$$

Using (45) and (46),

$$n_L^s > x^* - \left(1 + \frac{\rho\sigma\beta_H}{2}\right)\bar{\Omega} = x^* - \kappa_L\bar{\Omega}.$$

Therefore, for all $\pi \geq \beta_H$,

$$n_L^s > x^* - \kappa_L \bar{\Omega}.$$

Type H. Using

$$a_H = -\frac{1-\rho}{\rho}\theta k_L$$

and

$$m_H - \bar{m} = (1-\rho)(m_H - m_L),$$

seller-state labor is

$$n_H^s = x^* + \delta k_H - pf(k_H) + (\pi - 1)(1-\rho)(m_H - m_L) - \frac{1-\rho}{\rho}(1-\beta_H)\theta k_L.$$

Since $\delta k_H \geq 0$,

$$n_H^s \geq x^* - pf(k_H) + (\pi - 1)(1-\rho)(m_H - m_L) - \frac{1-\rho}{\rho}(1-\beta_H)\theta k_L.$$

If $\pi \geq 1$, the inflation term is nonnegative. If instead $\beta_H \leq \pi < 1$, then

$$(\pi - 1)(1-\rho)(m_H - m_L) > -(1-\beta_H)(1-\rho)m_H \geq -(1-\beta_H)(1-\rho)\bar{\Omega}.$$

Thus, in either case, using (46) and (47),

$$n_H^s > x^* - \left[\frac{1}{\rho} + (1-\rho)(1-\beta_H) + \frac{1-\rho}{\rho}\frac{(1-\beta_H)\theta}{A_L}\right]\bar{\Omega} = x^* - \kappa_H\bar{\Omega}.$$

Hence, if

$$x^* \geq \bar{\Omega}\max\{\kappa_H, \kappa_L\},$$

then

$$n_H^s > 0 \qquad \text{and} \qquad n_L^s > 0.$$

Since $n_j^s < n_j^o < n_j^b$, it follows that $n_j^z > 0$ for all $j \in \{H, L\}$ and $z \in \{b, s, o\}$. The bounds are uniform in π, so the result holds for all $\pi \geq \beta_H$. □

For the functional forms used in the quantitative analysis, $u(q) = \ln q$ and $f(k) = k^\alpha$, we have $\bar{\Omega} = 1$ and $f(0) = 0$. Under our benchmark calibration, $A_L \simeq 0.341$, $\kappa_L \simeq 1.133$, and $\kappa_H \simeq 2.652$. Since $x^* = B = 3.12$, the sufficient condition in Lemma 4 is satisfied.

Proof of Lemma 1

From the Euler equation in (23) we have that the following condition must hold:

$$\beta_L + \lambda_L^a = \beta_H + \lambda_H^a$$

Since $\beta_H > \beta_L$, it must be that $\lambda_L^a > \lambda_H^a \geq 0$. If $\lambda_L^a > \lambda_H^a > 0$, then by complementary slackness both collateral constraints bind, so that $a_j = \theta k_j \geq 0$ for $j = H, L$. The bonds market-clearing condition then implies $a_H = a_L = 0$, and hence there is no borrowing or lending. If instead $\lambda_L^a > \lambda_H^a = 0$, then $a_L = \theta k_L$ and given the bonds market-clearing condition:

$$\rho a_H + (1-\rho)a_L = 0 \tag{48}$$

we have that $a_H = -\theta k_L(1-\rho)/\rho$. From (23) and $\lambda_H^a = 0$ we have that $\pi p_a = \beta_H$, and therefore $p_a = \beta_H/\pi$. □

Proof of Lemma 2

We know from (15) that $u'(q_j) \geq p$ for $j = H, L$. This, together with (21), implies that $\pi \geq \beta_H$. □

Proof of Lemma 3

If $f(k)$ is linear and $f'(k) = \gamma$ with $\gamma > 0$, then the Euler equations for capital in (22) for $j = H, L$ become:

$$\frac{1}{\beta_L} = (1-\delta) + \frac{\sigma}{2}p\gamma + \frac{\mu_L^k}{\beta_L} + \frac{\theta(\beta_H - \beta_L)}{\beta_L} \tag{49}$$

$$\frac{1}{\beta_H} = (1-\delta) + \frac{\sigma}{2}p\gamma + \frac{\mu_H^k}{\beta_H} \tag{50}$$

From (49) and (50), we get that the following condition has to hold:

$$\frac{(\beta_H - \beta_L)(1-\theta\beta_H)}{\beta_L\beta_H} = \frac{\mu_L^k}{\beta_L} - \frac{\mu_H^k}{\beta_H}$$

which implies $\mu_L^k > 0$ and therefore $k_L = 0$. □

Proof of Proposition 1

Let $\pi \geq \beta_H$. Since $\beta_H > \beta_L$, from (21) we know that

$$u_q(q_H) < u_q(q_L)$$

and therefore $q_H > q_L$. Moreover, since $\pi \geq \beta_H > \beta_L$, (21) implies $u_q(q_L) > p$. From (15), this implies $\lambda_L^b > 0$, so the cash constraint of type L binds and

$$m_L = pq_L.$$

For type H, the cash constraint implies $m_H \geq pq_H$. The Inada condition $u_q(0) = +\infty$ implies $q_L > 0$. Hence,

$$m_H \geq pq_H > pq_L = m_L > 0,$$

and therefore

$$m_H > m_L > 0.$$

Next, the Inada condition $f_k(0) = +\infty$ implies that capital holdings are strictly positive for both types. Indeed, at $k_j = 0$ the marginal return to increasing capital is unbounded, whereas its marginal cost is finite. Hence $k_j = 0$ cannot be optimal for either $j = H, L$. Therefore,

$$k_H > 0 \qquad \text{and} \qquad k_L > 0,$$

and complementary slackness implies

$$\mu_H^k = \mu_L^k = 0.$$

From (23) and the proof of Lemma 1, we have

$$\lambda_H^a = 0 \qquad \text{and} \qquad \lambda_L^a = \beta_H - \beta_L.$$

Combining these conditions with (22) gives

$$\frac{(\beta_H - \beta_L)(1 - \theta\beta_H)}{\beta_H \beta_L} = \frac{\sigma}{2} p \left[f_k(k_L) - f_k(k_H) \right].$$

Since $\beta_H > \beta_L$ and $\theta\beta_H < 1$, the left-hand side is strictly positive. Hence

$$f_k(k_L) > f_k(k_H).$$

Since $f''(k) < 0$, f_k is strictly decreasing, and therefore

$$k_L < k_H.$$

Together with the positivity result above,

$$0 < k_L < k_H.$$

□

Proof of Proposition 2

If we differentiate (16), (21) and (22) with respect to θ for $j = H, L$ we get:

$$0 = \frac{\sigma}{2} \left[\frac{u''(q_j)}{p} \frac{dq_j}{d\theta} - \frac{u'(q_j)}{p^2} \frac{dp}{d\theta} \right] \quad \text{for } j = H, L \tag{51}$$

$$\frac{dk_H}{d\theta} = -\frac{dp}{d\theta} \frac{f'(k_H)}{p f''(k_H)} \tag{52}$$

$$\frac{dk_L}{d\theta} = -\frac{dp}{d\theta} \frac{f'(k_L)}{p f''(k_L)} - \frac{2(\beta_H - \beta_L)}{\sigma p \beta_L f''(k_L)} \tag{53}$$

$$\rho \frac{dq_H}{d\theta} + (1 - \rho) \frac{dq_L}{d\theta} = \rho f'(k_H) \frac{dk_H}{d\theta} + (1 - \rho) f'(k_L) \frac{dk_L}{d\theta} \tag{54}$$

If we plug (52) and (53) into (54), we find that the following conditions have to hold:

$$\frac{\sigma}{2}\left[\frac{u''(q_H)}{p}\frac{dq_H}{d\theta} - \frac{u'(q_H)}{p^2}\frac{dp}{d\theta}\right] = 0$$
$$\frac{\sigma}{2}\left[\frac{u''(q_L)}{p}\frac{dq_L}{d\theta} - \frac{u'(q_L)}{p^2}\frac{dp}{d\theta}\right] = 0$$
$$\rho\frac{dq_H}{d\theta} + (1-\rho)\frac{dq_L}{d\theta} + \left[\rho\frac{f'(k_H)^2}{pf''(k_H)} + (1-\rho)\frac{f'(k_L)^2}{pf''(k_L)}\right]\frac{dp}{d\theta} = \Upsilon(k_L)$$

The system of equations can be rewritten as:

$$\begin{bmatrix} \frac{\sigma}{2}\frac{u''(q_H)}{p} & 0 & -\frac{\sigma}{2}\frac{u'(q_H)}{p^2} \\ 0 & \frac{\sigma}{2}\frac{u''(q_L)}{p} & -\frac{\sigma}{2}\frac{u'(q_L)}{p^2} \\ \rho & 1-\rho & \Gamma(k_H,k_L) \end{bmatrix}\begin{bmatrix} \frac{dq_H}{d\theta} \\ \frac{dq_L}{d\theta} \\ \frac{dp}{d\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \Upsilon(k_L) \end{bmatrix}$$

where

$$\Gamma(k_H,k_L) = \rho\frac{f'(k_H)^2}{pf''(k_H)} + (1-\rho)\frac{f'(k_L)^2}{pf''(k_L)} \tag{55}$$

and

$$\Upsilon(k_L) = (1-\rho)f'(k_L)\frac{2(\beta_L-\beta_H)}{\sigma p\beta_L f''(k_L)}$$

so that $\Gamma(k_H,k_L) < 0$ and $\Upsilon(k_L) > 0$.

We can calculate the following determinants:

$$|D^\theta| = \frac{\sigma^2}{4p^2}\left[u''(q_H)u''(q_L)\Gamma(k_H,k_L) + \rho\frac{u''(q_L)u'(q_H)}{p} + (1-\rho)\frac{u''(q_H)u'(q_L)}{p}\right] \tag{56}$$

so that $|D^\theta| < 0$;

$$|D^\theta_H| = \Upsilon(k_L)\left(\frac{\sigma}{2}\right)^2\frac{u''(q_L)u'(q_H)}{p^3} \tag{57}$$

so that $|D^\theta_H| < 0$;

$$|D^\theta_L| = \Upsilon(k_L)\left(\frac{\sigma}{2}\right)^2\frac{u''(q_H)u'(q_L)}{p^3} \tag{58}$$

so that $|D^\theta_L| < 0$;

$$|D^\theta_p| = \Upsilon(k_L)\left(\frac{\sigma}{2p}\right)^2 u''(q_H)u''(q_L) \tag{59}$$

so that $|D^\theta_p| > 0$.

Using Cramer's rule, we know that:

$$\frac{dp}{d\theta} = \frac{|D_p^\theta|}{|D^\theta|}, \quad \frac{dk_H}{d\theta} = -\frac{dp}{d\theta}\frac{f'(k_H)}{pf''(k_H)}, \quad \frac{dq_H}{d\theta} = \frac{|D_H^\theta|}{|D^\theta|}, \quad \frac{dq_L}{d\theta} = \frac{|D_L^\theta|}{|D^\theta|}$$

Therefore, we have that $\frac{dp}{d\theta} < 0$, $\frac{dk_H}{d\theta} < 0$, $\frac{dq_H}{d\theta} > 0$ and $\frac{dq_L}{d\theta} > 0$. Since $\frac{dq_j}{d\theta} > 0$ for $j = H, L$ but $\frac{dk_H}{d\theta} < 0$, it must be that $\frac{dk_L}{d\theta} > 0$. □

Proof of Proposition 3

If we differentiate (16), (21) and (22) for $j = H, L$ with respect to π we get:

$$\frac{1}{\beta_j} = \frac{\sigma}{2}\left[\frac{u''(q_j)}{p}\frac{dq_j}{d\pi} - \frac{u'(q_j)}{p^2}\frac{dp}{d\pi}\right] \tag{60}$$

$$\frac{dk_j}{d\pi} = -\frac{dp}{d\pi}\frac{f'(k_j)}{pf''(k_j)} \tag{61}$$

$$\rho\frac{dq_H}{d\pi} + (1-\rho)\frac{dq_L}{d\pi} = \rho f'(k_H)\frac{dk_H}{d\pi} + (1-\rho)f'(k_L)\frac{dk_L}{d\pi} \tag{62}$$

If we plug (61) for $j = H, L$ into (62), we find the following system of equations has to hold:

$$\begin{bmatrix} \frac{\sigma}{2}\frac{u''(q_H)}{p} & 0 & -\frac{\sigma}{2}\frac{u'(q_H)}{p^2} \\ 0 & \frac{\sigma}{2}\frac{u''(q_L)}{p} & -\frac{\sigma}{2}\frac{u'(q_L)}{p^2} \\ \rho & 1-\rho & \Gamma(k_H, k_L) \end{bmatrix} \begin{bmatrix} \frac{dq_H}{d\pi} \\ \frac{dq_L}{d\pi} \\ \frac{dp}{d\pi} \end{bmatrix} = \begin{bmatrix} \frac{1}{\beta_H} \\ \frac{1}{\beta_L} \\ 0 \end{bmatrix}$$

for which the determinant is:

$$|D| = \frac{\sigma^2}{4p^2}\left[u''(q_H)u''(q_L)\Gamma(k_H, k_L) + \rho\frac{u''(q_L)u'(q_H)}{p} + (1-\rho)\frac{u''(q_H)u'(q_L)}{p}\right] \tag{63}$$

and $|D| < 0$. We also define the following determinants:

$$|D_H| = \frac{\sigma}{2p}\left[\frac{1}{\beta_H}u''(q_L)\Gamma(k_H, k_L) - (1-\rho)\frac{1}{\beta_L}\frac{u'(q_H)}{p} + (1-\rho)\frac{1}{\beta_H}\frac{u'(q_L)}{p}\right] \tag{64}$$

From (21) for $j = H, L$ we have that $(\beta_H - \beta_L)(1 - \sigma/2)2p/\sigma = \beta_L u'(q_L) - \beta_H u'(q_H)$ so that $u'(q_L)/\beta_H > u'(q_H)/\beta_L$ and $|D_H| > 0$.

$$|D_p| = -\rho\frac{\sigma}{2\beta_H}\frac{u''(q_L)}{p} - (1-\rho)\frac{\sigma}{2\beta_L}\frac{u''(q_H)}{p} \tag{65}$$

so that $|D_p| > 0$.

$$|D_L| = \frac{1}{\beta_L}\frac{\sigma u''(q_H)}{2p^2}\left(\rho\frac{f'(k_H)^2}{f''(k_H)} + (1-\rho)\frac{f'(k_L)^2}{f''(k_L)}\right) - \rho\frac{\sigma}{2p^2}\left(\frac{u'(q_L)}{\beta_H} - \frac{u'(q_H)}{\beta_L}\right) \tag{66}$$

Using Cramer's rule, we know that:

$$\frac{dp}{d\pi} = \frac{|D_p|}{|D|}, \quad \frac{dk_j}{d\pi} = -\frac{dp}{d\pi}\frac{f'(k_j)}{pf''(k_j)}, \quad \frac{dq_H}{d\pi} = \frac{|D_H|}{|D|}, \quad \frac{dq_L}{d\pi} = \frac{|D_L|}{|D|}$$

Therefore, we know that $\frac{dp}{d\pi} < 0$, $\frac{dk_H}{d\pi} < 0$, $\frac{dk_L}{d\pi} < 0$ and $\frac{dq_H}{d\pi} < 0$. □

Proof of Proposition 4

From (63) and (66), we know that $dq_L/d\pi = |D_L|/|D| < 0$ if $|D_L| > 0$. Assume that the production function is isoelastic, so that $\varepsilon f(k) = f'(k)k$. By differentiating this condition, we have that $\frac{f''(k)k}{f'(k)} = \varepsilon - 1$ and therefore:

$$\frac{f'(k)^2}{f''(k)} = \frac{f'(k)}{f''(k)}\frac{k}{k}\frac{f(k)}{f(k)}f'(k) = \frac{f'(k)}{f''(k)k}\frac{kf'(k)}{f(k)}f(k) = \frac{\varepsilon}{\varepsilon-1}f(k) \tag{67}$$

Then, from (66) and (67), $|D_L|$ can be written as:

$$|D_L| = \frac{\sigma}{2}\frac{1}{p^2}\frac{1}{\beta_L}\left(u''(q_H)\frac{\varepsilon}{\varepsilon-1}\left(\rho f(k_H) + (1-\rho)f(k_L)\right) - \rho\frac{1}{\beta_H}\left(u'(q_L)\beta_L - u'(q_H)\beta_H\right)\right)$$

Therefore:

$$|D_L| > \frac{\sigma}{2}\frac{1}{p^2}\frac{1}{\beta_L}\left(u''(q_H)\frac{\varepsilon}{\varepsilon-1}\rho q_H - \rho\frac{1}{\beta_H}\left(u'(q_L)\beta_L - u'(q_H)\beta_H\right)\right)$$

The condition above can be simplified to:

$$|D_L| > -\frac{\sigma}{2}\frac{1}{p}\frac{1}{\beta_L}\left(\frac{1}{p}u''(q_H)\frac{\varepsilon}{1-\varepsilon}\rho q_H + \rho\frac{\beta_H - \beta_L}{\beta_H}\left(1-\frac{\sigma}{2}\right)\frac{2}{\sigma}\right)$$

Therefore, $|D_L| > 0$ if:

$$\frac{1}{p}u''(q_H)\frac{\varepsilon}{1-\varepsilon}\rho q_H + \rho\frac{\beta_H - \beta_L}{\beta_H}\left(1-\frac{\sigma}{2}\right)\frac{2}{\sigma} < 0$$

and

$$-\frac{u''(q_H)\rho q_H}{p} > \frac{1-\varepsilon}{\varepsilon}\rho\frac{\beta_H - \beta_L}{\beta_H}\left(\frac{2-\sigma}{\sigma}\right).$$

Note that since $u'(q_H) \geq p$ from (15), then:

$$\frac{-u''(q_H)\rho q_H}{p} \geq \frac{-u''(q_H)\rho q_H}{u'(q_H)} = \rho\mu,$$

where μ denotes risk aversion. It follows that if:

$$\mu > \frac{1-\varepsilon}{\varepsilon}\frac{\beta_H - \beta_L}{\beta_H}\left(\frac{2-\sigma}{\sigma}\right),$$

then $|D_L| > 0$. □

Calibration of the baseline model

Here we report the calibration targets for the baseline model. First, we fit the ratio $L = M/PY$, i.e. the inverse of money velocity. The theoretical expression for the inverse of money velocity in the model is $L = \bar{m}/(\frac{\sigma}{2}p(\rho q_H + (1-\rho)q_L) + B + \delta(\rho k_H + (1-\rho)k_L))$, where the numerator denotes average money holdings and the denominator is the sum of output in the DM and the CM. Since average money holdings are $\bar{m} = p(\rho q_H + (1-\rho)q_L)$, the ratio in equilibrium becomes:

$$L = \frac{1}{\frac{\sigma}{2} + \frac{B + \delta(\rho k_H + (1-\rho)k_L)}{p(\rho q_H + (1-\rho)q_L)}} \tag{68}$$

Second, we match the theoretical share of DM consumption, with its empirical counterpart. The latter is calculated as the share of cash consumption transactions using data for the period 2015–2019 from the Diary of Consumer Payment Choice from the Federal Reserve Bank of Atlanta.

The theoretical expression for DM consumption is $\Lambda = (\frac{\sigma}{2}p(\rho q_H + (1-\rho)q_L))/(\frac{\sigma}{2}p(\rho q_H + (1-\rho)q_L) + B)$, where the numerator is DM consumption and the denominator is the sum of DM and CM consumption. Simplifying, we get:

$$\Lambda = \frac{1}{1 + \frac{2B}{\sigma p(\rho q_H + (1-\rho)q_L)}} \tag{69}$$

Last, we match the theoretical ratio of collateralized debt over GDP, with its empirical counterpart. The latter is computed as the ratio of mortgages and auto loans originations to GDP, using data for the period 1999–2019 from the Quarterly Report on Household Debt and Credit from the Federal Reserve Bank of New York. The theoretical ratio Ψ in equilibrium is:

$$\Psi = \frac{(1-\rho)\theta k_L}{\frac{\sigma}{2}p(\rho k_H^{\alpha} + (1-\rho)k_L^{\alpha}) + B + \delta(\rho k_H + (1-\rho)k_L)} \tag{70}$$

Model with two means of payment

The problem in the CM for an agent that participates in the DM cash market is analogous to (5). The problem for an agent that participates in the DM credit market is:

$$\begin{aligned} W_j(\hat{\zeta}_j^z) = & \max_{\hat{x}_j^z, \hat{n}_j^z, m_j', k_j', a_j'} U(\hat{x}_j^z) - \hat{n}_j^z + \beta_j V_j'(\zeta_j') \\ \text{s.t. } & \hat{x}_j^z = \hat{n}_j^z - k_j' + (1-\delta)k_j + m_j - \pi m_j' + \tau + p_a \pi a_j' - a_j - \ell_j^z \\ & a_j' \leq \theta k_j' \\ & k_j' \geq 0 \end{aligned} \tag{71}$$

where ℓ_j^z denotes the payment made in the CM for the loan taken during the DM by an agent of type j who received a trading shock z. Moreover, $\hat{\zeta}_j^z = m_j, k_j, a_j, \ell_j^z$. The problem in (71) yields the same first order conditions as in (6), (7), (8) and (9) and the envelope conditions

$$\frac{\partial W_j}{\partial m_j} = 1, \quad \frac{\partial W_j}{\partial a_j} = -1, \quad \frac{\partial W_j}{\partial k_j} = 1 - \delta \text{ and } \frac{\partial W_j}{\partial \ell_j^z} = -1$$

The DM problem for an agent of type j is:

$$\begin{aligned} V_j(\zeta_j) = & (1-\eta_j)\left[\frac{\sigma}{2}\left[u\left(q_j\right) + W_j(\zeta_j^b)\right] + \frac{\sigma}{2} W_j(\zeta_j^s) + (1-\sigma) W_j(\zeta_j^o)\right] \\ & + \eta_j \left[\frac{\sigma}{2}\left[u\left(\hat{q}_j\right) + W_j(\hat{\zeta}_j^b)\right] + \frac{\sigma}{2} W_j(\hat{\zeta}_j^s) + (1-\sigma) W_j(\hat{\zeta}_j^o)\right] \\ & \text{s.t. } pq_j \leq m_j \end{aligned}$$

where m_j^z is defined in (1) and loan balances evolve according to:

$$\ell_j^b = \hat{p}\hat{q}_j, \quad \ell_j^s = -\hat{p} f\left(k_j\right) \text{ and } \ell_j^o = 0$$

The first order condition for the DM problem in the cash market is the same as (15). In the loan market, instead, the first order condition is:

$$u_{\hat{q}}(\hat{q}_j) = -\hat{p} W_\ell = \hat{p}$$

The envelope conditions read:

$$\begin{aligned}
\frac{\partial V_j}{\partial m_j} &= 1 + \frac{\sigma}{2p}(1-\eta_j)\left[u'(q_j) - p\right] \\
\frac{\partial V_j}{\partial k_j} &= 1 - \delta + (1-\eta_j)\frac{\sigma}{2} p f_k(k_j) + \eta_j \frac{\sigma}{2}\hat{p} f_k(k_j) \\
\frac{\partial V_j}{\partial a_j} &= -1
\end{aligned}$$

We can combine envelope and first-order conditions to write the latter in a more compact form, as follows:

$$\begin{aligned}
&\frac{\pi}{\beta_j} = 1 + \frac{\sigma}{2}(1-\eta_j)\left(\frac{u'(q_j)}{p} - 1\right) \\
&1 = \beta_j\left[(1-\delta) + (1-\eta_j)\frac{\sigma}{2} p f_k(k_j) + \eta_j \frac{\sigma}{2}\hat{p} f_k(k_j)\right] + \mu_j^k + \theta\lambda_j^a \\
&\pi p_a = \beta_j + \lambda_j^a
\end{aligned}$$

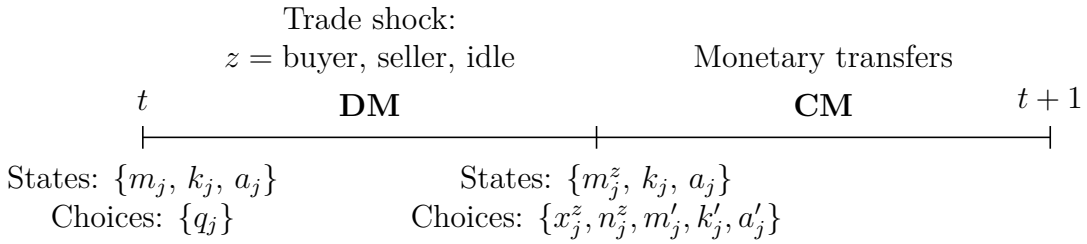
Finally, the market clearing conditions in the DM market are:

$$\begin{aligned}
(1-\eta_H)\rho q_H + (1-\eta_L)(1-\rho) q_L &= (1-\eta_H)\rho f(k_H) + (1-\eta_L)(1-\rho) f(k_L) \\
\eta_H \rho \hat{q}_H + \eta_L(1-\rho)\hat{q}_L &= \eta_H \rho f(k_H) + \eta_L(1-\rho) f(k_L)
\end{aligned}$$

We assume that only type H agents have access to credit, so that $\eta_H > \eta_L = 0$ and $\eta_H = \eta$.

B. Figures

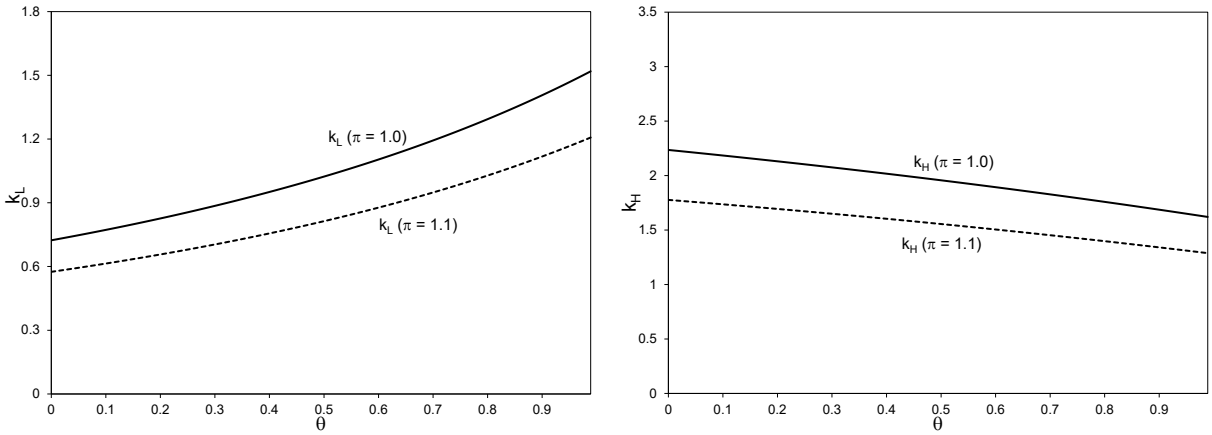
Figure 1: Timing of events in a period



Notes: The timing of events is the same in the baseline model and in the extended model.

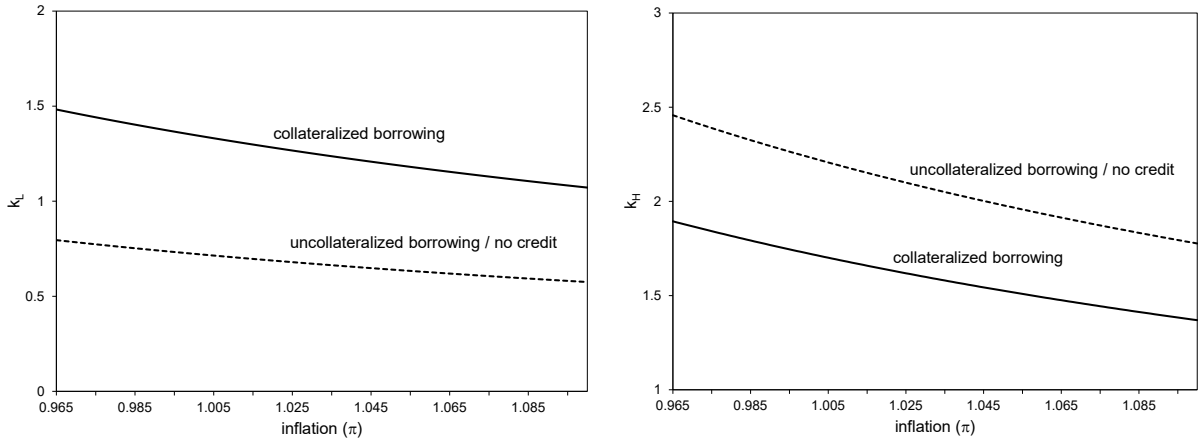
B1. Baseline Model

Figure 2: Capital holdings and pledgeability (baseline model)



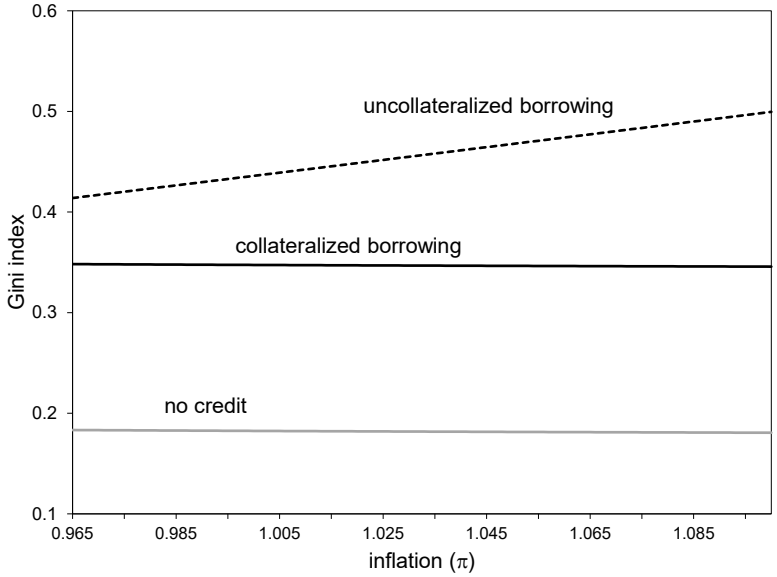
Notes: Capital holdings k_L and k_H against the pledgeability parameter θ in the baseline model with a collateral constraint. Solid and dotted lines correspond to the cases with gross inflation $\pi = 1$ and $\pi = 1.1$ respectively.

Figure 3: Capital holdings and inflation (baseline model)



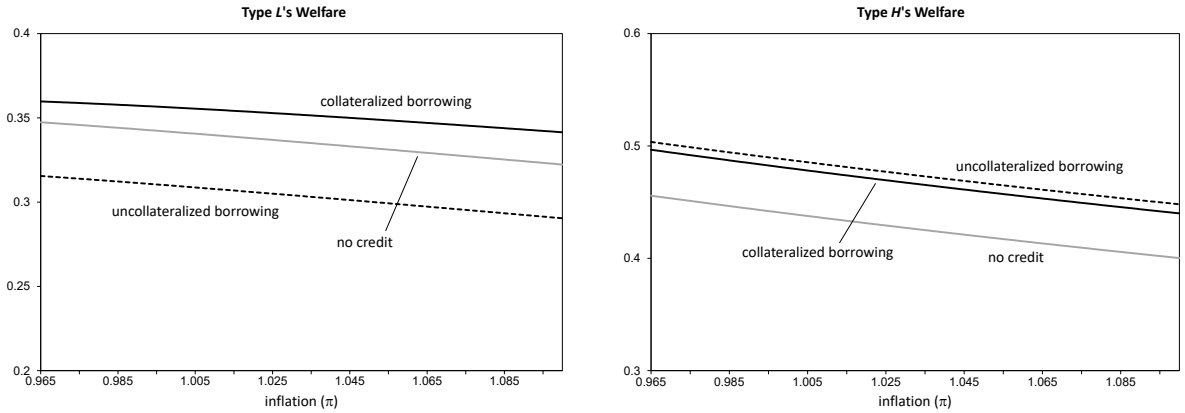
Notes: Capital holdings k_L and k_H against the gross inflation rate π. Solid line: baseline model with a collateral constraint. Dotted line: model with uncollateralized borrowing (i.e., with a fixed borrowing limit A) and model without borrowing and lending.

Figure 4: Wealth inequality and inflation (baseline model)



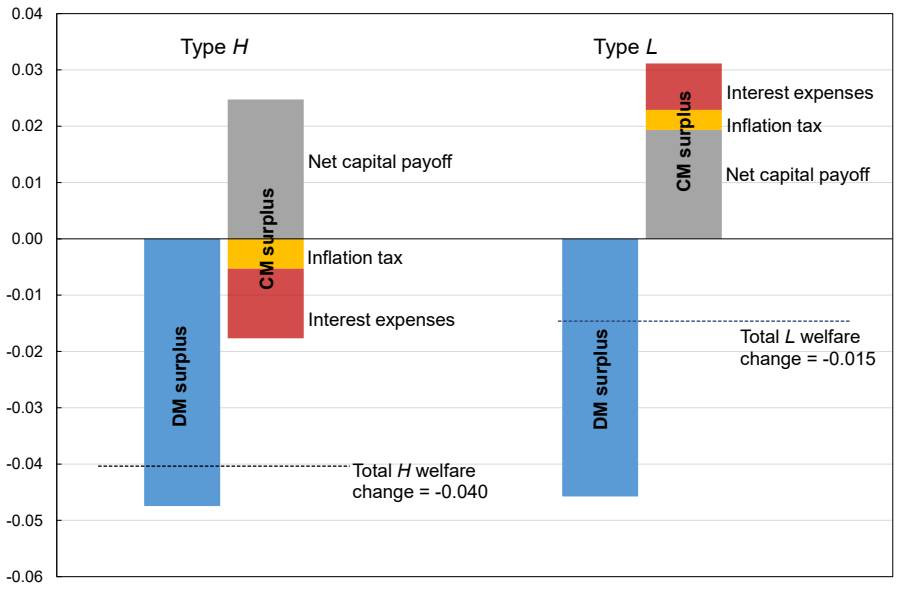
Notes: Wealth inequality as measured by the Gini index against the gross inflation rate π. Solid line: baseline model with a collateral constraint. Dotted line: model with uncollateralized borrowing (i.e., with a fixed borrowing limit A). Gray line: model without borrowing and lending.

Figure 5: Welfare and inflation (baseline model)



Notes: Welfare for L and H agents against the gross inflation rate π. Solid line: baseline model with a collateral constraint. Dotted line: model with uncollateralized borrowing (i.e., with a fixed borrowing limit A). Gray line: model without borrowing and lending.

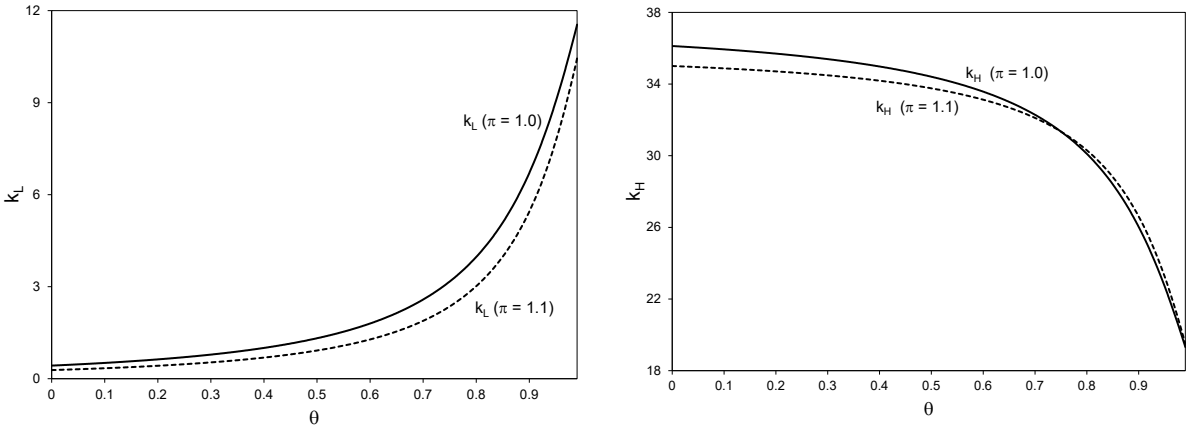
Figure 6: Decomposition of welfare changes when inflation increases from 0% to 10% (baseline model)



Notes: DM surplus is $\frac{\sigma}{2}[u(q_j)-pq_j+pf(k_j)]$; CM surplus is defined as the sum of the following components: net capital payoff $-\delta k_j$, inflation tax $(\pi-1)(\bar{m}-m_j)$ and interest expenses, $-a_j(1-\beta_H)$.

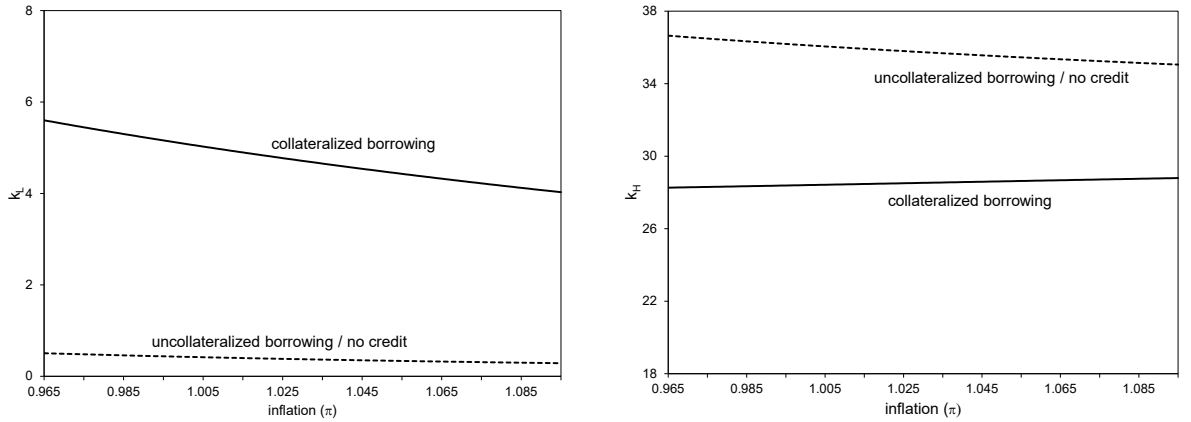
B2. Extended model

Figure 7: Capital holdings and pledgeability (extended model)



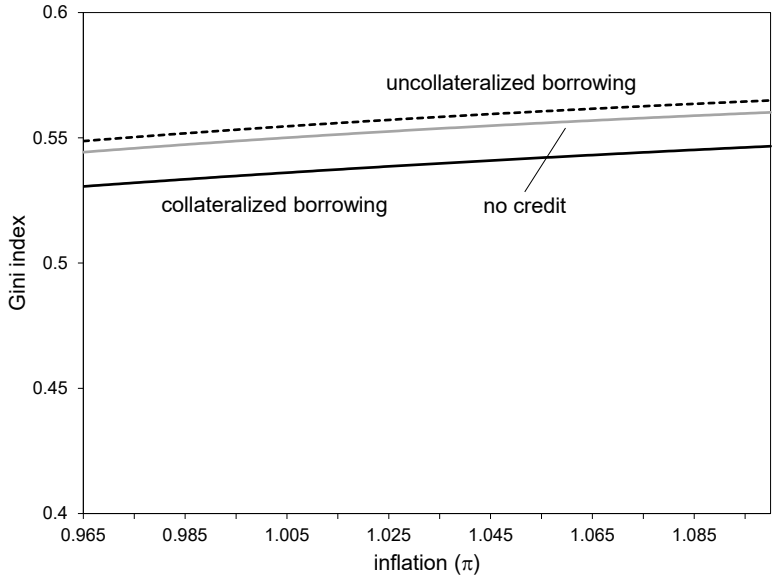
Notes: Capital holdings k_L and k_H against the pledgeability parameter θ in the extended model with a collateral constraint. Solid and dotted lines correspond to the cases with gross inflation $\pi = 1$ and with $\pi = 1.1$ respectively.

Figure 8: Capital holdings and inflation (extended model)



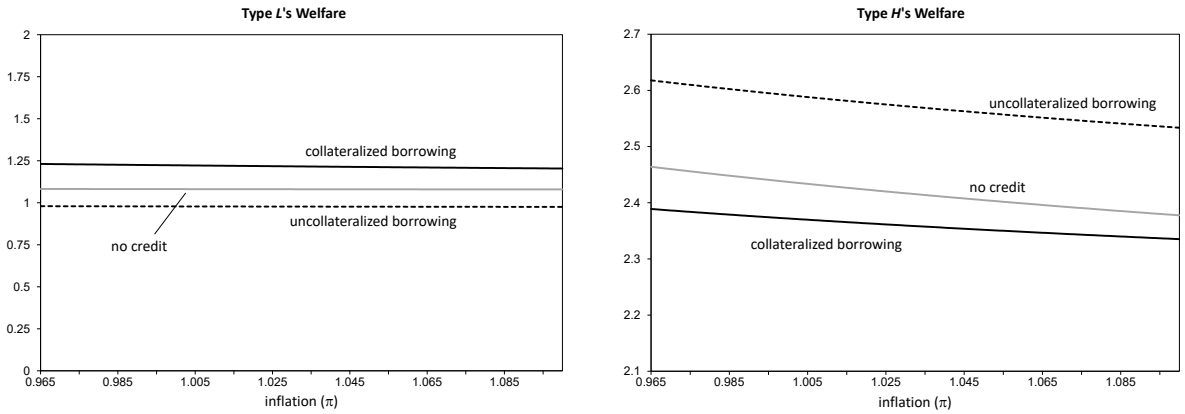
Notes: Capital holdings k_L and k_H against the gross inflation rate π. Solid line: extended model with a collateral constraint. Dotted line: model with uncollateralized borrowing (i.e., with a fixed borrowing limit A) and model without borrowing and lending.

Figure 9: Wealth inequality and inflation (extended model)



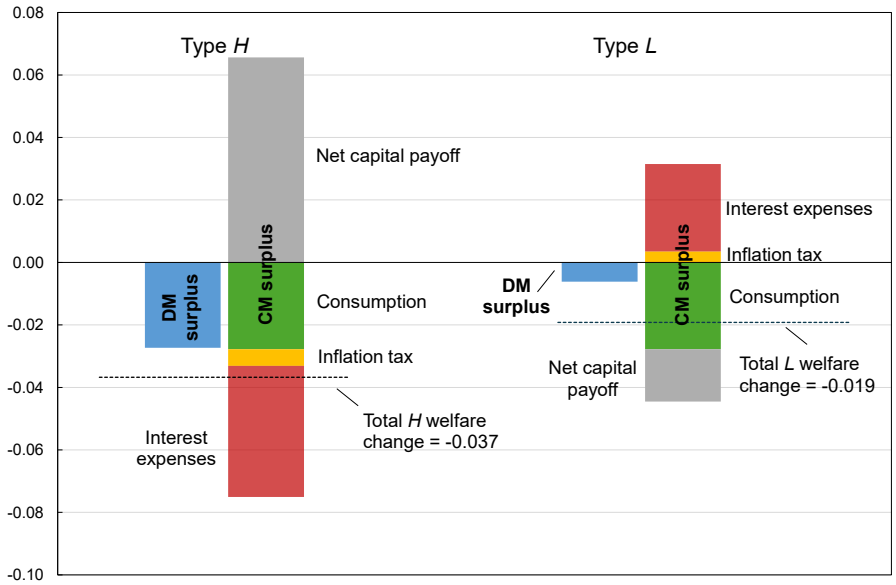
Notes: Wealth inequality as measured by the Gini index against the gross inflation rate π. Solid line: extended model with a collateral constraint. Dotted line: model with uncollateralized borrowing (i.e., with a fixed borrowing limit A). Gray line: model without borrowing and lending.

Figure 10: Welfare and inflation (extended model)



Notes: Welfare against the gross inflation rate π. Solid line: extended model with a collateral constraint. Dotted line: model with uncollateralized borrowing (i.e., with a fixed borrowing limit A). Gray line: model without borrowing and lending.

Figure 11: Decomposition of welfare changes when inflation increases from 0% to 10% (extended model)



Notes: DM surplus is $\frac{\sigma}{2}\left[u(q_j)+\frac{p}{w}\left(f(k_j)-q_j\right)\right]$; CM surplus is defined as the sum of the following components: net benefit from CM consumption $U(\bar{x})-\frac{\bar{x}}{w}$, net capital payoff $\frac{(r-\delta)k_j}{w}$, inflation tax $\frac{(\pi-1)(\bar{m}-m_j)}{w}$, and interest expenses $-\frac{a_j(1-\beta_H)}{w}$.

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